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Institute of Computer Science
Academy of Sciences of the Czech Republic

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Technical report No. V-1143

02.01.2012



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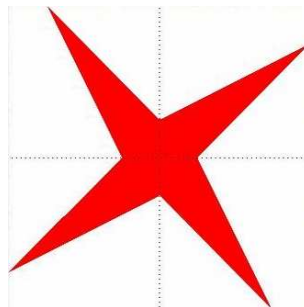
Jiří Rohn¹

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Abstract:

We disclose the file `ols.m` whose p-coded version is a part of the open source verification software package VERSOFT for computing verified eigenvalues and eigenvectors of a complex (or real) matrix.



Keywords:

Eigenvalue, eigenvector, verified result, interval arithmetic.²

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²Above: logo of interval computations and related areas (depiction of the solution set of the system $[2, 4]x_1 + [-2, 1]x_2 = [-2, 2]$, $[-1, 2]x_1 + [2, 4]x_2 = [-2, 2]$ (Barth and Nuding [1])).

1 Introduction

VERSOFT [2], a freely available verification software package written in INTLAB [3], a toolbox of MATLAB, contains as one of its pillars a p-coded (content-obscured) function `ols.p` for computing verified eigendecomposition of a complex (or real) matrix. We make it here publicly available as a function `ols.m` (OL Shortened) in a compact form consisting of only 23 lines of the source code. The original function has been stripped off the output error variable `E` only, the rest has been kept intact.

2 Description and examples

Here is the help of the function (not present in the compact source code):

```
function [L,X]=ols(A)
%   OLS       Verified eigenvalues and eigenvectors of a complex (or real) matrix.
%
%   This is an INTLAB file. It requires to have INTLAB installed under
%   MATLAB to function properly.
%
%   For a square complex (or real) matrix A,
%       [L,X]=ols(A)
%   computes (generally complex) interval matrices L and X, L diagonal,
%   that are verified to contain matrices Lo, Xo satisfying
%       A*Xo=Xo*Lo
%   in exact arithmetic, where diag(Lo) is the vector of ALL eigenvalues of A
%   and Xo is a matrix of corresponding eigenvectors; L, X are enclosures
%   of these quantities. Multiple eigenvalues are taken into account.
%
%   The vector
%       lam=diag(L)
%   has the following additional property: for each i, j, the intervals
%   lam(i) and lam(j) are either identical, or disjoint. Thus, if all of
%   them are disjoint, then each of them contains exactly one eigenvalue
%   of A.
%
%   If A is real and symmetric, then L, X are real. If A is Hermitian, then L is real.
%   In these cases both lam.inf and lam.sup are ordered in nondecreasing order.
%
%   EXAMPLE 1 (multiple eigenvalues). The following matrix has a six-tuple
%   eigenvalue 2 corresponding to three Jordan blocks of sizes 1, 2 and 3:
%   A =
%       -60     1    42    -3   -10     4
%       133     0   -92     6    23    -8
%       -186    3   128    -9   -30    12
%       252    -4  -171    14    41   -16
%       -310    5   210   -15   -48    20
```

```

%      372      -6  -252      18      60      -22
% >> [L,X]=ols(A); format long, lam=diag(L)
%
% intval lam =
% [ 1.99980487061652 - 0.00019256911560i, 2.00019000884616 + 0.00019256911404i]
% [ 1.99980487061652 - 0.00019256911560i, 2.00019000884616 + 0.00019256911404i]
% [ 1.99980487061652 - 0.00019256911560i, 2.00019000884616 + 0.00019256911404i]
% [ 1.99980487061652 - 0.00019256911560i, 2.00019000884616 + 0.00019256911404i]
% [ 1.99980487061652 - 0.00019256911560i, 2.00019000884616 + 0.00019256911404i]
% [ 1.99980487061652 - 0.00019256911560i, 2.00019000884616 + 0.00019256911404i]
%
% Enclosures of equal eigenvalues are equal, as explained above. Low
% accuracy is caused by multiplicity; the imaginary parts cannot be
% filtered out by the program here.
%
% EXAMPLE 2 (symmetric matrix; Hilbert 4x4). Output for a real symmetric
% matrix is always real:
% >> A=hilb(4), [L,X]=ols(A); format long, lam=diag(L), X
%
% A =
% Columns 1 through 3
% 1.000000000000000 0.500000000000000 0.333333333333333
% 0.500000000000000 0.333333333333333 0.250000000000000
% 0.333333333333333 0.250000000000000 0.200000000000000
% 0.250000000000000 0.200000000000000 0.166666666666667
% Column 4
% 0.250000000000000
% 0.200000000000000
% 0.166666666666667
% 0.142857142857143
% intval lam =
% [ 0.00009670230402, 0.00009670230403]
% [ 0.00673827360576, 0.00673827360577]
% [ 0.16914122022144, 0.16914122022146]
% [ 1.50021428005924, 1.50021428005925]
% intval X =
% Columns 1 through 2
% [ 0.02919332316478, 0.02919332316479] [ 0.17918629053545, 0.17918629053546]
% [ -0.32871205576320, -0.32871205576317] [ -0.74191779062846, -0.74191779062845]
% [ 0.79141114583312, 0.79141114583313] [ 0.10022813694718, 0.10022813694721]
% [ -0.51455274999717, -0.51455274999714] [ 0.63828252819360, 0.63828252819363]
% Columns 3 through 4
% [ -0.58207569949724, -0.58207569949723] [ 0.79260829116376, 0.79260829116377]
% [ 0.37050218506709, 0.37050218506710] [ 0.45192312090159, 0.45192312090160]
% [ 0.50957863450179, 0.50957863450180] [ 0.32241639858182, 0.32241639858183]
% [ 0.51404827222216, 0.51404827222217] [ 0.25216116968824, 0.25216116968825]
% Observe the high accuracy of the result.
%

```

```
% See also EIG, VERIFYEIG.  
%  
% Copyright 2008-2011 Jiri Rohn.  
%  
% Employs the routine VERIFYEIG by Siegfried M. Rump.
```

3 Download

The source file can be downloaded from
<http://uivtx.cs.cas.cz/~rohn/matlab/others/ols.m>

Dedication

Dedicated to O. L. after whom the file was named. ■

Bibliography

- [1] W. Barth and E. Nuding, *Optimale Lösung von Intervallgleichungssystemen*, Computing, 12 (1974), pp. 117–125. [1](#)
- [2] J. Rohn, *VERSOFT: Verification software in MATLAB/INTLAB*, 2009.
<http://uivtx.cs.cas.cz/~rohn/matlab>. [2](#)
- [3] S. Rump, *INTLAB - INTerval LABORatory*, in Developments in Reliable Computing, T. Csendes, ed., Kluwer Academic Publishers, Dordrecht, 1999, pp. 77–104.
<http://www.ti3.tu-harburg.de/rump/>. [2](#)