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Abstract:

The note presents a new definition of graded fuzzy logic, different from that of Běhounek et al. Some few properties of graded fuzzy logic (in our new sense) are proven.

Keywords:

graded theories, mathematical fuzzy logic

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What is graded fuzzy logic?

Petr Hájek

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This short remark wants to be a contribution to the discussion on various possibilities of defining graded fuzzy logic. The reader is assumed to know basics of the mathematical fuzzy logic as developed in [4], in particular the basic (propositional and predicate) mathematical fuzzy logic BL and $BL\forall$. Several definitions of graded notions related to (t-norm based) fuzzy logic are analyzed in papers by Běhounek et al., some of them being cited below. For example, in [1] the formula $\bigwedge_{\alpha,\beta}(T\alpha\beta \equiv T\beta\alpha)$ (with \equiv being fuzzy equivalence) is shown as a definition of degree of commutativity of T . The present approach is different from Běhounek's notions; several things are proven but a serious discussion of the offered notion is not contained here. Any comments on this are very welcome.

Let $\&, \rightarrow, 0$ be interpreted in the usual way using a continuous t-norm, let c be an element of the real interval $(0, 1]$. For $a, b, c \in [0, 1]$ let
 $(a \rightarrow^c b) \equiv (c \rightarrow (a \rightarrow b))$,
 $(a \equiv^c b) \equiv (c \rightarrow (a \equiv b))$.

Investigate connectives $\&^c, \rightarrow^c, 0^c$ satisfying
 $(a\&^c b) \equiv c \rightarrow (a\& b)$, $(a \rightarrow^c b) \equiv c \rightarrow (a \rightarrow b)$, $z \equiv 0^c \equiv^c z \equiv 0$.

Lemma. The following formulas are valid:

- (1) $((a \equiv^c b)\&(u \equiv^c v)) \rightarrow ((a\&u) \equiv^{c^2} (b\&v))$
- (2) $((a \equiv^{c^1} b)\&(b \equiv^{c^2} d)) \rightarrow (a \equiv^{c^1 \cdot c^2} d)$
- (3) $(x\&^c 0^c) \equiv^{c^3} 0^c$
- (4) $(1 \rightarrow^c y) \equiv^c y$
- (5) $(x\&^c(x \rightarrow^c y)) \rightarrow^{c^2} y$

Proof.

- (1) $((c \rightarrow (a \equiv b))\&(c \rightarrow (u \rightarrow v))) \rightarrow (c^2 \rightarrow ((a\&u) \equiv (b\&v)))$
- (2) Similar proof.
- (3) $(x\&^c 0^c) \equiv^c (x\& 0^c) \equiv^c (x\& 0) \equiv^1 0 \equiv^c 0^c$
- (4) easy;
- (5) $(x\&^c(x \rightarrow^c y)) \equiv^c (x\&(x \rightarrow^c y)) \equiv^c (x\&(x \rightarrow y)) \equiv^1 y$

Now let us analyze the axioms from [4] 2.2.4.

Lemma. The following formulas are valid:

- (A1^c) $(x \rightarrow^c y) \rightarrow^c ((y \rightarrow^c z) \rightarrow^c (x \rightarrow^c z))$
- (A2^c) $(x \&^c y) \rightarrow^c x$
- (A3^c) $(x \&^c y) \rightarrow^c (y \&^c x)$
- (A4^c) $(x \&^c (x \rightarrow^c y)) \rightarrow^c (y \&^c (y \rightarrow^c x))$
- (A51^c) $(x \rightarrow^c (y \rightarrow^c z)) \rightarrow^c (((x \&^c y) \rightarrow^c z)$
- (A6^c) $((x \rightarrow^c y) \rightarrow^c z) \rightarrow^c (((y \rightarrow^c x) \rightarrow^c z) \rightarrow^c z)$
- (A7^c) $0^c \rightarrow^c x$

Proof.

- (A1^c)
- $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z))$
- $(x \rightarrow y) \rightarrow ((c \rightarrow (y \rightarrow z)) \rightarrow (c \rightarrow (x \rightarrow z)))$
- $(c \rightarrow (x \rightarrow y)) \rightarrow (c \rightarrow ((c \rightarrow (y \rightarrow z)) \rightarrow (c \rightarrow (x \rightarrow z))))$
- $(x \rightarrow^c y) \rightarrow ((y \rightarrow^c z) \rightarrow^c (x \rightarrow^c z))$
- $c \rightarrow [(x \rightarrow^c y) \rightarrow ((y \rightarrow^c z) \rightarrow^c (x \rightarrow^c z))]$
- $(x \rightarrow^c y) \rightarrow^c ((y \rightarrow^c z) \rightarrow^c (x \rightarrow^c z))$

(A2^c)

- $(x \&^c y) \equiv^c (x \& y), (x \& y) \rightarrow^1 x$, thus
- $c \rightarrow ((x \&^c y) \rightarrow x)$
- $(x \&^c y) \rightarrow^c x$

(A3^c)

- $(c \rightarrow (x \& y)) \rightarrow (c \rightarrow (y \& x))$
- $(x \&^c y) \rightarrow (y \&^c x)$
- $(x \&^c y) \rightarrow^c (y \&^c x)$

(A4^c)

- cx is an abbreviation of $c \& x$.
- Each line implies the next one.
- $x \&^c (x \rightarrow^c y)$
- $c \rightarrow (x \& (x \rightarrow^c y))$
- $c \rightarrow (x \& (c \rightarrow (x \rightarrow y)))$
- $c^2 \rightarrow (cx \& (cx \rightarrow y))$
- $c^2 \rightarrow ((y \rightarrow cx) \& y)$
- $c \rightarrow (c \rightarrow ((y \rightarrow cx) \& y))$
- $c \rightarrow (((y \rightarrow^c cx) \& y))$
- $((y \rightarrow^c cx) \&^c y)$
- $((y \rightarrow^c x) \&^c y)$

(A51^c)

- $[x \rightarrow^c (y \rightarrow^c z)] \rightarrow [c \rightarrow (x \rightarrow (y \rightarrow (c \rightarrow z)))] \rightarrow$
- $\rightarrow [c \rightarrow (c \rightarrow (x \& y) \rightarrow z)] \rightarrow [c \rightarrow (c \rightarrow (x \& y) \rightarrow (c \rightarrow z))] \rightarrow$
- $\rightarrow [c \rightarrow ((x \&^c y) \rightarrow^c z)]$, thus
- $(x \rightarrow^c (y \rightarrow^c z)) \rightarrow^c (((x \&^c y) \rightarrow^c z))$.

$$\begin{aligned}
& (A6^c) \\
& ((x \rightarrow y) \rightarrow z) \rightarrow (((y \rightarrow x) \rightarrow z) \rightarrow z) \\
& ((x \rightarrow y) \rightarrow z) \rightarrow^c (((y \rightarrow x) \rightarrow z) \rightarrow^c z) \\
& ((x \rightarrow^c y) \rightarrow^c z) \rightarrow^c (((y \rightarrow^c x) \rightarrow^c z) \rightarrow^c z)
\end{aligned}$$

$$\begin{aligned}
& (A7^c) \\
& c \rightarrow (c \rightarrow 0) \rightarrow x, \text{ thus } 0^c \rightarrow^c x.
\end{aligned}$$

Lemma (modus ponens.) ϕ^c a $(\phi \rightarrow \psi)^c$ implies $c \rightarrow \psi^c$.

Proof. $(\phi \rightarrow \psi)^c$ is $\phi^c \rightarrow^c \psi^c$, thus $c \rightarrow (\phi^c \rightarrow \psi^c)$.

Corollary. If BL proves ϕ then for some n BL^c proves $c^n \rightarrow \phi^c$.

References

- [1] Běhounek L., Bodenhofer U., Cintula P., Saminger-Platz S., Sarkoci P.: Graded properties of t-norms. In: Abstracts of 10th Conference of Fuzzy Set Theory and Applications p. 30. February 2010.
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