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Abstract:

The note presents a new definition of graded fuzzy logic, different from that of Běhounek et al. Some few properties of graded fuzzy logic (in our new sense) are proven.

Keywords: graded theories, mathematical fuzzy logic

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This short remark wants to be a contribution to the discussion on various possibilities of defining graded fuzzy logic. The reader is assumed to know basics of the mathematical fuzzy logic as developed in [4], in particular the basic (propositional and predicate) mathematical fuzzy logic BL and $BL\forall$. Several definitions of graded notions related to (t-norm based) fuzzy logic are analyzed in papers by Běhounek et al., some of them being cited below. For example, in [1] the formula $\bigwedge_{\alpha,\beta}(T\alpha\beta \equiv T\beta\alpha)$ (with \equiv being fuzzy equivalence) is shown as a definition of degree of commutativity of T. The present approach is different from Běhounek's notions; several things are proven but a serious discussion of the offered notion is not contained here. Any comments on this are very welcome.

Let $\&, \to, 0$ be interpreted in the usual way using a continuous t-norm, let c be an element of the real interval (0, 1]. For $a, b, c \in [0, 1]$ let $(a \to ^c b) \equiv (c \to (a \to b)),$ $(a \equiv ^c b) \equiv (c \to (a \equiv b)).$

Investigate connectives $\&^c, \to^c, 0^c$ satisfying $(a\&^cb) \equiv c \to (a\&b), (a \to^c b) \equiv c \to (a \to b), z \equiv 0^c \equiv^c z \equiv 0.$

Lemma. The following formulas are valid:

- (1) $((a \equiv^{c} b)\&(u \equiv^{c} v)) \to ((a\&u) \equiv^{c^{2}} (b\&v))$
- (2) $((a \equiv^{c^1} b)\&(b \equiv c^2 d)) \rightarrow (a \equiv^{c^1 \cdot c^2} d)$
- (3) $(x\&^c 0^c) \equiv^{c^3} 0^c$
- $(4) (1 \to^c y) \equiv^c y$
- (5) $(x\&^c(x\to^c y))\to^{c^2} y$

Proof.

(1) $((c \rightarrow (a \equiv b))\&(c \rightarrow (u \rightarrow v))) \rightarrow (c^2 \rightarrow ((a\&u) \equiv (b\&v)))$ (2) Similar proof. (3) $(x\&^c0^c) \equiv^c (x\&0^c) \equiv^c (x\&0) \equiv^1 0 \equiv^c 0^c$ (4) easy; (5) $(x\&^c(x \rightarrow^c y)) \equiv^c (x\&(x \rightarrow^c y)) \equiv^c (x\&(x \rightarrow y)) \equiv^1 y$

Now let us analyze the axioms from [4] 2.2.4.

Lemma. The following formulas are valid: $(A1^c) (x \to {}^c y) \to^c ((y \to {}^c z) \to^c (x \to {}^c z))$ $(A2^c) (x\&^c y) \to^c x$ $(A3^c) (x\&^c y) \to^c (y\&^c x)$ $(A4^c) (x\&^c (x \to {}^c y)) \to^c (y\&^c (y \to {}^c x))$ $(A51^c) (x \to {}^c (y \to {}^c z)) \to^c (((x\&^c y) \to {}^c z))$ $(A6^c) ((x \to {}^c y) \to {}^c z) \to^c (((y \to {}^c x) \to {}^c z) \to {}^c z)$ $(A7^c) 0^c \to^c x$

Proof.

 $\begin{array}{l} (A1^c) \\ (x \to y) \to ((y \to z) \to (x \to z)) \\ (x \to y) \to ((c \to (y \to z) \to (c \to (x \to z))) \\ (c \to (x \to y)) \to (c \to ((c \to (y \to z) \to (c \to (x \to z)))) \\ (x \to^c y) \to ((y \to^c z) \to^c (x \to^c z)) \\ c \to [(x \to^c y) \to (((y \to^c z) \to^c (x \to^c z)))] \\ (x \to^c y) \to^c ((y \to^c z) \to^c (x \to^c z)) \end{array}$

 $(A2^c)$ $(x\&^c y) \equiv^c (x\&y), (x\&y) \to^1 x, \text{ thus }$ $c \to ((x\&^c y) \to x)$ $(x\&^c y) \to^c x$

$$\begin{array}{c} (A3^c) \\ (c \to (x\&y)) \to (c \to (y\&x)) \\ (x\&^c y) \to (y\&^c x) \\ (x\&^c y) \to^c (y\&^c x) \end{array}$$

 $(A4^c)$

 $\begin{array}{l} cx \text{ is an abbreviation of } c\&x.\\ \text{Each line implies the next one.}\\ x\&^c(x\to^c y))\\ c\to (x\&(x\to^c y))\\ c\to (x\&(c\to(x\to y)))\\ c^2\to (cx\&(cx\to y))\\ c^2\to ((y\to cx)\&y)\\ c\to (c\to((y\to cx)\&y))\\ c\to (((y\to^c cx)\&y))\\ c\to (((y\to^c cx)\&y))\\ ((y\to^c cx)\&^c y)\\ ((y\to^c x)\&^c y)\end{array}$

$$\begin{array}{l} (A51^c) \\ [x \rightarrow^c (y \rightarrow^c z)] \rightarrow [c \rightarrow (x \rightarrow (y \rightarrow (c \rightarrow z))] \rightarrow \\ \rightarrow [c \rightarrow (c \rightarrow (x\&y) \rightarrow z)] \rightarrow [c \rightarrow (c \rightarrow (x\&y) \rightarrow (c \rightarrow z))] \rightarrow \\ \rightarrow [c \rightarrow ((x\&^c y \rightarrow^c z), \text{ thus} \\ (x \rightarrow^c (y \rightarrow^c z)) \rightarrow^c (((x\&^c y) \rightarrow^c z)). \end{array}$$

$$(A6^c)$$

$$((x \to y) \to z) \to (((y \to x) \to z) \to z)$$

$$((x \to y) \to z) \to^c (((y \to x) \to z) \to^c z)$$

$$((x \to^c y) \to^c z) \to^c (((y \to^c x) \to^c z) \to^c z)$$

$$(A7^c)$$

$$(A7^c)$$

$$c \to (c \to 0) \to x, \text{ thus } 0^c \to^c x.$$

Lemma (modus ponens.) $\phi^c a (\phi \to \psi)^c$ implies $c \to \psi^c$. **Proof.** $(\phi \to \psi)^c$ is $\phi^c \to \psi^c$, thus $c \to (\phi^c \to \psi^c)$.

Corollary. If *BL* proves ϕ then for some $n BL^c$ proves $c^n \to \phi^c$.

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