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**Institute of Computer Science**  
**Academy of Sciences of the Czech Republic**

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Technical report No. V-1117

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## **INTLAB Primer**

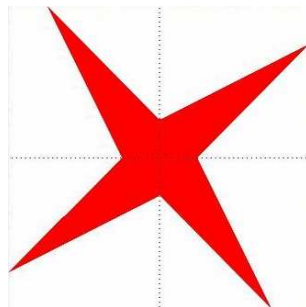
Jiří Rohn<sup>1</sup>

Technical report No. V-1117

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Abstract:

This report brings an extremely concise description of basic features of INTLAB, a MATLAB toolbox for verified computations. Only the very basics of INTLAB are presented; INTLAB itself contains much more.



Keywords:

INTLAB, MATLAB, verification software, primer.<sup>2</sup>

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<sup>1</sup>This work was supported by the Institutional Research Plan AV0Z10300504.

<sup>2</sup>Above: logo of interval computations and related areas (depiction of the solution set of the system  $[2, 4]x_1 + [-2, 1]x_2 = [-2, 2]$ ,  $[-1, 2]x_1 + [2, 4]x_2 = [-2, 2]$  (Barth and Nuding [1])).

## 1 INTRODUCTION

This is an extremely concise description of basic features of INTLAB aimed at newcomers to this area. Only the very basics of INTLAB are presented; INTLAB itself contains much more.

## 2 INTLAB'S AUTHOR

INTLAB, a MATLAB toolbox for self-validating algorithms, is a one-man work. It has been created by Prof. Dr. Siegfried M. Rump, Institute for Reliable Computing, Technical University of Hamburg-Harburg, Germany, [2].

## 3 INTLAB'S DOWNLOAD

INTLAB can be downloaded (free for academic use; observe the license) from <http://www.ti3.tu-harburg.de/rump/intlab/> .

## 4 INTLAB'S INSTALLATION

Follow the instructions in the file Readme.txt.

## 5 INTLAB'S DEMOS

Look at demointval.m, demointlab.m. Many important features contained therein are summarized below.

## 6 DEFAULT INITIALIZATIONS

There are two basic ways of displaying intervals:

`intvalinit('displayinfsup')` : display of intervals by endpoints,

`intvalinit('displaymidrad')` : display of intervals by midpoint and radius.

In what follows we use the first option. To invoke it, simply type

```
>> intvalinit('displayinfsup')
```

You will get the response

```
==> Default display of intervals by infimum/supremum (e.g. [ 3.14 , 3.15 ])
```

## 7 INTVAL TYPE

INTLAB uses newly defined MATLAB type “intval” for interval quantities.

## 8 BASIC INTERVAL ARITHMETIC PROPERTY

An operation always uses interval arithmetic if at least one of the operands is of type `intval`. The output is always rigorous (i.e., it encloses the real result within floating-point bounds).

## 9 INPUT

Given two real (of type `double`) matrices (vectors, numbers) `A`, `B` of the same size, the interval matrix (vector, number) `C=[A,B]` is inputted simply as `C=infsup(A,B)`.

Example. For real<sup>3</sup> (double) matrices `Ad`, `Ah` given by

```
>> Ad=[2 -2; -1 2]
Ad =
     2     -2
    -1      2
>> Ah=[4 1; 2 4]
Ah =
     4      1
     2      4
>> A=infsup(Ad,Ah)
```

yields

```
intval A =
      [ 2.0000, 4.0000] [ -2.0000, 1.0000]
      [ -1.0000, 2.0000] [ 2.0000, 4.0000]
```

`A` is the interval matrix `[Ad,Ah]`. Alternatively, the same result can be achieved by using

```
>> A=[infsup(2,4) infsup(-2,1); infsup(-1,2) infsup(2,4)]
intval A =
      [ 2.0000, 4.0000] [ -2.0000, 1.0000]
      [ -1.0000, 2.0000] [ 2.0000, 4.0000]
```

Similarly, `A=midrad(C,D)` gives the interval matrix `[C-D,C+D]` (i.e, the midpoint-radius representation). If you wish a real (double) matrix `Ao` to be handled as a thin interval matrix (so that the interval arithmetic could apply), use `Ao=intval(Ao)` (or, equivalently, `Ao=infsup(Ao,Ao)`):

```
>> Ao=[1 2;3 4]
Ao =
     1      2
     3      4
>> Ao=intval(Ao)
intval Ao =
      [ 1.0000, 1.0000] [ 2.0000, 2.0000]
      [ 3.0000, 3.0000] [ 4.0000, 4.0000]
```

---

<sup>3</sup>Throughout the text, “real” is meant as opposite of “interval”.

## 10 OUTPUT

Given an interval matrix  $A$  (of type `intval`), its bounds, midpoint and radius can be extracted as follows:

the lower bound is `inf(A)` or `A.inf`,  
the upper bound is `sup(A)` or `A.sup`,  
the midpoint is `mid(A)` or `A.mid`,  
the radius is `rad(A)` or `A.rad`.

Example. With the above  $A$ , we have

```
intval A =
      [ 2.0000, 4.0000] [ -2.0000, 1.0000]
      [-1.0000, 2.0000] [ 2.0000, 4.0000]
>> inf(A)
ans =
     2    -2
    -1     2
>> sup(A)
ans =
     4     1
     2     4
>> mid(A)
ans =
    3.0000  -0.5000
    0.5000   3.0000
>> rad(A)
ans =
    1.0000   1.5000
    1.5000   1.0000
```

## 11 DIFFERENCE BETWEEN `A.inf` AND `inf(A)`

`A.inf` and `inf(A)` are not entirely equivalent. `A.inf` allows indexing:

```
>> A.inf(1,1)
ans =
     2
```

whereas this does not work for `inf(A)`. The same holds for `A.sup` and `sup(A)`, etc.

## 12 ENCLOSURE OF THE INVERSE INTERVAL MATRIX

We shall demonstrate some typical INTLAB features on the function `B=inv(A)` which computes an enclosure of the interval inverse of  $A$ .

## 12.1 Interval input

```
>> A
intval A =
      [ 2.0000, 4.0000] [ -2.0000, 1.0000]
      [ -1.0000, 2.0000] [ 2.0000, 4.0000]
>> B=inv(A)
intval B =
      [ -2.9704, 3.6191] [ -3.2729, 3.3810]
      [ -3.3810, 3.2729] [ -2.9704, 3.6191]
```

This means that for each real (double) matrix  $A_0$  in  $A$ , its exact inverse is guaranteed to exist and to belong to  $B$ . The output uses outward rounding, so that the result is rigorous. The enclosure  $B$  is generally not optimal.

## 12.2 Real input

Inverse can also be used to obtain a rigorous output for a real (double) input:

```
Ao =
      1 2
      3 4
>> Ao=intval(Ao)
intval Ao =
      [ 1.0000, 1.0000] [ 2.0000, 2.0000]
      [ 3.0000, 3.0000] [ 4.0000, 4.0000]
>> B=inv(Ao)
intval B =
      [ -2.0001, -1.9999] [ 0.9999, 1.0001]
      [ 1.4999, 1.5001] [ -0.5001, -0.4999]
```

To see better the accuracy of the result, use

```
>> format long
>> B
intval B =
      [-2.000000000000001, -1.999999999999999] [ 0.999999999999999, 1.000000000000001]
      [ 1.499999999999999, 1.500000000000001] [-0.500000000000001, -0.499999999999999]
>> rad(B)
ans =
      1.0e-015 *
      0.44408920985006 0.33306690738755
      0.22204460492503 0.11102230246252
```

The very small radius matrix shows the high accuracy achieved.

## 12.3 Singularity

An attempt to invert a singular matrix

```

>> format short
Ao=[1 2;5 10]
Ao =
     1     2
     5    10
>> Ao=infsup(Ao,Ao)
intval Ao =
      [ 1.0000, 1.0000] [ 2.0000, 2.0000]
      [ 5.0000, 5.0000] [10.0000, 10.0000]
>> B=inv(Ao)

```

results in an interval matrix of NaN's:

```

intval B =
      [ NaN, NaN] [ NaN, NaN]
      [ NaN, NaN] [ NaN, NaN]

```

## 12.4 NaN output

Using NaN's for indication of an empty output is a typical feature of INTLAB. It enables feasibility of succeeding computations. E.g., in the above example

```

>> C=inv(Ao)*rand(2,2)

```

gives

```

intval C =
      [ NaN, NaN] [ NaN, NaN]
      [ NaN, NaN] [ NaN, NaN]

```

so that the computation does not break down despite the singularity of Ao.

# 13 SYSTEMS OF INTERVAL LINEAR EQUATIONS

A system of interval linear equations  $A*x=b$  ( $A$ ,  $b$  of type `intval`,  $A$  square) can be solved (i.e., an enclosure of the solution set can be obtained) using  $X=verifylss(A,b)$ .

## 13.1 Interval data

Example.

```

>> A
intval A =
      [ 2.0000, 4.0000] [ -2.0000, 1.0000]
      [ -1.0000, 2.0000] [ 2.0000, 4.0000]
>> bl=[-2 -2]'; bu=-bl;
>> b=infsup(bl,bu)
intval b =
      [ -2.0000, 2.0000] [ -2.0000, 2.0000]

```



```
>> X=verifylss(A,b)           % Barth-Nuding 1974 example, see [1]
intval X =
      [ -14.0001,  14.0001]
      [ -14.0001,  14.0001]
```

It is guaranteed that for each  $A_0$  in  $A$  and  $b_0$  in  $b$ ,  $A_0$  is nonsingular and the solution of  $A_0 x = b_0$  is contained in  $X$ . The enclosure  $X$  is generally not optimal: the optimal enclosure  $XX$  would be

```
intval XX =
      [ -4.0000,  4.0000]
      [ -4.0000,  4.0000]
```

(see the logo on the abstract page). The overestimation is caused by too wide input intervals here.

## 13.2 Real data

The procedure can be used for solving systems with thin data (without converting them to type `intval` first, contrary to the procedure “`inv`” above).

Example.

```
>> A=[ 1 2 3;4 5 6;7 8 10]
A =
      1      2      3
      4      5      6
      7      8     10
>> b=A*ones(3,1)           % so that the exact solution is [1 1 1]'
b =
      6
     15
     25
>> format long
>> X=verifylss(A,b)
intval X =
      [ 0.999999999999999,  1.000000000000001]
      [ 0.999999999999999,  1.000000000000001]
      [ 0.999999999999999,  1.000000000000001]
```

We can see that the degree of guaranteed accuracy is surprising.

## 14 DATA CONVERSION

Most numbers (like 0.1) cannot be exactly represented in binary finite precision arithmetic. To handle errors created by data conversion, INTLAB enables us to use

```
>> a=intval('0.1')         % notice the apostrophes
intval a =
```

```
          [ 0.0999999999999999, 0.1000000000000001]
>> rad(a)
ans =
    1.387778780781446e-017
```

(“rigorous representation”).

## 15 SOME ADDITIONAL INTERVAL FUNCTIONS

For  $x, y$  of type `intval`:

<code>intersect(x,y)</code>	intersection of $x, y$
<code>hull(x,y)</code>	union of $x, y$
<code>abss(x)</code>	absolute value of $x$
<code>mig(x)</code>	mignitude of $x$
<code>in(x,y)</code>	$x$ is included in $y$ (logical array)
<code>in0(x,y)</code>	$x$ is included in the interior of $y$ (logical array)

## 16 WEB VERSION

Web version of this text can be found at

[http://uivtx.cs.cas.cz/~rohn/matlab/primer/intlab\\_primer.html](http://uivtx.cs.cas.cz/~rohn/matlab/primer/intlab_primer.html) .

## 17 YOUR TURN NOW

Now it is your turn to explore INTLAB, this wonderful tool created by Siegfried Michael Rump. (Thanks also for his remarks on this text.)

## Bibliography

- [1] W. Barth and E. Nuding, *Optimale Lösung von Intervallgleichungssystemen*, Computing, 12 (1974), pp. 117–125. [1](#)
- [2] S. Rump, *INTLAB - INTerval LABORatory*, in Developments in Reliable Computing, T. Csendes, ed., Kluwer Academic Publishers, Dordrecht, 1999, pp. 77–104.  
<http://www.ti3.tu-harburg.de/rump/>. [2](#)