Asset prices in a production economy with long-run and idiosyncratic risk

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Abstract

This paper studies risk premia in an incomplete-markets economy with households facing idiosyncratic consumption risk. If the dispersion of idiosyncratic risk varies over the business cycle and households have a preference for early resolution of uncertainty, asset prices will be affected not only by news about current and expected future aggregate consumption (as in models with a representative agent), but also by news about current and future changes in the cross-sectional distribution of individual consumption. I investigate whether this additional effect can help explain high risk premia in a production economy where the aggregate consumption process is endogenous and thus can potentially be affected by the presence of idiosyncratic risk. Analyzing a neoclassical growth model combined with Epstein-Zin preferences and a tractable form of household heterogeneity, I find that countercyclical idiosyncratic risk increases the risk premium, but also effectively lowers the willingness of households to engage in intertemporal substitution and thus changes the dynamics of aggregate consumption. Nevertheless, with the added flexibility of Epstein-Zin preferences, it is possible both to increase risk premia and to maintain the same dynamics of quantities if we allow for higher intertemporal elasticity of substitution at the individual level.

Abstrakt

Tento článek zkoumá rizikovou prémii v ekonomice s nekompletními trhy a domácnostmi čelícími idiosynkratickému riziku ve spotřebě. Pokud se rozptyl idiosynkratického rizika mění v průběhu hospodářského cyklu a domácnosti preferují dřívější rozřešení nejistoty, ceny finančních aktiv budou ovlivněny nejen informacemi o současné a očekávané budoucí agregátní spotřebě (jako je tomu v modelech s reprezentativní domácností), ale také informacemi o současných a budoucích změnách v distribuci individuální spotřeby napříč domácnostmi. V článku zkoumám, jestli tento dodatečný efekt může pomoci vysvětlit vysokou rizikovou prémii v produkční ekonomice, ve které je proces pro agregátní spotřebu endogenní a potenciálně může být ovlivněn přítomností idiosynkratického rizika. Analýza neoklasického růstového modelu kombinovaného s Epstein-Zinovými preferencemi a snadno řešitelnou formou heterogenity mezi domácnostmi naznačuje, že proticyklický průběh idiosynkratického rizika zvyšuje rizikovou prémii, ale také snižuje efektivní ochotu domácností k intertemporální substituci, čímž se mění také dynamika agregátní spotřeby. Nicméně díky přidané flexibilitě Epstein-Zinových preferencí je možné zvýšit rizikovou prémii bez změny dynamiky množství, pokud připustíme zvýšenou intertemporální elasticitu substituce na individuální úrovni.

JEL Codes: E13, E21, E44, G12.
Keywords: Idiosyncratic risk, incomplete markets, production economy, risk premium.

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Nontechnical Summary

Financial markets serve the important task of facilitating saving and investment decisions, while prices and returns on financial assets capture the attitudes of market participants toward time and risk. However, explaining the joint behavior of macroeconomic fundamentals and financial returns in macroeconomic models has turned out to be a challenging problem, one that is still not fully resolved. According to standard asset-pricing theory, the spread between the average return on a risky asset and the risk-free rate, i.e., its risk premium, depends on how the risky return covaries with marginal utility, which is usually linked to consumption growth. More volatile and more procyclical returns offer a worse hedge against risk and are thus less desirable and require a higher return. Yet the observed fluctuations in aggregate consumption are too small to justify the observed levels of risk premia without requiring unrealistic levels of risk aversion, a result known as the equity premium puzzle.

While economists have come up with many potential explanations, this paper focuses on two particular approaches that have previously been proposed in the literature. On the one hand, allowing households to have a preference for early resolution of uncertainty (essentially a generalization of expected utility), combined with some degree of predictability in the macroeconomic environment, implies that changes in expected future consumption further away in the future (referred to as long-run risk) will affect marginal utility and risk premia. On the other hand, abandoning the representative household construct and accounting for heterogeneity between individual households facing idiosyncratic shocks might also drive up risk premia if the amount of risk faced by households varies over time in a cyclical way. When the two features are combined, an additional interaction term representing news about the future dispersion of idiosyncratic shocks faced by households will become relevant and can potentially help explain the high risk premia observed in real markets.

I investigate this mechanism in a version of a neoclassical growth model where aggregate consumption is determined endogenously through production and capital accumulation, while heterogeneity between households is incorporated on top of it in a tractable manner. More specifically, the model assumes that the spread between one household's individual consumption growth and aggregate consumption growth follows a random walk with idiosyncratic shocks whose distribution can vary over time. It is then possible to solve the model purely in terms of aggregate variables and time-varying moments of idiosyncratic shocks without the need to keep track of the cross-sectional distribution of wealth across households. Considering the variation in either the variance or the skewness of idiosyncratic shocks, I find that while in principle the combination of long-run risk and cyclical idiosyncratic risk can contribute nontrivially to the risk premium, it is important to consider its feedback to macroeconomic fundamentals as well.

Compared to the representative-agent model, the presence of cyclical individual risk will affect the incentives for intertemporal substitution at the aggregate level, essentially making the economy behave as if households were more averse to intertemporal substitution. This will, in turn, change the dynamics of aggregate consumption toward being less predictable, and this effect is stronger in the presence of a preference for early resolution of uncertainty. On the other hand, it turns out that it is possible to maintain high risk premia and keep consumption dynamics unchanged by suitably recalibrating the preference parameters, i.e., by making households' intertemporal substitution higher at the individual level. After doing so, I find that the model with heterogeneity generates risk premia about a third higher compared to a model with a representative agent. Decomposing the price of risk (i.e., the risk premium normalized by the return volatility) into the contributions of short-run and long-run risk in aggregate consumption and idiosyncratic shock dispersion, I find that the long-run idiosyncratic dispersion accounts for about 30 percent of the overall long-run channel, which, in turn, explains more than half of the overall price of risk.

1. Introduction

Explaining the joint dynamics of macroeconomic quantities and asset prices within the context of a microfounded general equilibrium model remains an active area of economic research. This paper contributes to that effort by constructing a tractable model of a production economy that combines recursive utility with a preference for early resolution of uncertainty and time-varying uninsurable idiosyncratic risk, and investigates its macroeconomic and asset-pricing properties.

Individually, both of these elements have been studied previously as a possible solution to the well-known failure of the standard representative-agent model with power utility in explaining the observed equity premium and interest rate.[1] When households have recursive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989), which break the link between risk aversion and the elasticity of intertemporal substitution and allow for a preference for early resolution of uncertainty, their marginal utility depends not only on current consumption, but also on the continuation value, which encodes expectations about future consumption. News regarding the level or volatility of future consumption thus becomes an additional priced factor, as in the long-run risk model of Bansal and Yaron (2004) or in the production economy[2] of Kaltenbrunner and Lochstoer (2010). Another line of research has shown that when agents face incomplete markets and uninsurable shocks, the amount of risk they face can also affect asset prices if it changes over time, as in Constantinides and Duffie (1996) and Krusell and Smith (1997).[3]

Therefore, if agents have a preference for early resolution of uncertainty and at the same time face idiosyncratic risk and incomplete markets, it follows that not only current change in the amount of idiosyncratic risk, but also news about future such changes enters the continuation value and thus affects asset prices. This presents a potential for interaction between the two mechanisms, studied in the context of an endowment economy in recent work by Constantinides and Ghosh (2017), Herskovic et al. (2016), and Schmidt (2014). However, matching asset prices is harder in a production economy than in endowment economies due to the endogenous consumption process and the need to match simultaneously the properties of quantities and prices. The main focus of this paper is therefore to look more closely at the interaction between the effects of varying idiosyncratic risk on macroeconomic dynamics and asset prices.

To illustrate the mechanism, I first construct a simple AK model with households having access to linear production technologies subject to heterogeneous rates of return on capital with time-varying variance. Assuming unit intertemporal elasticity of substitution, the model can be solved analytically and asset returns can be characterized by their exposure to news about current and future aggregate consumption and variance of idiosyncratic risk. A quantitative illustration suggests that omitting the last term could nontrivially underestimate the importance of overall long-run risk for determining risk premia.

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[1] See, for example, Mehra and Prescott (1985), Weil (1989), and Hansen and Singleton (1982). A review of the literature is provided, for example, in Cochrane (2008) and Ludvigson (2013).

[2] Regarding asset pricing in production/DSGE models, see, for example, the survey by Kogan and Papanikolaou (2012). Papers that study asset prices in production economies with recursive preferences include Tallarini (2000), Kaltenbrunner and Lochstoer (2010), Croce (2014), Rudebusch and Swanson (2012), van Binsbergen et al. (2012), and Campanale et al. (2010).

[3] See also Mankiw (1986), Telmer (1993), Heaton and Lucas (1996), Krebs and Wilson (2004), Storesletten et al. (2007), and Pijoan-Mas (2007). Gomes and Michaelides (2008) also study a model with heterogeneity, production, and recursive preferences, but their focus is primarily on the effects of limited participation and they do not model variation in either individual or aggregate risk over time. Empirical evidence is analyzed, for example, by Cogley (2002), Brav et al. (2002), and Balduzzi and Yao (2007), with somewhat mixed results.

Next, I construct a tractable model that embeds the Constantinides-Duffie framework within an otherwise standard real business cycle (RBC) model.[4] Individual households' consumption growth is determined, in a reduced-form way, by aggregate consumption growth and idiosyncratic shocks. With homothetic preferences and a random walk in individual consumption, the model has a no-trade equilibrium where each household consumes its income. The aggregate stochastic discount factor is determined by the cross-sectional average of the individual intertemporal marginal rates of substitution, and is used by a representative firm to make choices about investment and dividends, which, in turn, determine aggregate consumption growth. The distribution of the idiosyncratic shocks varies over time, possibly allowing for countercyclical variance (Storesletten et al., 2004) or procyclical skewness (Guvenen et al., 2014).

The fact that there is no trade among households is somewhat unattractive (and the resulting allocations should thus perhaps be interpreted rather as post-trade outcomes after households have already smoothed out transitory shocks), yet it allows us to solve the model without keeping track of the distribution over individual savings and thus to avoid the need for numerically intensive computation. The model can be solved by standard perturbation methods and its linearized dynamics can be characterized semi-analytically. I find that countercyclical idiosyncratic risk can raise risk premia, but also affects the aggregate dynamics through its impact on the saving and intertemporal smoothing incentives of households. The introduction of idiosyncratic risk leads to a lower "effective" intertemporal elasticity of substitution on the aggregate level, resulting in more volatile and less predictable aggregate consumption growth. Inspecting the linearized solution suggests that the strength of this feedback depends on the cyclicality of idiosyncratic risk and household risk aversion.

On the other hand, thanks to the flexibility of Epstein-Zin preferences, it is in principle possible to recalibrate the discount rate and the intertemporal elasticity of substitution (IES) parameters (to make households more willing to substitute consumption over time) in a way that compensates for the effect described above while risk premia remain higher. After suitable recalibration of the model, I find that introducing heterogeneity raises the price of risk (the Sharpe ratio) by about a third. Decomposing the price of risk by its source (aggregate consumption vs dispersion of individual shocks) and channel (short-run or long-run risk) shows that long-run idiosyncratic dispersion accounts for about 30 percent of the overall long-run channel, which, in turn, explains more than half of the overall Sharpe ratio. The results are quite similar regardless of whether the variation in individual risk unfolds through cyclical variance or skewness.

The paper is organized as follows: section 2 presents a simple example to motivate the introduction of recursive preferences, section 3 describes the model, section 4 discusses the calibration and results, and section 5 concludes.

2. Simple Model

Standard consumption-based asset pricing models explain the existence of risk premia by comovement of returns with consumption. Assets that pay off more in good times (i.e. states of the world with high consumption and low marginal utility) than in bad times (states with low consumption and high marginal utility), are less attractive for households wishing to smooth their consumption,

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[4] A similar approach is used for analyzing monetary policy in New-Keynesian models in recent papers by Braun and Nakajima (2012), Werning (2015), and Takahashi et al. (2016). In these setups, variation in idiosyncratic risk manifests itself in a similar way as discount rate shocks after aggregation. Relatedly, Albuquerque et al. (2016) study the role of discount rate shocks in asset pricing.

and thus must offer higher returns to be held in equilibrium. However, it is well established that the standard model with representative household and power utility has problems matching the observed level of risk premia quantitatively. This paper considers two modifications of the baseline model that have been previously studied as possible explanations of high risk premia.

First, a richer specification for the household utility function, which includes the preference for earlier resolution of uncertainty, implies that "bad times" happen not only when current consumption is low, but also when the household receives bad news about future consumption. This amplifies the sensitivity of the household to small but persistent changes in consumption, which helps to increase the price of risk through the so-called long run risk chanel. Second, households face not only aggregate risk, but also a large amount of individual variation in their consumption arising from idiosyncratic shocks and incomplete markets. If the amount of this idiosyncratic risk is larger when the aggregate consumption is already low, households will be again more sensitive to aggregate fluctuations and will require higher returns to hold assets with procyclical payoffs.

This paper considers these two features together. If households care about both the volatility of individual shocks and news about the future, it follows that persistent cyclical variation in idiosyncratic risk will also be amplified by the long run risk mechanism, and this interaction can potentially imply higher resk premia with smaller values of risk aversion. On the other hand, it is also important to consider whether such a story is consistent with the supply side of the economy, since the consumption process is, in the end, an endogeous outcome affected by the saving behavior of households. I will therefore study a production economy with idiosyncratic shocks and long run risk in the subsequent section. First, however, it may be useful to flesh out the intuition discussed above more formally in a setting where the consumption process is still effectively exogenous.

This section thus presents a simple AK-like model[5] in which the output is produced using a linear technology with capital as the only input. Each household operates such technology independently, subject to aggregate and individual productivity shocks with time-varying dispersion, and can spend the output on consumption, investment or a risk-free asset. If we assume that households have a unit intertemporal elasticity of substitution, the model has an analytical solution. Subsequently, the price of risk can be cleanly decomposed into four contributions, from short run and long run risk in aggregate productivity and level of idiosyncratic risk. I look into how these contributions depend on the parameters of the model, and argue that they can be quantitatively relevant.

2.1 Setup

Time t is discrete and there is a continuum of agents indexed by i. Each agent enters the period with some stock of capital $K_{i,t}$, which is used for production according to $Y_{i,t} = A_{i,t} K_{i,t}$, subject to an exogenous productivity process $A_{i,t}$ (which will have an idiosyncratic component and is thus indexed by i). Agents can also trade in risk-free one-period bonds, although the overall net supply of bonds is zero. Income obtained from production and bond holdings $B_{i,t}$ can be used for consumption $C_{i,t}$, stored as capital for the next period (for simplicity we will assume full depreciation) or spent on new bonds. The budget constraint thus reads

$$C_{i,t} + K_{i,t+1} + P_t^b B_{i,t+1} = A_{i,t} K_{i,t} + B_{i,t},$$

where P_t^b is the bond price.

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[5] Previous literature using AK models to analyze asset prices in the presence of idiosyncratic risk includes Krebs and Wilson (2004), who focused on the case of log utility, and Toda (2014), who provides theoretical analysis for a class of similar models.

Agents have identical Epstein-Zin preferences with unit intertemporal elasticity of substitution, so that their value function satisfies

$$V_{i,t} = C_{i,t}^{1-\beta} \left(E_t[V_{i,t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}} \right)^{\beta}.$$

Here, parameter β controls the time preference and γ is the coefficient of relative risk aversion. In the following, we will focus on the empirically relevant case $\gamma > 1$, so that agents have a preference for early resolution of uncertainty. Given the process for productivity, the bond price, and the initial capital, each household will make its consumption-savings and portfolio choice to maximize the value function defined above.

We will assume that productivity has an aggregate and an idiosyncratic component:

$$\log(A_{i,t}) = \log(A_t) + \sqrt{x_t}\,\eta_{i,t} - \frac{x_t}{2}, \ \eta_{i,t} \sim N(0,1),$$

where idiosyncratic shocks $\eta_{i,t}$ are independent both across time and across households. Another exogenous process x_t denotes the cross-sectional variance of log productivity, which will fluctuate over time, and the last term ensures that the normalization $A_t = \tilde{E}[A_{i,t}]$ holds ($\tilde{E}[\,]$ will denote cross-sectional averages, conditional on the realizations of aggregate variables).

2.2 Equilibrium

The equilibrium of this economy turns out to be particularly simple:

- Since preferences are homothetic and the value function is linear in wealth, there is a separation between the consumption-saving decision and the portfolio choice. Since the idiosyncratic shocks are uncorrelated over time, the only source of heterogeneity is in differing levels of wealth, so that all households make the same portfolio choice. Given the zero net supply of bonds, the equilibrium must thus involve no trade in them, so that $\forall i, \forall t : B_{i,t} = 0$.

- Without bonds, all the wealth comes from current production. With unit IES, the consumption choice will be a constant linear function of wealth, so that $C_{i,t} = \kappa Y_{i,t}$ and $K_{i,t} = (1-\kappa)Y_{i,t}$, where $\kappa = 1 - \beta$.

Defining aggregates straightforwardly as cross-sectional averages (e.g., $K_t = \tilde{E}[K_{i,t}]$), the aggregate dynamics can be summarized easily:

$$Y_t = A_t K_t,$$
$$C_t = \kappa Y_t,$$
$$K_{t+1} = (1-\kappa)Y_t.$$

Note that the aggregate dynamics of quantities depend only on the aggregate productivity process A_t, not on the cross-sectional variance process x_t. If we denote logs in lowercase, we can also derive aggregate and individual consumption growth as

$$\Delta c_t = \log(C_t/C_{t-1}) = \log((1-\kappa)A_t) = \log(1-\kappa) + a_t,$$
$$\Delta c_{i,t} = \log(C_{i,t}/C_{i,t-1}) = \log((1-\kappa)A_{i,t}) = \log(1-\kappa) + a_t + \sqrt{x_t}\,\eta_{i,t} - \frac{x_t}{2}.$$

The process for individual consumption thus has a similar form as in Constantinides and Duffie (1996).

2.3 Asset Prices

Moving on to asset prices, although strictly speaking there is no aggregate capital, we can naturally define the aggregate return on capital as the average payoff at time $t+1$ to one unit of good invested at time t, so that $R^k_{t+1} = A_{t+1}$. The return on bonds is then defined as $R^b_{t+1} = \frac{1}{P^b_t}$, and the difference between the two returns will be the equity premium. In this case, the return on capital is determined entirely by the linear technology, so the premium will be driven by adjusting the risk-free rate in accordance with the intertemporal marginal rate of substitution of households in the no-trade equilibrium, to which we turn next.

The intertemporal marginal rate of substitution (IMRS) of the i-th household is given by

$$M_{i,t+1} = \beta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-1} \left(\frac{V_{i,t+1}}{E_t[V^{1-\gamma}_{i,t+1}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma}$$

and includes the usual consumption growth term and the deviation of the next-period value function from its certainty-equivalent that would capture news about future consumption. In the equilibrium, each household's IMRS is a valid stochastic discount factor, and so will be their cross-sectional average $M_{t+1} = \tilde{E}[M_{i,t+1}]$. The returns on capital and bonds must satisfy the following equations:

$$1 = E_t[M_{t+1} R^k_{t+1}], \ 1 = E_t[M_{t+1} R^b_{t+1}].$$

Assuming (conditional) lognormality, we can express the conditional equity premium in terms of the logarithm of the stochastic discount factor (SDF) and log returns as

$$E_t[r^k_{t+1}] + \frac{1}{2} \mathrm{Var}_t[r^k_{t+1}] - r^b_{t+1} = -\mathrm{Cov}_t[m_{t+1}, r^k_{t+1}]. \tag{1}$$

Since the capital return is exogenous, the asset-pricing properties will depend mainly on the conditional distribution of the stochastic discount factor and its sensitivity to aggregate shocks.

To explicitly characterize the innovation to the logarithm of SDF, we need to find the innovation to the value function. To do so, we define the logarithm of the normalized value function $v_{i,t} = \log(V_{i,t}/C_{i,t})$ and rewrite the value function recursion as

$$v_{i,t} = \beta \frac{1}{1-\gamma} \log E_t \left[\exp \left((1-\gamma)(v_{i,t+1} + \Delta c_{i,t+1}) \right) \right]$$

$$= \beta \frac{1}{1-\gamma} \log E_t \left[\exp \left((1-\gamma)(v_{i,t+1} + \Delta c_{t+1} - \frac{1}{2} \gamma x_{t+1}) \right) \right],$$

where the second line follows from substituting for individual consumption growth and integrating out the idiosyncratic shock. Since the above expression involves only aggregate variables, clearly the normalized value function will be equalized across households: $v_{i,t} = v_t$. If we furthermore assume that a_t (and thus Δc_t) and x_t jointly follow a Gaussian homoskedastic process, we get

$$v_t = \beta \left(E_t \left[v_{t+1} + \Delta c_{t+1} - \frac{1}{2} \gamma x_{t+1} \right] + \frac{1-\gamma}{2} \Sigma \right),$$

with $\Sigma = \mathrm{Var}_t \left[v_{t+1} + \Delta c_{t+1} - \frac{1}{2} \gamma x_{t+1} \right]$ being a (constant) conditional variance. Iterating forward and imposing a proper terminal condition, the value function can be expressed as

$$v_t = \frac{\beta}{1-\beta} \frac{1}{2} (1-\gamma) \Sigma + \sum_{i=1}^{\infty} \beta^i \left(\mathbb{E}_t \left[\Delta c_{t+i} - \frac{1}{2} \gamma x_{t+i} \right] \right).$$

The log of the aggregate SDF in terms of v_{t+1} has the form of

$$m_{t+1} = \log(\beta) - \gamma \Delta c_{t+1} + (1-\gamma)(v_{t+1} - v_t/\beta) + \frac{1}{2}\gamma(1+\gamma)x_{t+1},$$

where the last term arises from integrating over cross-sectional consumption growth.

2.4 Price of Risk

The innovation to m_{t+1} can subsequently be shown to equal

$$m_{t+1} - E_t[m_{t+1}] = -\gamma \varepsilon_{t+1}^c + \frac{1}{2}\gamma(1+\gamma)\varepsilon_{t+1}^x - (\gamma-1)\eta_{t+1}^c + \frac{1}{2}\gamma(\gamma-1)\eta_{t+1}^x$$

where:

- $\varepsilon_{t+1}^c = \Delta c_{t+1} - E_t[\Delta c_{t+1}]$ is the short-run innovation to consumption growth,

- $\varepsilon_{t+1}^x = x_{t+1} - E_t[x_{t+1}]$ is the short-run innovation to the cross-sectional consumption growth variance,

- $\eta_{t+1}^c = (E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \beta^i \Delta c_{t+1+j}\right]$ is the innovation to long-run expected consumption growth,

- $\eta_{t+1}^x = (E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \beta^i x_{t+1+j}\right]$ is the innovation to the long-run expected cross-sectional variance.

Increases in current or future consumption growth reduce the marginal utility and thus carry a positive market price of risk, whereas increases in the current or future cross-sectional variance enter with the opposite sign and thus carry a negative price of risk. In other words, assets which pay well in those states of the world where a household receives bad news about current or *future* cross-sectional risk are less attractive and must offer higher returns.

In the above expression, the first term is standard and captures aggregate consumption growth. The second term is the same as in the Constantinides & Duffie model and captures the contemporaneous effects of idiosyncratic risk. The third term describes news about future consumption and has been studied in the long-run risk literature. The final term then captures news about future idiosyncratic risk and is present only with a preference for early resolution of uncertainty ($\gamma > 1$) and in a non-iid environment. The presence of this last term can potentially increase the equity premium if bad news about current and future consumption growth is accompanied by bad news about future levels of idiosyncratic risk.

As a more specific example, consider the following joint process for $\Delta c_t, x_t$:

$$\Delta c_t = (1-\rho_c)\mu_c + \rho_c \Delta c_{t-1} + \varepsilon_t, \ \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$x_t = \mu_x + \phi_x(\Delta c_t - \mu_c),$$

so that aggregate consumption growth follows an AR(1) process and the idiosyncratic risk level is its affine function. Setting $\phi_x < 0$ corresponds to the countercyclical cross-sectional variance

Figure 1: Comparative Static of Conditional Sharpe Ratio Decomposition
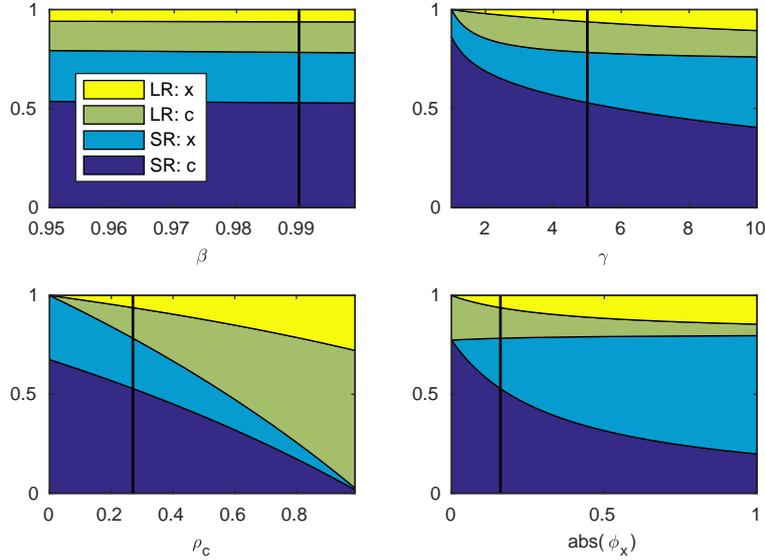
Note: The decomposition is defined according to equation (2). Filled areas show relative contribution of each channel (long- or short-run, aggregate consumption, or idiosyncratic risk). While one parameter is varied, others are kept fixed ($\beta = 0.99, \gamma = 5, \rho_c = 0.27, \phi_x = -0.16$, see also black lines in corresponding subplots).

emphasized by Constantinides & Duffie. Since there is just one aggregate shock, we can obtain the following expression for log SDF innovation:

$$m_{t+1} - E_t[m_{t+1}] = \left(-\gamma + \frac{1}{2}\gamma(1+\gamma)\phi_x - (\gamma-1)\frac{\beta\rho_c}{1-\beta\rho_c} + \frac{1}{2}\gamma(\gamma-1)\phi_x\frac{\beta\rho_c}{1-\beta\rho_c} \right) \varepsilon_{t+1}. \quad (2)$$

When $\gamma > 1$ and $\phi_x < 0$, all terms inside parentheses have the same sign and their magnitude can be interpreted as the contribution of individual channels to the overall price of risk.

For a quantitative illustration, we choose $\beta = 0.99, \gamma = 5$ (standard values), $\rho_c = 0.27$ (autocorrelation of quarterly US consumption growth), and $\phi_x = -0.16$ (see section 4.1). Following the above expression, we obtain that short-run consumption risk contributes 53.0%, short-run idiosyncratic risk 25.4%, long-run consumption risk 15.5%, and long-run idiosyncratic risk 6.2%. In relative terms, news about future idiosyncratic risk makes up 40% of the overall long-run risk. Figure 1 shows the sensitivity of this decomposition to each parameter. Varying the discount rate should in principle affect the weight households put on future events and thus also the relative importance of long-run risk, but for the range of values usually considered it does not seem to play a large role. Higher risk aversion raises the share of both long-run and idiosyncratic risk. Autocorrelation of consumption growth has a similar, although even stronger effect, as with more predictability a current shock to consumption causes a greater revision of expectations about the future. Finally, the degree of countercyclicality (plotted using its absolute value) makes the role of idiosyncratic risk larger.

The model presented in this section is too simplified in certain aspects. In a more standard production economy, the aggregate consumption process is endogenous and thus the introduction of

idiosyncratic risk may affect the asset-pricing results via general equilibrium effects. In addition, equity returns are also endogenous in the sense that the presence of idiosyncratic risk can affect the sensitivity of price-dividend ratios (and thus of returns themselves) to aggregate shocks, which might affect the predicted equity premium (although not the Sharpe ratio). For these reasons, in the next section I embed idiosyncratic risk in a version of a real business cycle model which will allow for both of these additional effects.

3. Full Model

This section describes the main model of a production economy with households facing idiosyncratic shocks. The model could be described as a variant of the standard stochastic growth model, similar to Kaltenbrunner and Lochstoer (2010), modified with a tractable form of heterogeneity on the household side modeled according to Constantinides and Duffie (1996).

3.1 Production

On the production side, there is a representative firm with standard Cobb-Douglas technology, producing output from capital K_t and labor H_t:

$$Y_t = K_t^\alpha (Z_t H_t)^{1-\alpha},\tag{3}$$

where Z_t is labor-augmenting productivity and its log growth rate $\Delta z_t = \log(Z_t) - \log(Z_{t-1})$ is a given exogenous stochastic process. The firm hires labor on a competitive market at wage rate W_t to the point where the wage equals the marginal product of labor:

$$W_t = (1-\alpha)\frac{Y_t}{H_t}.\tag{4}$$

Household labor supply is inelastic and fixed at unity, so in equilibrium

$$H_t = 1.\tag{5}$$

The firm owns its capital stock, uses part of its profits for investment I_t in the capital stock, and pays the residual as dividend D_t:

$$Y_t = W_t H_t + I_t + D_t.\tag{6}$$

Capital accumulation is standard:

$$K_{t+1} = (1-\delta)K_t + I_t.\tag{7}$$

Since the firm faces an intertemporal choice, it is necessary to discuss its objective. We will assume that the firm will choose investment policy to maximize the present value of its dividends evaluated with a one-period stochastic discount factor M_{t+1} (to be discussed later), which is taken as given by the firm. The multi-period SDF is then defined as $M_{t\to t+j} = \prod_{i=1}^{j} M_{t+i}$, and the firm's objective is to maximize the sum of the current dividend and the (ex-dividend) stock price P_t^s, with the latter equal to the present discounted value of future dividends:

$$\max D_t + \underbrace{\mathbb{E}_t\left[\sum_{j=1}^{\infty} M_{t\to t+j} D_{t+j}\right]}_{P_t^s}.$$

Under constant returns to scale, the return on the claim on the firm's equity (priced with the SDF referred to above) will be equal to the return on physical capital (Restoy and Rockinger, 1994), in this case given by

$$R^K_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \tag{8}$$

and by the standard variational argument, the firm's first-order condition is

$$1 = \mathbb{E}_t \left[M_{t+1} R^K_{t+1} \right]. \tag{9}$$

Finally, the resources left for aggregate consumption consist of wages and dividend payments, or equivalently of output less investment:

$$C_t = D_t + W_t H_t = Y_t - I_t. \tag{10}$$

Note that the production side of the model determines the dynamics of macroeconomic aggregates such as capital, output, and consumption once the stochastic discount factor is specified. Of course, in equilibrium the SDF process captures the attitudes of households toward intertemporal choice and risk, so we will discuss the household side of the model next.

3.2 Households

There is a continuum of households indexed by i, each having (the same) Epstein-Zin preferences over its own consumption stream $\{C_{i,t}\}$, summarized by a recursion for the value function

$$V_{i,t} = \left\{ (1-\beta) C_{i,t}^{1-\rho} + \beta \mathbb{E}_t \left[V_{i,t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}, \tag{11}$$

where β captures time preference, ρ is the inverse of the intertemporal elasticity of substitution, and γ is relative risk aversion. Each household also inelastically supplies one unit of labor.

The main object of interest on the household side of the model is the stochastic discount factor, which enters the firm's intertemporal decision. In a model with a representative household, we could drop the i subscript and the relevant SDF would be determined directly by the representative household's intertemporal marginal rate of substitution, the expression for which is known to be

$$M^{\text{RA}}_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t+1}}{\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}.$$

On the other hand, if households face idiosyncratic risks and markets are incomplete, so the risk cannot be insured away, we will observe dispersion in the individual consumption growth rates. In principle, individual consumption is an endogenous outcome, depending on the household's optimal decisions, which are themselves functions of individual and aggregate state variables. Generally, the aggregate state would include the cross-sectional distribution of wealth, necessitating the use of complex solution methods, such as those in Krusell and Smith (1998). Instead, I will follow

Constantinides and Duffie (1996) and assume directly[6] that the resulting dispersion of consumption growth rates can be described by a multiplicative shock to aggregate consumption growth:

$$\frac{C_{i,t+1}}{C_{i,t}} = \frac{C_{t+1}}{C_t} \exp(\eta_{i,t+1}),\qquad(12)$$

where innovations $\eta_{i,t+1}$ are uncorrelated across households and across time. However, since we are interested in idiosyncratic risk, whose severity varies over the business cycle, we will allow the distribution of $\eta_{i,t}$ to vary according to an exogenous parameter process x_t. It will turn out advantageous to summarize this dependence via a moment-generating function (MGF)

$$G(\tau;x) = \mathbb{E}\left[e^{\tau\eta}|x\right]\qquad(13)$$

and assume the parametrization satisfies the property $G(1,x) = 1$ for all possible x, ensuring that average consumption equals aggregate consumption. For example, if $\eta_{i,t}$ is normal with variance x_t and mean $-x_t/2$, the MGF would be $G(\tau;x) = e^{(x/2)(\tau^2-\tau)}$.

The main advantage of the above approach is that it allows us to define the aggregate stochastic discount factor as a cross-sectional average of the individual marginal rates of substitution in a tractable way, so that the resulting expression depends only on aggregate variables. For this purpose, we define the logarithm of the value function scaled by individual consumption $v_{i,t} = \log(V_{i,t}/C_{i,t})$ and the logarithm of the scaled certainty equivalent $\psi_{i,t} = \log\left(\mathbb{E}_t\left[V_{i,t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}/C_{i,t}\right)$, which satisfy the following:

$$v_{i,t} = \frac{1}{1-\rho}\log\left((1-\beta) + \beta\exp((1-\rho)\psi_{i,t})\right)$$
$$\psi_{i,t} = \frac{1}{1-\gamma}\log\left(\mathbb{E}_t\left[\exp((1-\gamma)(v_{i,t+1}+\Delta c_{i,t+1}))\right]\right)$$

Under the maintained assumption on individual consumption growth, we have $\Delta c_{i,t+1} = \Delta c_t + \eta_{i,t+1}$, and the distribution of $\eta_{i,t+1}$ is the same for each household from the point of view of period t. Using the law of iterated expectations to integrate over $\eta_{i,t+1}$ (conditional on the next-period parameters of its distribution x_{t+1}), we can rewrite the scaled value function recursion in terms of aggregates only, implying that these variables are equalized across households (so we can drop the i subscript):

$$v_t = \frac{1}{1-\rho}\log\left((1-\beta) + \beta\exp((1-\rho)\psi_t)\right)$$
$$\psi_t = \frac{1}{1-\gamma}\log\left(\mathbb{E}_t\left[\exp((1-\gamma)(v_{t+1}+\Delta c_{t+1}))\cdot G(1-\gamma,x_{t+1})\right]\right)\qquad(14)$$

Note the MGF term $G(1-\gamma,x_{t+1}) = \mathbb{E}[\exp((1-\gamma)\eta_{i,t+1})|x_{t+1}]$, which arises from integrating over individual shocks in the next period, conditional on its distribution, which depends on aggregate variables x_{t+1}.

An individual household's intertemporal marginal rate of substitution is

$$M_{i,t+1} = \beta\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\rho}\left(\frac{V_{i,t+1}}{\mathbb{E}_t\left[V_{i,t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma},\qquad(15)$$

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[6] See section 3.4 for a discussion of how such a result could be derived as a particular equilibrium outcome.

which can be equivalently expressed as

$$M_{i,t+1} = \beta \exp\left(-\gamma \Delta c_{i,t+1} + (\rho - \gamma)(v_{t+1} - \psi_t)\right),\qquad(16)$$

and subsequently the aggregate SDF is obtained by averaging over individual $M_{i,t+1}$ conditional on aggregate variables up to and including period $t+1$:

$$M_{t+1} = \beta \exp\left(-\gamma \Delta c_{t+1} + (\rho - \gamma)(v_{t+1} - \psi_t)\right) \cdot G(-\gamma, x_{t+1}),\qquad(17)$$

where again the term $G(-\gamma, x_{t+1})$ appears due to integration over individual shocks.

Although defining the aggregate SDF by averaging the individual rates of substitution may seem arbitrary, if we grant that individual consumption allocations are outcomes of some (as yet unspecified) equilibrium, and abstract from binding portfolio constraints, each household's intertemporal rate of substitution would in fact be a valid SDF, in the sense that it would be compatible with asset prices in the economy. Taking the cross-sectional average then results in the SDF, which is valid too, but does not depend directly on any individual-level variables.

The presence of idiosyncratic risk thus affects the resulting discount factor through the properties of its distribution: specifically, through the $G(1 - \gamma, x_{t+1})$ term in the value function recursion, provided that $\rho \neq \gamma$, and through the $G(-\gamma, x_{t+1})$ term in the SDF. Since the modifications are expressed in terms of moment-generating functions, all the higher moments of idiosyncratic risk could in principle affect the economy, although in the most commonly studied case of normal shocks, only the variance will matter. It is also clear that if the distribution of idiosyncratic shocks was time-invariant (i.e., x_t was constant), the only effect would be to introduce constant offsets into the value function and discount factor, while the risk premia would not be affected directly. Finally, making the distribution of η collapse to a constant would yield expressions identical to a representative-agent version of the model, which can thus be considered a special case of the setup presented above.

3.3 Quantity Dynamics and Asset Prices

To close the model, we need to further specify the exogenous process for productivity Z_t and the evolution of parameters x_t controlling the distribution of individual shocks (these could either be functions of other aggregate variables or follow their own exogenous process).

Productivity is assumed to be a random walk, so that

$$\Delta z_t = \mu_z + \sigma_z \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0,1).\qquad(18)$$

Regarding the form of individual risk, I will assume that the individual element of consumption growth is lognormal, so that

$$\eta_{i,t} \sim \mathcal{N}\left(-\frac{x_t}{2}, x_t\right)$$

and x_t represents its variance, which is exogenously given as an affine function of consumption growth

$$x_t = \mu_x + \phi_x(\Delta c_t - \mu_z).\qquad(19)$$

Equations (18) and (19) together with equations (3), (5), (7), (8), (9), (10), (14), and (17) and the functional form for $G(\tau,x)$ give us a sufficient number of relationships to solve the model. Since there is no need to track the cross-sectional distribution of assets, the model can be solved by standard perturbation methods after detrending.

In terms of asset prices, the unlevered return on capital has been defined in (8), and its logarithm will be denoted $r^k_{t+1} = \log R^k_{t+1}$. We will define the price of a one-period riskless bond that pays one unit in the following period in a standard way:

$$P^b_t = \mathbb{E}_t\left[M_{t+1}\cdot 1\right] \tag{20}$$

and define the log-return on the bond as $r^b_{t+1} = \log(1/P^b_t)$. The excess return is the difference between the return on capital and the return on bonds: $r^x_{t+1} = r^k_{t+1} - r^b_{t+1}$. The conditional equity premium and the Sharpe ratio are then defined as:

$$\begin{aligned} \text{EP}_t &= \mathbb{E}_t[r^x_{t+1}] \\ \text{SR}_t &= \frac{\mathbb{E}_t[r^x_{t+1}]}{\sqrt{\text{Var}_t[r^x_{t+1}]}} \end{aligned} \tag{21}$$

and their unconditional averages are $\text{EP} = \mathbb{E}[\text{EP}_t]$, $\text{SR} = \mathbb{E}[\text{SR}_t]$.

Recall the expression for the conditional equity premium in a lognormal setting (adjusted for Jensen's inequality) from equation (1):

$$\mathbb{E}_t[r^k_{t+1}] + \frac{1}{2}\text{Var}_t[r^k_{t+1}] - r^b_{t+1} = -\text{Cov}_t[m_{t+1}, r^k_{t+1}].$$

In the case of just one aggregate shock, so that $m_{t+1} - \mathbb{E}_t[m_{t+1}] = \eta_{m\varepsilon}\varepsilon_{t+1}$, the conditional Sharpe ratio and the equity premium are approximately

$$\text{SR}_t \approx |\eta_{m\varepsilon}|\sigma_z, \ \text{EP}_t = \text{SR}_t\text{Var}_t[r^x_{t+1}].$$

In the model, all the conditional volatility of the returns arises from fluctuations in the marginal product of capital, which is not volatile enough to match the observed variation in stock returns. This issue could in principle be fixed by introducing capital adjustment costs or leveraged equity, although in this paper I will focus mainly on the price rather than on the quantity of risk, i.e., on the Sharpe ratio.

3.4 No-trade Equilibrium

The model presented so far relies on a reduced-form way of incorporating idiosyncratic consumption risk. It is possible to support such an outcome as a no-trade equilibrium[7] of a model with households facing a particularly defined idiosyncratic additive shock to their budget constraints, which could represent unexpected expenditures, gains, or redistributive payments (which, however, cancel out in the aggregate) that cannot be insured against due to incomplete markets. Intuitively, given that households' utility function is homothetic and in the proposed equilibrium the deviation

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[7] The discussion here adapts the no-trade equilibrium setup of Constantinides and Duffie (1996) from endowment to a production economy with EZ preferences. A close, although not identical aggregation approach is offered in Braun and Nakajima (2012), who allow for elastic labor supply, but also consider only a time-separable utility function.

of individual consumption from the aggregate is a geometric random walk with shocks uncorrelated in time, all the households behave essentially symmetrically in their consumption/saving and portfolio decisions, thus implying no trade in assets. No trade, together with symmetric initial portfolios, leads, in turn, to individual consumption heterogeneity of the form described in previous sections. For completeness, this section will present such an equilibrium in more detail.

An individual household receives labor income and can trade firm shares and bonds. Its budget constraint reads:

$$C_{i,t} + P_t^s A_{i,t+1} + P_t^b B_{i,t+1} = W_t + (P_t^s + D_t) A_{i,t} + B_{i,t} + \Upsilon_{i,t} C_t,$$

where P_t^s, P_t^b are prices of firm equity and a risk-free one-period bond, respectively, $A_{i,t}, B_{i,t}$ are the household's beginning-of-period portfolio positions, and the other variables are as defined previously. The household also faces an additive shock $\Upsilon_{i,t}$ to its wealth, scaled by the current level of aggregate consumption. We will require the cross-sectional average of $\Upsilon_{i,t}$ to equal zero, so that individual shocks do not add or subtract resources to or from the economy.

The evolution of the idiosyncratic shocks is specified as:

$$\Upsilon_{i,t} = (1 + \Upsilon_{i,t-1}) \exp(\eta_{i,t}) - 1,$$

where $\eta_{i,t}$ are the same shocks which were previously characterized in equation (13). Since we assumed $\int \exp(\eta_{i,t}) \mathrm{d}i = 1$, the above law of motion maintains a zero cross-sectional mean of $\Upsilon_{i,t}$. For example, if $\eta_{i,t}$ is normally distributed, $\Upsilon_{i,t}$ will have a lognormal distribution shifted by a negative constant.

The household takes asset prices, wages, dividends, aggregate consumption, and idiosyncratic shocks as given, and chooses its consumption and portfolio positions to maximize its value function (11). Given the allocation of consumption across households, the rest of the model functions as previously described, although we will also require stock and bond prices to be consistent with market clearing in financial markets, so that in the aggregate households own the whole firm ($\int A_{i,t} \mathrm{d}i = 1$) and bonds are in zero net supply ($\int B_{i,t} \mathrm{d}i = 0$). Given the specification of exogenous shocks $Z_t, \Upsilon_{i,t}$, the equilibrium of the economy can thus be defined as:

- a stochastic process for aggregate output Y_t, consumption C_t, investment I_t, capital K_t, the wage W_t, the return on capital R_t^k, and the dividend D_t

- the firm equity price P_t^s and the bond price P_t^b

- individual household consumption $C_{i,t}$, portfolio positions $A_{i,t}, B_{i,t}$, the value function $V_{i,t}$, and the IMRS $M_{i,t+1}$

- the aggregate SDF M_{t+1}

such that

- given the aggregate SDF, $Y_t, I_t, K_t, C_t, D_t, R_t^k, and W_t$ are consistent with the firm optimality condition (9), the production function (3), capital accumulation (7), resource constraints (6), (10), and marginal products (4), (8).

- markets for financial assets clear

- $C_{i,t}, A_{i,t}, B_{i,t}, V_{i,t}$ and $M_{i,t+1}$ are consistent with optimal decisions by households

- M_t is consistent with the cross-sectional aggregation of household intertemporal rates of substitution $M_{i,t}$ as described in (17).

Next, notice that if households held symmetric market-clearing portfolios, i.e., $\forall t, \forall i : A_{i,t} = 1, B_{i,t} = 0$, their consumption growth would in fact be described by (12), since in such case their consumption is $C_{i,t} = W_t + D_t + \Upsilon_{i,t} C_t = (1 + \Upsilon_{i,t}) C_t$ and their consumption growth thus satisfies

$$\frac{C_{i,t+1}}{C_{i,t}} = \frac{C_{t+1}}{C_t} \frac{1 + \Upsilon_{i,t+1}}{1 + \Upsilon_{i,t}} = \frac{C_{t+1}}{C_t} \cdot \exp(\eta_{i,t+1})$$

The following result shows that an outcome where households hold symmetric portfolios at all times, embedded within the rest of the model described previously, is in fact an equilibrium:

Claim: Consider an allocation where

- the firm stock price is given by $P_t^s = K_{t+1}$ and the bond price is determined by the aggregate SDF as in (20),

- households hold symmetric portfolios $A_{i,t} = 1, B_{i,t} = 0$,

- and the rest of the model functions as described previously;

then such an allocation is an equilibrium. Moreover, households are in agreement in terms of the firm's investment policy.

To see why the above holds, we need to check whether the first-order conditions of individual households are satisfied. The intertemporal rate of substitution of household i between two consecutive periods (implicitly taking the current aggregate state of the economy as given; I also suppress the time indices for clarity) can be generally written as a function of some first-period individual state s_i and second-period individual shock η_i' and aggregate shock ε': $M_i(s_i, \eta_i', \varepsilon')$. In our case, however, the individual IMRS given by (16) depends on the individual state only through the household's consumption growth, which is assumed to be uncorrelated over time and determined by future idiosyncratic shocks η_i'. Therefore, the individual IMRS does not depend on the initial individual state and can be written as $M(\eta_i', \varepsilon')$. Intuitively, if individual consumption behaves like a multiplicative random walk and households have homothetic preferences, any differences in wealth are simply a matter of scale.

The aggregate stochastic discount factor is obtained by averaging over the individual shocks: $M(\varepsilon') = \mathbb{E}\left[M_i(\eta_i', \varepsilon') \mid \varepsilon'\right]$ (since the distribution of shocks is symmetric across households, this does not actually depend on i). We can then show that the aggregate optimality condition $\mathbb{E}\left[M(\varepsilon')R(\varepsilon')\right]$ for some return R also implies individual optimality $\mathbb{E}\left[M_i(\eta_i', \varepsilon')R(\varepsilon')\right]$, since here this follows directly from the law of iterated expectations. The aggregate optimality is satisfied by the return on bonds by assumption, and it is easy to show it also holds for the return on stocks held by

households.[8] It then follows that the household individual optimality conditions are also satisfied and the no-trade equilibrium is consistent with the optimal consumption and portfolio choice by households.

The same argument also ensures that households do not differ in their preferred investment policy (see also Carceles-Poveda and Coen-Pirani (2009) for a more general discussion of when this is true): in equilibrium, each household receives a stream of dividends from the firm, so its preferred policy is to maximize the present value of future dividends, using its own IMRS as a discount factor. This would lead to a first-order condition for investment $1 = \mathbb{E}[M_i(\eta_i', \varepsilon')R^K(\varepsilon')]$, but by the same logic of iterated expectations this is equivalent to the assumed firm's condition (9). Another possible question is whether a different choice of weights across households when defining the aggregate SDF might affect the results. In general, this is possible in models with incomplete markets (Carceles-Poveda, 2009), but it turns out that in the current model the weighting does not matter. Any weights corresponding to some reasonable corporate governance mechanism should depend only on the current states of firm owners, not on realizations of next-period shocks. A weighted SDF $\tilde{M}(\varepsilon') = \mathbb{E}\left[w(s)M_i(s, \eta_i', \varepsilon') \mid \varepsilon'\right]$ will not make a difference when M_i is independent of s.

4. Results

To evaluate how the addition of idiosyncratic risk affects the behavior of the neoclassical growth model, I first calibrate most of the parameters based on a representative-agent version of the model, then solve the model with and without idiosyncratic risk and inspect its properties. In the second part of this section, I proceed by describing a loglinear approximate solution to the model, which is helpful in illustrating the interplay between idiosyncratic risk and the dynamics of the macroeconomic aggregates in the model. Finally, I will consider an alternative way of modeling cyclical variation in the distribution of idiosyncratic risk by way of cyclical skewness rather than variance.

4.1 Calibration

The model calibration is summarized in Table 1. The frequency is quarterly. Starting with the representative-agent version of the model, most parameters are chosen close to standard values in the literature, as in Campbell (1994), for example. α is set to match a capital share of income of one third, while δ implies an annual depreciation rate of 10%. The discount rate β and the inverse of the IES ρ are set so as to match a steady-state return on capital of 6% per annum and a relative volatility of consumption and output growth of one half. Trend productivity growth is set at 2% per year. The volatility of productivity shocks matches a standard deviation of quarterly output growth of 1%, roughly corresponding to the postwar US data. Finally, risk aversion is set to 5, a relatively standard value.

Following Storesletten et al. (2007), who use a process for the variance of idiosyncratic shocks of the same form, I set $\mu_x = 0.0036$ (i.e., their value 0.014 rescaled to a quarterly setting) and $\phi_x = -0.16$. The average level μ_x corresponds to an annualized standard deviation of individual consumption growth of about 12%. The value of sensitivity ϕ_x captures the sensitivity of idiosyncratic risk to the business cycle, with negative values representing countercyclical variation. Given that the quarterly

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[8] This can be verified by plugging in the proposed expression for the stock price into the definition of the return and using the fact that $D_{t+1} = Y_{t+1} - W_{t+1} - I_{t+1} = \alpha Y_{t+1} - I_{t+1}$. After some rearranging, we obtain that the stock return is equal to the return on capital defined in (8) and thus satisfies the condition due to the firm's optimality condition (9).

Table 1: Parameter Values

Parameter	Value	Description
β	0.988	discount factor
ρ	0.7	inverse of IES
γ	5	risk aversion
α	0.33	capital share
δ	0.025	depreciation rate
μ_z	0.005	mean productivity growth
σ_z	0.015	volatility of productivity shock
μ_x	0.0036	mean level of ind. risk
ϕ_x	-0.16	cyclicality of ind. risk

(non-annualized) standard deviation of consumption growth will be approximately half a percent, and assuming a normal distribution, the chosen value implies that the fluctuations in x_t correspond to an annualized standard deviation of individual consumption growth ranging from approximately 9% to 15% with 95% probability (in terms of the ergodic distribution).

After detrending by productivity (a list of the detrended equations can be found in the appendix), I solve the model by a third-order perturbation method using Dynare (Adjemian et al., 2011), as higher-order approximation is necessary to obtain non-zero risk premia when the perturbation approach is used for numerical solution. Model-implied moments for various variables are then computed from a pruned representation of the system, using the approach and code presented by Andreasen et al. (2013). In a recent study, Pohl et al. (2018) argue that models with long-run risk can exhibit nonlinearities that make local approximations potentially unreliable, and suggest using global solution methods. It turns out that in the model presented here, the nonlinearities are quite mild, so that local and global solutions yield very similar results, as documented in the appendix.

4.2 Quantitative Results

Table 2 displays selected unconditional moments from three versions of the model and also from US quarterly macroeconomic data. The representative-agent variant of the model (column RA) matches the variances of output and consumption growth (which were the targets of the calibration) and the autocorrelation of consumption growth. The implied Sharpe ratio of about 12% is lower than observed, yet still quite substantial compared to its value in a model with separable utility (approximately 0.6%). The second variant (column HA1) is a model with idiosyncratic risk parameters calibrated as described above and otherwise the same as the representative-agent model, with the exception of the discount factor β, which has been adjusted to obtain the same steady state. Looking at our main object of interest, we see that the presence of countercyclical idiosyncratic risk has increased the market price of risk (proxied here by the Sharpe ratio of excess returns) by approximately a third, but the dynamics of the macroeconomic quantities have also changed significantly: with idiosyncratic risk, aggregate consumption growth has volatility closer to that of output growth and autocorrelation closer to zero, which worsens the empirical fit of the model. In the third version (column HA2), both the discount factor and the intertemporal elasticity of substitution are modified to maintain the same dynamics of output and consumption as in the RA variant of the model. We

Table 2: Comparison of Model-implied Annualized Moments

	Data	Model: RA	Model: HA1	Model: HA2
Moments:				
$\sigma[\Delta y_t]$	1.90%	2.02%	2.01%	2.02%
$\sigma[\Delta c_t]/\sigma[\Delta y_t]$	0.56	0.50	0.74	0.49
$\sigma[\Delta i_t]/\sigma[\Delta y_t]$	2.58	2.65	1.81	2.63
$\text{cor}(\Delta y_t, \Delta y_{t-1})$	0.37	0.03	0.02	0.03
$\text{cor}(\Delta c_t, \Delta c_{t-1})$	0.27	0.21	0.06	0.21
Sharpe ratio	0.39	0.121	0.163	0.161
Risk price decomposition:				
short run, Δc	-	39.1%	45.8%	29.7%
short run, x	-	0.0%	22.0%	14.2%
long run, Δc	-	60.9%	23.0%	40.1%
long run, x	-	0.0%	9.2%	16.0%

Note: Data: US quarterly series 1947–2016, see appendix for definitions. Model RA: calibrated as in Table 1, but setting $\mu_x = \phi_x = 0$. Model HA1: as in Table 1, but setting $\beta = 0.973$ to match RA model steady state. Model HA2: as in Table 1, but setting $\beta = 0.975, \rho = 0.214$ to match RA model steady state and quantity dynamics. Standard deviations and Sharpe ratio are annualized by doubling from quarterly values. Bottom part shows relative contributions to price of risk based on loglinear approximation.

Figure 2: Impulse Responses of Log Consumption and Output
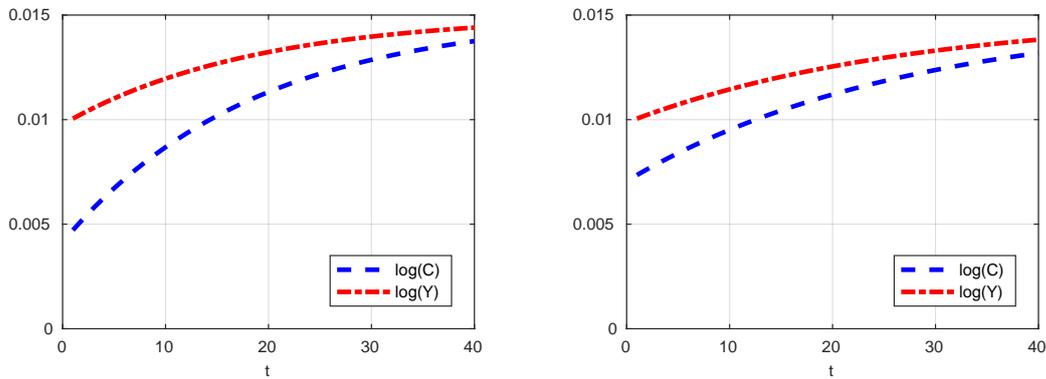
Note: Responses are to 1 s.d. (permanent) productivity shock. Right panel: models RA (representative agent) and HA2 (het. agents, with β and ρ adjusted to match RA model dynamics). Left panel: model HA1 (het. agents, with β adjusted to match RA model steady state).

can see that the market price of risk still remains high, so that with a suitable choice of preference parameters the model can be relatively successful along both dimensions.

Even though the model with idiosyncratic risk has a higher price of risk relative to the representative agent model, the overall level of the Sharpe ratio still does not achieve the observed values. In principle, one could achieve a higher Sharpe ratio by cranking up the risk aversion. However, high values of γ are often considered unrealistic, as they imply implausibly conservative behavior by agents faced with a risky choice. In addition, higher risk aversion would make it harder to match the behavior of consumption in a model with idiosyncratic risk by requiring excessive adjustment of the intertemporal elasticity parameter (see also the discussion in the following subsection). The results presented here should then be interpreted as offering a partial resolution of the equity premium puzzle in a model with a moderate amount of risk aversion, but to explain the observed Sharpe ratio fully would likely require a richer model.

The bottom part of the table presents the decomposition of the risk premium based on loglinear approximation, similar to the discussion in section 2 (see also the next subsection and the appendix for more details about the loglinear solution). The dispersion of the idiosyncratic shocks makes up rather less than a third of the overall long-run risk contribution and around a third of the overall short-run risk contribution. The overall contribution of long-run risk is 61% in the representative-agent model and 56% in the HA2 model, but it is only 32% in the HA1 model, due to the overall amount of predictability in the economy being lower (aggregate consumption is closer to a random walk).

To better understand how the introduction of idiosyncratic risk affects the behavior of output and consumption, Figure 2 plots the impulse responses to a productivity shock to the output and consumption (log) levels for both the RA and HA1 variants of the model (the impulse responses in the HA2 calibration are by construction close to the RA variant). The representative-agent version shows both consumption and output growing over time toward their new permanently higher values implied by the permanent increase in productivity, but the response of consumption on impact is about half of that of output (in line with the calibration targeting consumption growth volatility at half of output growth volatility). Households are thus willing to spread the increase in consumption over a longer period and accept variation in future consumption growth rates in order to accumulate capital stock more quickly and thus obtain more benefit from the increased productivity. However, in the model with idiosyncratic risk the response of consumption on impact is much stronger and essentially consumes the whole productivity gain straight away, at the cost of slower accumulation of capital, as if households were much more averse to intertemporal substitution of consumption.

This consumption smoothing effect also complicates the analysis of asset prices, since the price of risk can be affected by the presence of idiosyncratic risk – in addition to its direct impact on the stochastic discount factor described in section 2 – through changes in the endogenous process for aggregate consumption caused by the lower steady-state interest rate and the lower "aggregate" intertemporal elasticity of substitution. Specifically, with consumption growth less predictable, the long-run consumption risk emphasized by Kaltenbrunner and Lochstoer (2010) becomes less important, although the overall market price of risk has in our case gone up. On the other hand, as can be seen from the final column of Table 2, it is possible to counteract such impacts by increasing the IES (i.e., reducing the ρ) of individual households, although in general the size of the adjustment will depend on both the level and the cyclicality of idiosyncratic risk and on households' risk aversion, as discussed in more detail in the next subsection.

4.3 Qualitative Analysis

To gain better intuition about the implications of idiosyncratic risk, we will inspect a loglinear approximation of the model solution along the lines of Campbell (1994). Since the productivity process is a random walk, the detrended model has just one relevant state variable – the (log) ratio of capital to productivity $k_t^* = \log(K_t/Z_t)$ (in terms of notation, lowercase symbols will denote logs and starred variables are detrended by productivity). The dynamics of capital, output, and consumption are determined by the deterministic steady state and by the sensitivity of detrended consumption to detrended capital: $\tilde{c}_t^* = \eta_{ck}\tilde{k}_t^*$, with tilde denoting deviation from the steady-state value.

A complete derivation can be found in the appendix, but it is possible to show that the steady state depends on preferences and idiosyncratic risk parameters only through their effect on the steady-state return on capital $\bar{r}^k = -\log(\beta) + \rho\mu_z - \frac{1}{2}\gamma(1+\rho)\mu_x$. The coefficient η_{ck} depends on the steady state and on the "effective" inverse of the IES $\hat{\rho} = \rho - \frac{1}{2}\gamma(1+\rho)\phi_x$. In other words, any combinations of parameters $\beta, \rho, \gamma, \mu_x, \phi_x$ which imply the same $\hat{r}^k$ and $\hat{\rho}$ will lead to identical dynamics of output and consumption growth.

More specifically, if we start with a representative-agent model with some parameters $\beta^{RA}, \rho^{RA}, \gamma^{RA}$ and then introduce idiosyncratic risk by setting $\mu_x > 0, \phi_x \neq 0$, we can maintain the same quantity dynamics in the heterogeneous-agent model by choosing parameters $\beta^{HA}, \rho^{HA}, \gamma^{HA}$ such that

$$-\log(\beta^{RA}) + \rho^{RA}\mu_z = -\log(\beta^{HA}) + \rho^{HA}\mu_z - \frac{1}{2}\gamma^{HA}(1+\rho^{HA})\mu_x$$
$$\rho^{RA} = \rho^{HA} - \frac{1}{2}\gamma^{HA}(1+\rho^{HA})\phi_x$$
.

If, for example, we decide to keep risk aversion the same: $\gamma^{HA} = \gamma^{RA}$, the above two equations pin down the new values of the discount rate and the intertemporal elasticity of substitution. If the individual risk is acyclical ($\phi_x = 0$), the only necessary adjustment is in the discount rate, which should be set lower to counteract the precautionary saving effect pushing interest rates down. In the presence of countercyclical individual risk ($\phi_x < 0$), we would additionally need to make ρ^{HA} lower[9] to counteract the greater aversion of agents to intertemporal substitution.

Why do agents exhibit the latter? We can gain some intuition by looking at the power utility case ($\gamma = \rho$). The individual Euler's equation can then be written approximately as

$$\log(\beta) + \rho E_t[\Delta c_{i,t+1}] - \frac{1}{2}\rho^2 Var_t[\Delta c_{i,t+1}] = r_{t+1}^b.$$

Since $\Delta c_{i,t+1} = \Delta c_{t+1} + \eta_{i,t+1}$, if we ignore the small normalization shift in $\eta_{i,t+1}$, expected individual consumption growth moves one to one with aggregate expected consumption growth. However, with countercyclical risk, the conditional variance of individual consumption growth will vary inversely to Δc_{t+1}, and thus the whole left-hand side will be more sensitive to $E_t[\Delta c_{t+1}]$. As a result, if we consider only aggregate data, the agent behaves as if he had higher ρ (lower intertemporal substitution) than he really does, which is consistent with empirical estimates of the IES finding higher values when estimated on micro data compared with findings from aggregate time series (Havranek, 2015).

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[9] A similar expression for "effective" intertemporal substitution in the CRRA case was derived in Constantinides and Duffie (1996).

Moreover, if the agent has Epstein-Zin preferences with risk aversion differing from the inverse of the IES, the above result suggests that the degree of required adjustment in ρ depends on risk aversion as well, or alternatively, that risk aversion affects the dynamics of macroeconomic aggregates even at first-order approximation. The separation property described by Tallarini (2000) (i.e., that risk aversion affects risk premia but not the behavior of quantities) thus does not hold outside the representative-agent model. A related issue with the proposed adjustment might be that if idiosyncratic risk is strongly cyclical (ϕ_x has large magnitude) or households are very risk averse (γ is high), the adjustment might imply parameter values for ρ that are too low or even negative. It is possible that introducing other extensions affecting intertemporal choice, such as habit formation, might counteract this tendency, although I do not follow this direction in the current paper.

Even though the above discussion would suggest that the effect of idiosyncratic risk (at least as modeled here) does not affect the qualitative properties of the representative-agent model conditional on suitable recalibration of the preference parameters, the equivalence does not carry over to asset prices. Up to a linear approximation, the log of the scaled value function $v_t = \log(V_{i,t}/C_{i,t})$ can also be solved for as a function of the capital stock, so that in terms of deviations from the steady state $\tilde{v}_t = \eta_{vk}\tilde{k}_t^*$. The coefficient η_{vk} is a function of the steady state and η_{ck}, but also depends on both μ_x and ϕ_x. With countercyclical risk ($\phi_x < 0$), the value function will be more sensitive to the detrended capital stock and thus also to the productivity shock. The innovation to log SDF can be written as

$$m_{t+1} - E_t[m_{t+1}] = -\underbrace{\left[\left(\gamma - \frac{1}{2}\gamma(1+\gamma)\phi_x \right) \eta_{cz} + (\gamma-\rho)(-\eta_{vk}) \right]}_{\eta_{m\varepsilon}} \varepsilon_{t+1} = \eta_{m\varepsilon}\varepsilon_{t+1},$$

implying the conditional Sharpe ratio

$$\frac{\log\left(E_t[R_{t+1}^k] \right) - r_{t+1}^b}{sd_t[r_{t+1}^k]} = -\eta_{m\varepsilon}\sigma_z.$$

Therefore, even if we recalibrate the parameters to maintain the same dynamics of aggregate consumption, the market price of risk will still differ from the one implied by the representative-agent model with the same dynamics.

The left panel of Figure 3 plots the (annualized) conditional Sharpe ratio as a function of μ_x, ϕ_x when the preference parameters are recalibrated to match the quantity dynamics of the representative-agent model solved previously. Each point on the graph thus implies the same consumption process so that we can distinguish the pure effects of idiosyncratic risk on the risk premium. If the risk was acyclical ($\phi_x = 0$), the price of risk would actually go down slightly due to the required lower discount rate, which, in turn, weakens the impact of long-run consumption risk (this effect is present only when consumption growth is not iid, otherwise acyclical idiosyncratic risk would have no impact, as in Krueger and Lustig (2010)). However, making the risk countercyclical increases the price of risk substantially. Note that Epstein-Zin preferences are crucial for this result, since if we imposed $\gamma = \rho$, we would obtain $\eta_{m\varepsilon} = -\hat{\rho}\eta_{cz}$ and thus the recalibration procedure would imply the same price of risk for any combination of parameters.

The right panel of Figure 3 plots the dependence of the risk premium on the risk aversion parameter for the representative-agent model and for the model with idiosyncratic risk calibrated as in the previous section, again keeping the quantity dynamics the same. We can observe that the presence of idiosyncratic risk not only makes the risk premium rise faster with higher risk aversion, but makes

Figure 3: Comparative Static for Conditional Sharpe Ratio
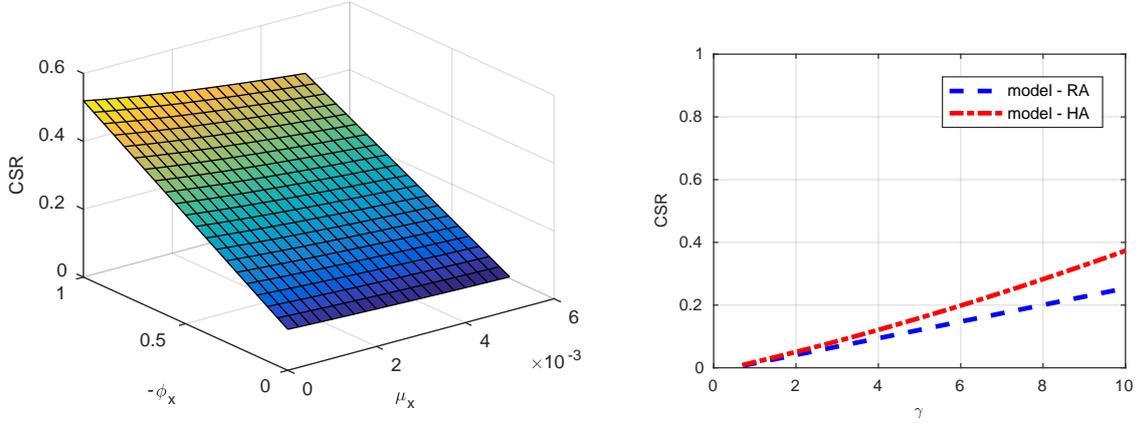
Note: Left: dependence on idiosyncratic risk parameters. Right: dependence on risk aversion. At each point, ρ and β are recalibrated to imply same dynamics of aggregate quantities.

it do so at an increasing rate, leading to a convex relationship (whereas the dependence is linear in the RA model). This confirms that the combination of Epstein-Zin preferences with idiosyncratic risk leads to an interaction that makes it easier to match observed risk premia with lower levels of risk aversion.

4.4 Cyclical Skewness

Recent research (Guvenen et al., 2014) suggests that it is cyclical variation in the skewness, rather than the variance, of idiosyncratic shocks that is more consistent with the data. Although cyclical variance, as analyzed in the previous sections, is especially tractable given the loglinear form of the moment-generating function for the Gaussian distribution, the model allows us to use other distributions as well, as long as their moment-generating function can be expressed in closed form. To see how much the results described above depend on the specific form of idiosyncratic risk, I solve the model with $\eta_{i,t}$ following a mixture of three normal distributions with time-varying means, as proposed by McKay (2017).[10] Specifically, I will assume that

$$\eta_{i,t} \sim \text{constant} + \begin{cases} N(\mu_{1,t}, \sigma_1^2) & \text{with prob. } p_1 \\ N(\mu_{2,t}, \sigma_2^2) & \text{with prob. } p_2 \\ N(\mu_{3,t}, \sigma_3^2) & \text{with prob. } p_3, \end{cases}$$

where the constant captures normalization so that $E[\exp(\eta_{i,t})] = 1$, the means are given by

$$\mu_{1,t} = 0$$
$$\mu_{2,t} = \mu_2 - x_t, \ \mu_2 < 0$$
$$\mu_{3,t} = \mu_3 - x_t, \ \mu_3 > 0,$$

and like before, x_t is a function of aggregate consumption growth:

$$x_t = \phi_x (\Delta c_t - \mu_z).$$

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[10] To be precise, I use the distribution of the permanent component of the income shock faced by employed agents in the model described in that paper.

Table 3: Comparison of Model-implied Annualized Moments under Cyclical Skewness

	Data	Model: RA	Model: HA3	Model: HA4
Moments:				
$\sigma[\Delta y_t]$	1.90%	2.02%	2.01%	2.02%
$\sigma[\Delta c_t]/\sigma[\Delta y_t]$	0.56	0.50	0.76	0.53
$\sigma[\Delta i_t]/\sigma[\Delta y_t]$	2.58	2.65	1.69	2.47
corr$(\Delta y_t, \Delta y_{t-1})$	0.37	0.03	0.02	0.03
corr$(\Delta c_t, \Delta c_{t-1})$	0.27	0.21	0.05	0.17
SR	0.39	0.121	0.189	0.184
Risk price decomposition:				
short run, Δc	-	39.1%	41.1%	26.4%
short run, x	-	0.0%	26.1%	16.8%
long run, Δc	-	60.9%	20.8%	36.1%
long run, x	-	0.0%	12.0%	20.8%

Note: Data: US quarterly series 1947–2016, see appendix for definitions. Model RA: calibrated as in Table 1 without idiosyncratic risk. Model HA3: as in Table 1 and section 4.4, but setting $\beta = 0.974$ to match RA model steady state. Model HA4: as in Table 1 and section 4.4, but setting $\beta = 0.974, \rho = 0.255$ to match RA model steady state and quantity dynamics. Standard deviations and Sharpe ratio are annualized by doubling from quarterly values. Bottom part shows relative contributions to price of risk based on loglinear approximation.

Individual consumption growth can belong either to the first mixture component, which stands for the "normal" experience faced by the majority of households, or to one of the other two components, which represent negative or positive jumps. Movements in x_t then shift the position of the second and third components relative to first one, making the size of negative jumps larger during recessions (provided $\phi_x < 0$) and thus making the cross-sectional distribution of consumption growth more negatively skewed.

The calibration of the means, variances, and probabilities of the mixture elements follows McKay (2017), although I scale the overall size of the shock (i.e., the means and standard deviations of the mixture components) by one half to achieve variance comparable to the lognormal calibration used in previous sections. The sensitivity of x_t is estimated by regressing the time series for x_t provided by Alisdair McKay on his website[11] on US consumption growth, and the resulting coefficient is also scaled by one half. The chosen parameters are thus: $\mu_2 = -0.835$, $\mu_3 = 0.1970$, $\sigma_1 = 0.0319$, $\sigma_2 = \sigma_3 = 0.1668$, $p_1 = 94.87\%$, $p_2 = 3.24\%$, $p_3 = 1.89\%$, and $\phi_x = -7.285$. In the steady state, the standard deviation of η with the given parameters is 6.1%, or around 12.2% annualized, while the coefficient of skewness is 1.05 and that of kurtosis 27.6, so the distribution is slightly positively skewed and fat-tailed. Measured in terms of plus/minus two standard deviations of aggregate consumption growth, the skewness ranges from -1.5 to 3.1 over the business cycle.

Table 3, organized similarly as Table 2, contains unconditional moments from two versions of a model with cyclical skewness. Again, I compare a version of the model with β recalibrated to match the steady-state return on capital (column HA3) and another (column HA4) with β and ρ re-

[11] http://people.bu.edu/amckay/files/risk_time_series.csv

calibrated to match the dynamics of output and consumption.[12] The results are largely comparable with those in Table 2, although the Sharpe ratio of 18% under skewed idiosyncratic shocks is somewhat higher than the 16% under lognormal shocks. Without adjusting the individual intertemporal elasticity of substitution, we again observe a change in the behavior of aggregate consumption, although it is not as strong as in the lognormal case. The decomposition of the risk premium is also qualitatively similar to the lognormal case, but quantitatively the role of idiosyncratic risk is slightly higher.

5. Conclusion

In this paper, I have studied how preferences for early resolution of uncertainty and idiosyncratic, uninsurable risk affect risk premia in a tractable macroeconomic model with production. On the one hand, the combination of the two elements implies that households care about direct shocks and about news about both aggregate consumption and the amount or shape of individual risk, and if the latter varies cyclically over time, both can increase the price of risk more than each element would in isolation. On the other hand, when households can shift consumption intertemporally by investing in productive capital, countercyclical risk affects their incentive to do so and on the aggregate level the economy behaves as if households had a lower intertemporal elasticity of substitution, leading potentially to different behavior of macroeconomic quantities. Nevertheless, at least in the setting analyzed here, one can maintain the same quantity dynamics by suitably recalibrating the preference parameters. Specifically, if we are willing to assume that individual agents have a higher intertemporal elasticity of substitution, it is possible to compensate for the effect of cyclical risk on aggregate consumption while keeping the price of risk higher.

Several directions could be pursued in further research. Introducing elastic labor supply or habit formation would allow for greater flexibility in matching macroeconomic dynamics. It might be also interesting to investigate independent shocks to the process describing the distribution of idiosyncratic risk, either as a source of macroeconomic fluctuations or as an asset-pricing factor, although identifying such shocks might present a challenge. Another direction to consider would be to include stochastic volatility of aggregate shocks, which is another channel of time-varying uncertainty often analyzed in the literature, in order to compare and contrast the effects of "macro" and "micro" uncertainty on the economy. Finally, a closer comparison with models with a more realistic structure of household heterogeneity and trade between households would be useful in establishing the validity of the modeling approach used in the present paper.

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[12] It is possible to derive approximate formulas for adjusting the parameters like in the previous section, although they are somewhat more involved due to the necessity of loglinearizing the MGF terms. Qualitatively, however, the direction of adjustment is the same as before.

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Appendix

Detrended Model Equations

Notation:

Lowercase variable names usually denote logarithms, e.g., $k_t = \log(K_t)$. Starred variables denote variables detrended by productivity, i.e., $y_t^* = \log(Y_t/Z_t) = y_t - z_t$. Delta denotes 1st difference, e.g., $\Delta c_t = c_t - c_{t-1}$.

List of variables:

Variable	Description
Δz_t	productivity growth rate
y_t^*	log detrendend output
k_t^*	log detrended capital
c_t^*	log detrended agg. consumption
Δc_t	growth rates of output, consumption
r_t^k	log return on capital
p_t^b	log bond price
r_t^b	log return on risk-free bond
m_t	log aggregate SDF
v_t	log scaled value function
ψ_t	log scaled certainty equivalent
x_t	variance of individual consumption growth rates
ε_t	productivity shock

Equations:

- The production block contains equations describing productivity growth, the production function, capital accumulation, the marginal product of capital, the Euler equation for investment, and the definition of consumption growth:

$$\Delta z_t = \mu_z + \varepsilon_t$$
$$y_t^* = \alpha k_t^*$$
$$\exp\left(k_{t+1}^* + \Delta z_{t+1}\right) = (1-\delta)\exp\left(k_t^*\right) + \exp\left(y_t^*\right) - \exp\left(c_t^*\right)$$
$$\exp\left(r_t^K\right) = \alpha \exp\left((\alpha-1)k_t^*\right) + 1 - \delta$$
$$1 = E_t\left[\exp\left(m_{t+1} + r_{t+1}^K\right)\right]$$
$$\Delta c_{t+1} = c_{t+1}^* - c_t^* + \Delta z_{t+1}$$

- The household block contains equations describing the scaled value function, the certainty equivalent, the process of variance of individual consumption growth rates, and the stochastic

discount factor:

$$v_t = \frac{1}{1-\rho}\log\left(1-\beta+\beta\exp((1-\rho)\psi_t)\right)$$
$$\exp((1-\gamma)\psi_t) = E_t\left[\exp((1-\gamma)(v_{t+1}+\Delta c_{t+1}-(\gamma/2)x_{t+1}))\right]$$
$$x_{t+1} = \mu_x + \phi_x(\Delta c_{t+1}-\mu_z)$$
$$m_{t+1} = \log(\beta) - \rho\Delta c_{t+1} + (\rho-\gamma)(v_{t+1}-\psi_t+\Delta c_{t+1}) + (1/2)\gamma(1+\gamma)x_{t+1}$$

- The remaining equations describe the price and the return on the risk-free bond:

$$\exp(p_t^b) = E_t\left[\exp(m_{t+1})\right]$$
$$r_t^b = -p_{t-1}^b$$

Steady state:

Setting the productivity shocks to zero allows us to find the stationary steady state which corresponds to the balanced growth path in terms of the original, undetrended variables. We will denote steady-state values by dropping the time index and the bars over the variables.

- Along the balanced growth path, productivity and consumption grow at the same rate, so $\overline{\Delta z} = \overline{\Delta c} = \mu_z$. Idiosyncratic risk is at its average level: $\bar{x} = \mu_x$.

- Given the constant consumption growth, we can solve for the value function and the steady-state SDF:

$$\bar{v} = \frac{1}{1-\rho}\log\left(\frac{1-\beta}{1-\beta e^{(1-\rho)(\mu_z-(\gamma/2)\mu_x)}}\right)$$
$$\bar{\psi} = \bar{v} + \mu_z - (\gamma/2)\mu_x$$
$$\bar{m} = \log(\beta) - \rho\mu_z + \frac{1}{2}\gamma(1+\rho)\mu_x$$

- The steady-state SDF determines the return on capital, which, in turn, allows us to solve for steady-state capital, output, and consumption:

$$\bar{r}^k = -\log(\beta) + \rho\mu_z - \frac{1}{2}\gamma(1+\rho)\mu_x$$
$$\bar{k}^* = \frac{1}{\alpha-1}\log\left(\frac{\exp(\bar{r}^k)-1+\delta}{\alpha}\right)$$
$$\bar{y}^* = \alpha\bar{k}^*$$
$$\bar{c}^* = \log\left(\exp(\bar{y}^*) - (\exp(\mu_z)-1+\delta)\exp(\bar{k}^*)\right)$$

- Finally, the SDF determines the bond price and the return on bonds, which equals the return on capital:

$$\bar{p}^b = \log(\beta) - \rho\mu_z + \frac{1}{2}\gamma(1+\rho)\mu_x$$
$$\bar{r}^b = -\log(\beta) + \rho\mu_z - \frac{1}{2}\gamma(1+\rho)\mu_x$$

Local vs. Global Solution

To find out whether solving the model numerically with perturbation omits any substantial nonlinearities, I solve the version of the model with countercyclical variance also by using a projection method. I approximate consumption and the value functions as combinations of Chebyshev polynomials up to the 10th degree and solve for the polynomial coefficients such that the forward-looking conditions (i.e., the definition of the value function and the Euler equation, with expectations evaluated by 5-point Gauss-Hermite quadrature) hold exactly at a set of corresponding collocation nodes. The following table shows the resulting Sharpe ratios (obtained as averages from a simulation with each solution), which are very similar. The other moments are omitted, as they were virtually identical up to three decimal places. It thus seems that for the model and calibration studied here, nonlinearities do not matter very much.

	Model: RA	Model: HA1	Model: HA2
3rd order perturbation			
Sharpe ratio	0.121	0.163	0.161
Projection			
Sharpe ratio	0.119	0.160	0.160

Linearized Solution

The model summarized above has a single state variable, detrended capital k_t^*, so its linearized solution can be found explicitly. We will denote deviations from a steady-state value by tilde, e.g., $\tilde{k}_t^* = k_t^* - \bar{k}^*$. First, we linearize the key equations around the steady state:

$$\tilde{k}_{t+1}^* = \lambda_1 \tilde{k}_t^* - \lambda_2 \tilde{c}_t^* - \varepsilon_{t+1}$$
$$\tilde{r}_t^K = \lambda_3 \tilde{k}_t^*$$
$$E_t[\tilde{r}_{t+1}^K] = -E_t[\tilde{m}_{t+1}]$$
$$\tilde{m}_{t+1} = -\gamma\widetilde{\Delta c}_{t+1} + (\rho - \gamma)(\tilde{v}_{t+1} - \tilde{\psi}_t) + (1/2)\gamma(1+\gamma)\tilde{x}_{t+1}$$
$$\tilde{v}_t = \kappa\tilde{\psi}_t$$
$$\tilde{\psi}_t = E_t\left[\tilde{v}_{t+1} + \widetilde{\Delta c}_{t+1} - (\gamma/2)\tilde{x}_{t+1}\right]$$
$$\widetilde{\Delta c}_{t+1} = \tilde{c}_{t+1}^* - \tilde{c}_t^* + \widetilde{\Delta z}_{t+1}$$
$$\widetilde{\Delta z}_{t+1} = \varepsilon_{t+1}$$
$$\tilde{x}_{t+1} = \phi_x\widetilde{\Delta c}_{t+1},$$

where $\lambda_1, \lambda_2, \lambda_3$, and κ are defined as

$$\lambda_1 = \exp\left(\bar{r}^k - \mu_z\right)$$
$$\lambda_2 = \exp\left(\bar{c}^* - \bar{k}^* - \mu_z\right)$$
$$\lambda_3 = \alpha(\alpha - 1)\exp\left((\alpha - 1)\bar{k}^* - \bar{r}^K\right)$$
$$\kappa = \beta\exp((1 - \rho)(\mu_z - (\gamma/2)\mu_x)).$$

We are looking for consumption policy in the form of $\tilde{c}_t^* = \eta_{ck}\tilde{k}_t^*$.

Claim: If we can write expected log SDF as $E_t[\tilde{m}_{t+1}] = -\hat{\rho}E_t\left[\widetilde{\Delta c}_{t+1}\right]$ for some $\hat{\rho}$, then η_{ck} can be found by the method of undetermined coefficients as a (positive) solution to the quadratics

$$\hat{\rho}\lambda_2\eta_{ck}^2 + (\hat{\rho} - \lambda_2\lambda_3 - \hat{\rho}\lambda_1)\eta_{ck} + \lambda_1\lambda_3 = 0.$$

Proof: Substitute the law of motion for capital and consumption policy into the linearized Euler equation, take the expectation (which simply cancels the shock), and rearrange. There will be two real roots, one positive, one negative (since $\hat{\rho}\lambda_2 > 0$ and $\lambda_1\lambda_3 < 0$). The positive one is required for a stable solution. $\square$

Claim: Our model satisfies the above with

$$\hat{\rho} = \rho - \frac{1}{2}\gamma(1+\rho)\phi_x.$$

Proof: Since

$$\tilde{v}_{t+1} - \tilde{\psi}_t = \tilde{v}_{t+1} - E_t[\tilde{v}_{t+1}] - E_t[\widetilde{\Delta c}_{t+1}] + (\gamma/2)E_t[\tilde{x}_{t+1}]$$

and

$$E_t[\tilde{v}_{t+1} - \tilde{\psi}_t] = -E_t[\widetilde{\Delta c}_{t+1}] + (\gamma/2)E_t[\tilde{x}_{t+1}],$$

after a bit of algebra we get

$$E_t[\tilde{m}_{t+1}] = -\left(\rho - \frac{1}{2}\gamma(1+\rho)\phi_x\right)E_t\left[\widetilde{\Delta c}_{t+1}\right].$$

$\square$

Finally, we can also solve for the value function in the form of $\tilde{v}_t = \eta_{vk}\tilde{k}_t^*$ by the method of undetermined coefficients. The result is

$$\eta_{vk} = \frac{\kappa\left(1 - \frac{\gamma}{2}\phi_x\right)\eta_{ck}(\lambda_1 - \lambda_2\eta_{ck} - 1)}{1 - \kappa(\lambda_1 - \lambda_2\eta_{ck})}.$$

Having solved for consumption and the value function, an innovation to log SDF can be expressed as

$$m_{t+1} - E_t[m_{t+1}] = \left(\gamma(1-\eta_{ck}) + (\gamma-\rho)(-\eta_{vk}) + \frac{1}{2}\gamma(1+\gamma)(-\phi_x)(1-\eta_{ck})\right)(-\varepsilon_t).$$

Since typically $\gamma > \rho$, $\eta_{vk} < 0$, and $\phi_x < 0$, each of the three added terms inside large parentheses is positive and can be understood as standing for short-run aggregate consumption risk, long-run risk, and short-run idiosyncratic risk, respectively. To further decompose long-run risk, we iterate forward on the definition of $\tilde{v}_t$ to obtain

$$\tilde{v}_t = \sum_{i=1}^{\infty}\kappa^i\left(E_t[\widetilde{\Delta c}_{t+i}] - \frac{1}{2}\gamma E_t[\tilde{x}_{t+i}]\right) = \left(1 + \frac{1}{2}\gamma(-\phi_x)\right)\sum_{i=1}^{\infty}\kappa^i E_t[\widetilde{\Delta c}_{t+i}]$$

so that the share of long-run risk attributable to news about x can be taken as $\frac{\frac{1}{2}\gamma(-\phi_x)}{(1+\frac{1}{2}\gamma(-\phi_x))}$.

Linearized Solution with a General MGF

The previous derivation of the loglinear approximation can be relatively easily extended to the case of a general moment-generating function describing the distribution of idiosyncratic shocks. Specifically, let $G(t,x)$ be the MGF as described in the main text (normalized so that $G(1,x) = 1$) and denote the cumulant generating function as $g(t,x) = \log(G(t,x))$. We will continue to assume that x is a scalar following $x_t = \mu_x + \phi_x \widetilde{\Delta c_t}$. The relevant equations for the value function and log SDF are modified as follows:

$$\exp((1-\gamma)\psi_t) = E_t\left[\exp\left((1-\gamma)\left(v_{t+1} + \Delta c_{t+1} + \frac{1}{1-\gamma}g(1-\gamma,x_{t+1})\right)\right)\right]$$

$$m_{t+1} = \log(\beta) - \rho\Delta c_{t+1} + (\rho-\gamma)(v_{t+1} - \psi_t + \Delta c_{t+1}) + g(-\gamma,x_{t+1})$$

and their steady-state values, given that $\bar{x} = \mu_x$, are

$$\bar{v} = \frac{1}{1-\rho}\log\left(\frac{1-\beta}{1-\beta e^{(1-\rho)\left(\overline{\Delta c} + \frac{1}{1-\gamma}g(1-\gamma,\bar{x})\right)}}\right)$$

$$\bar{\psi} = \bar{v} + \overline{\Delta c} + \frac{1}{1-\gamma}g(1-\gamma,\bar{x})$$

$$\bar{m} = \log(\beta) - \rho\overline{\Delta c} + \frac{\gamma-\rho}{1-\gamma}g(1-\gamma,\bar{x}) + g(-\gamma,\bar{x}).$$

To solve for the dynamics, we linearize g wrt x at $t = -\gamma$ and $t = 1 - \gamma$:

$$g(-\gamma,x) \approx g(-\gamma,\bar{x}) + \theta_{(-\gamma)}\tilde{x}$$

$$g(1-\gamma,x) \approx g(1-\gamma,\bar{x}) + \theta_{(1-\gamma)}\tilde{x},$$

where $\theta_{(t)} = \frac{\partial g(t,\bar{x})}{\partial x}$. The linearized equations then become

$$\tilde{v}_t = \kappa\tilde{\psi}_t$$

$$\tilde{\psi}_t = E_t\left[\tilde{v}_{t+1} + \widetilde{\Delta c}_{t+1} + (1/(1-\gamma))\theta_{(1-\gamma)}\tilde{x}_{t+1}\right]$$

$$\tilde{m}_{t+1} = -\gamma\widetilde{\Delta c}_{t+1} + (\rho-\gamma)(\tilde{v}_{t+1} - \tilde{\psi}_t) + \theta_{(-\gamma)}\tilde{x}_{t+1},$$

where $\kappa = \beta\exp\left((1-\rho)\left(\overline{\Delta c} + \frac{1}{1-\gamma}g(1-\gamma,\mu_x)\right)\right)$. Everything else is the same as in the previous case, and following the same argument we can derive the effective inverse IES:

$$\hat{\rho} = \rho + \frac{\gamma-\rho}{\gamma-1}\theta_{(1-\gamma)}\phi_x - \theta_{(-\gamma)}\phi_x$$

and then η_{ck} is (the positive) solution to

$$\hat{\rho}\lambda_2\eta_{ck}^2 + (\hat{\rho} - \lambda_2\lambda_3 - \hat{\rho}\lambda_1)\eta_{ck} + \lambda_1\lambda_3 = 0.$$

By the method of undetermined coefficients, η_{vk} can be derived to be

$$\eta_{vk} = \frac{\kappa\left(1 + \frac{1}{1-\gamma}\theta_{(1-\gamma)}\phi_x\right)\eta_{ck}(\lambda_1 - \lambda_2\eta_{ck} - 1)}{1 - \kappa(\lambda_1 - \lambda_2\eta_{ck})}.$$

Then one can show the innovation to log SDF is

$$m_{t+1} - E_t[m_{t+1}] = \left(\gamma(1 - \eta_{ck}) + (\gamma - \rho)(-\eta_{vk}) + \theta_{(-\gamma)}(-\phi_x)(1 - \eta_{ck}) \right)(-\varepsilon_{t+1}),$$

which can again be used to decompose the risk premium, with the share of long-run risk attributable to news about x being $\dfrac{\left(\frac{1}{1-\gamma}\theta_{(1-\gamma)}\phi_x \right)}{\left(1 + \frac{1}{1-\gamma}\theta_{(1-\gamma)}\phi_x \right)}$.

Data Sources

The data moments in Table 2 for macroeconomic variables are obtained from the quarterly national accounts data constructed by the Bureau of Economic Analysis and published in the St. Louis Fed FRED database. The sample period is 1947Q1–2016Q2. Output and investment growth (Δy, Δi) are computed as logarithmic growth rates of the GDP and gross private domestic fixed investment quantity indices (NIPA Table 1.1.3) divided by the population (NIPA Table 7.1). Consumption growth (Δc) is computed as a weighted average of the logarithmic growth rates in quantity indices for nondurables and services consumption (NIPA Table 1.1.3) divided by the population, with weights determined by the nominal shares of both consumption components in combined nominal non-durable+services consumption (NIPA Table 1.1.5), i.e., using the Tornqvist index method (although simply summing the two series in real chained dollars yields almost identical results).

Data for financial returns are constructed from the monthly dataset on Fama-French 3 factors published on Kenneth French's website.[13] In place of the return on capital/firm stock (R^s) I use the market return (i.e., the return on the value-weighted portfolio of all firms listed on the NYSE, AMEX, or NASDAQ), while the risk-free rate (R^b) is represented by the return on the 1-month Treasury Bill. Returns are expressed in real terms by subtracting CPI inflation (series CPIAUCSL from FRED) and aggregated to quarterly frequency by summing the monthly returns over the given quarter. The resulting sample period is 1947Q1–2016Q3.

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[13] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

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