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Jiri Rohn

Technical report No. V-1147

02.01.2012



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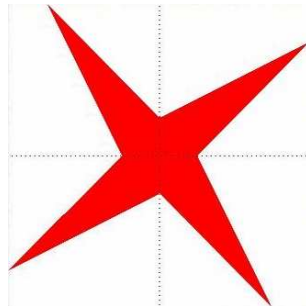
Vahideh Hooshyarbakhsh,<sup>1</sup> Raena Farhadsefat<sup>2</sup> and Jiri Rohn<sup>3</sup>

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Abstract:

We describe a not-a-priori-exponential necessary and sufficient condition for regularity of interval matrices which is an easy consequence of an earlier result on interval linear equations.



Keywords:

Interval matrix, regularity, necessary and sufficient condition.<sup>4</sup>

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<sup>4</sup>Above: logo of interval computations and related areas (depiction of the solution set of the system  $[2, 4]x_1 + [-2, 1]x_2 = [-2, 2]$ ,  $[-1, 2]x_1 + [2, 4]x_2 = [-2, 2]$  (Barth and Nuding [1])).

Checking regularity of interval matrices is a known NP-hard problem. Forty necessary and sufficient regularity conditions are summed up in [2]; all of them are exponential because they explicitly or implicitly contain the quantifier “for each  $z \in \{-1, 1\}^n$ ”. The condition given below is, to these authors’ knowledge, the *first* ever published not-a-priori-exponential regularity condition because instead of  $\{-1, 1\}^n$  it employs only a subset  $Z$  of it. Cardinality of the set  $Z$  varies with the data, but its minimal value is 1. Notation used:  $e_j$  is the  $j$ th column of the  $n \times n$  identity matrix,  $e = (1, 1, \dots, 1)^T \in \mathbb{R}^n$ ,  $\text{diag}(z)$  is the  $n \times n$  diagonal matrix with diagonal vector  $z$  and for an  $x \in \mathbb{R}^n$ ,  $\text{sgn}(x)$  is defined by  $(\text{sgn}(x))_i = 1$  if  $x_i \geq 0$  and  $(\text{sgn}(x))_i = -1$  otherwise.

**Theorem 1.** *An  $n \times n$  interval matrix  $\mathbf{A} = [A_c - \Delta, A_c + \Delta]$  is regular if and only if  $A_c$  is nonsingular and there exists a subset  $Z$  of  $\{-1, 1\}^n$  having the following properties:*

(a)  $\text{sgn}(A_c^{-1}e) \in Z$ ,

(b) for each  $z \in Z$  the inequalities

$$(QA_c - I) \text{diag}(z) \geq |Q|\Delta, \quad (0.1)$$

$$(QA_c - I) \text{diag}(-z) \geq |Q|\Delta \quad (0.2)$$

have matrix solutions  $Q_z$  and  $Q_{-z}$ , respectively,

(c) if  $z \in Z$ ,  $Q_{-z}e \leq Q_z e$ , and  $(Q_{-z}e)_j(Q_z e)_j \leq 0$  for some  $j$ , then  $z - 2z_j e_j \in Z$ .

*Proof.* “If”: The assumptions (a)-(c) imply that the three assumptions of Theorem 3 in [3] are met for the system of interval linear equations  $\mathbf{A}x = [e, e]$  whose solution set in virtue of the same theorem is bounded, hence  $\mathbf{A}$  is regular. “Only if”: If  $\mathbf{A}$  is regular, then (a) and (c) are satisfied for  $Z = \{-1, 1\}^n$  and for each  $z \in \{-1, 1\}^n$  the equations

$$(QA_c - I) \text{diag}(z) = |Q|\Delta,$$

$$(QA_c - I) \text{diag}(-z) = |Q|\Delta$$

have (even unique) solutions, see [2]. ▀

Hence we can also formulate the theorem in the following way:

**Theorem 2.** *An  $n \times n$  interval matrix  $\mathbf{A} = [A_c - \Delta, A_c + \Delta]$  is regular if and only if  $A_c$  is nonsingular and there exists a subset  $Z$  of  $\{-1, 1\}^n$  having the following properties:*

(a)  $\text{sgn}(A_c^{-1}e) \in Z$ ,

(b) for each  $z \in Z$  the equations

$$(QA_c - I) \text{diag}(z) = |Q|\Delta, \quad (0.3)$$

$$(QA_c - I) \text{diag}(-z) = |Q|\Delta \quad (0.4)$$

have matrix solutions  $Q_z$  and  $Q_{-z}$ , respectively,

(c) if  $z \in Z$ ,  $Q_{-z}e \leq Q_z e$ , and  $(Q_{-z}e)_j(Q_z e)_j \leq 0$  for some  $j$ , then  $z - 2z_j e_j \in Z$ .

Notice that if  $z \in \{-1, 1\}^n$ , then  $z - 2z_j e_j \in \{-1, 1\}^n$  (in (c)), so that  $Z \subseteq \{-1, 1\}^n$ ; thus  $Z$  is defined recursively by (a) and (c). In practical computations, equations (0.3), (0.4) are solved instead of inequalities (0.1), (0.2) as it was done in the function `qzmatrix`, using the subfunction `absvaleqn`, in [4].

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