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**Institute of Computer Science**  
**Academy of Sciences of the Czech Republic**

## **UFO 2008**

# **Interactive System for Universal Functional Optimization**

L.Lukšan, M.Tůma, J.Víček, N.Ramešová,  
M.Šiška, J.Hartman, C.Matonoha

Technical report No. 1040

December 2008



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**Abstract:**

This report contains a description of the interactive system for universal functional optimization UFO, version 2008.

**Keywords:**

Numerical optimization, nonlinear programming, nonlinear approximation, algorithms, software systems.

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# 1 Introduction to the UFO system

The universal functional optimization (UFO) system is an interactive modular system for solving both dense medium-size and sparse large-scale optimization problems. The UFO system can be used for the following applications:

1. Formulation and solution of particular optimization problems that are described in Chapter 2.
2. Preparation of specialized optimization routines (or subroutines) based on methods described in Chapter 3.
3. Designing and testing new optimization methods. The UFO system is a very useful tool for the development of optimization algorithms.

The special realization of the UFO system described in the subsequent text makes this system portable and extensible and we continue with its further development.

## 1.1 Philosophy of the UFO system

The UFO system is an open software system for solving a broad class of optimization problems. An optimization problem solution is processed in three phases. In the first phase the optimization problem is specified and an optimization method is selected. This can be done in three different ways:

1. The full dialogue mode: The problem specification and the method selection are realized by using a conversation between the user and the UFO system.
2. The batch mode: The problem specification and the method selection are realized by using the UFO control language, which is a generalization and enlargement of the batch editing language (BEL) (see Appendix B). An input file written in the UFO control language has to be prepared and stored.
3. The combined mode: Only a part of the specification is written in the input file. The rest of the specification is obtained as in the dialogue mode. This possibility is usually the best one since the problem functions can be defined beforehand by using a convenient text editor.

The first phase is realized by using the UFO control language preprocessor (UFOCLP). This preprocessor consists of the BEL interpreter controlled by input template UZDCLP.I (see Appendix B.7). The BEL interpreter is written in Fortran 77 and its output is a Fortran 77 source program (P.FOR in the MS DOS and Windows versions or P.F in the UNIX and LINUX systems). This conception is very advantageous for the following reasons:

1. Fortran 77 (full ANSI norm) is a sufficiently high and portable programming language. Fortran 77 is very suitable for numerical computations, and a large number of subroutines realizing various numerical methods is available in this language.
2. A source program, generated by the UFO control language preprocessor, calls for necessary modules only and its specification is very easy. Moreover, its global declarations are determined by the problem size, which decreases storage requirements. This overcomes an impossibility of dynamical declarations in Fortran 77.
3. The UFO system is open. When a new class of optimization problems or optimization methods is included, one only needs to change some system templates and prepare new modules. The UFO source program is composed of individual modules by using specifications given by the user. This fact allows us to create a great number of various optimization methods and their modifications.



In the second phase, the UFO source program `P.FOR` (or `P.F`) is compiled by using a suitable Fortran 77 compiler and a final executable program is linked by using library modules. In the third phase, the final executable program is started and thus results which can be viewed by using extensive output means are obtained.

The above conception is enabled by a special form of source modules. These modules usually consist of two parts, the interface template and the Fortran 77 realization. The interface template is used by the UFO preprocessor only and it serves for the UFO source program generation (the part of the source program corresponding to a given module is coded in the template). These templates also contain knowledge bases for automatic selection of the optimization method. If the UFO system is extended then usually only templates, which do not need to be compiled, are changed. Besides interface templates, which are a part of source modules, special templates controlling the UFO preprocessor exist. A batch input file written in the UFO control language is the first of these special templates.

The UFO macroprocessor works in two passes. In the first pass, the file `P.TMP` is created. This file is a predecessor of the UFO source program. It contains macroinstructions and macrovariables which are processed in the second pass. The source program `P.FOR` (or `P.F`) is the result of the second pass.

## 1.2 Execution of the UFO system

The UFO system contains three basic procedures `GENER`, `COMPIL` and `UFOGO`. The UFO preprocessor is called if the statement

```
GENER input_name
```

is typed. Then the source program, written in Fortran 77, is obtained. Furthermore, the compilation of the source program, followed by its loading and executing, is started if the statement

```
COMPIL output_name
```

is typed. Finally, all the UFO system phases are performed if the statement

```
UFOGO problem_name
```

is typed. Here `input_name` is the first part of the batch file name `input_name.UFO` that is used as a batch input for the source program generation, `output_name` is the first part of the text file name `output_name.OUT` that is used as a text output from the UFO system and `problem_name` is the first part of both the batch file name `problem_name.UFO` and the text file name `problem_name.OUT`. All these names have to be typed with capital letters in the UNIX and LINUX versions. If `GENER` and `UFOGO` statements do not contain a file name specification, then a full dialogue mode is considered (the batch file name is `STANDARD.UFO` in this case) and the standard text file name is `P.OUT`. If `COMPIL` statement does not contain a file name specification, then the standard text file name is `P.OUT`. The `UFOGO` statement has the same meaning as two consecutive statements `GENER` and `COMPIL`.

First we show how the batch mode proceeds. We suppose that the model function has the form

$$f^F(x) = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2$$

(the Rosenbrock function) and the starting point is  $x_1 = -1.2$  and  $x_2 = 1.0$ . If we prepare the batch input file `P.UFO` of the form

```
$REM : model specifications
$SET(INPUT)
  X(1)=-1.2D0; X(2)= 1.0D0
$ENDSET
$SET(FMODEL)
  FF=1.0D2*(X(1)**2-X(2))**2+(X(1)-1.0D0)**2
$ENDSET
$NF=2
```

```

$REM : the default method is used
$REM : print specifications
$MOUT=1
$NOUT=1
$REM : the batch mode is used
$BATCH
$REM : the standard form of the source program is used
$STANDARD

```

and type the statement `UFOGO P`, then the following results appear in the output file `P.OUT`

```

CLASS = VM - LI1   UPDATE = B   MODEL = FF   HESF = D   NF =       2
      0 NIT=   40 NfV=  138 NfG=    0 NDC=    0 NCG=    0 F= .504D-13 G= .828D-05
FF =   .5038712822D-13
X  =   .1000000098D+01   .1000000177D+01

```

(in a PC computer). Batch files are written in the UFO control language. This language is described in Section 4.1 and in Appendix B. Here we note that a certain experience with the UFO control language can be obtained by using the demo-files `PROB01.UFO`, ..., `PROB20.UFO`. These demo-files contain 20 test problems described in Chapter 7. We can solve them by using the statements `UFOGO PROB01`, ..., `UFOGO PROB20`.

Besides the batch mode, we can use the full dialogue mode. The full dialogue mode is started if we use the statement `UFOGO` (without a batch input file specification). Full dialogue modes (text and graphic) are described in Sections 4.3 and 4.4. An example which demonstrates the text dialogue mode applied to the Rosenbrock function is given in Appendix A.

Besides basic output files, the UFO system produces additional files which can contain some useful information. A list of the most important UFO files follows:

```

P.UFO      - Batch input file.
P.TMP      - Temporary file containing a source program ancestor generated in the first pass of the UFO
preprocessor.
P.FOR      - UFO source program generated in the second pass of the UFO preprocessor (the UNIX
and LINUX versions use file P.F in this case).
P.OUT      - Text output file.
P.DAT      - Stored values of problem variables.
P.DIM      - Dimensions of basic problem vectors and matrices.
P.SIF      - Messages of the SIF decoder.
P.I        - Template given by the SIF decoder.

```

### 1.3 The UFO versions for PC computers

There are four UFO versions for PC computers. The MS DOS version `UFOD1` requires the Microsoft Fortran Power Station compiler version 1 and it uses the DOS graphic system for the graphic dialogue (Section 4.4) and the graphic screen output (Section 5.3). The Windows 2000 version `UFOW4` (which can be used also in other Windows systems) requires the Microsoft Visual Fortran compiler version 4 and it uses the QuickWin graphic system with simplified control (the mouse cannot be used). The Windows 2000 version `UFOW6` (which can be used also in other Windows systems) requires the Digital Visual Fortran compiler version 6 and it uses the QuickWin graphic system with simplified control (the mouse cannot be used). The LINUX version `UFOL8` requires SUSE LINUX version 8.0 system with GNU Fortran 77 compiler, but it does not contain graphical possibilities at present. Of course, the UFO system can also be used for PC computers with other compilers. In this case, the UNIX version of the UFO system can be applied with procedures `GENER`, `COMPIL` and `UFOGO` modified for the compiler used.

The MS DOS and Windows 2000 versions are distributed by using the files UFOD1.ZIP, UFOR4.ZIP and UFOR6.ZIP (page <http://www.cs.cas.cz/~luksan/ufodis.html>), which contain templates \*.I, sample input files \*.UFO, sample output files \*.OUT, batch procedures \*.BAT, programs \*.EXE and other important files together with the subdirectory LIB, which contains libraries \*.LIB. The installation of the UFO system is carried out by putting the \*.ZIP file into the directory UFO and by using the routine PKUNZIP. Furthermore, paths and settings in procedures SET1.BAT, SET4.BAT and SET6.BAT have to be arranged to correspond to directories used. More information concerning installation are given in text files UFOD1.TXT, UFOR4.TXT and UFOR6.TXT. If connection to the CUTE test environment is required, then the subdirectory SIF has to be created and the \*.SIF files from the CUTE collection (page <http://www.cse.clrc.ac.uk/nag/cuter/>) have to be copied into this subdirectory.

The MS DOS and Windows 2000 versions of the UFO system are provided with the special UFO environment, which makes the use of the UFO system easy. The UFO environment is called by using the statement UFO (procedure UFO.BAT) and controlled by using the pull-down menu. The main menu is activated by pressing key <F10>. The UFO environment contains a text editor whose control is similar to commonly used text editors under the MS DOS system. All significant statements of the source program editor are available from the UFO environment menu.

Since the UFO environment menu is clearly understandable we do not describe it (the description is given in [264]). We only show the usual way for operating input files. When the batch mode input file is prepared by using the text editor we press key <F10> and find the command Run! in the UFO environment menu. This command starts the UFO preprocessor and its action corresponds to the statement UFOGO (with the input file which is loaded in the text editor). An easier possibility is pressing keys <Alt-1>. Similarly, pressing keys <Alt-9> has the same effect as the statement GENER (with the input file which is loaded in the text editor). Furthermore, if the source program P.FOR is loaded in the text editor, pressing keys <Alt-3> has the same effect as the statement COMPIL and pressing keys <Alt-5> causes an exit from the UFO environment.

The PC LINUX version is distributed by using the file ufo18.tar.gz (page <http://www.cs.cas.cz/~luksan/ufodis.html>), which contains source modules \*.F, templates \*.I, sample input files \*.UFO, sample output files \*.OUT, library ufolib.a, batch procedures gener, compil, ufogo, program ufobel and other important files. The installation of the UFO system is carried out by putting this file into the directory ufo and typing gunzip \*.gz and tar xvf \*.tar. If connection to the CUTE test environment is required, then the subdirectory ufo/sif has to be created and the \*.SIF files from the CUTE collection (page <http://www.cse.clrc.ac.uk/nag/cuter/>) have to be copied into this subdirectory.

#### 1.4 The UFO versions for UNIX workstations

The UNIX versions of the UFO system will be distributed by the file ufo.tar.gz. The installation of the UFO system is carried out by putting this file into the directory ufo, typing gunzip \*.gz, tar xvf \*.tar and using the statements instal SGI, instal DIG, instal SUN and instal HP, for Silicon Graphic, Digital Equipment, Sun and Hewlett Packard workstations with standard operation systems, respectively (the Fortran 77 compiler is assumed). For other workstations or operation systems, the procedure INSTAL.SGI should be changed to INSTAL.XXX containing relevant statements and the statement instal XXX should be used. The UNIX versions of the UFO system do not contain graphical possibilities at present.

#### 1.5 Distribution of the UFO system

The UFO system is intended for the academic use. The full UNIX version of the UFO system (with source modules) is not freely distributed at present. The following free versions of the UFO system for PC computers can be downloaded from page <http://www.cs.cas.cz/~luksan/ufodis.html>.

UFOW6.ZIP	- Windows version. It requires Digital Visual Fortran compiler, version 6.
UFOL6.ZIP	- LINUX version. It requires GNU Fortran 95 compiler.

The installation of the UFO UNIX version `ufosgi` is carried out by the same way as the installation of the UFO LINUX version `ufo18`.

## 1.6 Suggestions for beginners

If we want to solve a particular optimization problem, then the best way to understanding the UFO system is to find a similar problem in the list of sample problems (Chapter 7). This sample problem can be solved by typing `UFOGO PROBxx` (`xx` is the number of the sample problem). After solving the sample problem, we can modify the batch input file `PROBxx.UFO` to describe our problem. Basic suggestions concerning the description of optimization problems are given in Chapter 2 and the most important features of the UFO control language are described in Chapter 4. Optimization methods need not be selected by the user, the system automatically chooses a suitable method. On the other hand, if the user is familiar with optimization methods, then the method can be selected by using suggestions given in Chapter 3. Information about output possibilities can be found in Chapter 5 and special tools of the UFO system are described in Chapter 6.

The following list refers to basic problems solved by the UFO system and to sections containing more detailed description.

Minimization of a general objective function: Sections 2.2 – 2.3.

Minimization of a linear objective function: Section 2.2.

Minimization of a quadratic objective function: Section 2.2.

Minimization of a partially separable function (the sum of approximating functions): Sections 2.4 – 2.6.

Minimization of the sum of squares (or powers) of approximating functions: Sections 2.4 – 2.6.

Minimization of the maximum of approximating functions: Sections 2.4 – 2.6.

Minimization of the sum of absolute values of approximating functions: Sections 2.4 – 2.6.

Optimization of systems described by ordinary differential equations: Sections 2.7 – 2.11.

Solution of systems of nonlinear functional equations: Section 2.15.

Solution of systems of ordinary differential equations: Section 2.16.

Moreover, all optimization problems can contain box constraints (Section 2.1) and general linear or nonlinear constraints (Sections 2.12 – 2.14).

## 1.7 Authors of the UFO system

The UFO system was developed in the Institute of Computer Science, Czech Academy of Sciences, Prague. In addition, the UFO system contains two direct solvers for sparse linear systems based on UMFPAK code of T.A.Davis [63] and MA27 code of I.S.Duff [74]. Moreover, some modules for solving ordinary differential equations are based on subroutines proposed in the book of E.Hairer, S.P.Norsett and G.Wanner [105] (they were quite rewritten).

The following table contains names of individual authors, their letter codes introduced in source modules and their contribution in percent.

Author	code	contribution
L.Lukšan	LU	73.83
M.Tůma	TU	9.35
J.Vlček	VL	4.16
N.Ramešová	RA	3.94
M.Šiška	SI	3.18
J.Hartman	HA	1.58
C.Matonoha	MA	1.17
T.A.Davis	DA	1.72
I.S.Duff	DU	1.07

## 2 Problems solved using the UFO system

The most general problem which can be solved by using the UFO system is a minimization of an objective function  $F : R^n \rightarrow R$  over a set  $X \subset R^n$ . The objective function can have several forms determined using the macrovariable \$MODEL:

\$MODEL='FF' - General optimization. In this case

$$F(x) = \pm f^F(x)$$

where  $f^F : R^n \rightarrow R$  is a real valued, so-called model function

\$MODEL='FL' - Linear optimization. In this case

$$F(x) = \pm (f^F + \sum_{i=1}^n g_i^F x_i)$$

where  $f^F, g_i^F, 1 \leq i \leq n$ , are real coefficients.

\$MODEL='FQ' - Quadratic optimization. In this case

$$F(x) = \pm (f^F + \sum_{i=1}^n (g_i^F + \frac{1}{2} \sum_{j=1}^n h_{ij}^F x_j) x_i)$$

where  $f^F, g_i^F, 1 \leq i \leq n, h_{ij}^F, 1 \leq i, j \leq n$ , are real coefficients.

\$MODEL='AF' - Sum of functions minimization. In this case

$$F(x) = \sum_{k=1}^{n_A} f_k^A(x)$$

where  $f_k^A : R^n \rightarrow R, 1 \leq k \leq n_A$ , are real valued, so-called approximating functions.

\$MODEL='AQ' - Sum of squares minimization. In this case

$$F(x) = \frac{1}{2} \sum_{k=1}^{n_A} |f_k^A(x)|^2$$

where  $f_k^A : R^n \rightarrow R, 1 \leq k \leq n_A$ , are real valued, so-called approximating functions.

\$MODEL='AP' - Sum of powers minimization. In this case

$$F(x) = \frac{1}{p} \sum_{k=1}^{n_A} |f_k^A(x)|^p$$

where  $f_k^A : R^n \rightarrow R, 1 \leq k \leq n_A$ , are real valued, so-called approximating functions and  $1 < p < \infty$  is a real exponent.

\$MODEL='AA' - Sum of absolute values minimization. In this case

$$F(x) = \sum_{k=1}^{n_A} |f_k^A(x)|$$

where  $f_k^A : R^n \rightarrow R, 1 \leq k \leq n_A$ , are real valued, so-called approximating functions.

\$MODEL='AM' - Minimization of maximum (minimax). In this case

$$F(x) = \max_{1 \leq k \leq n_A} |f_k^A(x)|$$

where  $f_k^A : R^n \rightarrow R$ ,  $1 \leq k \leq n_A$ , are real valued, so-called approximating functions.

\$MODEL='DF' - Minimization of the general integral criterion with respect to the state equations. In this case

$$F(x) = \int_{t_A^{min}}^{t_A^{max}} f^A(x, y_A(x, t_A), t_A) dt_A + f^F(x, y_A(x, t_A^{max}), t_A^{max})$$

and

$$\frac{dy_A(x, t_A)}{dt_A} = f^E(x, y_A(x, t_A), t_A), \quad y^A(x, t_A^{min}) = f^Y(x)$$

where  $f^A : R^{n+n_E+1} \rightarrow R$  is a real valued, smooth, so-called subintegral function,  $f^F : R^{n+n_E+1} \rightarrow R$  is a real valued, smooth, so-called terminal function,  $f^E : R^{n+n_E+1} \rightarrow R^{n_E}$  is a real valued, smooth, so-called state function and  $f^Y : R^n \rightarrow R^{n_E}$  is a real valued, smooth, so-called initial function.

\$MODEL='DQ' - Minimization of the sum of square integral criterion with respect to the state equations. In this case

$$F(x) = \frac{1}{2} \int_{t_A^{min}}^{t_A^{max}} \sum_{i=1}^{n_E} w_i^E(t_A) (y_i^A(x, t_A) - y_i^E(t_A))^2 dt_A + \frac{1}{2} \sum_{i=1}^{n_E} w_i^E(y_i^A(x, t_A^{max}) - y_i^E)^2$$

and

$$\frac{dy_A(x, t_A)}{dt_A} = f^E(x, y_A(x, t_A), t_A), \quad y^A(x, t_A^{min}) = f^Y(x)$$

where  $f^E : R^{n+n_E+1} \rightarrow R^{n_E}$  is a real valued, smooth, so-called state function and  $f^Y : R^n \rightarrow R^{n_E}$  is a real valued, smooth, so-called initial function.

\$MODEL='NE' - Solving a system of nonlinear functional equations

$$f_k^A(x) = 0, \quad 1 \leq k \leq n_A$$

where  $n_A = n$  (\$MODEL='NE' is equivalent to \$MODEL='AQ' if  $n_A = n$ ).

\$MODEL='DE' - Solving an initial value problem for a system of ordinary differential equations. In this case

$$\frac{dy_A(t_A)}{dt_A} = f^E(y_A(t_A), t_A), \quad y^A(t_A^{min}) = y_A^{min}$$

where  $f^E : R^{n_E+1} \rightarrow R^{n_E}$  is a real valued, smooth, so-called state function.

The model function  $f^F : R^n \rightarrow R$  can have several types of Hessian matrices specified by the macrovariable \$HESF:

- \$HESF='D' - Dense Hessian matrix.
- \$HESF='S' - Sparse Hessian matrix with a general pattern.
- \$HESF='N' - Hessian matrix is not used.

The default option is \$HESF='D'. The approximating functions  $f_k^A : R^n \rightarrow R$ ,  $1 \leq k \leq n_A$ , can have

several types of Jacobian matrices specified by the macrovariable \$JACA:

- \$JACA='D'\$ - Dense Jacobian matrix.
- \$JACA='S'\$ - Sparse Jacobian matrix with a general pattern.
- \$JACA='N'\$ - Jacobian matrix is not used.

If the approximating functions are used then we can choose several types of the Hessian matrix representation. These types are again specified by the macrovariable \$HESF:

- \$HESF='D'\$ - Dense Hessian matrix.
- \$HESF='S'\$ - Sparse Hessian matrix with a general pattern.
- \$HESF='B'\$ - Sparse Hessian matrix with a partitioned pattern.
- \$HESF='N'\$ - Hessian matrix is not used.

If \$JACA='D'\$, then it must be either \$HESF='D'\$ or \$HESF='N'\$. If \$JACA='S'\$, then we can specify all types of Hessian matrices (\$HESF='D'\$, \$HESF='S'\$, \$HESF='B'\$, \$HESF='N'\$). The representation \$HESF='B'\$ usually leads to more expensive matrix operations. Therefore, we recommend to prefer the choice \$HESF='S'\$ against the choice \$HESF='B'\$.

The subintegral function, the terminal function, the state function and the initial function, which appear in the case of dynamical systems optimization, are considered to be dense. Therefore we cannot use the specifications \$HESF='S'\$ or \$HESF='B'\$ in this case.

The set  $X \subset R^n$  can be the whole  $R^n$  (unconstrained case) or defined by box constraints

$$\begin{aligned} x_i^L &\leq x_i && \text{if } i \in I_1 \\ &x_i &\leq x_i^U & \text{if } i \in I_2 \\ x_i^L &\leq x_i \leq x_i^U && \text{if } i \in I_3 \\ x_i^L &= x_i && \text{if } i \in I_5 \end{aligned}$$

where  $I_1 \cup I_2 \cup I_3 \cup I_5 \subset \{i \in N : 1 \leq i \leq n\}$ , or by general linear constraints

$$\begin{aligned} c_k^L &\leq \sum_{i=1}^n g_{ki}^C x_i && \text{if } k \in L_1 \\ &\sum_{i=1}^n g_{ki}^C x_i &\leq c_k^U & \text{if } k \in L_2 \\ c_k^L &\leq \sum_{i=1}^n g_{ki}^C x_i \leq c_k^U && \text{if } k \in L_3 \\ c_k^L &= \sum_{i=1}^n g_{ki}^C x_i && \text{if } k \in L_5 \end{aligned}$$

where  $g_{ki}^C$ ,  $1 \leq k \leq n_C$ ,  $1 \leq i \leq n$ , are real coefficients and  $L_1 \cup L_2 \cup L_3 \cup L_5 \subset \{k \in N : 1 \leq k \leq n_C\}$ , or by general nonlinear constraints

$$\begin{aligned} c_k^L &\leq f_k^C(x) && \text{if } k \in N_1 \\ &f_k^C(x) &\leq c_k^U & \text{if } k \in N_2 \\ c_k^L &\leq f_k^C(x) \leq c_k^U && \text{if } k \in N_3 \\ c_k^L &= f_k^C(x) && \text{if } k \in N_5 \end{aligned}$$

where  $f_k^C : R^n \rightarrow R$ ,  $1 \leq k \leq n_C$ , are real valued, smooth, so-called constraint functions and  $N_1 \cup N_2 \cup N_3 \cup N_5 \subset \{k \in N : 1 \leq k \leq n_C\}$ . The constraint functions  $f_k^C : R^n \rightarrow R$ ,  $1 \leq k \leq n_C$ , can have several types of Jacobian matrices specified by the macrovariable \$JACC:

\$JACC='D' - Dense Jacobian matrix.  
 \$JACC='S' - Sparse Jacobian matrix with a general pattern.

If \$JACC='D', then it must be \$HESF='D' or \$HESF='N'. If \$JACC='S', then it must be \$HESF='S' or \$HESF='N'.

There are several limitations in the current version of the UFO system:

1. Minimization of dynamical systems is not implemented in the sparse case.
2. Usually the UFO system serves for local optimization. Global optimization can be used only for small-size ( $n \leq 100$ ) dense problems that are unconstrained or contain box constraints.

These limitations will be consecutively removed in subsequent versions of the UFO system.

In the rest of this report we will use the notation NF, NA, NC instead of  $n$ ,  $n_A$ ,  $n_C$  and X, F(X), FF(X), GF(X), HF(X), FA(KA;X), GA(KA;X), FC(KC;X), GC(KC;X) instead of  $x$ ,  $F(x)$ ,  $f^F(x)$ ,  $g^F(x)$ ,  $h^F(x)$ ,  $f_k^A(x)$ ,  $g_k^A(x)$ ,  $f_k^C(x)$ ,  $g_k^C(x)$ . This notation corresponds to names of variables and fields in the UFO system, which is listed in Section 2.16.

## 2.1 Specification of variables and the box constraints

First we must specify the number of variables using the statement \$NF=number\_of\_variables. If there are no box constraints we set \$KBF=0. In the opposite case we set \$KBF=1 or \$KBF=2. If \$KBF=1 or \$KBF=2, then

X(I) -	unbounded	,	if IX(I) = 0
XL(I) ≤	X(I)	,	if IX(I) = 1
	X(I) ≤ XU(I)	,	if IX(I) = 2
XL(I) ≤	X(I) ≤ XU(I)	,	if IX(I) = 3
X(I) -	constant	,	if IX(I) = 5

where  $1 \leq I \leq NF$ . The option \$KBF=2 must be chosen if IX(I)=3 for at least one index  $1 \leq I \leq NF$ . Then two different fields XL(I) and XU(I),  $1 \leq I \leq NF$  are declared. In the opposite case we set \$KBF=1 and only one common field XL(I)=XU(I),  $1 \leq I \leq NF$  is declared.

The initial values of variables X(I),  $1 \leq I \leq NF$ , types of box constraints IX(I),  $1 \leq I \leq NF$ , and lower and upper bounds XL(I) and XU(I),  $1 \leq I \leq NF$ , can be specified using macrovariable \$INPUT. The default values are IX(I)=0 and XL(I)=XU(I)=0,  $1 \leq I \leq NF$ . For example:

```

$KBF=2; $NF=4
$SET(INPUT)
  X(1)=x1
  X(2)=x2; IX(2)=1; XL(2)=x2L
  X(3)=x3; IX(3)=3; XL(3)=x3L; XU(3)=x3U
  X(4)=x4; IX(4)=5
$ENDSET

```

The UFO system allows us to use a scaling of variables (for instance if the values of variables differ very much in their magnitude). We set the option \$NORMF:

\$NORMF=1 - Scaling parameters XN(I),  $1 \leq I \leq NF$ , are determined automatically so that  $X(I)/XN(I)=1$ ,  $1 \leq I \leq NF$ , for the initial values of variables.



`$NORMF=2` - Scaling parameters must be specified by the user by means of the macrovariable `$INPUT`.

The scaling of variables is recommended only in exceptional cases since it increases the computational time and storage requirements. The scaling of variables is suppressed if `$NORMF=0` (this value is default). The scaling of variables is not permitted in the case of general constraints (if `$KBC>0`).

## 2.2 Specification of the model function (dense problems)

If the macrovariable `$MODEL` is not specified or if `$MODEL='FF'`, then the objective function is defined by the formula

$$F(X) = + FF(X) \text{ if } \$IEXT = 0 \text{ (minimization)}$$

or

$$F(X) = - FF(X) \text{ if } \$IEXT = 1 \text{ (maximization)}$$

Option `$IEXT=0` is default.

The model function `FF(X)` must be defined by the user either directly in the full dialogue mode, or by using corresponding macrovariables in the batch (or mixed) mode. The value of the model function is specified by using the macrovariable `$FMODEL`:

```
$SET(FMODEL)
  FF = value FF(X)
  (for given values of variables X(I), 1≤I≤NF)
$ENDSET
```

The first derivatives of the model function are specified by using the macrovariable `$GMODEL`:

```
$SET(GMODEL)
  GF(1) = derivative ∂FF(X)/∂X(1)
  GF(2) = derivative ∂FF(X)/∂X(2)
  —
  GF(NF) = derivative ∂FF(X)/∂X(NF)
  (for given values of variables X(I), 1≤I≤NF)
$ENDSET
```

The second derivatives of the model function are specified by using the macrovariable `$HMODEL`. If `$HESF='D'`, the Hessian matrix is assumed to be dense and we specify only its upper half:

```
$SET(HMODEL)
  HF(1) = derivative ∂²FF(X)/∂X(1)²
  HF(2) = derivative ∂²FF(X)/∂X(1)∂X(2)
  HF(3) = derivative ∂²FF(X)/∂X(2)²
  HF(4) = derivative ∂²FF(X)/∂X(1)∂X(3)
  HF(5) = derivative ∂²FF(X)/∂X(2)∂X(3)
  HF(6) = derivative ∂²FF(X)/∂X(3)²
  —
  HF(NF*(NF+1)/2) = derivative ∂²FF(X)/∂X(NF)²
  (for given values of variables X(I), 1≤I≤NF)
$ENDSET
```

If the macrovariables `$GMODEL` or `$HMODEL` are not defined, we suppose that the first or the second derivatives of the model function are not given analytically. In this case, they are computed numerically by using the UFO system routines whenever it is required. If it is advantageous to compute the first derivatives of model function `FF(X)` together with its value, we can replace the set of macrovariables `$FMODEL`, `$GMODEL` by the common macrovariable `$FGMODEL`. Similarly we can replace the set

of macrovariables \$FMODEL\$, \$GMODEL\$, \$HMODEL\$ by the common macrovariable \$FGHMODEL\$.

To improve the efficiency of the computation, we can specify additional information about the model function \$FF(X)\$. The first piece of information, useful for an automatic choice of the optimization method, is the computational complexity specified by the macrovariable \$KCF\$:

- \$KCF=1\$ - Evaluation of the model function \$FF(X)\$ is very easy (it requires \$O(n)\$ simple operations at most).
- \$KCF=2\$ - Evaluation of the model function \$FF(X)\$ is of medium complexity (it requires \$O(n)\$ complicated operations at least and \$O(n^2)\$ simple operations at most).
- \$KCF=3\$ - Evaluation of the model function \$FF(X)\$ is extremely difficult (it requires \$O(n^2)\$ complicated or \$O(n^3)\$ simple operations at least).

The option \$KCF=2\$ is default. An additional useful piece of information is the analytical complexity (differentiability and conditioning), which is specified by the macrovariable \$KSF\$:

- \$KSF=1\$ - The model function \$FF(X)\$ is smooth and well-conditioned.
- \$KSF=2\$ - The model function \$FF(X)\$ is smooth but ill-conditioned.
- \$KSF=3\$ - The model function \$FF(X)\$ is nonsmooth.

The option \$KSF=1\$ is default. Other specifications which can improve the computational efficiency and robustness of optimization methods are a lower bound of the objective function values and an upper bound of the stepsize. Both these values depend on the definition of the objective function and can be specified by the statements \$FMIN=lower\\_bound\$ (for the objective function) and \$XMAX=upper\\_bound\$ (for the stepsize). We recommend a definition of \$FMIN\$ whenever it is possible and a definition of \$XMAX\$ whenever the objective function contains exponentials.

If \$MODEL='FL'\$, we suppose the model function to be linear of the form

$$FF(X) = FF + \sum_{I=1}^{NF} GF(I) * X(I)$$

In this case we need not specify the value and the first derivatives of the model function by the macrovariables \$FMODEL\$ and \$GMODEL\$ as in the general case. Instead, we must specify the coefficients \$FF\$ (constant value) and \$GF(I)\$, \$1 \le I \le NF\$, (constant gradient) by using the macrovariable \$INPUT\$:

```

$ADD(INPUT)
  FF = constant value
  GF(1) = constant derivative $\partial FF(X)/\partial X(1)$
  GF(2) = constant derivative $\partial FF(X)/\partial X(2)$
  —
  GF(NF) = constant derivative $\partial FF(X)/\partial X(NF)$
$ENDADD

```

If \$MODEL='FL'\$, we usually assume that either box constraints or general linear constraints are given. In this case the optimization problem is the linear programming problem.

If \$MODEL='FQ'\$, we suppose the model function to be quadratic of the form

$$FF(X) = FF + \sum_{I=1}^{NF} GF(I) * X(I) + \frac{1}{2} \sum_{I=1}^{NF} \sum_{J=1}^{NF} HF(K) * X(I) * X(J)$$

where \$K=MAX(I,J)\*(MAX(I,J)-1)/2+MIN(I,J)\$. In this case we need not specify the value, the first and second derivatives of the model function by the macrovariables \$FMODEL\$, \$GMODEL\$ and \$HMODEL\$ as in the general case. The coefficients \$FF\$ (constant value) and \$GF(I)\$, \$1 \le I \le NF\$, (constant gradient)

are specified in the same way as in the linear case. The coefficients  $HF(K)$ ,  $1 \leq K \leq NF*(NF+1)/2$ , (the constant Hessian matrix) must be specified by using the macrovariable \$INPUT. If \$HESF='D', the Hessian matrix is assumed to be dense and we specify only its upper half:

```

$ADD(INPUT)
  HF(1) = constant derivative  $\partial^2 FF(X)/\partial X(1)^2$ 
  HF(2) = constant derivative  $\partial^2 FF(X)/\partial X(1)\partial X(2)$ 
  HF(3) = constant derivative  $\partial^2 FF(X)/\partial X(2)^2$ 
  HF(4) = constant derivative  $\partial^2 FF(X)/\partial X(1)\partial X(3)$ 
  HF(5) = constant derivative  $\partial^2 FF(X)/\partial X(2)\partial X(3)$ 
  HF(6) = constant derivative  $\partial^2 FF(X)/\partial X(3)^2$ 
  —
  HF(NF*(NF+1)/2) = constant derivative  $\partial^2 FF(X)/\partial X(NF)^2$ 
$ENDADD

```

If \$MODEL='FQ', we usually assume that either box constraints or general constraints are given. In this case the optimization problem is the quadratic programming problem.

If the model function is linear or quadratic, then the options \$KCF and \$KSF need not be defined since they are not used.

### 2.3 Specification of the model function (sparse problems)

The UFO system contains optimization methods which take into account the sparsity pattern of the Hessian matrix HF. This possibility decreases the computational time and storage requirements for large-scale optimization problems. In this case we use the option \$HESF='S' which means that the sparsity pattern is specified. All other specifications remain the same as in the case of dense problems. The sparsity pattern of the Hessian matrix is specified by using the macrovariable \$INPUT. Two integer vectors IH and JH are used where IH(I),  $1 \leq I \leq NF+1$ , are pointers and JH(K),  $1 \leq K \leq M$ , are indices of nonzero elements. Only the upper half of the Hessian matrix is assumed and the nonzero elements are ordered in rows. The number of nonzero elements must be specified using the statement \$M=number\_of\_elements. The number of nonzero elements could be greater than is required (twice say) since it is used for the declaration of working fields. For example, if we have the Hessian matrix

$$HF = \begin{pmatrix} h_{11}^F & h_{12}^F & h_{13}^F & 0 & h_{15}^F \\ h_{21}^F & h_{22}^F & 0 & h_{24}^F & 0 \\ h_{31}^F & 0 & h_{33}^F & 0 & h_{35}^F \\ 0 & h_{42}^F & 0 & h_{44}^F & 0 \\ h_{51}^F & 0 & h_{53}^F & 0 & h_{55}^F \end{pmatrix}$$

then we have to set:

```

$NF=5
$M=20 (the minimum required value is M=10)
$ADD(INPUT)
  IH(1)=1; IH(2)=5; IH(3)=7
  IH(4)=9; IH(5)=10; IH(6)=11
  JH(1)=1; JH(2)=2; JH(3)=3; JH(4)=5; JH(5)=2
  JH(6)=4; JH(7)=3; JH(8)=5; JH(9)=4; JH(10)=5
$ENDADD

```

All diagonal elements of the sparse Hessian matrix are assumed to be nonzero.

As in the case of the dense problem, the second derivatives of the model function can be specified by using the macrovariable \$HMODEL. If \$HESF='S', only nonzero elements of the upper half (including the diagonal) of the Hessian matrix are specified. For the above example the specification has the form:

```

$SET(HMODEL F)
HF(1)=h11F; HF(2)=h12F; HF(3)=h13F; HF(4)=h15F
HF(5)=h22F; HF(6)=h24F; HF(7)=h33F; HF(8)=h35F
HF(9)=h44F; HF(10)=h55F
$ENDSET

```

If the model function is quadratic (i.e. if \$MODEL='FQ') and if \$HESF='S', then the coefficients HF(K), 1≤K≤M, (constant sparse Hessian matrix) must be specified by using the macrovariable \$INPUT. If the matrix given in the above example is the constant sparse Hessian matrix, we use the following specification:

```

$ADD(INPUT)
HF(1)=h11F; HF(2)=h12F; HF(3)=h13F; HF(4)=h15F
HF(5)=h22F; HF(6)=h24F; HF(7)=h33F; HF(8)=h35F
HF(9)=h44F; HF(10)=h55F
$ENDADD

```

## 2.4 Objective functions for discrete approximation

If we set \$MODEL='AF', then we suppose that the objective function F(X) has this form:

$$F(X) = \sum_{KA=1}^{NA} FA(KA; X) \quad \text{if } \$KBA = 0$$

or

$$F(X) = \sum_{KA=1}^{NA} AW(KA) * (FA(KA; X) - AM(KA)) \quad \text{if } \$KBA = 1$$

where FA(KA;X), 1≤KA≤NA, are approximating functions. This form of the objective function is very useful in large-scale optimization when the approximating functions FA(KA;X), 1≤KA≤NA, are assumed to have sparse gradients.

If we set \$MODEL='AP', then we suppose that the objective function F(X) has this form:

$$F(X) = \frac{1}{R} \sum_{KA=1}^{NA} |FA(KA; X)| * *R \quad \text{if } \$KBA = 0$$

or

$$F(X) = \frac{1}{R} \sum_{KA=1}^{NA} |AW(KA) * (FA(KA; X) - AM(KA))| * *R \quad \text{if } \$KBA = 1$$

where FA(KA;X), 1≤KA≤NA, are approximating functions, and R>1 is a real exponent. The value of the exponent is specified by the choice \$REXP=R (default value is \$REXP=2). Since the most used value of the exponent is R=2, and since the computations are the simplest and the most efficient for such a choice, we can use the specification \$MODEL='AQ' in this case (minimization of the sum of squares). Moreover, \$MODEL='AQ' is formally set whenever we choose \$MODEL='AP' and \$REXP=2.

If we set \$MODEL='AA', then we suppose that the objective function F(X) has this form:

$$F(X) = \sum_{KA=1}^{NA} |FA(KA; X)| \quad \text{if } \$KBA = 0$$

or

$$F(X) = \sum_{KA=1}^{NA} |AW(KA) * (FA(KA; X) - AM(KA))| \quad \text{if } \$KBA = 1$$

where  $FA(KA;X)$ ,  $1 \leq KA \leq NA$ , are approximating functions.

If we set  $\$MODEL='AM'$ , then we suppose that the objective function  $F(X)$  has the form:

$$F(X) = \max_{1 \leq KA \leq NA} (+FA(KA; X)) \quad \text{if } \$IEXT = -1$$

$$F(X) = \max_{1 \leq KA \leq NA} (|FA(KA; X)|) \quad \text{if } \$IEXT = 0$$

$$F(X) = \max_{1 \leq KA \leq NA} (-FA(KA; X)) \quad \text{if } \$IEXT = +1$$

for  $\$KBA=0$ , or

$$F(X) = \max_{1 \leq KA \leq NA} (+AW(KA) * (FA(KA; X) - AM(KA))) \quad \text{if } \$IEXT = -1$$

$$F(X) = \max_{1 \leq KA \leq NA} (|AW(KA) * (FA(KA; X) - AM(KA))|) \quad \text{if } \$IEXT = 0$$

$$F(X) = \max_{1 \leq KA \leq NA} (-AW(KA) * (FA(KA; X) - AM(KA))) \quad \text{if } \$IEXT = +1$$

for  $\$KBA=1$ , where  $FA(KA;X)$ ,  $1 \leq KA \leq NA$ , are approximating functions. The default value is  $\$IEXT=0$  (the minimax or the Chebyshev approximation).

The option  $\$KBA$  serves as a decision between a simple objective function and a more complicated one. The simple objective function uses no additional fields while the more complicated one uses two additional fields at most,  $AM$  and  $AW$ . Vector  $AM$  usually contains frequently used observations which can be included into the functions  $FA(KA;X)$ ,  $1 \leq KA \leq NA$ , in the case of the simple objective function. Observations  $AM(KA)$ ,  $1 \leq KA \leq NA$ , are specified by using the macrovariable  $\$INPUT$ . Their default values are  $AM(KA)=0$ ,  $1 \leq KA \leq NA$ . Vector  $AW$  serves for possible scaling specified by the option  $\$NORMA$ :

- $\$NORMA=0$  - No scaling is performed. In this case  $AW(KA)=1$ ,  $1 \leq KA \leq NA$ .
- $\$NORMA=1$  - Scaling parameters are determined automatically so that  $AW(KA)=|AM(KA)|$ ,  $1 \leq KA \leq NA$ .
- $\$NORMA=2$  - Scaling parameters must be specified by the user by means of the macrovariable  $\$INPUT$ .

The number of approximating functions  $NA$  must be specified, in all the above cases, by using the statement  $\$NA=number\_of\_functions$ .

## 2.5 Specification of the approximating functions (dense problems)

The approximating functions  $FA(KA;X)$ ,  $1 \leq KA \leq NA$ , must be defined by the user either directly in the full dialogue mode or by using corresponding macrovariables in the batch (or mixed) mode. The values of the approximating functions are specified by using the macrovariables  $\$FMODELA$  or  $\$FMODELAS$ :

```

$SET(FMODELA)
  FA = value FA(KA;X)
      (for a given index KA and given values of variables X(I), 1 ≤ I ≤ NF)
$ENDSET

```

or

```

$SET(FMODELAS)
  AF(1) = value FA(1;X)
  AF(2) = value FA(2;X)
  —
  AF(NA) = value FA(NA;X)
$ENDSET

```

The first derivatives of the approximating functions are specified by using the macrovariables \$GMODEL A or \$GMODELAS:

```

$SET(GMODEL A)
  GA(1) = derivative  $\partial FA(KA;X)/\partial X(1)$ 
  GA(2) = derivative  $\partial FA(KA;X)/\partial X(2)$ 
  —
  GA(NF) = derivative  $\partial FA(KA;X)/\partial X(NF)$ 
  (for a given index KA and given values of variables X(I),  $1 \leq I \leq NF$ )
$ENDSET

```

or

```

$SET(GMODELAS)
  AG(1) = derivative  $\partial FA(1;X)/\partial X(1)$ 
  AG(2) = derivative  $\partial FA(1;X)/\partial X(2)$ 
  —
  AG(NF) = derivative  $\partial FA(1;X)/\partial X(NF)$ 
  AG(NF+1) = derivative  $\partial FA(2;X)/\partial X(1)$ 
  AG(NF+2) = derivative  $\partial FA(2;X)/\partial X(2)$ 
  —
  AG(NA*NF) = derivative  $\partial FA(NA;X)/\partial X(NF)$ 
$ENDSET

```

The second derivatives of the approximating functions are specified by using the macrovariables \$HMODEL A or \$HMODELAS. If \$JACA='D', the Hessian matrices are assumed to be dense and we specify only their upper half:

```

$SET(HMODEL A)
  HA(1) = derivative  $\partial^2 FA(KA;X)/\partial X(1)^2$ 
  HA(2) = derivative  $\partial^2 FA(KA;X)/\partial X(1)\partial X(2)$ 
  HA(3) = derivative  $\partial^2 FA(KA;X)/\partial X(2)^2$ 
  HA(4) = derivative  $\partial^2 FA(KA;X)/\partial X(1)\partial X(3)$ 
  HA(5) = derivative  $\partial^2 FA(KA;X)/\partial X(2)\partial X(3)$ 
  HA(6) = derivative  $\partial^2 FA(KA;X)/\partial X(3)^2$ 
  —
  HA(NF*(NF+1)/2) = derivative  $\partial^2 FA(KA;X)/\partial X(NF)^2$ 
  (for a given index KA and given values of variables X(I),  $1 \leq I \leq NF$ )
$ENDSET

```

or

```

$SET(HMODELAS)
  AH(1) = derivative  $\partial^2 FA(1;X)/\partial X(1)^2$ 
  AH(2) = derivative  $\partial^2 FA(1;X)/\partial X(1)\partial X(2)$ 
  AH(3) = derivative  $\partial^2 FA(1;X)/\partial X(2)^2$ 
  AH(4) = derivative  $\partial^2 FA(1;X)/\partial X(1)\partial X(3)$ 

```

AH(5) = derivative  $\partial^2\text{FA}(1;X)/\partial X(2)\partial X(3)$

AH(6) = derivative  $\partial^2\text{FA}(1;X)/\partial X(3)^2$

AH(NF\*(NF+1)/2) = derivative  $\partial^2\text{FA}(1;X)/\partial X(\text{NF})^2$

AH(NF\*(NF+1)/2+1) = derivative  $\partial^2\text{FA}(2;X)/\partial X(1)^2$

AH(NF\*(NF+1)/2+2) = derivative  $\partial^2\text{FA}(2;X)/\partial X(1)\partial X(2)$

AH(NF\*(NF+1)/2+3) = derivative  $\partial^2\text{FA}(2;X)/\partial X(2)^2$

AH(NA\*NF\*(NF+1)/2) = derivative  $\partial^2\text{FA}(\text{NA};X)/\partial X(\text{NF})^2$

SENDSET

If the macrovariables \$GMODEL A and \$GMODEL AS or \$HMODEL A and \$HMODEL AS are not defined, we suppose that the first or the second derivatives of the approximating functions are not given analytically. In this case, they are computed numerically by using the UFO system routines, whenever they are required. If it is advantageous to compute the first derivatives of approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , together with their values, we can replace the set of macrovariables \$FMODEL A, \$GMODEL A by the common macrovariable \$FGMODEL A and the set of macrovariables \$FMODEL AS, \$GMODEL AS by the common macrovariable \$FGMODEL AS. Similarly we can replace the set of macrovariables \$FMODEL A, \$GMODEL A, \$HMODEL A by the common macrovariable \$FGHMODEL A and the set of macrovariables \$FMODEL AS, \$GMODEL AS, \$HMODEL AS by the common macrovariable \$FGHMODEL AS.

To improve the efficiency of the computation, we can specify additional information about the approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ . The first piece of information, useful for an automatic choice of the optimization method, is the computational complexity specified by the macrovariable \$KCA:

\$KCA=1 - Evaluations of the approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , are very easy (they require  $O(n)$  simple operations at most).

\$KCA=2 - Evaluations of the approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , are of medium complexity (they at least require  $O(n)$  complicated operations and  $O(n^2)$  simple operations at most).

\$KCA=3 - Evaluations of the approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , are extremely difficult (they at least require  $O(n^2)$  complicated or  $O(n^3)$  simple operations).

The option \$KCA=2 is default. An additional useful piece of information is the analytical complexity (conditioning) which is specified by the macrovariable \$KSA:

\$KSA=1 - The approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , are smooth and well-conditioned.

\$KSA=2 - The approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , are smooth but ill-conditioned.

\$KSA=3 - The approximating functions FA(KA;X),  $1 \leq \text{KA} \leq \text{NA}$ , are nonsmooth.

The option \$KSA=1 is default.

If some of the approximating functions are linear and have the form

$$\text{FA}(\text{KA}; X) = \sum_{I=1}^{\text{NF}} \text{AG}((\text{KA} - 1) * \text{NF} + I) * X(I)$$

we can specify them separately. Then the number of linear approximating functions must be specified by using the statement \$NAL=number\_of\_linear\_functions (default value is \$NAL=0). We always suppose that the first NAL approximating functions are linear. Then the coefficients AG((KA-1)\*NF+I),  $1 \leq \text{KA} \leq \text{NAL}$ ,  $1 \leq I \leq \text{NF}$ , are specified by using the macrovariable \$INPUT, and the macrovariables \$FMODEL A or \$FMODEL AS, \$GMODEL A or \$GMODEL AS, \$HMODEL A or \$HMODEL AS are used only for the specification of the nonlinear approximating functions FA(KA;X),  $\text{NAL} < \text{KA} \leq \text{NA}$ .

## 2.6 Specification of the approximating functions (sparse problems)

The UFO system contains optimization methods which take into account the sparsity pattern of the Jacobian matrix AG. This possibility decreases the computational time and storage requirements for large-scale optimization problems. In this case we use the option \$JACA='S' which means that the sparsity pattern is specified. All other specifications remain the same as in the case of dense problems. The sparsity pattern of the Jacobian matrix is specified by using the macrovariable \$INPUT. Two integer vectors IAG and JAG are used where IAG(KA),  $1 \leq KA \leq NA+1$ , are pointers and JAG(K),  $1 \leq K \leq IAG(NA+1)-1$ , are indices of nonzero elements. Nonzero elements are ordered by the gradients of the approximating functions. The number of nonzero elements must be specified by using the statement \$MA=number\_of\_elements. For example, if we have the gradients

$$GA(1; X) = [g_{11}^A, 0, 0, g_{14}^A],$$

$$GA(2; X) = [0, g_{22}^A, 0, g_{24}^A],$$

$$GA(3; X) = [0, 0, g_{33}^A, 0],$$

$$GA(4; X) = [g_{41}^A, g_{42}^A, g_{43}^A, 0],$$

$$GA(5; X) = [0, 0, g_{53}^A, g_{54}^A],$$

and the Jacobian matrix

$$AG(X) = \begin{pmatrix} g_{11}^A & ,0 & ,0 & ,g_{14}^A \\ 0 & ,g_{22}^A & ,0 & ,g_{24}^A \\ 0 & ,0 & ,g_{33}^A & ,0 \\ g_{41}^A & ,g_{42}^A & ,g_{43}^A & ,0 \\ 0 & ,0 & ,g_{53}^A & ,g_{54}^A \end{pmatrix}$$

then we have to set:

```
$NA=5
$MA=10
$ADD(INPUT)
  IAG(1)=1; IAG(2)=3; IAG(3)=5
  IAG(4)=6; IAG(5)=9; IAG(6)=11
  JAG(1)=1; JAG(2)=4; JAG(3)=2; JAG(4)=4; JAG(5)=3
  JAG(6)=1; JAG(7)=2; JAG(8)=3; JAG(9)=3; JAG(10)=4
$ENDADD
```

As in the case of the dense problem, the first derivatives of the approximating functions can be specified by using the macrovariables \$GMODEL or \$GMODELAS. If \$JACA='S', only nonzero elements of the gradients are specified. There are two possibilities distinguished by the macrovariable \$ARED (the default value is \$ARED='N'). If \$ARED='N', indices of nonzero elements remains unchanged. If \$ARED='Y', indices of nonzero elements are replaced by indices of elements of reduced gradients. If KA-th approximating function depends on LA variables, we use indices 1, 2, ..., LA.

For the above example the specifications have the form:



```

$SET(GMODELA)
  IF (KA.EQ.1) THEN
    GA(1) =  $\partial FA(1;X)/\partial X(1)$ 
    GA(4) =  $\partial FA(1;X)/\partial X(4)$ 
  ELSE IF (KA.EQ.2) THEN
    GA(2) =  $\partial FA(2;X)/\partial X(2)$ 
    GA(4) =  $\partial FA(2;X)/\partial X(4)$ 
  ELSE IF (KA.EQ.3) THEN
    GA(3) =  $\partial FA(3;X)/\partial X(3)$ 
  ELSE IF (KA.EQ.4) THEN
    GA(1) =  $\partial FA(4;X)/\partial X(1)$ 
    GA(2) =  $\partial FA(4;X)/\partial X(2)$ 
    GA(3) =  $\partial FA(4;X)/\partial X(3)$ 
  ELSE
    GA(3) =  $\partial FA(5;X)/\partial X(3)$ 
    GA(4) =  $\partial FA(5;X)/\partial X(4)$ 
  ENDIF
$ENDSET

```

if \$ARED='N',

```

$SET(GMODELA)
  IF (KA.EQ.1) THEN
    GA(1) =  $\partial FA(1;X)/\partial X(1)$ 
    GA(2) =  $\partial FA(1;X)/\partial X(4)$ 
  ELSE IF (KA.EQ.2) THEN
    GA(1) =  $\partial FA(2;X)/\partial X(2)$ 
    GA(2) =  $\partial FA(2;X)/\partial X(4)$ 
  ELSE IF (KA.EQ.3) THEN
    GA(1) =  $\partial FA(3;X)/\partial X(3)$ 
  ELSE IF (KA.EQ.4) THEN
    GA(1) =  $\partial FA(4;X)/\partial X(1)$ 
    GA(2) =  $\partial FA(4;X)/\partial X(2)$ 
    GA(3) =  $\partial FA(4;X)/\partial X(3)$ 
  ELSE
    GA(1) =  $\partial FA(5;X)/\partial X(3)$ 
    GA(2) =  $\partial FA(5;X)/\partial X(4)$ 
  ENDIF
$ENDSET

```

if \$ARED='Y' or

```

$SET(GMODELAS)
  AG(1) =  $\partial FA(1;X)/\partial X(1)$ 
  AG(2) =  $\partial FA(1;X)/\partial X(4)$ 
  AG(3) =  $\partial FA(2;X)/\partial X(2)$ 
  AG(4) =  $\partial FA(2;X)/\partial X(4)$ 
  AG(5) =  $\partial FA(3;X)/\partial X(3)$ 
  AG(6) =  $\partial FA(4;X)/\partial X(1)$ 
  AG(7) =  $\partial FA(4;X)/\partial X(2)$ 
  AG(8) =  $\partial FA(4;X)/\partial X(3)$ 
  AG(9) =  $\partial FA(5;X)/\partial X(3)$ 
  AG(10) =  $\partial FA(5;X)/\partial X(4)$ 
$ENDSET

```

in both cases.

As in the case of the dense problem, the second derivatives of the approximating functions can be specified by using the macrovariables \$HMODEL A or \$HMODEL AS. If \$JACA='S', only nonzero elements of the Hessian matrices are specified. The use of macrovariable \$HMODEL A is allowed only if \$ARED='Y'. For the above example the specifications have the form:

```

$SET(HMODEL A)
  IF (KA.EQ.1) THEN
    HA(1) =  $\partial^2 FA(1;X)/\partial X(1)^2$ 
    HA(2) =  $\partial^2 FA(1;X)/\partial X(1)\partial X(4)$ 
    HA(3) =  $\partial^2 FA(1;X)/\partial X(4)^2$ 
  ELSE IF (KA.EQ.2) THEN
    HA(1) =  $\partial^2 FA(2;X)/\partial X(2)^2$ 
    HA(2) =  $\partial^2 FA(2;X)/\partial X(2)\partial X(4)$ 
    HA(3) =  $\partial^2 FA(2;X)/\partial X(4)^2$ 
  ELSE IF (KA.EQ.3) THEN
    HA(1) =  $\partial^2 FA(3;X)/\partial X(3)^2$ 
  ELSE IF (KA.EQ.4) THEN
    HA(1) =  $\partial^2 FA(4;X)/\partial X(1)^2$ 
    HA(2) =  $\partial^2 FA(4;X)/\partial X(1)\partial X(2)$ 
    HA(3) =  $\partial^2 FA(4;X)/\partial X(2)^2$ 
    HA(4) =  $\partial^2 FA(4;X)/\partial X(1)\partial X(3)$ 
    HA(5) =  $\partial^2 FA(4;X)/\partial X(2)\partial X(3)$ 
    HA(6) =  $\partial^2 FA(4;X)/\partial X(3)^2$ 
  ELSE
    HA(1) =  $\partial^2 FA(5;X)/\partial X(3)^2$ 
    HA(2) =  $\partial^2 FA(5;X)/\partial X(3)\partial X(4)$ 
    HA(3) =  $\partial^2 FA(5;X)/\partial X(4)^2$ 
  ENDIF
$ENDSET

```

if \$ARED='Y' or

```

$SET(HMODEL AS)
  AH(1) =  $\partial^2 FA(1;X)/\partial X(1)^2$ 
  AH(2) =  $\partial^2 FA(1;X)/\partial X(1)\partial X(4)$ 
  AH(3) =  $\partial^2 FA(1;X)/\partial X(4)^2$ 
  AH(4) =  $\partial^2 FA(2;X)/\partial X(2)^2$ 
  AH(5) =  $\partial^2 FA(2;X)/\partial X(2)\partial X(4)$ 
  AH(6) =  $\partial^2 FA(2;X)/\partial X(4)^2$ 
  AH(7) =  $\partial^2 FA(3;X)/\partial X(3)^2$ 
  AH(8) =  $\partial^2 FA(4;X)/\partial X(1)^2$ 
  AH(9) =  $\partial^2 FA(4;X)/\partial X(1)\partial X(2)$ 
  AH(10) =  $\partial^2 FA(4;X)/\partial X(2)^2$ 
  AH(11) =  $\partial^2 FA(4;X)/\partial X(1)\partial X(3)$ 
  AH(12) =  $\partial^2 FA(4;X)/\partial X(2)\partial X(3)$ 
  AH(13) =  $\partial^2 FA(4;X)/\partial X(3)^2$ 
  AH(14) =  $\partial^2 FA(5;X)/\partial X(3)^2$ 
  AH(15) =  $\partial^2 FA(5;X)/\partial X(3)\partial X(4)$ 
  AH(16) =  $\partial^2 FA(5;X)/\partial X(4)^2$ 
$ENDSET

```

in both cases. Note that the dimensions of arrays HA or AH must be specified by the statement \$MHA=dimension\_of\_HA or \$MAH=dimension\_of\_AH.

If some of the approximating functions are linear (i.e. if \$NAL>0) and if \$JACA='S', then the coefficients AG(K), 1≤K≤IAG(NAL+1)−1 (constant part of the sparse Jacobian matrix), must be specified by using the macrovariable \$INPUT. If the matrix given in the above example is the constant sparse Jacobian matrix, we use this specification:

```
$ADD(INPUT)
  AG(1)=g11A; AG(2)=g14A; AG(3)=g22A; AG(4)=g24A
  AG(5)=g33A; AG(6)=g41A; AG(7)=g42A; AG(8)=g43A
  AG(9)=g53A; AG(10)=g54A
$ENDADD
```

There is another possibility which can be useful when all approximating functions are linear. It is based on the usage of special procedure UKMAI1 which serves for a direct input of individual Jacobian matrix elements. The procedure UKMAI1 is formally called by using the statement

```
CALL $UKMAI1(K,I,GAKI)    or    $SETAG(K,I,GAKI)
```

where K is an index of a given approximating function (a row of the Jacobian matrix), I is an index of a given variable (a column of the Jacobian matrix), and GAKI is the numerical value of the element ∂FA(K;X)/∂X(I). For the example given above we can write:

```
$ADD(INPUT)
  $SETAG(1,1,g11A)
  $SETAG(1,4,g14A)
  $SETAG(2,2,g22A)
  $SETAG(2,4,g24A)
  $SETAG(3,3,g33A)
  $SETAG(4,1,g41A)
  $SETAG(4,2,g42A)
  $SETAG(4,3,g43A)
  $SETAG(5,3,g53A)
  $SETAG(5,4,g54A)
$ENDADD
```

The main advantage of the last possibility is the fact that it is not necessary to specify the fields IAG and JAG beforehand.

If we use the option \$JACA='S', then we can specify a form of the objective function sparse Hessian matrix. There are four possibilities:

```
$HESF='D'      - Dense Hessian matrix.
$HESF='B'      - Partitioned sparse Hessian matrix. This matrix is a sum of simple Hessian matrices
                which correspond to the individual approximating functions. Only nonzero blocks are
                stored.
$HESF='S'      - General sparse Hessian matrix (the same as the model function Hessian matrix
                corresponding to the option $HESF='S').
$HESF='N'      - Hessian matrix is not used.
```

This specification only serves for an internal realization of optimization methods and has no influence on the user's input. The default option is \$HESF='D'.

## 2.7 Objective functions for optimization of dynamical systems

If we set \$MODEL='DF', then we suppose that the objective function F(X) has this form:

$$F(X) = \int_{TAMIN}^{TAMAX} FA(X, YA(TA), TA) dTA + FF(X, YA(TAMAX), TAMAX)$$

where FA(X, YA(TA), TA) is a smooth subintegral function and FF(X, YA(TAMAX), TAMAX) is a smooth terminal function. At the same time

$$\frac{dYA(KE; TA)}{dTA} = FE(KE; X, YA(TA), TA), \quad YA(KE; TAMIN) = FY(KE; X)$$

where FE(KE; X, YA(TA), TA),  $1 \leq KE \leq NE$ , are smooth state functions and FY(KE; X),  $1 \leq KE \leq NE$ , are smooth initial functions.

If we set \$MODEL='DQ', then we suppose the objective function F(X) has the form:

$$F(X) = \frac{1}{2} \int_{TAMIN}^{TAMAX} \sum_{KE=1}^{NE} WE(KE; TA) * (YA(KE; TA) - YE(KE; TA))^2 dTA \\ + \frac{1}{2} \sum_{KE=1}^{NE} EW(KE) * (YA(KE; TAMAX) - EY(KE))^2$$

At the same time

$$\frac{dYA(KE; TA)}{dTA} = FE(KE; X, YA(TA), TA), \quad YA(KE; TAMIN) = FY(KE; X)$$

where FE(KE; X, YA(TA), TA),  $1 \leq KE \leq NE$ , are smooth state functions and FY(KE; X),  $1 \leq KE \leq NE$ , are smooth initial functions.

In all the above cases, the statement \$NE=number\_of\_differential\_equations must be used for the specification of number of differential equations NE. Moreover, values TAMIN and TAMAX have to be specified by using the macrovariable \$INPUT.

```
$ADD(INPUT)
  TA = initial (minimum) value of the independent variable (TA = TAMIN)
  TAMAX = maximum value of the independent variable
$ENDADD
```

## 2.8 Specification of the state functions

The state functions FE(KE; X, YA(TA), TA),  $1 \leq KE \leq NE$ , must be defined by the user either directly in the full dialogue mode or by using corresponding macrovariables in the batch (or mixed) mode. The values of the state functions are specified by using the macrovariables \$FMODELE or \$FMODELES:

```
$SET(FMODELE)
  FE = value FE(KE; X, YA(TA), TA)
      (for a given index KE, a given vector of variables X,
      a given vector of state variables YA(TA) and a given time TA)
$ENDSET
```

or

```
$SET(FMODELES)
  EF(1) = value FE(1; X, YA(TA), TA)
  EF(2) = value FE(2; X, YA(TA), TA)
```

---

```

EF(NE) = value FE(NE;X,YA(TA),TA)
$ENDSET

```

The first derivatives of the state functions according to the variables are specified by using the macrovariables \$GMODELE or \$GMODELES:

```

$SET(GMODELE)
GE(1) = derivative ∂FE(KE;X,YA(TA),TA)/∂X(1)
GE(2) = derivative ∂FE(KE;X,YA(TA),TA)/∂X(2)


---


GE(NF) = derivative ∂FE(KE;X,YA(TA),TA)/∂X(NF)
(for a given index KE, a given vector of variables X,
a given vector of state variables YA(TA) and a given time TA)
$ENDSET

```

or

```

$SET(GMODELES)
EG(1) = derivative ∂FE(1;X,YA(TA),TA)/∂X(1)
EG(2) = derivative ∂FE(1;X,YA(TA),TA)/∂X(2)


---


EG(NF) = derivative ∂FE(1;X,YA(TA),TA)/∂X(NF)
EG(NF+1) = derivative ∂FE(2;X,YA(TA),TA)/∂X(1)
EG(NF+2) = derivative ∂FE(2;X,YA(TA),TA)/∂X(2)


---


EG(NE*NF) = derivative ∂FE(NE;X,YA(TA),TA)/∂X(NF)
$ENDSET

```

The first derivatives of the state functions according to the state variables are specified by using the macrovariables \$DMODELE or \$DMODELES:

```

$SET(DMODELE)
DE(1) = derivative ∂FE(KE;X,YA(TA),TA)/∂YA(1)
DE(2) = derivative ∂FE(KE;X,YA(TA),TA)/∂YA(2)


---


DE(NE) = derivative ∂FE(KE;X,YA(TA),TA)/∂YA(NE)
(for a given index KE, a given vector of variables X,
a given vector of state variables YA(TA) and a given time TA)
$ENDSET

```

or

```

$SET(DMODELES)
ED(1) = derivative ∂FE(1;X,YA(TA),TA)/∂YA(1)
ED(2) = derivative ∂FE(1;X,YA(TA),TA)/∂YA(2)


---


ED(NE) = derivative ∂FE(1;X,YA(TA),TA)/∂YA(NE)
ED(NE+1) = derivative ∂FE(2;X,YA(TA),TA)/∂YA(1)
ED(NE+2) = derivative ∂FE(2;X,YA(TA),TA)/∂YA(2)


---


ED(NE*NE) = derivative ∂FE(NE;X,YA(TA),TA)/∂YA(NE)
$ENDSET

```

If it is advantageous to compute the first derivatives of the state functions  $FE(KE;X,YA(TA),TA)$ ,  $1 \leq KE \leq NE$ , together with their values, we can replace the set of macrovariables  $\$FMODELE$ ,  $\$GMODELE$ ,  $\$DMODELE$  by the common macrovariable  $\$FGDMODELE$  and the set of macrovariables  $\$FMODELES$ ,  $\$GMODELES$ ,  $\$DMODELES$  by the common macrovariable  $\$FGDMODELES$ . Partially we can replace the macrovariables  $\$FMODELE$ ,  $\$GMODELE$  or  $\$FMODELE$ ,  $\$DMODELE$  or  $\$GMODELE$ ,  $\$DMODELE$  by the common macrovariables  $\$FGMODELE$  or  $\$FDMODELE$  or  $\$GDMODELE$ , respectively. Similarly we can replace the macrovariables  $\$FMODELES$ ,  $\$GMODELES$  or  $\$FMODELES$ ,  $\$DMODELES$  or  $\$GMODELES$ ,  $\$DMODELES$  by the common macrovariables  $\$FGMODELES$  or  $\$FDMODELES$  or  $\$GDMODELES$ , respectively.

If  $\$MODEL='DQ'$ , we have to define the functions  $WE(KE;TA)$  and  $YE(KE;TA)$ ,  $1 \leq KE \leq NE$ , for a given index  $KE$  and a given time  $TA$ . These functions can be specified by using the macrovariable  $\$FMODELE$  together with the state function  $FE(KE;X,YA(TA),TA)$ :

```

$SET(FMODELE)
    FE = value FE(KE;X,YA(TA),TA)
    WE = value WE(KE;TA)
    YE = value YE(KE;TA)
    (for a given index KE, a given vector of variables X,
     a given vector of state variables YA(TA) and a given time TA)
$ENDSET

```

The default values  $WE(KE;TA)=1$  and  $YE(KE;TA)=0$  cannot be specified, they are supposed automatically.

## 2.9 Specification of the initial functions

The initial functions  $FY(KE;X)$ ,  $1 \leq KE \leq NE$ , must be defined by the user either directly in the full dialogue mode or by using corresponding macrovariables in the batch (or mixed) mode. The values of the initial functions are specified by using the macrovariables  $\$FMODELY$  or  $\$FMODELYS$ :

```

$SET(FMODELY)
    FE = value FY(KE;X)
    (for a given index KE and a given vector of variables X)
$ENDSET

```

or

```

$SET(FMODELYS)
    EF(1) = value FY(1;X)
    EF(2) = value FY(2;X)
    —
    EF(NE) = value FY(NE;X)
$ENDSET

```

The first derivatives of the initial functions according to the variables are specified by using the macrovariables  $\$GMODELY$  or  $\$GMODELYS$ :

```

$SET(GMODELY)
    GE(1) = derivative  $\partial FY(KE;X)/\partial X(1)$ 
    GE(2) = derivative  $\partial FY(KE;X)/\partial X(2)$ 
    —
    GE(NF) = derivative  $\partial FY(KE;X)/\partial X(NF)$ 
    (for a given index KE and a given vector of variables X)

```

\$ENDSET

or

```
$SET(GMODELYS)
  EG(1) = derivative  $\partial FY(1;X)/\partial X(1)$ 
  EG(2) = derivative  $\partial FY(1;X)/\partial X(2)$ 
  —
  EG(NF) = derivative  $\partial FY(1;X)/\partial X(NF)$ 
  EG(NF+1) = derivative  $\partial FY(2;X)/\partial X(1)$ 
  EG(NF+2) = derivative  $\partial FY(2;X)/\partial X(2)$ 
  —
  EG(NE*NF) = derivative  $\partial FY(NE;X)/\partial X(NF)$ 
$ENDSET
```

If it is advantageous to compute the first derivatives of initial functions  $FY(KE;X)$ ,  $1 \leq KE \leq NE$ , together with their values, we can replace the set of macrovariables \$FMODELY, \$GMODELY by the common macrovariable \$FGMODELY and the set of macrovariables \$FMODELYS, \$GMODELYS by the common macrovariable \$FGMODELYS.

If the initial values  $YA(KE;TAMIN)$ ,  $1 \leq KE \leq NE$ , do not depend on the variables  $X(I)$ ,  $1 \leq I \leq NF$ , they can be specified by using the macrovariable \$INPUT:

```
$ADD(INPUT)
  YA(1) = initial value YA(1,TAMIN)
  YA(2) = initial value YA(2,TAMIN)
  —
  YA(NE) = initial value YA(NE,TAMIN)
$ENDADD
```

## 2.10 Specification of the subintegral function

If \$MODEL='DF', the subintegral function  $FA(X, YA(TA), TA)$  must be defined by the user either directly in the full dialogue mode, or by using corresponding macrovariables in the batch (or mixed) mode. The value of the subintegral function is specified by using the macrovariable \$FMODELA:

```
$SET(FMODELA)
  FA = value  $FA(X, YA(TA), TA)$ 
  (for a given vector of variables X, a given vector of state variables YA(TA)
  and a given time TA)
$ENDSET
```

The first derivatives of the subintegral function according to the variables are specified by using the macrovariable \$GMODELA:

```
$SET(GMODELA)
  GA(1) = derivative  $\partial FA(X, YA(TA), TA)/\partial X(1)$ 
  GA(2) = derivative  $\partial FA(X, YA(TA), TA)/\partial X(2)$ 
  —
  GA(NF) = derivative  $\partial FA(X, YA(TA), TA)/\partial X(NF)$ 
  (for a given vector of variables X, a given vector of state variables YA(TA)
  and a given time TA)
$ENDSET
```

The first derivatives of the subintegral function according to the state variables are specified by using the macrovariable \$DMODELA:

```

$SET(DMODELA)
  DA(1) = derivative  $\partial FA(X, YA(TA), TA) / \partial YA(1)$ 
  DA(2) = derivative  $\partial FA(X, YA(TA), TA) / \partial YA(2)$ 
  —
  DA(NF) = derivative  $\partial FA(X, YA(TA), TA) / \partial YA(NF)$ 
  (for a given vector of variables X, a given vector of state variables YA(TA)
  and a given time TA)
$ENDSET

```

If it is advantageous to compute the first derivatives of subintegral function  $FA(X, YA(TA), TA)$  together with its value, we can replace the set of macrovariables \$FMODELA, \$GMODELA, \$DMODELA by the common macrovariable \$FGDMODELA. Partially we can replace the macrovariables \$FMODELA, \$GMODELA or \$FMODELA, \$DMODELA or \$GMODELA, \$DMODELA by the common macrovariables \$FGMODELA or \$FDMODELA or \$GDMODELA, respectively.

If \$MODEL='DQ' and the objective function contains an integral part, then we have to set \$MODELA='YES' and define the functions  $WE(KE;TA)$  and  $YE(KE;TA)$ ,  $1 \leq KE \leq NE$ , by using the macrovariable \$FMODELE.

## 2.11 Specification of the terminal function

If \$MODEL='DF', the terminal function  $FF(X, YA(TAMAX), TAMAX)$  must be defined by the user either directly in the full dialogue mode, or by using corresponding macrovariables in the batch (or mixed) mode. The value of the terminal function is specified by using the macrovariable \$FMODEL F:

```

$SET(FMODEL F)
  FF = value  $FF(X, YA(TAMAX), TAMAX)$ 
  (for a given vector of variables X, a given vector of state variables YA(TAMAX)
  and a given time TAMAX)
$ENDSET

```

The first derivatives of the terminal function according to the variables are specified by using the macrovariable \$GMODEL F:

```

$SET(GMODEL F)
  GF(1) = derivative  $\partial FF(X, YA(TAMAX), TAMAX) / \partial X(1)$ 
  GF(2) = derivative  $\partial FF(X, YA(TAMAX), TAMAX) / \partial X(2)$ 
  —
  GF(NF) = derivative  $\partial FF(X, YA(TAMAX), TAMAX) / \partial X(NF)$ 
  (for a given vector of variables X, a given vector of state variables YA(TAMAX)
  and a given time TAMAX)
$ENDSET

```

The first derivatives of the terminal function according to the state variables are specified by using the macrovariable \$DMODEL F:

```

$SET(DMODEL F)
  DF(1) = derivative  $\partial FF(X, YA(TAMAX), TAMAX) / \partial YA(1)$ 
  DF(2) = derivative  $\partial FF(X, YA(TAMAX), TAMAX) / \partial YA(2)$ 
  —

```



DF(NE) = derivative  $\partial FF(X, YA(TAMAX), TAMAX) / \partial YA(NE)$   
(for a given vector of variables X, a given vector of state variables YA(TAMAX)  
and a given time TAMAX)

\$ENDSET

If it is advantageous to compute the first derivatives of terminal function  $FF(X, YA(TAMAX), TAMAX)$  together with its value, we can replace the set of macrovariables \$FMODEL, \$GMODEL, \$DMODEL by the common macrovariable \$FGDMODEL. Partially we can replace the macrovariables \$FMODEL, \$GMODEL or \$FMODEL, \$DMODEL or \$GMODEL, \$DMODEL by the common macrovariables \$FGDMODEL or \$FDMODEL or \$GDMODEL, respectively.

If \$MODEL='DQ' and the objective function contains a terminal part, then we have to set \$MODEL='YES' and define the coefficients EW(KE) and EY(KE),  $1 \leq KE \leq NE$ , by using the macrovariable \$INPUT:

```
$ADD(INPUT)
  EW(1) = value EW(1); EY(1) = value EY(1)
  EW(2) = value EW(2); EY(2) = value EY(2)
  —
  EW(NE) = value EW(NE); EY(NE) = value EY(NE)
$ENDADD
```

## 2.12 Optimization with general constraints

If there are no general constraints we set \$KBC=0. In the opposite case we set \$KBC=1 or \$KBC=2. If \$KBC=1 or \$KBC=2, then

FC(KC;X) - unbounded	, if IC(KC) = 0
CL(KC) ≤ FC(KC;X)	, if IC(KC) = 1
FC(KC;X) ≤ CU(KC)	, if IC(KC) = 2
CL(KC) ≤ FC(KC;X) ≤ CU(KC)	, if IC(KC) = 3
CL(KC) = FC(KC;X) = CU(KC)	, if IC(KC) = 5

where  $1 \leq KC \leq NC$ . The option \$KBC=2 must be chosen if  $IC(KC)=3$  for at least one index  $1 \leq KC \leq NC$ . Then two different fields CL(K) and CU(KC),  $1 \leq KC \leq NC$  are declared. In the opposite case we set \$KBC=1 and only one common field  $CL(KC)=CU(KC)$ ,  $1 \leq KC \leq NC$  is declared. The number of constraints NC must be specified by using the statement \$NC=number\_of\_functions.

The types of general constraints IC(KC),  $1 \leq KC \leq NC$ , and lower and upper bounds CL(KC) and CU(KC),  $1 \leq KC \leq NC$ , can be specified by using the macrovariable \$INPUT. The default values are  $IC(KC)=3$  and  $CL(KC)=CU(KC)=0$ ,  $1 \leq KC \leq NC$ . For example:

```
$KBC=2; $NC=3
$ADD(INPUT)
  IC(1)=1; CL(1)=c1L
  IC(2)=1; CL(2)=c2L
  IC(3)=3; CL(3)=c3L; CU(3)=c3L
$ENDADD
```

## 2.13 Specification of the constraint functions (dense problems)

The constraint functions  $FC(KC;X)$ ,  $1 \leq KC \leq NC$ , must be defined by the user either directly in the full dialogue mode, or by using corresponding macrovariables in the batch (or mixed) mode. The values of the constraint functions are specified by using the macrovariables \$FMODEL or \$FMODELCS:

```

$SET(FMODEL C)
    FC = value FC(KC;X)
    (for a given index KC and given values of variables X(I), 1≤I≤NF)
$ENDSET

```

or

```

$SET(FMODEL CS)
    CF(1) = value FC(1;X)
    CF(2) = value FC(2;X)
    —
    CF(NC) = value FC(NC;X)
$ENDSET

```

The first derivatives of the constraint functions are specified by using the macrovariables \$GMODEL C or \$GMODEL CS:

```

$SET(GMODEL C)
    GC(1) = derivative ∂FC(KC;X)/∂X(1)
    GC(2) = derivative ∂FC(KC;X)/∂X(2)
    —
    GC(NF) = derivative ∂FC(KC;X)/∂X(NF)
    (for a given index KC and given values of variables X(I), 1≤I≤NF)
$ENDSET

```

or

```

$SET(GMODEL CS)
    CG(1) = derivative ∂FC(1;X)/∂X(1)
    CG(2) = derivative ∂FC(1;X)/∂X(2)
    —
    CG(NF) = derivative ∂FC(1;X)/∂X(NF)
    CG(NF+1) = derivative ∂FC(2;X)/∂X(1)
    CG(NF+2) = derivative ∂FC(2;X)/∂X(2)
    —
    CG(NC*N F) = derivative ∂FC(NC;X)/∂X(NF)
$ENDSET

```

The second derivatives of the constraint functions are specified by using the macrovariables \$HMODEL C or \$HMODEL CS. If \$JACC='D', the Hessian matrices are assumed to be dense and we only specify their upper half:

```

$SET(HMODEL C)
    HC(1) = derivative ∂2FC(KC;X)/∂X(1)2
    HC(2) = derivative ∂2FC(KC;X)/∂X(1)∂X(2)
    HC(3) = derivative ∂2FC(KC;X)/∂X(2)2
    HC(4) = derivative ∂2FC(KC;X)/∂X(1)∂X(3)
    HC(5) = derivative ∂2FC(KC;X)/∂X(2)∂X(3)
    HC(6) = derivative ∂2FC(KC;X)/∂X(3)2
    —
    HC(NF*(NF+1)/2) = derivative ∂2FC(KC;X)/∂X(NF)2
    (for a given index KC and given values of variables X(I), 1≤I≤NF)
$ENDSET

```

or

```

$SET(HMODELCS)
  CH(1) = derivative  $\partial^2FC(1;X)/\partial X(1)^2$ 
  CH(2) = derivative  $\partial^2FC(1;X)/\partial X(1)\partial X(2)$ 
  CH(3) = derivative  $\partial^2FC(1;X)/\partial X(2)^2$ 
  CH(4) = derivative  $\partial^2FC(1;X)/\partial X(1)\partial X(3)$ 
  CH(5) = derivative  $\partial^2FC(1;X)/\partial X(2)\partial X(3)$ 
  CH(6) = derivative  $\partial^2FC(1;X)/\partial X(3)^2$ 
  —
  CH(NF*(NF+1)/2) = derivative  $\partial^2FC(1;X)/\partial X(NF)^2$ 
  CH(NF*(NF+1)/2+1) = derivative  $\partial^2FC(2;X)/\partial X(1)^2$ 
  CH(NF*(NF+1)/2+2) = derivative  $\partial^2FC(2;X)/\partial X(1)\partial X(2)$ 
  CH(NF*(NF+1)/2+3) = derivative  $\partial^2FC(2;X)/\partial X(2)^2$ 
  —
  CH(NC*NF*(NF+1)/2) = derivative  $\partial^2FC(NC;X)/\partial X(NF)^2$ 
$ENDSET

```

If the macrovariables \$GMODEL C and \$GMODEL CS or \$HMODEL C and \$HMODEL CS are not defined, we suppose that the first or the second derivatives of the constraint functions are not given analytically. In this case, they are computed numerically, by using the UFO system routines whenever they are required. If it is advantageous to compute the first derivatives of constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , together with their values, we can replace the set of macrovariables \$FMODEL C, \$GMODEL C by the common macrovariable \$FGMODEL C and the set of macrovariables \$FMODEL CS, \$GMODEL CS by the common macrovariable \$FGMODEL CS. Similarly we can replace the set of macrovariables \$FMODEL C, \$GMODEL C, \$HMODEL C by the common macrovariable \$FGHMODEL C and the set of macrovariables \$FMODEL CS, \$GMODEL CS, \$HMODEL CS by the common macrovariable \$FGHMODEL CS.

To improve the efficiency of the computation, we can specify some additional information about the constraint functions FC(KC;X),  $1 \leq KC \leq NC$ . The first piece of information, useful for an automatic choice of the optimization method, is the computational complexity specified by the macrovariable \$KCC:

```

$KCC= 1      - Evaluations of the constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , are very easy (they
              require  $O(n)$  simple operations at most).
$KCC= 2      - Evaluations of the constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , are of medium com-
              plexity (they at least require  $O(n)$  complicated operations and  $O(n^2)$  simple opera-
              tions at most).
$KCC= 3      - Evaluations of the constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , are extremely diffi-
              cult (they at least require  $O(n^2)$  complicated or  $O(n^3)$  simple operations).

```

The option \$KCC=2 is default. An additional useful piece of information is the analytical complexity (conditioning) which is specified by the macrovariable \$KSC:

```

$KSC=1      - The constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , are smooth and well-conditioned.
$KSC=2      - The constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , are smooth but ill-conditioned.
$KSC=3      - The constraint functions FC(KC;X),  $1 \leq KC \leq NC$ , are nonsmooth.

```

The option \$KSC=1 is default.

If some of the constraint functions are linear and have the form

$$FC(KC; X) = \sum_{I=1}^{NF} CG((KC - 1) * NF + I) * X(I)$$

we can specify them separately. Then the number of linear constraint functions must be specified by using the statement \$NCL=number\_of\_linear\_functions (default value is \$NCL=0). We always suppose that the

first NCL constraint functions are linear. Then the coefficients  $CG((KC-1)*NF+I)$ ,  $1 \leq KC \leq NCL$ ,  $1 \leq I \leq NF$ , are specified by using the macrovariable \$INPUT and the macrovariables \$FMODEL C or \$FMODELCS, \$GMODEL C or \$GMODELCS, \$HMODEL C or \$HMODELCS are used only for the specification of the nonlinear constraint functions  $FC(KC;X)$ ,  $NCL < KC \leq NC$ .

## 2.14 Specification of the constraint functions (sparse problems)

The UFO system contains optimization methods which take into account the sparsity pattern of the Jacobian matrix CG. This possibility decreases the computational time and storage requirements for large-scale optimization problems. In this case, we use option \$JACC='S' which means that the sparsity pattern is specified. All other specifications remain the same as in the case of dense problems. The sparsity pattern of the Jacobian matrix is specified by using the macrovariable \$INPUT. Two integer vectors ICG and JCG are used where  $ICG(KC)$ ,  $1 \leq KC \leq NC+1$ , are pointers and  $JCG(K)$ ,  $1 \leq K \leq ICG(NC+1)-1$ , are indices of nonzero elements. Nonzero elements are ordered by the gradients of the constraint functions. The number of nonzero elements must be specified by using the statement \$MC=number\_of\_elements. The number of nonzero elements could be greater than is needed (twice say) since it is used for the declaration of working fields. For example, if we have the gradients

$$GC(1; X) = [g_{11}^C, 0, 0, g_{14}^C],$$

$$GC(2; X) = [0, g_{22}^C, 0, g_{24}^C],$$

$$GC(3; X) = [0, 0, g_{33}^C, 0],$$

$$GC(4; X) = [g_{41}^C, g_{42}^C, g_{43}^C, 0],$$

$$GC(5; X) = [0, 0, g_{53}^C, g_{54}^C],$$

and the Jacobian matrix

$$CG(X) = \begin{pmatrix} g_{11}^C & ,0 & ,0 & ,g_{14}^C \\ 0 & ,g_{22}^C & ,0 & ,g_{24}^C \\ 0 & ,0 & ,g_{33}^C & ,0 \\ g_{41}^C & ,g_{42}^C & ,g_{43}^C & ,0 \\ 0 & ,0 & ,g_{53}^C & ,g_{54}^C \end{pmatrix}$$

then we have to set:

```
$NC=5
$MC=20 (the minimum required value is MC=10)
$ADD(INPUT)
  ICG(1)=1; ICG(2)=3; ICG(3)=5
  ICG(4)=6; ICG(5)=9; ICG(6)=11
  JCG(1)=1; JCG(2)=4; JCG(3)=2; JCG(4)=4; JCG(5)=3
  JCG(6)=1; JCG(7)=2; JCG(8)=3; JCG(9)=3; JCG(10)=4
$ENDADD
```

As in the case of the dense problem, the first derivatives of the constraint functions can be specified by using the macrovariables \$GMODEL C or \$GMODELCS. If \$JACC='S', only nonzero elements of the gradients are specified. There are two possibilities distinguished by the macrovariable \$CRED (the default

value is \$CRED='N'). If \$CRED='N', indices of nonzero elements remains unchanged. If \$CRED='Y', indices of nonzero elements are replaced by indices of elements of reduced gradients. If KC-th approximating function depends on LC variables, we use indices 1, 2, ..., LC.

For the above example the specifications have the form:

```

$SET(GMODEL C)
  IF (KC.EQ.1) THEN
    GC(1) = ∂FC(1;X)/∂X(1)
    GC(4) = ∂FC(1;X)/∂X(4)
  ELSE IF (KC.EQ.2) THEN
    GC(2) = ∂FC(2;X)/∂X(2)
    GC(4) = ∂FC(2;X)/∂X(4)
  ELSE IF (KC.EQ.3) THEN
    GC(3) = ∂FC(3;X)/∂X(3)
  ELSE IF (KC.EQ.4) THEN
    GC(1) = ∂FC(4;X)/∂X(1)
    GC(2) = ∂FC(4;X)/∂X(2)
    GC(3) = ∂FC(4;X)/∂X(3)
  ELSE
    GC(3) = ∂FC(5;X)/∂X(3)
    GC(4) = ∂FC(5;X)/∂X(4)
  ENDIF
$ENDSET

```

if \$CRED='N',

```

$SET(GMODEL C)
  IF (KC.EQ.1) THEN
    GC(1) = ∂FC(1;X)/∂X(1)
    GC(2) = ∂FC(1;X)/∂X(4)
  ELSE IF (KC.EQ.2) THEN
    GC(1) = ∂FC(2;X)/∂X(2)
    GC(2) = ∂FC(2;X)/∂X(4)
  ELSE IF (KC.EQ.3) THEN
    GC(1) = ∂FC(3;X)/∂X(3)
  ELSE IF (KC.EQ.4) THEN
    GC(1) = ∂FC(4;X)/∂X(1)
    GC(2) = ∂FC(4;X)/∂X(2)
    GC(3) = ∂FC(4;X)/∂X(3)
  ELSE
    GC(1) = ∂FC(5;X)/∂X(3)
    GC(2) = ∂FC(5;X)/∂X(4)
  ENDIF
$ENDSET

```

if \$CRED='Y' or

```

$SET(GMODEL CS)
  CG(1) = ∂FC(1;X)/∂X(1)
  CG(2) = ∂FC(1;X)/∂X(4)
  CG(3) = ∂FC(2;X)/∂X(2)
  CG(4) = ∂FC(2;X)/∂X(4)
  CG(5) = ∂FC(3;X)/∂X(3)
  CG(6) = ∂FC(4;X)/∂X(1)

```

```

CG(7) = ∂FC(4;X)/∂X(2)
CG(8) = ∂FC(4;X)/∂X(3)
CG(9) = ∂FC(5;X)/∂X(3)
CG(10) = ∂FC(5;X)/∂X(4)
$ENDSET

```

in both cases.

As in the case of the dense problem, the second derivatives of the constraint functions can be specified by using the macrovariables \$HMODEL C or \$HMODEL CS. If \$JACC='S', only nonzero elements of the Hessian matrices are specified. The use of macrovariable \$HMODEL C is allowed only if \$CRED='Y'. For the above example the specifications have the form:

```

$SET(HMODEL C)
  IF (KC.EQ.1) THEN
    HC(1) = ∂2FC(1;X)/∂X(1)2
    HC(2) = ∂2FC(1;X)/∂X(1)∂X(4)
    HC(3) = ∂2FC(1;X)/∂X(4)2
  ELSE IF (KC.EQ.2) THEN
    HC(1) = ∂2FC(2;X)/∂X(2)2
    HC(2) = ∂2FC(2;X)/∂X(2)∂X(4)
    HC(3) = ∂2FC(2;X)/∂X(4)2
  ELSE IF (KC.EQ.3) THEN
    HC(1) = ∂2FC(3;X)/∂X(3)2
  ELSE IF (KC.EQ.4) THEN
    HC(1) = ∂2FC(4;X)/∂X(1)2
    HC(2) = ∂2FC(4;X)/∂X(1)∂X(2)
    HC(3) = ∂2FC(4;X)/∂X(2)2
    HC(4) = ∂2FC(4;X)/∂X(1)∂X(3)
    HC(5) = ∂2FC(4;X)/∂X(2)∂X(3)
    HC(6) = ∂2FC(4;X)/∂X(3)2
  ELSE
    HC(1) = ∂2FC(5;X)/∂X(3)2
    HC(2) = ∂2FC(5;X)/∂X(3)∂X(4)
    HC(3) = ∂2FC(5;X)/∂X(4)2
  ENDIF
$ENDSET

```

if \$CRED='Y' or

```

$SET(HMODEL CS)
CH(1) = ∂2FC(1;X)/∂X(1)2
CH(2) = ∂2FC(1;X)/∂X(1)∂X(4)
CH(3) = ∂2FC(1;X)/∂X(4)2
CH(4) = ∂2FC(2;X)/∂X(2)2
CH(5) = ∂2FC(2;X)/∂X(2)∂X(4)
CH(6) = ∂2FC(2;X)/∂X(4)2
CH(7) = ∂2FC(3;X)/∂X(3)2
CH(8) = ∂2FC(4;X)/∂X(1)2
CH(9) = ∂2FC(4;X)/∂X(1)∂X(2)
CH(10) = ∂2FC(4;X)/∂X(2)2
CH(11) = ∂2FC(4;X)/∂X(1)∂X(3)
CH(12) = ∂2FC(4;X)/∂X(2)∂X(3)
CH(13) = ∂2FC(4;X)/∂X(3)2

```

```

CH(14) =  $\partial^2 FC(5;X)/\partial X(3)^2$ 
CH(15) =  $\partial^2 FC(5;X)/\partial X(3)\partial X(4)$ 
CH(16) =  $\partial^2 FC(5;X)/\partial X(4)^2$ 
$ENDSET

```

in both cases. Note that the dimensions of arrays HC or CH must be specified by the statement \$MHC=dimension\_of\_HC or \$MCH=dimension\_of\_CH.

If some of the constraint functions are linear (i.e. if \$NCL>0) and if \$JACC='S', then the coefficients CG(K),  $1 \leq K \leq ICG(NCL+1)-1$  (constant part of the sparse Jacobian matrix), must be specified by using the macrovariable \$INPUT. If the matrix given in the above example is the constant sparse Jacobian matrix, we use this specification:

```

$ADD(INPUT)
  CG(1)= $g_{11}^C$ ; CG(2)= $g_{14}^C$ ; CG(3)= $g_{22}^C$ ; CG(4)= $g_{24}^C$ 
  CG(5)= $g_{33}^C$ ; CG(6)= $g_{41}^C$ ; CG(7)= $g_{42}^C$ ; CG(8)= $g_{43}^C$ 
  CG(9)= $g_{53}^C$ ; CG(10)= $g_{54}^C$ 
$ENDADD

```

There is another possibility which can be useful when all constraint functions are linear. It is based on the usage of a special procedure UKMCI1 which serves for a direct input of individual Jacobian matrix elements. The procedure UKMCI1 is formally called by using the statement

```
CALL $UKMCI1(K,I,GCKI)    or    $SETCG(K,I,GCKI)
```

where K is an index of a given constraint function (a row of the Jacobian matrix), I is an index of a given variable (a column of the Jacobian matrix), and GCKI is a numerical value of the element  $\partial FC(K;X)/\partial X(I)$ . For the example given above we can write:

```

$ADD(INPUT)
  $SETCG(1,1, $g_{11}^C$ )
  $SETCG(1,4, $g_{14}^C$ )
  $SETCG(2,2, $g_{22}^C$ )
  $SETCG(2,4, $g_{24}^C$ )
  $SETCG(3,3, $g_{33}^C$ )
  $SETCG(4,1, $g_{41}^C$ )
  $SETCG(4,2, $g_{42}^C$ )
  $SETCG(4,3, $g_{43}^C$ )
  $SETCG(5,3, $g_{53}^C$ )
  $SETCG(5,4, $g_{54}^C$ )
$ENDADD

```

The main advantage of the last possibility is the fact that it is not necessary to specify the fields ICG and JCG beforehand. If the number of the constraints is very large, then we can use a slightly more complicated procedure UKMCI2 which uses dynamic structures and therefore works more quickly. The procedure UKMCI2 is formally called by using the statement

```
CALL $UKMCI2(K,I,GCKI)
```

where K is an index of a given constraint function (a row of the Jacobian matrix), I is an index of a given variable (a column of the Jacobian matrix), GCKI is the numerical value of the element  $\partial FC(K;X)/\partial X(I)$ .

## 2.15 Solution of systems of nonlinear functional equations

If we set `$MODEL='NE'`, then we suppose that the system of nonlinear functional equations

$$FA(KA; X) = 0, \quad 1 \leq KA \leq NF$$

is solved. This possibility is equivalent to minimization of sum of squares when `$MODEL='AQ'`, but now `NA=NF` so that special methods for solving systems of nonlinear functional equations can be used. Functions  $FA(KA; X)$ ,  $1 \leq KA \leq NF$ , are in fact approximating functions. Their specification is described in Sections 2.5 – 2.6.

## 2.16 Solution of systems of ordinary differential equations

If we set `$MODEL='DE'`, then we suppose that the system of ordinary differential equations

$$\frac{dYA(KE; TA)}{dTA} = FE(KE; YA(TA), TA), \quad YA(KE; TAMIN) = FY(KE) \quad 1 \leq KE \leq NE$$

is solved in the interval  $TAMIN \leq TA \leq TAMAX$ , where  $FE(KE; YA(TA), TA)$ ,  $1 \leq KE \leq NE$ , are smooth state functions and  $FY(KE)$ ,  $1 \leq KE \leq NE$ , are smooth initial functions. In this case, the statement `$NE=number_of_differential_equations` must be used for the specification of number of differential equations `NE`. Moreover, values `TAMIN` and `TAMAX` have to be specified by using the macrovariable `$INPUT`.

```
$ADD(INPUT)
  TA = initial (minimum) value of the independent variable (TA = TAMIN)
  TAMAX = maximum value of the independent variable
$ENDADD
```

Specification of state functions is described in Section 2.8. Specification of initial functions is described in Section 2.9. The resulting state variables (solution of system of ordinary differential equations) can be stored in all integration steps (if `$MED=1`) or in prescribed mesh points (if `$MED=2`). In the second case, the macrovariable `$NA` specifies the number of mesh points equidistantly distributed in the interval  $TAMIN \leq TA \leq TAMAX$ .

## 2.17 Additional specifications concerning optimization problems

Useful specifications, which can improve the computational efficiency and robustness of the optimization methods, are a lower bound for the objective function value and an upper bound for the stepsize. Both of these values depend on the definition of the objective function and can be specified by the statements `$FMIN=lower_bound` (for the objective function value) and `$XMAX=upper_bound` (for the stepsize). We recommend a definition of `$FMIN` whenever it is possible, and a definition of `$XMAX` whenever the objective function contains the exponential functions. If the objective function is a sum of powers (or a sum of squares), then automatically `$FMIN=0`. The default option for the maximum stepsize is `$XMAX=1000`.

If there are no general constraints and if the number of variables is not greater than 100, then we can use global optimization methods. A decision between local and global optimization is effected by means of macrovariable `$EXTREM`:

`$EXTREM='L'` - A local extremum is found, which usually contains the starting point in its basin of attraction.

`$EXTREM='G'` - All extremum points in the given region are found and a global extremum is determined.

The default option is `$EXTREM='L'`. If `$EXTREM='G'`, we cannot use the common macrovariables `$FGMODELF` and `$FGHMODELF` for a common specification of the value, the gradient and the Hessian matrix of the model function. Similarly we cannot use the macrovariables `$FGMODELA` or `$FGMODELAS` and `$FGHMODELA` or `$FGHMODELAS` for a common specification of the approximating functions.



The global optimization is performed over a bounded region specified by lower and upper bounds  $XL(I)$  and  $XU(I)$ ,  $1 \leq I \leq NF$ . If these bounds are not specified (using the macrovariable \$INPUT), they are computed from initial values of variables and from the given maximum stepsize, so that  $XL(I)=X(I)-XMAX$  and  $XU(I)=X(I)+XMAX$ ,  $1 \leq I \leq NF$ . The maximum stepsize is specified, as in the case given above, using the statement  $\$XMAX=maximum\_stepsize$ . The default option is again  $\$XMAX=1000$ .

Additional useful specifications, concerning the solution precision, are bounds used in termination criteria. These bounds can be specified by the macrovariables \$TOLX, \$TOLF, \$TOLB, \$TOLG, \$TOLC and \$MIT, \$MFV, \$MFG:

- \$TOLX - lower bound for a relative change of variables
- \$TOLF - lower bound for a relative change of function values
- \$TOLB - lower bound for the objective function value
- \$TOLG - lower bound for the objective function gradient norm
- \$TOLC - lower bound for the violated constraint functions
- \$MIT - maximum number of iterations
- \$MFV - maximum number of function evaluations
- \$MFG - maximum number of gradient evaluations

The default values are  $\$TOLX='1.0D-8'$ ,  $\$TOLF='1.0D-16'$ ,  $\$TOLB='-1.0D60'$ ,  $\$TOLG='1.0D-6'$ ,  $\$TOLC='1.0D-6'$  and  $\$MIT=500$ ,  $\$MFV=1000$ ,  $\$MFG=10000$ .

If a direct solver ( $\$NUMBER=1$ ) is used for direction determination in methods for sparse constrained problems, the statement  $\$MMAX=space\_for\_sparse\_factor$  should be given. The default value  $\$MMAX=M$  can be insufficient in these cases.

## 2.18 Conventional names of basic variables and arrays

Basic variables and arrays of optimization problems solved by the UFO system require the use of the following prescribed names.

- NF - Number of variables.
- X - Vector of variables.
- XN - Scaling coefficients.
- XL - Lower bounds of variables.
- XU - Upper bounds of variables.
- IX - Types of box constraints.
- NX - Number of box constraints.
- FF - Value of the objective function.
- GF - Gradient of the objective function.
- HF - Hessian matrix of the objective function (only the upper half is stored).
- IH - Pointers of diagonal elements of sparse matrix HF.
- JH - Column indices of nonzero elements of sparse matrix HF.
- NA - Number of approximating functions.
- KA - Index of the approximating function.
- FA - Value of the KA-th approximating function.
- AF - Vector containing values of all approximating functions.
- GA - Gradient of the KA-th approximating function.
- AG - Jacobian matrix containing gradients of all approximating functions.
- IAG - Pointers of row-start elements of sparse matrix AG.
- JAG - Column indices of nonzero elements of sparse matrix AG.
- MA - Number of nonzero elements of sparse matrix AG.
- HA - Hessian matrix of the KA-th approximating function (only the upper half is stored).
- AH - Tensor containing Hessian matrices of all approximating functions.

AM	- Vector containing fitted values of all approximating functions.
AW	- Vector containing weights of all approximating functions.
NC	- Number of constraint functions.
KC	- Index of the constraint function.
FC	- Value of the KC-th constraint function.
CF	- Vector containing values of all constraint functions.
GC	- Gradient of the KC-th constraint function.
CG	- Jacobian matrix containing gradients of all constraint functions.
ICG	- Pointers of row-start elements of sparse matrix CG.
JCG	- Column indices of nonzero elements of sparse matrix CG.
MC	- Number of nonzero elements of sparse matrix CG.
HC	- Hessian matrix of the KC-th constraint function (only the upper half is stored).
CH	- Tensor containing Hessian matrices of all constraint functions.
CL	- Lower bounds of constraint functions.
CU	- Upper bounds of constraint functions.
IC	- Types of general constraints.
CW	- Vector containing weights of all constraint functions.
NE	- Number of state variables and state functions (number of differential equations).
KE	- Index of the state function.
FE	- Value of the KE-th state function.
EF	- Vector containing values of all state functions.
GE	- Gradient of the KE-th state function.
EG	- Jacobian matrix containing gradients of all state functions.
DE	- Matrix containing derivatives of the KE-th state function with respect to state variables.
ED	- Tensor containing derivatives of all state functions with respect to state variables.

Note that the names of arrays given above can be changed by the user using statement \$NAME1='NAME2' in the macrovariable \$INPUT. However, the same names cannot be used for different objects, i.e., NAME2 cannot be taken from the above list. For instance, the example demonstrated in Section 1.2 can be rewritten in the form

```

$SET(INPUT)
  $X='U'
  U(1)=-1.2D0; U(2)= 1.0D0
$ENDSET
$SET(FMODEL)
  FF=1.0D2*(U(1)**2-U(2))**2+(U(1)-1.0D0)**2
$ENDSET
$NF=2
$MOUT=1
$NOUT=1
$BATCH
$STANDARD

```

## 2.19 Automatic differentiation

If derivatives of the model function FF are not given, i.e., if only macrovariable \$FMODEL is defined, then either numerical or automatic differentiation is used if it is necessary. The choice of the kind of differentiation is specified by the macrovariable \$IADF:

- `$IADF=0` - Derivatives of the model function are computed by numerical differentiation.
- `$IADF=1` - The first order derivatives of the model function are computed by using the reverse mode of automatic differentiation. New macrovariable `$FGMODEL`, which defines the value `FF` and the gradient `GF` of the model function is created and the original macrovariable `$FMODEL` is cancelled. The second order derivatives are computed numerically if they are required.
- `$IADF=2` - The first order derivatives of the model function are computed by using the reverse mode of automatic differentiation. New macrovariable `$FGMODEL`, which defines the value `FF` and the gradient `GF` of the model function is created. The reverse mode is followed by the forward mode for computation of the second order derivatives. New macrovariable `$HMODEL`, which defines the Hessian matrix `HF` of the model function is created. Finally, the original macrovariable `$FMODEL` is cancelled.

Automatic differentiation is realized in the first phase by the UFO control language preprocessor. All variables contained in macrovariable `$FMODEL` are redefined and all expressions are transformed in the way that also the sequence of elementary operations and their parameters are stored. The list of elementary operations is used in the reverse mode of automatic differentiation by the subroutine `RVRSWP` (if `$IADF=1`) or `RVRSWPH` (if `$IADF=2`). The subroutines, which realize elementary operations are included to the UFO source program.

If the automatic differentiation is chosen (`$IADF>0`), there are limitations concerning Fortran 77 expressions in the macrovariable `$FMODEL`:

- If an expression contains array `$FLOAT W(100)`, say, with elements depending on the vector of variables, then the user has to declare corresponding array `INTEGER IAD_W(100)` in the input file `*.UFO` (before statements `$GLOBAL` or `$STANDARD`).
- Calls of functions and subroutines (with exceptions of intrinsic Fortran 77 functions in the generic form, i.e., `SQRT`, `EXP`, `LOG`, `LOG10`, `SIN`, `COS`, `TAN`, `ASIN`, `ACOS`, `ATAN`, `SINH`, `COSH`, `TANH`) are not permitted.
- Names of variables cannot contain digits.
- Statements containing scalar variables, e.g., `W = W * X(I)`, cannot be used in a cycle. It is necessary to declare array `W(*)` and write `W(I+1) = W(I) * X(I)` instead.
- Blanks cannot be used in `WRITE` statement. Arguments of functions are not replaced by transformed variables in `WRITE` statement.
- Blanks cannot be used in `IF` and `ELSE IF` statements. Only comparisons of numbers and variables are permitted (expressions are forbidden). Arguments of functions and indices of arrays are not replaced by transformed variables in `IF` and `ELSE IF` statements.
- Computed `GO TO` statement cannot be used.

Automatic differentiation can also be used for computation of derivatives of approximating functions and constraint functions described by macrovariables `$FMODEL` and `$FMODEL`, respectively. In these cases, the kind of differentiation is specified by macrovariables `$IADA` and `$IADC`. The meaning of these macrovariables and the limitations concerning Fortran 77 expressions in macrovariables `$FMODEL` and `$FMODEL` are the same as those for macrovariables `$IADF` and `$FMODEL` described above.

Automatic differentiation can be used for dense problems at present, i.e., it does not work if `$JACA='S'`, `$JACC='S'`, `$HESF='S'`, `$HESF='B'`.

### 3 Optimization methods in the UFO system

The UFO system has a modular structure. All optimization methods can be set up using the individual simple modules. For example, the sequential quadratic programming variable metric methods for nonlinearly constrained optimization problems are set up by using the modules for an objective function evaluation, penalty function definition, direction determination, quadratic programming solution, stepsize selection, and variable metric update. The optimization methods contained in the UFO system can be roughly divided into two groups. The first group contains methods for unconstrained and linearly constrained optimization problems, while the second group contains methods for general nonlinear programming problems (or problems which are transformed to general nonlinear programming problems, e.g.  $l_1$ ,  $l_\infty$  and minimax problems). Methods for general nonlinear programming problems, i.e. for problems with nonlinear constraints, are classified by their realization form which is determined by using the macrovariable \$FORM:

- \$FORM='SQ' - Recursive quadratic programming methods for dense nonlinear programming problems or dense unconstrained minimax problems.
- \$FORM='SL' - Recursive linear programming methods for dense unconstrained minimax problems.
- \$FORM='SE' - Recursive quadratic programming methods for sparse equality constrained problems.
- \$FORM='SI' - Primal-dual interior point methods for sparse nonlinear programming problems.
- \$FORM='SF' - Nonsmooth equation methods for sparse nonlinear programming problems.
- \$FORM='SP' - Primal interior point methods for sparse nonlinear programming problems, sparse unconstrained minimax problems and sparse unconstrained  $l_\infty$  or  $l_1$  approximation problems.
- \$FORM='SM' - Smoothing methods for sparse unconstrained minimax problems.

Sections 3.1 – 3.29 concern methods for unconstrained and linearly constrained problems. These methods do not use the macrovariable \$FORM for a classification. Methods for general nonlinear programming problems are described in Sections 3.30 – 3.33. The basic parts of optimization methods are described in Sections 3.34 – 3.38. Section 3.39 is devoted to global optimization methods.

Methods for unconstrained and linearly constrained problems contained in the UFO system can be partitioned into several classes which are specified by using the macrovariable \$CLASS:

- \$CLASS='HM' - Heuristic methods for small-size problems. This class contains the pattern search method and the simplex method.
- \$CLASS='CD' - Conjugate direction methods which use no matrices. This class contains conjugate direction methods and variable metric methods with limited memory based on the Strang recursions.
- \$CLASS='VM' - Variable metric methods which use an approximation of the Hessian matrix which is updated in each iteration.
- \$CLASS='VL' - Variable metric methods with limited memory based on compact updates.
- \$CLASS='VS' - Variable metric methods with limited memory based on shifted product-form updates.
- \$CLASS='VP' - Variable metric methods with limited memory based on corrected product-form updates.
- \$CLASS='VN' - Variable metric methods with limited memory based on projections.
- \$CLASS='VR' - Variable metric methods with limited memory based on reduced Hessians.
- \$CLASS='MN' - Modified Newton methods which use the Hessian matrix computed either analytically or numerically.
- \$CLASS='TN' - Truncated Newton methods based on the difference approximation of directional derivatives.

- \$CLASS='GN' - Modified Gauss-Newton methods for nonlinear least squares problems which use the normal equation matrix as an approximation of the Hessian matrix. These methods are also realized by using the Jacobian matrix representation.
- \$CLASS='QN' - Quasi-Newton methods for nonlinear least squares problems and nonlinear equations.
- \$CLASS='QL' - Quasi-Newton methods with limited memory for sparse nonlinear least squares problems and sparse nonlinear equations.
- \$CLASS='QB' - Quasi-Newton and Brent methods for nonlinear equations.
- \$CLASS='LP' - Simplex type methods for linear programming problems.
- \$CLASS='LI' - Interior point methods for linear programming problems.
- \$CLASS='QP' - Simplex type methods for quadratic programming problems.
- \$CLASS='QI' - Interior point methods for quadratic programming problems.
- \$CLASS='BM' - Proximal bundle methods for nonsmooth optimization.
- \$CLASS='BN' - Bundle-Newton methods for nonsmooth optimization.
- \$CLASS='BV' - Bundle variable metric methods for nonsmooth optimization.
- \$CLASS='BL' - Bundle limited-memory variable metric methods for large-scale nonsmooth optimization.

Individual methods from the above classes can be chosen by using additional specifications. The most important ones, concerning direction determination and stepsize selection, are the type of the method, the kind of the matrix decomposition and the number of the method. The type of the method is specified by the macrovariable \$TYPE:

- \$TYPE='L' - Line search methods.
- \$TYPE='G' - General trust region methods.
- \$TYPE='T' - Special trust region methods for nonlinear least squares problems.
- \$TYPE='M' - Modified Marquardt methods for nonlinear least squares problems.
- \$TYPE='P' - Pattern search method of Hooke and Jeeves.
- \$TYPE='S' - Simplex method of Nelder and Mead.

The kind of the matrix decomposition is specified by the macrovariable \$DECOMP:

- \$DECOMP='M' - The original symmetric matrix is used as an input for the direction determination.
- \$DECOMP='G' - The  $LDL^T$  decomposition without permutations is used as an input for the direction determination. This decomposition is usually obtained by the Gill-Murray algorithm [94].
- \$DECOMP='S' - The  $LDL^T$  decomposition with permutations is used as an input for the direction determination. This decomposition is usually obtained by the Schnabel-Eskow algorithm [245].
- \$DECOMP='B' - The block  $LDL^T$  decomposition with permutations is used as an input for the direction determination. This decomposition is usually obtained by the Bunch-Parlett algorithm [27].
- \$DECOMP='I' - The inverse of a symmetric matrix is used as an input for the direction determination.
- \$DECOMP='R' - the  $R^T R$  decomposition without permutations is used as an input for the direction determination. This decomposition is usually obtained by the recursive  $QR$  factorization [130].
- \$DECOMP='C' - The  $R^T R$  decomposition with permutations is used as an input for the direction determination. This decomposition is usually obtained by an application of the rank revealing algorithm [34].
- \$DECOMP='A' - The rectangular matrix is used as an input for the direction determination.
- \$DECOMP='Q' - The  $QR$  decomposition of a rectangular matrix without permutations is used as an input for the direction determination. This decomposition is usually obtained by using the Householder reflection with the explicitly stored orthogonal matrix  $Q$ .

`$DECOMP='E'` - The general square matrix is used as an input for the direction determination in the case `NA=NF` (system of nonlinear equations).

If `$FORM='SE'`, we have additional possibilities for a representation of matrices in the direction determination:

`$DECOMP='K'` - The indefinite Karush-Kuhn-Tucker matrix is used as an input for the direction determination.

`$DECOMP='Z'` - The null space representation based on orthogonal projection is used as an input for the direction determination.

`$DECOMP='G'` - The range space representation based on the Schur complement is used as an input for the direction determination.

If `$FORM='SI'`, then the following possibility for a representation of matrices in the direction determination is used implicitly:

`$DECOMP='I'` - The interior point Karush-Kuhn-Tucker matrix is used as an input for the direction determination.

If `$FORM='SF'`, then the following possibility for a representation of matrices in the direction determination is used implicitly:

`$DECOMP='F'` - The nonsmooth equation Karush-Kuhn-Tucker matrix is used as an input for the direction determination.

Macrovariable `$DECOMP` is also used for the selection of conjugate direction methods. In this case it does not concern the kind of matrix decomposition.

The serial number of the method is specified by the macrovariable `$NUMBER`. It determines an individual realization of the direction determination.

Additional information about specifications `$TYPE`, `$DECOMP`, `$NUMBER` is given in Section 3.33.

All options used for the method selection have default values, which follow from the knowledge bases coded in the individual templates. Therefore they need not be specified by the user. The possibilities we describe can be of service to users who are familiar with optimization methods.

Almost all optimization methods have different realizations for the three different representations of the objective function. If `$HESF='D'`, dense variants can be used for either unconstrained problems or box constrained problems or linearly constrained problems (with dense linear constraints specified by `$JACC='D'`). If `$HESF='S'`, sparse variants can be used for either unconstrained problems or box constrained problems or linearly constrained problems (with sparse linear constraints specified by `$JACC='S'`). If `$JACA='S'` and `$HESF='B'`, partitioned variants can be used for either unconstrained problems or box constrained problems. Partitioned variants of optimization methods are usually less efficient due to the more expensive matrix operations. Therefore we recommend preferring sparse variants to the partitioned ones.

### 3.1 Heuristic methods

Heuristic (or comparative) methods are specified by the statement `$CLASS='HM'`. These methods should be used only for small-size problems (with 10 variables at most). The main advantage of heuristic methods is that they do not require continuity of the objective function.

The individual heuristic methods are specified by the macrovariable `$TYPE`:

`$TYPE='P'` - Pattern search method of Hooke and Jeeves [118].

`$TYPE='S'` - Simplex method of Nelder and Mead [219].

The default value is `$TYPE='P'`.

## 3.2 Conjugate direction methods

Conjugate direction methods are specified by the statement `$CLASS='CD'`. These methods are very efficient for large problems with computationally simple objective functions (`$KCF=1` or `$KCA=1`). The main advantage of conjugate direction methods is that no matrices are used (implicitly `$HESF='N'`). This fact highly decreases storage requirements.

The individual conjugate direction methods are specified by the macrovariable `$DECOMP`:

- `$DECOMP='C'` - Conjugate gradient methods. These methods are the simplest ones of all conjugate direction methods and they require the fewest storage requirements. However, they usually consume a greater number of function evaluations than other conjugate direction methods.
- `$DECOMP='V'` - Variable metric methods with limited memory based on the Strang recursions. These methods allow us to prescribe storage requirements using the number of VM steps (the number of necessary used vectors is approximately twice as great a number of VM steps). The number of VM steps is specified by the macrovariable `$MF`. Variable metric methods with limited memory usually consume fewer function evaluations than conjugate gradient methods.

The default value is `$DECOMP='C'` if `$KCF=1` or `$KCA=1` and `$DECOMP='V'` otherwise.

There are two families of conjugate gradient methods implemented in the UFO system:

- `$NUMBER=1` - Basic conjugate gradient methods described in [156].
- `$NUMBER=2` - Generalized conjugate gradient methods introduced in [139].

If `$NUMBER=1`, the individual methods and their modifications are specified by using the macrovariables `$MET`, `$MET1`, `$MET2`, `$MET3`, `$MET4`, and `$MET5`.

Macrovariable `$MET` determines the conjugate gradient method:

- `$MET=0` - The steepest descent method is used.
- `$MET=±1` - The Hestenes-Stiefel method [116] is used.
- `$MET=±2` - The Polak-Ribiere method [227] is used.
- `$MET=±3` - The Liu-Storey method [139] is used.
- `$MET=±4` - The Dai-Yuan method [54] is used.
- `$MET=±5` - The Fletcher-Reeves method [84] is used.
- `$MET=±6` - The conjugate descent method [80] is used.
- `$MET=±7` - The hybrid Dai-Liao method [49] is used.
- `$MET=±8` - The hybrid Hager-Zhang method [109] is used.

If `$MET<0`, the previous direction vector is used. If `$MET>0`, the vector of variables difference is used. The default value is `$MET=1`.

Macrovariable `$MET1` specifies a modification of the conjugate gradient method.

- `$MET1=±1` - The basic CG method is used.
- `$MET1=±2` - A combined CG method [52] is used.
- `$MET1=±3` - The basic CG method with scaled gradients [305] is used.
- `$MET1=±4` - A modified CG method with scaled gradients [52], [305] is used.
- `$MET1=±5` - A three term CG method [307] is used.
- `$MET1=±6` - An improved CG method with scaled gradients [306] is used.
- `$MET1=±7` - A modified CG method [296] is used.
- `$MET1=±8` - A convex combination [288] of CG methods is used.

If `$MET1<0`, the CG method with a general parameter is used. If `$MET1>0`, the CG method with a nonnegative parameter is used. The default value is `$MET1=3` if `|$MET|=4,5,6` and `$MET1=1` otherwise.

Macrovariable \$MET2 specifies a test on conjugacy or orthogonality described in [156].

- \$MET2=1 - The basic CG method is used.
- \$MET2=2 - A test on conjugacy is used.
- \$MET2=3 - A test on orthogonality is used.

The default value is \$MET2=2 if |\$MET|=4,5,6 and \$MET2=1 otherwise.

Macrovariable \$MET3 specifies the scaling parameter as is described in [156].

- \$MET3=1 - No scaling is used.
- \$MET3=2 - The BFGS scaling in every iteration is used.
- \$MET3=3 - The DFP scaling in every iteration is used.
- \$MET3=4 - The Hoshino scaling in every iteration is used.

The default value is \$MET3=1.

Macrovariable \$MET4 specifies a nonquadratic correction.

- \$MET4=1 - No correction is used.
- \$MET4=2 - The Wei nonquadratic model [134] is used.
- \$MET4=3 - The Deng nonquadratic model [301] is used.

The default value is \$MET4=1.

Macrovariable \$MET5 specifies a periodic restart.

- \$MET5=1 - Periodic restart after  $n$  iterations used.
- \$MET5=2 - The Periodic restart after  $2n$  iterations used.
- \$MET5=3 - The Periodic restart after  $12n$  iterations used.

The default value is \$MET5=1.

If \$NUMBER=2, modifications of generalized conjugate gradient methods are specified by using the macrovariable \$MET1 which specifies the restart procedure as it is described in [156].

- \$MET1=1 - The basic CG method is used.
- \$MET1=2 - The CG method with a positive parameter is used.
- \$MET1=3 - The CG method with a bounded positive parameter is used.
- \$MET1=4 - The CG method with a bounded positive parameter and the Storey restart is used.
- \$MET1=5 - The modified CG method with a bounded and restricted parameter is used.
- \$MET1=6 - The CG method with the Powell restart is used.

The default value is \$MET1=4.

They are four variable metric methods with limited memory implemented in the UFO system:

- \$NUMBER=1 - The BFGS method with limited memory described in [220]. The default number of VM steps is \$MF=10.
- \$NUMBER=2 - The extended BFGS method with limited memory described in [123]. The default number of VM steps is \$MF=3.
- \$NUMBER=3 - Methods with limited memory from the Broyden class transformed to the BFGS form with Strang recurrences described in [284] – a general version. The default number of VM steps is \$MF=10.
- \$NUMBER=4 - Methods with limited memory from the Broyden class transformed to the BFGS form with Strang recurrences described in [284] – a simplified version. The default number of VM steps is \$MF=10.

If \$NUMBER=1, the limited memory method is realized by using various scaling techniques [138] specified by the macrovariable \$MET1.



- \$MET1=1 - The scaling is suppressed.
- \$MET1=2 - The scalar scaling is used.
- \$MET1=3 - The diagonal scaling is used.
- \$MET1=4 - The scalar and diagonal scalings are used simultaneously.

The default value is \$MET1=2. If \$NUMBER=2, only the scalar scaling is used, which is specified by the macrovariable \$MET1.

- \$MET1=1 - The scaling is suppressed.
- \$MET1=2 - The scalar scaling is used.

The default value is \$MET1=2.

Possible specifications (type-decomposition-number) for the conjugate direction methods in the unconstrained case are these:

L-C-1, L-V-1,  
L-C-2, L-V-2,  
L-V-3,  
L-V-4.

Conjugate direction methods can also be used for sparse linear constraints when \$JACC='S'.

### 3.3 Variable metric methods

Variable metric methods are specified by the statement \$CLASS='VM'. These methods are most commonly used for either unconstrained or linearly constrained optimizations. Variable metric methods use a symmetric (usually positive definite) matrix which is updated in every iteration in such a way that it approximates the Hessian matrix of the objective function as precisely as possible. In the UFO system, the variable metric methods are realized in the three different forms (for \$HESF='D', \$HESF='S' and \$HESF='B') depending on the Hessian matrix specification.

There are three families of variable metric methods for dense problems (\$HESF='D') which are distinguished using the macrovariable \$UPDATE:

- \$UPDATE='B' - The Broyden family [22]. Variable metric methods from this family are the most commonly used ones since they are very robust and efficient.
- \$UPDATE='D' - The Davidon family [61]. Variable metric methods from this family are similar to the previous ones. The only difference is that projections into the new subspace are computed. This guarantees the quadratic termination property even in the case of an imperfect line search.
- \$UPDATE='S' - The shifted Broyden family [279]. Variable metric methods from this family are efficient without scaling strategies.

The default value is \$UPDATE='B'.

Individual variable metric methods are specified by using macrovariables \$MET, \$MET1, \$MET2, \$MET3 and \$MET4. Macrovariable \$MET determines the variable metric update.

- \$MET=1 - The BFGS method [22], [78], [97], [247] is used.
- \$MET=2 - The DFP method [62], [83] is used.
- \$MET=3 - The Hoshino method [119] is used.
- \$MET=4 - The safeguarded rank-one method [153] is used.
- \$MET=5 - The optimally conditioned method [61] is used.
- \$MET=6 - The rank-one based method [153] from the perfect Broyden subclass is used.
- \$MET=7 - The variationally derived method [157] from the perfect Broyden subclass is used.

- \$MET=8 - The heuristic method [160] is used.
- \$MET=9 - The method [302] derived from the matrix decomposition is used.
- \$MET=10 - The method [303] which minimizes the angle between the direction vector and the negative gradient is used.
- \$MET=11 - The method [160] which minimizes the norm of the direction vector is used.
- \$MET=12 - The least prior deviation method [206] is used.

The default value is \$MET=1. If we specify \$DECOMP='M', only values \$MET=1,2,3,4 can be used. If we specify \$UPDATE='M', only values \$MET=1,2,3,4,5,6,7,8 can be used. If we specify \$UPDATE='S', only values \$MET=1,2,3,4,5,6 can be used (\$MET=5 corresponds to the variationally derived method [157] from the perfect Broyden subclass and \$MET=5 corresponds to the hybrid globally convergent update [279] in this case).

Macrovariable \$MET1 determines the Oren (scaling) parameter [224].

- \$MET1=±1 - No scaling is used.
- \$MET1=±2 - The initial scaling [251] is used.
- \$MET1=±3 - The controlled scaling [157] is used.
- \$MET1=±4 - The interval scaling [180] is used.
- \$MET1=±5 - The scaling in each iteration is used.

If \$MET1>0, the basic initial scaling is used. If \$MET2<0, the modified initial scaling is used. The default value is \$MET1=4.

Macrovariable \$MET2 determines the value of the Biggs (nonquadratic model) parameter [11].

- \$MET2=±1 - The unit value is used.
- \$MET2=±2 - The Spedicato value [255] is used.
- \$MET2=±3 - The modified Spedicato value [160] is used.
- \$MET2=±4 - The value determined from the homogeneous model [160] is used.
- \$MET2=±5 - The value determined from the Bigs cubic model [10] is used.
- \$MET2=±6 - The value determined from the Zhang tensor model [304] is used.
- \$MET2=±7 - The Deng curvilinear model [301] is used.

If \$MET2>0, the basic update is used. If \$MET2<0, the modified update [235] is used. The default value is \$MET2=2.

Macrovariable \$MET3 determines the Powell correction [230].

- \$MET3=1 - The Powell correction is suppressed (the strong update elimination).
- \$MET3=2 - The Powell correction is suppressed (the weak update elimination).
- \$MET3=3 - The Powell correction is applied.

The default value is \$MET3=1. If \$UPDATE='S', then macrovariable \$MET3 determines the value of the shift parameter [279].

- \$MET3=0 - The parameter from the previous iteration is used.
- \$MET3=1 - A simple choice with the constant relative parameter is used.
- \$MET3=2 - A choice defined by a quadratic equation is used.
- \$MET3=3 - A heuristic choice with the relative parameter not greater than half is used.
- \$MET3=4 - A heuristic choice with the optimally conditioned first iteration (the first formula) is used.
- \$MET3=5 - A heuristic choice with the optimally conditioned first iteration (the second formula) is used.
- \$MET3=6 - A heuristic choice derived from the lowest eigenvalue is used.

The default value is \$MET3=4.

Macrovariable \$MET4 specifies a rule for the determination of the scaling parameter [178].

- \$MET4=1 - The BFGS scaling is used.
- \$MET4=2 - The DFP scaling is used.
- \$MET4=3 - The geometric mean is used.
- \$MET4=4 - The harmonic mean is used.
- \$MET4=5 - The arithmetic mean is used.

The default value depends on the value of macrovariable \$MET (values \$MET4=1 or \$MET4=3 are most frequently used).

Possible specifications (type-decomposition-number) for dense variable metric methods in the unconstrained case are these:

L-G-1,	L-S-1,	L-B-1,	L-I-1,	L-M-1,
				L-M-3,
G-G-1,	G-S-1,	G-B-1,		G-M-1,
G-G-2,	G-S-2,	G-B-2,		G-M-2,
				G-M-3,
				G-M-4,
				G-M-5,
				G-M-7.

The default choice is L-I-1. In both the box constrained and the linearly constrained cases we cannot use specifications with \$DECOMP='B'.

If the Hessian matrix is sparse with a general pattern (\$HESF='S'), the sparse variable metric methods, which preserve this pattern, are used. If \$DECOMP='M', the individual variable metric updates (or families) are specified by using the macrovariable \$UPDATE:

- \$UPDATE='M' - The simple Marwil projection update [203]. This update can only be used if \$DECOMP='M'.
- \$UPDATE='G' - The fractioned Marwil projection update [272]. This update can only be used if \$DECOMP='M' and \$NUMBER=3.
- \$UPDATE='T' - The fractioned Toint projection update (the best method given in [272]). This update can only be used if \$DECOMP='M' and \$NUMBER=3.
- \$UPDATE='B' - The partitioned variable metric updates from the Broyden family [102]. These updates can only be used if \$MODEL='AF' or \$MODEL='AQ' or \$MODEL='AP'.

The default value is \$UPDATE='M'.

Fractioned updates with specifications \$UPDATE='G' or \$UPDATE='T' can only be used in the unconstrained case. If \$UPDATE='B', the particular update is specified by using the macrovariable \$MET.

- \$MET=1 - The BFGS method [22], [78], [97], [247] is used.
- \$MET=2 - The DFP method [62], [83] is used.
- \$MET=3 - The Hoshino method [119] is used.
- \$MET=4 - The safeguarded rank-one method [153] is used.

The default value is \$MET=1.

If \$DECOMP='G', less efficient sparse product form updates from the Broyden family are used. In this case, the particular update is specified by using the macrovariable \$MET as in the previous case. The default value is \$MET=1.

Possible specifications (type-decomposition-number) for sparse variable metric methods in the unconstrained case are these:

L-G-1, L-M-1,  
 L-M-3,  
 G-G-1, G-M-1,  
 G-M-2,  
 G-M-3,  
 G-M-4,  
 G-M-5,  
 G-M-7.

The default choice is L-M-3. In the box constrained case, only the choice \$DECOMP='M' is permitted. The fractioned updates (\$UPDATE='T' and \$UPDATE='G') can only be used if \$DECOMP='M' and \$NUMBER=3.

If the Hessian matrix is sparse with a partitioned pattern (\$HESF='B'), only the partitioned variable metric updates, specified by the choice \$UPDATE='B', can be used. These updates are the same as in the case when the Hessian matrix is sparse with a general pattern, but the partitioned realization is usually less efficient than the general one due to the more expensive matrix operations.

Possible specifications (type-decomposition-number) for partitioned variable metric methods in the unconstrained case are these:

L-M-3,  
 G-M-3.

The default choice is L-M-3.

### 3.4 Variable metric methods with limited memory based on compact updates

Variable metric methods with limited memory based on compact updates are specified by the statement \$CLASS='VL'. The number of VM steps is specified by the macrovariable \$MF (the default value is \$MF=5). Variable metric methods with limited memory based on compact updates use several small-size matrices which are updated in every iteration in such a way that their product approximates the Hessian matrix as precisely as possible [31].

Individual variable metric methods with limited memory based on compact updates are specified by using the macrovariables \$MET and \$MET1. Macrovariable \$MET determines the variable metric update.

\$MET=1 - The BFGS method [22], [78], [97], [247] is used.  
 \$MET=4 - The safeguarded rank-one method [153] is used.

The default value is \$MET=1.

Macrovariable \$MET1 determines the scaling technique.

\$MET1=1 - The scaling is suppressed.  
 \$MET1=2 - The scalar scaling is used.

The default value is \$MET1=2.

Possible specifications (type-decomposition-number) for variable metric methods with limited memory based on compact updates are these:

L-I-1,  
 L-M-3.

### 3.5 Variable metric methods with limited memory based on shifted product-form updates

Variable metric methods with limited memory based on shifted product-form updates are specified by the statement `$CLASS='VS'`. The number of VM steps is specified by the macrovariable `$MF` (the default value is `$MF=10`). Variable metric methods with limited memory based on shifted product-form updates use a rectangular matrix containing `$MF` columns with `$NF` elements, which is updated in every iteration in such a way that the shifted product of this matrix with its transposition approximates the Hessian matrix as precisely as possible [279], [280], [281].

Individual variable metric methods with limited memory based on shifted product-form updates are specified by using the macrovariables `$MET`, `$MET3` and `$MET5`. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The the BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The DFP method [62], [83] is used.
- `$MET=3` - The Hoshino method [119] is used.
- `$MET=4` - The safeguarded rank-one method [153] is used.
- `$MET=5` - The variationally derived method [157] from the preconvex part of the Broyden family is used.

The default value is `$MET=1`.

Macrovariable `$MET3` determines the shift parameter.

- `$MET3=0` - The parameter from the previous iteration is used.
- `$MET3=1` - A simple choice with the constant relative parameter is used.
- `$MET3=2` - A choice defined by a quadratic equation is used.
- `$MET3=3` - A heuristic choice with the relative parameter not greater than half is used.
- `$MET3=4` - A heuristic choice with the optimally conditioned first iteration (the first formula) is used.
- `$MET3=5` - A heuristic choice with the optimally conditioned first iteration (the second formula) is used.
- `$MET3=6` - A heuristic choice derived from the lowest eigenvalue is used.

The default value is `$MET3=4`.

Macrovariable `$MET5` determines the individual limited-memory method.

- `$MET5=±1` - A rank-one limited memory method is used.
- `$MET5=±2` - A simple rank-two limited memory method is used.
- `$MET5=±3` - The the rank-two limited memory method derived from the shifted Broyden class (based on a simple simple choice) is used.
- `$MET5=±4` - The rank-two limited memory method derived from the shifted Broyden class (based on a minimization of the Frobenius norm) is used.
- `$MET5=±5` - The rank-two limited memory method derived from the shifted Broyden class (based on a comparison of direction vectors) is used.
- `$MET5=±6` - The rank-two limited memory method derived from the standard Broyden class (based on a minimization of the Frobenius norm) is used.

Two different implementations (`$MET5>0` and `$MET5<0`) can be used. The default value is `$MET5=5`.

Possible specifications (type-decomposition-number) for variable metric methods with limited memory based on shifted product-form updates are these:

L-I-1.

### 3.6 Variable metric methods with limited memory based on corrected product-form updates

Variable metric methods with limited memory based on corrected product-form updates are specified by the statement `$CLASS='VP'`. The number of VM steps is specified by the macrovariable `$MF` (the default value is `$MF=10`). Variable metric methods with limited memory based on corrected product-form updates use a rectangular matrix containing `$MF` columns with `$NF` elements, which is updated in every iteration in such a way that the product of this matrix with its transposition satisfies the quasi-Newton condition. This product is augmented by the scaled unit matrix and the resulting matrix is corrected by using a limited number of the Strang recurrences to approximate the Hessian matrix as precisely as possible [282].

Individual variable metric methods with limited memory based on corrected product-form updates are specified by using the macrovariables `$MET2`, `$MET3` and `$MET5`. Macrovariable `$MET2` determines a nonquadratic correction.

- `$MET2=±1` - The unit value is used.
- `$MET2=±2` - The Spedicato value [255] is used.
- `$MET2=±3` - The modified Spedicato value [160] is used.
- `$MET2=±4` - The value determined from the homogeneous model [160] is used.
- `$MET2=±5` - The value determined from the Bigs cubic model [10] is used.

If `$MET2<0`, the basic variable metric update is used. If `$MET2>0`, a modified variable metric update is used. The default value is `$MET2=1`.

Macrovariable `$MET3` determines a variable metric method used for correction.

- `$MET3=1` - The variable metric correction with a constant Broyden parameter is used.
- `$MET3=2` - The variable metric correction with a variable Broyden parameter computed by a special formula is used.

The default value is `$MET3=1`.

Macrovariable `$MET5` determines the number of the Strang recurrences used in the correction phase. This number has to be nonnegative. The default value is `$MET5=2`.

Possible specifications (type-decomposition-number) for variable metric methods with limited memory based on corrected product-form updates are these:

L-I-1.

### 3.7 Variable metric methods with limited memory based on projections

Variable metric methods with limited memory based on projections are specified by statement `$CLASS='VN'`. The number of VM steps is specified by the macrovariable `MF` (the default value is `$MF=10`). Variable metric methods with limited memory based on projections use two rectangular matrices containing `$MF` columns with `$NF` elements, which are updated in every iteration in such a way that the difference of products of these matrices with their transpositions augmented by the scaled unit matrix satisfies the quasi-Newton condition [283].

Possible specifications (type-decomposition-number) for variable metric methods with limited memory based on projections are these:

L-I-1.

### 3.8 Variable metric methods with limited memory based on reduced Hessians

Variable metric methods with limited memory based on reduced Hessians are specified by the statement `$CLASS='VR'`. The number of VM steps is specified by the macrovariable `$MF` (the default value is `$MF=10`). Variable metric methods with limited memory based on reduced Hessians use a small-size matrix which is updated in every iteration in such a way that it approximates the reduced Hessian matrix as precisely as possible [92].

There are two families of variable metric methods with limited memory based on reduced Hessians implemented in the UFO system:

- \$NUMBER=1 - Basic variable metric methods with limited memory based on reduced Hessians described in [92].
- \$NUMBER=2 - Variable metric methods with limited memory based on reduced Hessians and orthogonal transformations described in [279].

Individual variable metric methods with limited memory based on reduced Hessians are specified by using the macrovariables \$MET, \$MET1, \$MET2, \$MET3, \$MET4 and \$MET5. Macrovariable \$MET determines the variable metric update.

- \$MET=1 - The BFGS method [22], [78], [97], [247] is used.
- \$MET=2 - The DFP method [62], [83] is used.
- \$MET=3 - The Hoshino method [119] is used.
- \$MET=4 - The safeguarded rank-one method [153] is used.
- \$MET=5 - The optimally conditioned method [61] is used.
- \$MET=6 - The rank-one based method [153] from the perfect Broyden subclass is used.
- \$MET=7 - The variationally derived method [157] from the perfect Broyden subclass is used.
- \$MET=8 - The heuristic method [160] is used.
- \$MET=9 - The method [302] derived from the matrix decomposition is used.
- \$MET=10 - The method [303] which minimizes the angle between the direction vector and the negative gradient is used.

The default value is \$MET=1.

Macrovariable \$MET1 determines the Oren (scaling) parameter [224].

- \$MET1=1 - No scaling is used.
- \$MET1=2 - The initial scaling [251] is used.
- \$MET1=3 - The controlled scaling [157] is used.
- \$MET1=4 - The interval scaling [180] is used.
- \$MET1=5 - The scaling in each iteration is used.

The default value is \$MET1=3.

Macrovariable \$MET2 determines the value of the Biggs (nonquadratic model) parameter [11].

- \$MET2=1 - The unit value is used.
- \$MET2=2 - The Spedicato value [255] is used.
- \$MET2=3 - The modified Spedicato value [160] is used.
- \$MET2=4 - The value determined from the homogeneous model [160] is used.
- \$MET2=5 - The value determined from the Biggs cubic model [10] is used.

Moreover, if \$MET2>0, the basic update is used and if \$MET2<0, the modified update [235] is used. The default value is \$MET2=2.

Macrovariable \$MET3 determines the Powell correction [230].

- \$MET3=1 - The Powell correction is suppressed (the strong update elimination).
- \$MET3=2 - The Powell correction is suppressed (the weak update elimination).
- \$MET3=3 - The Powell correction is applied.

The default value is \$MET3=1.

Macrovariable \$MET4 specifies a rule for the determination of the scaling parameter [178].

- \$MET4=1 - The BFGS scaling is used.
- \$MET4=2 - The DFP scaling is used.





If \$MODEL='AQ' (sum of squares), the combination [163] of both the modified Newton and the modified Gauss-Newton methods can be used. This choice is possible by using the macrovariable \$MET.

\$MET=1               - The modified Newton method is used.  
\$MET=2               - The combined method is used.

The default value is \$MET=2.

Possible specifications (type-decomposition-number) for sparse modified Newton methods in the unconstrained case are these:

L-G-1,   L-M-1,  
          L-M-3,  
G-G-1,   G-M-1,  
          G-M-2,  
          G-M-3,  
          G-M-4,  
          G-M-5,  
          G-M-7.

The default choice is G-M-3. In the box constrained case, only the choice \$DECOMP='M' is permitted.

If the Hessian matrix is sparse with a partitioned pattern (\$HESF='B'), a computation of the second order derivatives is the same as in the case when the Hessian matrix is sparse with a general pattern, but the partitioned realization is usually less efficient than the general one due to the more expensive matrix operations.

If \$MODEL='AQ' (sum of squares), the combination of both the modified Newton and the modified Gauss-Newton methods can be used. This choice is possible by using the macrovariable \$MET in the same way that is used if \$HESF='S'. Possible specifications (type-decomposition-number) for partitioned modified Newton methods in the unconstrained case are these:

L-M-3,  
G-M-3.

The default choice is G-M-3.

### 3.10 Truncated Newton methods

Truncated Newton methods are specified by the statement \$CLASS='TN'. These methods differ from modified Newton methods in that the directional derivatives are determined by the numerical differentiation instead of the sparse Hessian matrix multiplication. Truncated Newton methods are very efficient for large problems with computationally simple objective functions (\$KCF=1 or \$KCA=1). The main advantage of truncated Newton methods is that no matrices are used (implicitly \$HESF='N'). This fact highly decreases storage requirements.

Truncated Newton methods are implemented either as line search methods or as trust region methods and are based on the conjugate gradient subalgorithm. Possible specifications (type-decomposition-number) for truncated Newton methods are these:

L-M-3,  
G-M-3,  
G-M-4,  
G-M-5.

The default choice is G-M-3.

### 3.11 Modified Gauss-Newton methods for nonlinear least squares and nonlinear equations

Modified Gauss-Newton methods are specified by the statement `$CLASS='GN'`. These methods are special optimization methods for either nonlinear least squares (`$MODEL='AQ'`) or nonlinear least powers (`$MODEL='AP'`) problems. Modified Gauss-Newton methods are based on the fact that the first term in the Hessian matrix expression, the so-called normal equation matrix, depending on the first derivatives of the approximating functions only is a good approximation of the whole Hessian matrix. The second term in the Hessian matrix expression can be approximated by using the variable metric updates.

Modified Gauss-Newton methods are realized in four different forms (for `$HESF='D'`, `$HESF='S'`, `$HESF='B'`, `$HESF='N'`) depending on the Hessian matrix specification. Although the modified Gauss-Newton methods can be realized as the line search methods (`$TYPE='L'`), it is more advantageous to realize them as the trust region methods (`$TYPE='G'`).

If the Hessian matrix is specified to be dense (`$HESF='D'`), then the normal equation matrix is also dense. In this case, we can use hybrid methods with dense updates:

- `$UPDATE='N'` - No update is used. The method utilizes the normal equation matrix (the first part of the Hessian matrix expression).
- `$UPDATE='S'` - The Dennis structured approach [67] is used. The second part of the Hessian matrix is approximated by using modified variable metric updates. This part is added to the normal equation matrix if the conditions for leaving the modified Gauss-Newton method are satisfied.
- `$UPDATE='F'` - The Fletcher hybrid approach [1], [85] is used. The Hessian matrix is approximated either by the normal equation matrix or by the matrix obtained by using the variable metric updates. The decision between the two cases is based on the rate of the function value decrease and on the normal equation matrix conditioning.
- `$UPDATE='B'` - A variable metric update from the Broyden class is applied either to the normal equation matrix or to the previous approximation of the Hessian matrix if conditions for leaving the modified Gauss-Newton method are satisfied [163].

The default value is `$UPDATE='N'`.

Individual variable metric updates from the above families are specified by using the macrovariable `$MET`.

- `$MET=1` - The BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The DFP method [62], [83] is used.
- `$MET=3` - The Hoshino method [119] is used.
- `$MET=4` - The original (unsafeguarded) rank-one method is used.

The value `$MET=4` is allowed only if `$UPDATE='S'` and is the default in this case. The value `$MET=1` is the default in the other cases.

Variable metric updates (`$UPDATE='F'` or `$UPDATE='B'`) can be realized either as simple updates (normal equation matrix is updated) or as cumulative updates (previous approximation of the Hessian matrix is updated), as is described in [163]. A decision between these possibilities is mediated by the macrovariable `$MOT1`.

- `$MOT1=0` - The cumulative update is used.
- `$MOT1=1` - The simple update is used.

The default value is `MOT1=1`.

In the dense case, the modified Gauss-Newton methods can be realized with additional special matrix decompositions which cannot be used in other cases. If `$DECOMP='R'`, the recursive QR decomposition [227] is used with an additional correction of the upper triangular matrix  $R$ . If `$DECOMP='C'`, this matrix  $R$  can moreover be changed by using the rank revealing algorithm [34] which can improve its conditioning.

Possible specifications (type-decomposition-number) for dense modified Gauss-Newton methods in the unconstrained case are these:

L-G-1,	L-S-1,	L-B-1,	L-R-1,	L-C-1,	L-M-1,
					L-M-3,
G-G-1,	G-S-1,	G-B-1,	G-R-1,	G-C-1,	G-M-1,
G-G-2,	G-S-2,	G-B-2,	G-R-2,	G-C-2,	G-M-2,
					G-M-3,
					G-M-4,
					G-M-5,
					G-M-7,
T-G-1,	T-S-1,		T-R-1,	T-C-1,	T-M-1,
T-G-2,					
	T-S-7,			T-C-7,	T-M-7,
					M-M-1.

The default choice is G-M-7. In both the box constrained and the linearly constrained cases we cannot use specifications \$DECOMP='S', \$DECOMP='R', \$DECOMP='C'. If \$DECOMP='S' or \$DECOMP='C', then variable metric updates cannot be used (\$UPDATE='N'). The specification \$UPDATE='S' can only be used if \$DECOMP='M'.

If the Hessian matrix is specified to be sparse with a general pattern (\$HESF='S'), the normal equation matrix has the same structure. In this case, we can use hybrid methods with sparse updates:

- \$UPDATE='N' - No update is used. The method utilizes the normal equation matrix (the first part of the Hessian matrix expression).
- \$UPDATE='S' - The Dennis structured approach [67] is used. The second part of the Hessian matrix is approximated by using modified variable metric updates. This part is added to the normal equation matrix if conditions for leaving the modified Gauss-Newton method are satisfied.
- \$UPDATE='D' - The Brown-Dennis structured approach [23] is used. The Hessian matrices of approximating functions are approximated by using variable metric updates. These matrices serve for approximating the second part of the Hessian matrix which is added to the normal equation matrix if conditions for leaving the modified Gauss-Newton method are satisfied.
- \$UPDATE='B' - A variable metric update from the Broyden class is applied either to the normal equation matrix or to the previous approximation of the Hessian matrix if conditions for leaving the modified Gauss-Newton method are satisfied [163].
- \$UPDATE='M' - A sparse update based on the Marwil projection is applied either to the normal equation matrix or to the previous approximation of the Hessian matrix if conditions for leaving the modified Gauss-Newton method are satisfied [163].

The default value is \$UPDATE='N'.

Individual variable metric updates from the above families are specified by using the macrovariable \$MET as in the dense case. The value \$MET=4 is allowed only if either \$UPDATE='S' or \$UPDATE='D', and is the default in this case. The value \$MET=1 is the default in the other cases excepting the case \$UPDATE='M' in which the macrovariable \$MET is not utilized.

Variable metric updates (\$UPDATE='M' or \$UPDATE='B') can be realized either as simple updates (normal equation matrix is updated) or as cumulative updates (previous approximation of the Hessian matrix is updated). A decision between these possibilities is mediated by the macrovariable \$MOT1 similarly as in the dense case.

If \$UPDATE='D', we can use several switches for utilizing variable metric updates specified by the macrovariable \$MOT2.

\$MOT2=0           - The Fletcher and Xu switch [85] is used.  
 \$MOT2=1           - A modification of the Fletcher and Xu switch is used.  
 \$MOT2=2           - The Dennis and Welsch switch [71] is used.  
 \$MOT2=3           - The Ramsin and Wedin switch [239] is used.

The default value is \$MOT2=0.

Possible specifications (type-decomposition-number) for sparse Gauss-Newton methods in the unconstrained case are these:

L-G-1,   L-M-1,  
           L-M-3,  
 G-G-1,   G-M-1,  
 G-G-2,   G-M-2,  
           G-M-3,  
           G-M-4,  
           G-M-5,  
           G-M-7,  
 T-G-1,   T-M-1,  
           T-M-7,  
           M-M-1.

The default choice is G-M-3. In the box constrained case, only the choice \$DECOMP='M' is permitted.

If the Hessian matrix is specified to be sparse with a partitioned pattern (\$HESF='B'), the normal equation matrix has the same structure. If this is the case, then we can use hybrid methods with partitioned updates \$UPDATE='N', \$UPDATE='S', \$UPDATE='D', \$UPDATE='F', \$UPDATE='B', whose details have already been explained above. Note that the partitioned realization is usually less efficient than the general one due to the more expensive matrix operations.

Possible specifications (type-decomposition-number) for partitioned Gauss-Newton methods are these:

L-M-3,  
 G-M-3.

The default choice is G-M-3.

If the Hessian matrix is not specified (\$HESF='N'), the normal equation matrix is not used. The Jacobian matrix, defining a linear least squares problem, is utilized in each iteration instead. Such so-called normal equation free Gauss-Newton methods are realized in the two different forms (for \$JACA='D' and \$JACA='S') depending on the Jacobian matrix specification.

If the Jacobian matrix is specified to be dense (\$JACA='D'), then we cannot use hybrid methods with variable metric updates (only the specification \$UPDATE='N' is permitted). Moreover, dense, normal equation free Gauss-Newton methods can only be used in the unconstrained case.

Possible specifications (type-decomposition-number) for dense, normal equation free, Gauss-Newton methods are these:

L-Q-1,   L-A-1,   L-E-1,  
           L-A-3,   L-E-3,  
           L-A-4,   L-E-4,  
                   L-E-5,  
 G-Q-1,   G-A-1,   G-E-1,  
 G-Q-2,            G-E-2,  
           G-A-3,   G-E-3,  
           G-A-4,   G-E-4,  
                   G-E-5,  
           G-A-7.

The default choice is G-A-3 for least squares problems and G-E-3 for systems of nonlinear equations. The specification \$DECOMP='E' can only be used if NA=NF (system of nonlinear equations).

If the Jacobian matrix is specified to be sparse (\$JACA='S'), we can use hybrid methods with simple variable metric updates:

- \$UPDATE='N' - No update is used. The method utilizes the original Jacobian matrix.
- \$UPDATE='V' - The simple factorized BFGS update [163] is used. The second order information is approximated by the unsymmetric rank-one update of the Jacobian matrix.
- \$UPDATE='R' - The simple factorized rank-one update [163] is used. The second order information is approximated by the addition of a dense row to the Jacobian matrix.

If \$UPDATE='V' or \$UPDATE='R', we can use several switches for utilizing variable metric updates, specified by the macrovariable \$MOT2 as in the case of the specification \$HESF='S' described above. The default value is \$MOT2=0.

The main advantage of sparse, normal equation free, Gauss-Newton methods consists in the fact that the normal equation matrix is dense if the sparse Jacobian matrix has at least one dense row. If this is the case, then the classical Gauss-Newton methods cannot be used. On the other hand, the normal equation matrix often has a lower number of nonzero elements than the Jacobian one. Consequently, the classical Gauss-Newton methods are more efficient in this case.

Possible specifications (type-decomposition-number) for sparse, normal equation free, Gauss-Newton methods are these:

L-A-1, L-E-1,  
 L-A-3, L-E-3,  
 L-A-4, L-E-4,  
           L-E-5,  
 G-A-1, G-E-1,  
           G-E-2,  
 G-A-3, G-E-3,  
 G-A-4, G-E-4,  
           G-E-5,  
 G-A-7.

The default choice is G-A-3 for least squares problems and G-E-3 for systems of nonlinear equations. The specification \$DECOMP='E' can only be used if NA=NF (system of nonlinear equations). The choice L-E-1 differs from the choice L-E-2. The last one corresponds to the incomplete LU decomposition.

### 3.12 Quasi-Newton methods for nonlinear least squares and nonlinear equations

Quasi-Newton methods are specified by the statement \$CLASS='QN'. These methods are special optimization methods for nonlinear least squares (\$MODEL='AQ') problems including systems of nonlinear equations (\$MODEL='NE') when the first derivatives are not specified analytically (the macrovariable \$GMODEL is not defined). Quasi-Newton methods use a rectangular matrix which is updated in every iteration in such a way that it approximates the Jacobian matrix as precisely as possible. In the UFO system, the quasi-Newton methods are realized in the two different forms (for \$JACA='D' and \$JACA='S') depending on the Jacobian matrix specification.

There are two possibilities for dense problems (\$JACA='D') which are distinguished by using the macrovariable \$UPDATE:

- \$UPDATE='N' - No update is used. An approximation of the Jacobian matrix is computed numerically by using differences.
- \$UPDATE='B' - The Broyden family [21] of rank-one updates is used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.

When \$UPDATE='B', the individual quasi-Newton methods are specified by using the macrovariable \$MET.

- \$MET=1 - The first (good) Broyden update [21] is used.
- \$MET=2 - The second Broyden update [21] is used.
- \$MET=3 - The the second Greenstadt update [256] is used.
- \$MET=4 - The first Greenstadt update [256] is used.
- \$MET=5 - The first Todd OC update [125] is used.
- \$MET=6 - The first Todd OCX update [125] is used.
- \$MET=7 - The second Todd OC update [125] is used.
- \$MET=8 - The second Todd OCX update [125] is used.

The default value is \$MET=1.

Dense quasi-Newton methods can only be used in the unconstrained case.

Possible specifications (type-decomposition-number) for dense quasi-Newton methods are these:

- L-Q-1, L-A-1, L-E-1,
- L-A-3, L-E-3,
- L-A-4, L-E-4,
- L-E-5,
- G-Q-1, G-A-1, G-E-1,
- G-Q-2, G-E-2,
- G-A-3, G-E-3,
- G-A-4, G-E-4,
- G-E-5,
- G-A-7.

The default choice is G-Q-3. The specification \$DECOMP='E' can only be used if NA=NF (system of nonlinear equations).

If the Jacobian matrix is sparse with a general pattern (\$JACA='S'), there are two possibilities for computing an approximation of the Jacobian matrix by the differences. These possibilities are distinguished by using the macrovariable \$NUMDER:

- \$NUMDER=1 - Derivatives of individual approximating functions are computed.
- \$NUMDER=2 - The Coleman-More [40] graph coloring algorithm is used.

Moreover, various sparse quasi-Newton updates which preserve the pattern of the Jacobian matrix can be used.

If \$NUMDER=1, there are three choices of the quasi-Newton updates which are specified by the macrovariable \$UPDATE:

- \$UPDATE='N' - No update is used. An approximation of the Jacobian matrix is computed numerically by using differences.
- \$UPDATE='B' - Sparse quasi-Newton updates are used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.
- \$UPDATE='S' - Modified Newton methods such as the row scaling update are used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.

If \$NUMDER=2, there are four choices of the quasi-Newton updates which are specified by the macrovariable \$UPDATE:

- \$UPDATE='N' - No update is used. An approximation of the Jacobian matrix is computed numerically by using differences.

- \$UPDATE='B' - Sparse quasi-Newton updates [246] are used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.
- \$UPDATE='S' - Modified Newton methods such as the row scaling update are used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.
- \$UPDATE='C' - Cyclic column determination methods are used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.

Individual quasi-Newton methods are specified by using the macrovariable \$MET. If \$UPDATE='B', the following specifications are possible.

- \$MET=1 - The Schubert update [246] is used.
- \$MET=2 - The Bogle-Perkins update [18] is used.
- \$MET=3 - The column update [201] is used.

If \$UPDATE='S', the following specifications are possible.

- \$MET=0 - The modified Newton method is used.
- \$MET=1 - The the row scaling update [201] is used.

If \$UPDATE='C', the following specifications are possible.

- \$MET=0 - The cyclic column determination method [133] is used.
- \$MET=1 - The cyclic column determination method [133] is used followed by the Schubert update [246].

Possible specifications (type-decomposition-number) for sparse quasi-Newton methods are these:

L-A-1, L-E-1,  
 L-A-3, L-E-3,  
 L-A-4, L-E-4,  
           L-E-5,  
 G-A-1, G-E-1,  
           G-E-2,  
 G-A-3, G-E-3,  
 G-A-4, G-E-4,  
           G-E-5,  
 G-A-7.

The default choice is G-A-3 for the least squares problems and G-E-3 for systems of nonlinear equations. The specification \$DECOMP='E' can only be used if NA=NF (system of nonlinear equations). The choice L-E-1 differs from the choice L-E-2. The latter corresponds to the incomplete LU decomposition.

### 3.13 Quasi-Newton methods with limited memory for nonlinear equations

Quasi-Newton methods with limited memory are specified by the statement \$CLASS='QL'. The number of QN steps is specified by the macrovariable \$MF (the default value is \$MF=5). These methods are special methods for solving sparse systems of nonlinear equations (\$MODEL='NE') when the first derivatives are not specified analytically (the macrovariable \$GMODEL is not defined). Therefore only the case NA=NF is permitted. Quasi-Newton methods with limited memory use an initial approximation of the sparse Jacobian matrix together with several small-size matrices which are updated in every iteration in such a way that their product approximates the Jacobian matrix as precisely as possible [31]. There are two possibilities which are distinguished by using the macrovariable \$UPDATE:

- `$UPDATE='N'` - No update is used. Every approximation of the Jacobian matrix is computed numerically by using differences.
- `$UPDATE='B'` - The Broyden good update of rank-one with limited memory [31] is used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.

Possible specifications (type-decomposition-number) for quasi-Newton methods with limited memory are these:

L-A-3, L-E-3,  
 L-A-4, L-E-4,  
           L-E-5,  
 G-A-3, G-E-3,  
 G-A-4, G-E-4,  
           G-E-5,

The default choice is G-E-3.

Besides the quasi-Newton methods with limited memory, this class contains inverse column scaling methods which are chosen by using the specification `$DECOMP='I'`. There are two possibilities which are distinguished by using the macrovariable `$UPDATE`:

- `$UPDATE='N'` - No update is used. Every approximation of the Jacobian matrix is computed numerically by using differences.
- `$UPDATE='B'` - The inverse column scaling update [202] is used in almost all iterations. After restart, the Jacobian matrix is approximated numerically by using differences.

Possible specifications (type-decomposition-number) for inverse column scaling methods are these:

L-I-1,  
 L-I-3.

If `$NUMBER=1`, then a complete LU decomposition is used. If `$NUMBER=3`, then a combination of direct and iterative methods is used. The default value is `$NUMBER=3`.

### 3.14 Truncated Newton methods for nonlinear equations

Truncated Newton methods are specified by the statement `$CLASS='TN'`. These methods are special methods for solving systems of nonlinear equations (`$MODEL='NE'`) when the first derivatives are not specified analytically (the macrovariable `$GMODEL` is not defined). Therefore only the case `NA=NF` is permitted. Truncated Newton methods differ from quasi-Newton methods in that the sparse Jacobian matrix multiplication is replaced by the numerical differentiation. These methods are very efficient for large problems with computationally simple functions in nonlinear equations (`$KCA=1`). The main advantage of the truncated Newton methods is that no matrices are used (implicitly `$JACA='N'`). This fact highly decreases storage requirements.

Truncated Newton methods are implemented either as the line search methods or as the trust region methods and are based on the smoothed CGS subalgorithm. This subalgorithm can be preconditioned by using the tridiagonal decomposition. This possibility is determined by the macrovariable `$MOS2`.

- `$MOS2=0` - The tridiagonal decomposition is not used.
- `$MOS2=1` - The tridiagonal decomposition is used before the iterative process is started.
- `$MOS2=2` - The tridiagonal decomposition is used as a preconditioner.
- `$MOS2=3` - Both previous cases are assumed.

The default value is `$MOS2=0`.

Possible specifications (type-decomposition-number) for truncated Newton methods are these:



L-E-3,  
L-E-4,  
L-E-5,  
G-E-3,  
G-E-4,  
G-E-5.

The default choice is G-E-3.

### 3.15 Quasi-Newton and Brent methods for nonlinear equations

Quasi-Newton and Brent methods are specified by the statement `$CLASS='QB'`. These methods are special simple methods for solving dense systems of nonlinear equations (`$MODEL='NE'`) when the first derivatives are not specified analytically (the macrovariable `$GMODEL` is not defined). Therefore, only the case `NA=NF` is permitted. Individual methods are selected using the macrovariable `$NUMBER`:

`$NUMBER=1` - The Brent method described in [20].  
`$NUMBER=3` - The simple Newton method (this method can also be used if the macrovariable `$GMODEL` is defined).

The default value is `$NUMBER=3`.

### 3.16 Simplex type methods for linear programming problems

Simplex type methods for linear programming problems are specified by the statement `$CLASS='LP'`. These methods are realized in the two different forms (for `$JACC='D'` and `$JACC='S'`) depending on the constraint Jacobian matrix specification.

If the constraint Jacobian matrix is dense (`$JACC='D'`), we can use two different linear programming methods based on the active set strategy:

`$NUMBER=1` - Primal reduced gradient (null-space) method (like the method proposed in [93]), which is a special implementation of the steepest descent reduced gradient method.  
`$NUMBER=2` - Primal projected gradient (range-space) method which is a special implementation of the steepest descent projected gradient method.

Possible specifications (type-number) for dense linear programming methods are L-1 and L-2. The default choice is L-1.

If the constraint Jacobian matrix is sparse (`$JACC='S'`), we can use two different linear programming methods based on the active set strategy:

`$NUMBER=1` - Primal reduced gradient (null-space) simplex type method described in [270].  
`$NUMBER=2` - Primal projected steepest descent (range-space) method.

A possible specification (type-number) for simplex type linear programming methods are L-1 and L-2. The default choice is L-1, but this choice is not suitable for problems with equality constraints.

### 3.17 Interior point methods for linear programming problems

Interior point methods for linear programming problems are specified by the statement `$CLASS='LI'`. These methods, based on an infeasible primal-dual predictor-corrector strategy, can be used only in the sparse case when `$JACC='S'`. Moreover, only the standard LP constraints  $Ax = b$ ,  $x \geq 0$  can be considered at present. Individual methods are chosen by using the macrovariable `$MLP`:

- \$MLP=1 - The first algorithm of Miao [204].
- \$MLP=2 - The second algorithm of Miao [204].
- \$MLP=3 - The Mizuno algorithm [207].

All these methods can be realized in the three forms depending on the way of solving the linear generalized Karush-Kuhn-Tucker system:

- \$NUMBER=1 - Direct solution based on the Gill-Murray decomposition applied to the Schur complement.
- \$NUMBER=2 - Direct solution based on the Bunch-Parlett decomposition applied to the original Karush-Kuhn-Tucker system.
- \$NUMBER=3 - Iterative solution based on the conjugate gradient method applied to the Schur complement.

Possible specifications (type-number) for interior point linear programming methods are L-1, L-2 and L-3. The default choice is L-1.

### 3.18 Simplex type methods for quadratic programming problems

Simplex type methods for quadratic programming problems are specified by the statement \$CLASS='QP'. These methods are realized in the two different forms (for \$JACC='D' and \$JACC='S') depending on the constraint Jacobian matrix specification.

If the constraint Jacobian matrix is dense (\$JACC='D'), we can use three different quadratic programming methods based on the active set strategy:

- \$NUMBER=1 - Primal reduced gradient (null-space) method (like the method proposed in [95]) which is a special implementation of the Newton reduced gradient method.
- \$NUMBER=2 - Primal projected gradient (range-space) method (like the method proposed in [76]) which is a special implementation of the Newton projected gradient method.
- \$NUMBER=3 - Dual projected gradient (range-space) method (like the method proposed in [98]).

Possible specifications (type-number) for dense quadratic programming methods are L-1, L-2, and L-3. The default choice is L-1.

If the constraint Jacobian matrix is sparse (\$JACC='S'), we can use two different quadratic programming methods based on the active set strategy:

- \$NUMBER=1 - Primal reduced gradient (null-space) simplex type method described in [270].
- \$NUMBER=2 - Primal projected conjugate gradient (range-space) method.

A possible specification (type-number) for sparse simplex type quadratic programming methods are L-1 and L-2. The default choice is L-2. Choice L-1 is not suitable for problems with equality constraints.

### 3.19 Interior point methods for quadratic programming problems

Interior point methods for quadratic programming problems are specified by the statement \$CLASS='QI'. These primal-dual methods, based on the logarithmic barrier function and iterative solution of the indefinite Karush-Kuhn-Tucker system, can be used only in the sparse case when \$JACC='S'. Interior point methods for quadratic programming problems are in fact the same as methods with the choices \$TYPE='L' and \$DECOMP='I' described in Section 3.31.

Two realizations are possible, which are specified by the macrovariable \$NUMBER:

- \$NUMBER=1 - An exact sparse Bunch-Parlett (BP) decomposition [74] of the indefinite Karush-Kuhn-Tucker system is used.

`$NUMBER=3` - An inexact preconditioned conjugate gradient (PCG) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. The particular realization of the inexact preconditioned conjugate gradient method depends on specifications given by the macrovariables `$MOS1`, `$MOS2` and `$MOS3`.

The default value is `$NUMBER='3'`.

Macrovariable `$MOS1` specifies the precision control and the choice of the penalty parameter.

`$MOS1=0` - The precision control is suppressed.  
`$MOS1=1` - A precision guaranteeing descent direction is used together with the basic choice of the penalty parameter.  
`$MOS1=2` - A precision guaranteeing descent direction is used together with an extended choice of the penalty parameter.

The default value is `$MOS1=0`.

Macrovariable `$MOS2` specifies a preconditioning technique.

`$MOS2=0` - Preconditioning is suppressed.  
`$MOS2=±1` - The indefinite preconditioner [177] based on a diagonal approximation of the Hessian matrix is used in the normal equation form.  
`$MOS2=±1` - The indefinite preconditioner [177] based on a diagonal approximation of the Hessian matrix is used in the augmented system form.

If `$MOS2>0`, a complete Gill-Murray decomposition is used. If `$MOS2<0`, an incomplete Gill-Murray decomposition is used. The default value is `$MOS2=1`.

Macrovariable `$MOS3` specifies residual smoothing of the conjugate gradient method.

`$MOS2=0` - The residual smoothing is suppressed.  
`$MOS2=0` - A simple one-dimensional residual smoothing is used.

The default value is `$MOS3=0`.

Possible specifications (type-number) for sparse interior point quadratic programming methods are L-1 and L-3. The default choice is L-3.

### 3.20 Proximal bundle methods for nonsmooth optimization

Proximal bundle methods for nonsmooth optimization problems are specified by the statement `$CLASS='BM'`. These methods use a bundle of gradients computed in trial points and updated in every iteration. The size of this bundle is specified by the macrovariable `$MB` (the default value is `$MB='NF+3'`). Proximal bundle methods solve a special quadratic programming subproblem derived from the cutting plane approach [277]. This subproblem is in fact the same as in the recursive quadratic programming methods for minimax problems. Proximal bundle methods are realized only for unconstrained or linearly constrained dense problems (`$JACA='D'`). The special quadratic programming subproblem can be solved by using the following methods:

`$NUMBER=1` - Dual projected gradient (range-space) method proposed in [145].  
`$NUMBER=2` - Primal projected gradient (range-space) method which is a special implementation of the Newton projected gradient method.

The special quadratic programming subproblem is defined in such a way that it has a diagonal Hessian matrix. There are several methods for computing the diagonal weight coefficients, which are selected by using the macrovariables `$MOS` and `$MES2`.

- \$MOS=1           - If \$MES2=1, the weights are updated by using curvature of the one-dimensional quadratic function.
- \$MOS=1           - If \$MES2=2, the weights are updated by using the minimum position estimate (suitable for polyhedral and nearly polyhedral functions).
- \$MOS=2           - The weights are updated by using the quasi-Newton condition.

The default values are \$MOS=1 and \$MES2=1.

Proximal bundle methods are only realized as line search methods in two modifications which are specified by the macrovariable \$MEX.

- \$MEX=0           - A convex version is assumed.
- \$MEX=1           - A nonconvex version is assumed and we can define a measure of nonconvexity by using the macrovariable \$ETA5. The default value is \$ETA5=0.25.

Another important parameter is the maximum stepsize defined by the macrovariable \$XMAX. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is \$XMAX=1000. Proximal bundle methods are sensitive to the values of parameters \$ETA5 and \$XMAX. Therefore, these values should be carefully chosen.

Possible specifications (type-number) for proximal bundle methods are L-1 and L-2. The default choice is L-1. Proximal bundle methods can be used when \$KSF=3 or \$KSA=3. They can be also used for minimizing sums of absolute values (\$MODEL='AA') and for solving minimax problems (\$MODEL='AM').

### 3.21 Bundle Newton methods for nonsmooth optimization

Bundle Newton methods for nonsmooth optimization problems are specified by the statement \$CLASS='BN'. These methods use a bundle of gradients and Hessian matrices computed in trial points and updated in every iteration. The size of this bundle is specified by the macrovariable \$MB (the default value is \$MB='NF+3'). Bundle Newton methods solve a special quadratic programming subproblem derived from the cutting plane approach which contains second order information [185]. This subproblem is in fact the same as in recursive quadratic programming methods for minimax problems. Bundle Newton methods are only realized for unconstrained or linearly constrained dense problems (\$JACA='D'). The special quadratic programming subproblem can be solved by using the following methods:

- \$NUMBER=1       - Dual projected gradient (range-space) method proposed in [145].
- \$NUMBER=2       - Primal projected gradient (range-space) method which is a special implementation of the Newton projected gradient method.

The special quadratic programming subproblem has a general (dense) Hessian matrix which is a bundle approximation of the second-order matrix of the original nonsmooth problem.

Bundle Newton methods are only realized as line search methods. A nonconvex version is assumed and we can define a measure of nonconvexity by using the macrovariable \$ETA5. The default value is \$ETA5=0.25. Another important parameter is the maximum stepsize defined by the macrovariable \$XMAX. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is \$XMAX=1000. Bundle Newton methods are sensitive to the values of parameters \$ETA5 and \$XMAX. Therefore, these values should be carefully chosen.

Possible specifications (type-number) for bundle Newton methods are L-1 and L-2. The default choice is L-1. Bundle Newton methods can be used when \$KSF=3 or \$KSA=3. They can be also used for minimizing sums of absolute values (\$MODEL='AA') and for solving minimax problems (\$MODEL='AM').

### 3.22 Bundle variable metric methods for nonsmooth optimization

Bundle variable metric methods for nonsmooth optimization problems are specified by the statement \$CLASS='BV'. These methods are based on a special realization of variable metric method updates. This realization uses special null steps and restarts. Stepsize selection is based on the polyhedral approximation

obtained by using bundles of points and subgradients. Bundle variable metric methods are realized only for unconstrained or linearly constrained dense problems ( $\$JACA='D'$ ). They need not solve any quadratic programming subproblem.

Bundle variable metric methods are realized as line search methods in two modifications which are specified by the macrovariable  $\$MEX$ .

- $\$MEX=0$  - A convex version [188] is assumed.
- $\$MEX=1$  - A nonconvex version [278] is assumed and we can define a measure of nonconvexity by using the macrovariable  $\$ETA5$ . The default value is  $\$ETA5=0.25$ .

Another important parameter is the maximum stepsize defined by the macrovariable  $\$XMAX$ . The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is  $\$XMAX=1000$ . Variable metric bundle methods are sensitive to the values of parameters  $\$ETA5$  and  $\$XMAX$ . Therefore, these values should be carefully chosen.

Bundle variable metric methods use an auxiliary bundle of gradients, which serve for the initial stepsize determination in the line search subalgorithm. The size of this bundle is specified by the macrovariable  $\$MB$  (the default value is  $\$MB='NF+3'$ ).

Possible specifications (type-number) for bundle variable metric methods are L-1 and L-2. The default choice is L-1. Bundle variable metric methods can be used when  $\$KSF=3$  or  $\$KSA=3$ . They can be also used for minimizing sums of absolute values ( $\$MODEL='AA'$ ) and for solving minimax problems ( $\$MODEL='AM'$ ).

### 3.23 Bundle variable metric methods with limited memory for nonsmooth optimization

Bundle variable metric methods with limited memory for nonsmooth optimization problems are specified by the statement  $\$CLASS='BL'$ . The number of VM steps is specified by the macrovariable  $\$MF$  (the default value is  $\$MF=5$ ). These methods are based on special realization of limited memory variable metric updates. This realization uses special null steps and restarts. Stepsize selection is based on the polyhedral approximation obtained by using bundles of points and subgradients. Bundle variable metric methods are realized only for unconstrained or box constrained sparse problems ( $\$JACA='S'$  and  $\$HESF='S'$ ). They need not solve any quadratic programming subproblem.

Bundle variable metric methods with limited memory are realized as line search methods in several modifications which are specified by the macrovariables  $\$MEX$ ,  $\$MEX1$ ,  $\$MEX2$ ,  $\$MEX3$ .

- $\$MEX=0$  - A convex version [188] is assumed.
- $\$MEX=1$  - A nonconvex version [278] is assumed and we can define a measure of nonconvexity by using the macrovariable  $\$ETA5$ . The default value is  $\$ETA5=0.25$ .
- $\$MEX1=0$  - The basic algorithm is used.
- $\$MEX1=1$  - An additional restart is used if the direction vector is short.
- $\$MEX1=2$  - The controlled correction of the direction vector is used. The correction parameter is specified by the macrovariable  $\$ETA3$  The default value is  $\$ETA3=10^{-12}$ .
- $\$MEX1=2$  - The permanent correction of the direction vector is used. The correction parameter is specified by the macrovariable  $\$ETA3$  The default value is  $\$ETA3=10^{-12}$ .
- $\$MEX2=0$  - The basic termination criterion is used.
- $\$MEX2=1$  - The extended termination criterion for large-scale problems is used.
- $\$MEX3=0$  - The termination is used after a zero difference of gradients.
- $\$MEX3=1$  - The extrapolation with a constant stepsize is used after a zero difference of gradients.
- $\$MEX3=2$  - The extrapolation with an increasing stepsize is used after a zero difference of gradients.

The default values are  $\$MEX=1$ ,  $\$MEX1=0$ ,  $\$MEX2=0$ ,  $\$MEX3=0$ .

Variable metric updates are controlled by the macrovariables  $\$MET$ ,  $\$MET2$ ,  $\$MET3$ ,  $\$MET4$ .

- \$MET=0 - Limited memory BFGS matrix is updated only in descent steps.
- \$MET=1 - Limited memory BFGS matrix is updated in all steps.
- \$MET2=0 - No updates are used in null steps.
- \$MET2=1 - Rank-one updates are used in \$MET3 consecutive null steps.
- \$MET2=2 - Rank-one updates are used in all null steps.
- \$MET2=3 - BFGS updates are used in \$MET4 consecutive null steps.

The default values are \$MET=1 \$MET2=1, \$MET3=5, \$MET4=5.

Another important parameter is the maximum stepsize defined by the macrovariable \$XMAX. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is \$XMAX=1000. Bundle variable metric methods for large-scale sum of absolute values are sensitive to the values of parameters \$ETA5 and \$XMAX. Therefore, these values should be carefully chosen.

Bundle variable metric methods with limited memory use an auxiliary bundle of gradients, which serve for the initial stepsize determination in the line search subalgorithm. The size of this bundle is specified by the macrovariable \$MB (the default value is \$MB=20).

A possible specification (type-number) for bundle variable metric methods for large-scale sum of absolute values is L-1.

### 3.24 Bundle variable metric methods for sparse sum of absolute values

If \$MODEL='AA', then the objective function is the sum of absolute values. In this case, bundle variable metric methods [197] can be chosen by using the statement \$CLASS='BV'. These methods are based on a special realization of partitioned variable metric updates. This realization uses special null steps and restarts. Step size selection is based on the polyhedral approximation obtained by using bundles of points and subgradients. Bundle variable metric methods are realized only for unconstrained or box constrained sparse problems (\$JACA='S' and \$HESF='S'). They need not solve any quadratic programming subproblem.

Bundle variable metric methods for sparse sum of absolute values are realized as line search methods in several modifications which are specified by the macrovariables \$MEX, \$MEX1, \$MEX2, \$MEX3.

- \$MEX=0 - A convex version [188] is assumed.
- \$MEX=1 - A nonconvex version [278] is assumed and we can define a measure of nonconvexity by using the macrovariable \$ETA5. The default value is \$ETA5=0.25.
- \$MEX1=0 - The basic algorithm is used.
- \$MEX1=1 - An additional restart is used if the direction vector is short.
- \$MEX1=2 - The controlled correction of the direction vector is used. The correction parameter is specified by the macrovariable \$ETA3 The default value is \$ETA3=10<sup>-12</sup>.
- \$MEX1=3 - The permanent correction of the direction vector is used. The correction parameter is specified by the macrovariable \$ETA3 The default value is \$ETA3=10<sup>-12</sup>.
- \$MEX2=0 - The basic termination criterion is used.
- \$MEX2=1 - The extended termination criterion for large-scale problems is used.
- \$MEX3=0 - The termination is used after a zero difference of gradients.
- \$MEX3=1 - The extrapolation with a constant stepsize is used after a zero difference of gradients.
- \$MEX3=2 - The extrapolation with an increasing stepsize is used after a zero difference of gradients.

The default values are \$MEX=1, \$MEX1=0, \$MEX2=0, \$MEX3=0.

variable metric updates can be scaled. This possibility is specified by the macrovariable \$MET1.

- \$MET1=0 - Scaling is suppressed.
- \$MET1=1 - Scaling is performed.

The default value is `$MET1=0`.

Another important parameter is the maximum stepsize defined by the macrovariable `$XMAX`. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is `$XMAX=1000`. Bundle variable metric methods for large-scale sum of absolute values are sensitive to the values of parameters `$ETA5` and `$XMAX`. Therefore, these values should be carefully chosen.

Bundle variable metric methods for sparse sum of absolute values use an auxiliary bundle of gradients, which serve for the initial stepsize determination in the line search subalgorithm. The size of this bundle is specified by the macrovariable `$MB` (the default value is `$MB=20`).

A possible specification (type-number) for bundle variable metric methods for large-scale sum of absolute values is L-1.

### 3.25 Primal interior point methods for sparse sum of absolute values

If `$MODEL='AA'`, then the objective function is a sum of absolute values. In this case, primal interior point methods [173] can be chosen by using the statement `$FORM='SP'`. These methods, which are intended for large problems, belong to the following classes:

- `$CLASS='VM'` - Primal interior point variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates. The partitioned variable metric updates from the Broyden family [102] are used
- `$CLASS='MN'` - Primal interior point modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically.

The default value is `$CLASS='MN'`.

If `$CLASS='VM'`, the particular variable metric method is specified by using macrovariables `$MET`, `$MET1`, `$MET5`. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The partitioned BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The partitioned DFP method [62], [83] is used.
- `$MET=3` - The partitioned Hoshino method [119] is used.
- `$MET=4` - The partitioned safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

Macrovariable `$MET1` determines scaling of variable metric updates [224].

- `$MET1=1` - No scaling is used.
- `$MET1=2` - The initial scaling [251] is used.
- `$MET1=3` - The controlled scaling [157] is used.
- `$MET1=4` - The interval scaling [180] is used.
- `$MET1=5` - The scaling in each iteration is used.

The default value is `$MET1=3`.

Macrovariable `$MET5` determines subjects of variable metric updates.

- `$MET5=0` - Updates concerns approximating functions.
- `$MET5=2` - Updates concerns approximating functions multiplied by the signs of the Lagrange multipliers.
- `$MET5=3` - Updates concerns terms of the Lagrangian function.

The default value is `$MET5=1`.

The macrovariable `$MEP3` determines a strategy for computation of the barrier parameter.

- \$MEP3=1 - The geometric sequence is used.
- \$MEP3=2 - A special monotone sequence is used.
- \$MEP3=3 - A special retarded monotone sequence is used.
- \$MEP3=4 - A simple retarded monotone sequence with stagnations is used.

The default value is \$MEP3=2.

Primal interior point methods for sparse sum of absolute values use several parameters specified by macrovariables \$ETA4, \$ETA5. The macrovariable \$ETA4 determines the reduction of the barrier parameter. The default value is \$ETA4=0.95. The macrovariable \$ETA5 determines the minimum value of the barrier parameter. The default value is \$ETA5=10<sup>-8</sup>. Another important parameter is the maximum stepsize defined by the macrovariable \$XMAX. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is \$XMAX=1000.

Primal interior point methods for sparse sum of absolute values uses either line search (if \$TYPE='L') or trust region (if \$TYPE='G') strategies. Possible specifications (type-decomposition-number) are these:

L-G-1,		L-M-1,
G-G-1,	G-B-1,	G-M-1,
		G-M-7.

The default choice is L-G-1.

### 3.26 Recursive linear programming methods for dense minimax problems.

If \$MODEL='AM', then the objective function is the maximum of approximating functions or their absolute values. In this case, recursive linear programming methods [198] can be chosen by using the statement \$FORM='LP'. Recursive linear programming methods are realized as trust region methods with box constrained subproblems. The special linear programming subproblem, which is derived from the minimax problem, is solved by a primal projected gradient (range-space) method which is a special implementation of the steepest descent method. A possible specification (type-number) for recursive linear programming methods is G-1.

### 3.27 Recursive quadratic programming methods for dense minimax problems.

If \$MODEL='AM', then the objective function is the maximum of approximating functions or their absolute values. In this case, recursive quadratic programming methods [112], [146] can be chosen by using the statement \$FORM='QP'. These methods belong to the following classes:

- \$CLASS='VM' - Recursive quadratic programming variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates belonging to the Broyden family.
- \$CLASS='MN' - Recursive quadratic programming modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically.

The default value is \$CLASS='VM'.

Recursive quadratic programming methods are realized in three different forms:

- \$TYPE='L' - Line search methods.
- \$TYPE='G' - General trust region methods.
- \$TYPE='C' - General trust region methods with second order corrections [81].

If \$TYPE='L', the special line search method (\$MES=5), described in [146], can be used.

The special quadratic programming subproblem, which is derived from the minimax problem, can be solved by using the two different methods:



- \$NUMBER=1 - Dual projected gradient (range-space) method proposed in [145].
- \$NUMBER=2 - Primal projected gradient (range-space) method which is a special implementation of the Newton projected gradient method.

Recursive quadratic programming variable metric methods use the same updates as methods with the choices \$DECOMP='G' and \$UPDATE='B' described in Section 3.3 (values \$MET=1 - \$MET=12 can be used). Similarly, recursive quadratic programming modified Newton methods correspond to the methods with the choice \$DECOMP='G' described in Section 3.8 (the Gill-Murray decomposition is used).

Even if the minimax problems can be solved by using bundle methods described in Sections 3.19 – 3.21, it is more efficient to use the recursive quadratic programming methods that utilize a special structure of the minimax problem.

All of the above methods can be used only for dense unconstrained or linearly constrained problems. A possible specifications (type-number) for recursive quadratic programming methods are these:

L-1, G-1, C-1,  
L-2, G-2, C-2.

The default choice is L-1. A possible specification (type-number) for recursive linear programming methods is G-1.

### 3.28 Primal interior point methods for sparse minimax problems

If \$MODEL='AM', then the objective function is the maximum of approximating functions or its absolute values. In this case, primal interior point methods [171] can be chosen by using the statement \$FORM='SP'. These methods, which are intended for large problems, belong to the following classes:

- \$CLASS='VM' - Primal interior point variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates. The partitioned variable metric updates from the Broyden family [102] are used
- \$CLASS='MN' - Primal interior point modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically.

The default value is \$CLASS='MN'.

If \$CLASS='VM', the particular variable metric method is specified by using macrovariables \$MET, \$MET1, \$MET5. Macrovariable \$MET determines the variable metric update.

- \$MET=1 - The partitioned BFGS method [22], [78], [97], [247] is used.
- \$MET=2 - The partitioned DFP method [62], [83] is used.
- \$MET=3 - The partitioned Hoshino method [119] is used.
- \$MET=4 - The partitioned safeguarded rank-one method [153] is used.

The default value is \$MET=1.

Macrovariable \$MET1 determines scaling of variable metric updates [224].

- \$MET1=1 - No scaling is used.
- \$MET1=2 - The initial scaling [251] is used.
- \$MET1=3 - The controlled scaling [157] is used.
- \$MET1=4 - The interval scaling [180] is used.
- \$MET1=5 - The scaling in each iteration is used.

The default value is \$MET1=3.

Macrovariable \$MET5 determines subjects of variable metric updates.

- \$MET5=0 - Updates concerns approximating functions.
- \$MET5=2 - Updates concerns approximating functions multiplied by the signs of the Lagrange multipliers.
- \$MET5=3 - Updates concerns terms of the Lagrangian function.

The default value is \$MET5=1.

The macrovariable \$MEP determines the particular barrier function:

- \$MEP=0 - Minimax function is used.
- \$MEP=1 - The logarithmic barrier function is used.
- \$MEP=2 - The Ben-Tal barrier function is used.
- \$MEP=3 - The composite barrier function is used.
- \$MEP=4 - The Carrol barrier function is used.

The default value is \$MEP=1.

The macrovariable \$MEP3 determines a strategy for computation of the barrier parameter.

- \$MEP3=1 - The geometric sequence is used.
- \$MEP3=2 - A special monotone sequence is used.
- \$MEP3=3 - A special retarded monotone sequence is used.
- \$MEP3=4 - A simple retarded monotone sequence with stagnations is used.

The default value is \$MEP3=2.

Primal interior point methods for sparse minimax problems use several parameters specified by macrovariables \$ETA3, \$ETA4, \$ETA5. The macrovariable \$ETA3 determines the precision of the solution to inner nonlinear equation. The default value is \$ETA3=10<sup>-6</sup>. The macrovariable \$ETA4 determines the reduction of the barrier parameter. The default value is \$ETA4=0.85. The macrovariable \$ETA5 determines the minimum value of the barrier parameter. The default value is \$ETA5=10<sup>-10</sup>. Another important parameter is the maximum stepsize defined by the macrovariable \$XMAX. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is \$XMAX=1000.

Primal interior point methods for sparse minimax problems use either line search (if \$TYPE='L') or trust region (if \$TYPE='G') strategies. Possible specifications (type-decomposition-number) are these:

L-G-1,		L-M-1,
G-G-1,	G-B-1,	G-M-1,
		G-M-7.

The default choice is L-G-1.

### 3.29 Smoothing methods for sparse minimax problems

If \$MODEL='AM', then the objective function is the maximum of approximating functions or their absolute values. In this case, smoothing methods [228], [291] can be chosen by using the statement \$FORM='SM'. These methods, which are intended for large problems, belong to the following classes:

- \$CLASS='VM' - Smoothing variable metric methods. An approximation of the Lagrangian function. Hessian matrix is updated in each iteration by using variable metric updates. The partitioned variable metric updates from the Broyden family [102] are used
- \$CLASS='MN' - Smoothing modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically.

The default value is \$CLASS='MN'.

If `$CLASS='VM'`, the particular variable metric method is specified by using macrovariables `$MET`, `$MET1`, `$MET5`. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The partitioned BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The partitioned DFP method [62], [83] is used.
- `$MET=3` - The partitioned Hoshino method [119] is used.
- `$MET=4` - The partitioned safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

Macrovariable `$MET1` determines scaling of variable metric updates [224].

- `$MET1=1` - No scaling is used.
- `$MET1=2` - The initial scaling [251] is used.
- `$MET1=3` - The controlled scaling [157] is used.
- `$MET1=4` - The interval scaling [180] is used.
- `$MET1=5` - The scaling in each iteration is used.

The default value is `$MET1=3`.

Macrovariable `$MET5` determines subjects of variable metric updates.

- `$MET5=0` - Updates concerns approximating functions.
- `$MET5=2` - Updates concerns approximating functions multiplied by the signs of the Lagrange multipliers.
- `$MET5=3` - Updates concerns terms of the Lagrangian function.

The default value is `$MET5=1`.

The macrovariable `$MEP2` specifies whether values of exponentials are saved.

- `$MEP2=0` - Values of exponentials are not saved.
- `$MEP2=1` - Values of exponentials are saved.

The default value is `$MEP2=0`.

The macrovariable `$MEP3` determines a strategy for computation of the barrier parameter.

- `$MEP3=1` - A geometric sequence is used.
- `$MEP3=2` - A special monotone sequence is used.
- `$MEP3=3` - A special retarded monotone sequence is used.
- `$MEP3=4` - A simple retarded monotone sequence with stagnations is used.

The default value is `$MEP3=2`.

Smoothing methods for sparse minimax problems use parameters specified by macrovariables `$ETA4`, `$ETA5`. The macrovariable `$ETA4` determines the reduction of the barrier parameter. The default value is `$ETA4=0.85`. The macrovariable `$ETA5` specifies the minimum value of the barrier parameter. The default value is `$ETA5=10-8`. Another important parameter is the maximum stepsize defined by the macrovariable `$XMAX`. The maximum stepsize is a safeguard, which guarantees that the new point lies in the region where the bundle model is valid. The default value is `$XMAX=1000`.

Primal interior point methods for sparse minimax problems use a line search strategy. Possible specification (type-decomposition-number) is L-G-2.

### 3.30 Recursive quadratic programming methods for dense nonlinear programming problems

Recursive quadratic programming methods for dense general nonlinear programming problems are specified by the statement `$FORM='SQ'`. These methods belong to the two following classes:

- `$CLASS='VM'` - Recursive quadratic programming variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates.
- `$CLASS='MN'` - Recursive quadratic programming modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically.

The default value is `$CLASS='VM'`. Variable metric methods are the same as in Section 3.3 with the choice `$DECOMP='G'` and `$UPDATE='B'` (values `$MET=1` - `$MET=12` can be used). Similarly, modified Newton methods are the same as in Section 3.8 with the choice `$DECOMP='G'` (the Gill-Murray decomposition is used).

Recursive quadratic programming methods for dense general nonlinear programming problems are realized as line search methods (`$TYPE='L'`) with the  $l_1$ -exact penalty function. They are like the methods proposed in [230]. The special line search method (`$MES=5`) for  $l_1$ -exact penalty function can be used successfully. The quadratic programming subproblem can be solved by using the two different methods:

- `$NUMBER=1` - Dual projected gradient (range-space) method (like the method proposed in [98]).
- `$NUMBER=2` - Primal projected gradient (range-space) method (like the method proposed in [76]) which is a special implementation of the Newton projected gradient method.

Possible specifications (type-number) for recursive quadratic programming methods for dense general nonlinear programming problems are L-1 and L-2. The default choice is L-1.

Recursive quadratic programming methods can be used for dense nonlinear programming problems with various objective functions. If we set `$MODEL='AA'` (sum of absolute values) or `$MODEL='AM'` (minimax) then an extended nonlinear programming problem containing extra variables is defined and solved.

### 3.31 Recursive quadratic programming methods for sparse equality constrained problems

Recursive quadratic programming methods for sparse equality constrained nonlinear programming problems are specified by the statement `$FORM='SE'`. These methods, which are intended for large problems, belong to the following classes:

- `$CLASS='VM'` - Recursive quadratic programming variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates.
- `$CLASS='VL'` - Recursive quadratic programming variable metric methods with limited memory based on compact representations of variable metric updates. The number of VM steps is specified by the macrovariable `$MF` (the default value is `$MF=5`). Variable metric methods with limited memory use several small-size matrices which are updated in every iteration in such a way that their product approximates the Lagrangian function Hessian matrix as precisely as possible [31].
- `$CLASS='MN'` - Recursive quadratic programming modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically. The sparsity pattern is required
- `$CLASS='TN'` - Recursive quadratic programming truncated Newton methods. These methods differ from modified Newton methods in that the directional derivatives are determined by the numerical differentiation instead of the sparse Hessian matrix multiplication. The sparsity pattern is not required

The default value is `$CLASS='MN'`.

If `$CLASS='VM'` and `$HESF='D'`, dense variable metric updates are used. Macrovariable `$MET` determines the variable metric update.

- \$MET=1 - The BFGS method [22], [78], [97], [247] is used.
- \$MET=2 - The DFP method [62], [83] is used.
- \$MET=3 - The Hoshino method [119] is used.
- \$MET=4 - The safeguarded rank-one method [153] is used.

The default value is \$MET=1.

If \$CLASS='VM' and \$HESF='S', the individual variable metric updates are specified by using the macrovariable \$UPDATE:

- \$UPDATE='M' - The simple Marwil projection update [203].
- \$UPDATE='B' - The partitioned variable metric updates from the Broyden family [102]. The partitioned variable metric updates can only be used if \$MODEL='AF' or \$MODEL='AQ' or \$MODEL='AP'.

The default value is \$UPDATE='B' if \$MODEL='AF' or \$MODEL='AQ' or \$MODEL='AP' and \$UPDATE='M' in the opposite case. If \$UPDATE='B', the particular variable metric method is specified by using macrovariables \$MET, \$MET1, \$MET5. Macrovariable \$MET determines the variable metric update.

- \$MET=1 - The partitioned BFGS method [22], [78], [97], [247] is used.
- \$MET=2 - The partitioned DFP method [62], [83] is used.
- \$MET=3 - The partitioned Hoshino method [119] is used.
- \$MET=4 - The partitioned safeguarded rank-one method [153] is used.

The default value is \$MET=1.

Macrovariable \$MET1 determines scaling of variable metric updates [224].

- \$MET1=1 - No scaling is used.
- \$MET1=2 - The initial scaling [251] is used.
- \$MET1=3 - The controlled scaling [157] is used.
- \$MET1=4 - The interval scaling [180] is used.
- \$MET1=5 - The scaling in each iteration is used.

The default value is \$MET1=3.

Macrovariable \$MET5 determines subjects of variable metric updates.

- \$MET5=0 - Updates concerns approximating functions.
- \$MET5=2 - Updates concerns approximating functions multiplied by the signs of the Lagrange multipliers.
- \$MET5=3 - Updates concerns terms of the Lagrangian function.

The default value is \$MET5=1.

Recursive quadratic programming methods for sparse equality constrained nonlinear programming problems are realized in the three different ways which are specified by using the macrovariable \$TYPE:

- \$TYPE='L' - Line search methods based on various merit functions or filter structures.
- \$TYPE='G' - Trust region methods. These methods use two direction determination subproblems [70], [129], [184]. The vertical subproblem, solved by using the dog-leg method, serves for a sufficient decrease of constraint violations. The horizontal subproblem, solved by a special realization of the conjugate gradient method, serves for minimization of a quadratic approximation of a particular merit function.

The default value is \$TYPE='L'.

If \$TYPE='L', various penalty functions or filter structures can be used for step-size selection. The corresponding choice is determined by the macrovariable \$MERIT:

- \$MERIT='P' - A penalty function is used.
- \$MERIT='M' - The Markov filter [275], [170] is used.
- \$MERIT='F' - The Fletcher-Leyffer filter [82], [275] is used.

The default value is \$MERIT='P'. If \$MERIT='P', additional choices specified by macrovariables \$MEP, \$MEP1, \$MEP2 are possible.

Macrovariable \$MEP determines the particular merit function:

- \$MEP=0 - No merit function is used.
- \$MEP=1 - The Powell  $l_1$  exact penalty function is used.
- \$MEP=2 - The  $l_2$  augmented Lagrangian function is used.
- \$MEP=3 - The  $l_1$  augmented Lagrangian function is used.
- \$MEP=4 - The Han  $l_1$  exact penalty function is used.
- \$MEP=5 - The Schittkowski augmented Lagrangian function is used.

The default value is \$MEP=2.

Macrovariable \$MEP1 specifies the second order correction for overcoming the Maratos effect.

- \$MEP1=1 - The second order correction is suppressed.
- \$MEP1=2 - The second order correction is determined as a least squares solution of the shifted constraint system.

The default value is \$MEP1=1.

Macrovariable \$MEP2 specifies estimates of Lagrange multipliers at the beginning of each iteration.

- \$MEP2=1 - The initial estimate is taken from the previous iteration.
- \$MEP2=2 - The initial estimate is determined as a least squares solution of the first part of the Karush-Kuhn-Tucker system.

The default value is \$MEP2=1.

If \$TYPE='G', only the default specification \$MERIT='P' is possible (with penalty functions determined by macrovariable \$MET as above). However, default values \$MEP1=1 and \$MEP2=1 are used in this case.

If \$TYPE='L', the direction vector can be computed in the three different ways, which are specified by using the macrovariable \$DECOMP:

- \$DECOMP='K' - The direction vector is determined as a solution of the indefinite Karush-Kuhn-Tucker system [187].
- \$DECOMP='Z' - The direction vector is decomposed into two parts. The vertical part is computed directly from the constraint violation. The horizontal part, lying in the null-space, is computed iteratively by using a special realization of the conjugate gradient method. Instead of projecting into the null-space, either the augmented system or an orthogonal projection matrix, both determined from a range-space basis, are used [120].
- \$DECOMP='G' - The direction vector is determined directly from the Lagrangian multipliers, which are determined iteratively by using the conjugate gradient method in the range space using the Schur complement. This choice can be used only if \$CLASS='MN' or \$CLASS='VM' and \$HESF='S' (if a sparsity pattern is available).

The default value is \$DECOMP='K'.

If \$DECOMP='K', five realizations are possible, which are specified by the macrovariable \$NUMBER:

- \$NUMBER=1 - An exact sparse Bunch-Parlett (BP) decomposition [74] of the indefinite Karush-Kuhn-Tucker system is used. This choice can be used only if \$CLASS='MN' or \$CLASS='VM' and \$HESF='S' (if a sparsity pattern is available).

- \$NUMBER=3 - An inexact preconditioned conjugate gradient (PCG) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. The particular realization of the inexact preconditioned conjugate gradient method depends on specifications given by the macrovariables \$MOS1, \$MOS2, \$MOS3 and \$MOS4.
- \$NUMBER=4 - An inexact preconditioned conjugate residual (PCR) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. The particular realization of the inexact preconditioned conjugate residual method depends on specifications given by the macrovariables \$MOS1 and \$MOS2.
- \$NUMBER=5 - An inexact symmetric preconditioned quasi-minimum residual (PQMR) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. The particular realization of the inexact symmetric preconditioned quasi-minimum residual method depends on specifications given by the macrovariables \$MOS1 and \$MOS2.
- \$NUMBER=6 - An inexact nonsymmetric preconditioned conjugate gradient squared (PCGS) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. The particular realization of the inexact nonsymmetric preconditioned conjugate gradient squared method depends on specifications given by the macrovariables \$MOS1, \$MOS2 and \$MOS3.

The default value is \$NUMBER='3'.

Macrovariable \$MOS1 specifies the precision control and the choice of the penalty parameter.

- \$MOS1=0 - The precision control is suppressed.
- \$MOS1=1 - A precision guaranteeing descent direction is used together with the basic choice of the penalty parameter.
- \$MOS1=2 - A precision guaranteeing descent direction is used together with an extended choice of the penalty parameter.

The default value is \$MOS1=0.

Macrovariable \$MOS2 specifies a preconditioning technique.

- \$MOS2=0 - Preconditioning is suppressed.
- \$MOS2=±1 - The indefinite preconditioner [187] based on a diagonal approximation of the Hessian matrix is used in the normal equation form.
- \$MOS2=2 - The indefinite preconditioner [187] based on a diagonal approximation of the Hessian matrix is used in the augmented system form.
- \$MOS2=±3 - The indefinite preconditioner [187] based on a diagonal perturbation of the Schur complement is used.

If \$MOS2>0, a complete Gill-Murray decomposition is used. If \$MOS2<0, an incomplete Gill-Murray decomposition is used. The default value is \$MOS2=1.

Macrovariable \$MOS3 specifies residual smoothing of the conjugate gradient method.

- \$MOS3=0 - The residual smoothing is suppressed.
- \$MOS3=1 - A simple one-dimensional residual smoothing is used.

The default value is \$MOS3=0.

Macrovariable \$MOS4 specifies the choice of the initial direction.

- \$MOS4=0 - The zero initial direction is used.
- \$MOS4=1 - The vertical initial direction is used.

The default value is \$MOS4=0.

If \$DECOMP='Z', only one realization is possible, which is specified by the macrovariable \$NUMBER:

\$NUMBER=3 - An inexact null-space preconditioned conjugate gradient (NPCG) method for the determination of the horizontal direction is applied which uses a special determination of the required precision. A particular realization of the null-space preconditioned conjugate gradient method depends on the specifications given by the macrovariables \$MOS1 and \$MOS2.

Macrovariable \$MOS1 specifies the precision control and the choice of the penalty parameter.

\$MOS1=0 - The precision control is suppressed.  
\$MOS1=1 - A precision guaranteeing descent direction is used together with the basic choice of the penalty parameter.  
\$MOS1=2 - A precision guaranteeing descent direction is used together with an extended choice of the penalty parameter.

The default value is \$MOS1=0.

Macrovariable \$MOS2 specifies a way for computing the preconditioner.

\$MOS2=±1 - The preconditioner is computed by using the orthogonal projection matrix determined from a range-space basis.  
\$MOS2=±2 - The preconditioner is computed by using the augmented system determined from a range-space basis.

If \$MOS2>0, a diagonal approximation of the Hessian matrix is used. If \$MOS2<0, the unit approximation of the Hessian matrix is used. The default value is \$MOS2=1.

If \$DECOMP='G', two realizations are possible, which are specified by the macrovariable \$NUMBER:

\$NUMBER=3 - The sparse Gill-Murray decomposition of the Lagrangian function Hessian matrix followed by a range-space smoothed conjugate gradient (RSCG) method for a positive definite range space system is applied which uses a special determination of the required precision. The particular realization of the preconditioned conjugate gradient method depends on specifications given by the macrovariables \$MOS1, \$MOS2 and \$MOS3.

\$NUMBER=4 - The sparse Bunch-Parlett decomposition of the Lagrangian function Hessian matrix followed by a range-space smoothed conjugate gradient (RSCG) method for an indefinite range space system is applied which uses a special determination of the required precision. The particular realization of the smoothed conjugate gradient method depends on specifications given by the macrovariables \$MOS1 and \$MOS3.

The default value is \$NUMBER='3'.

Macrovariable \$MOS1 specifies the precision control and the choice of the penalty parameter.

\$MOS1=0 - The precision control is suppressed.  
\$MOS1=1 - A precision guaranteeing descent direction is used.

The default value is \$MOS1=1.

Macrovariable \$MOS2 specifies a preconditioning technique.

\$MOS2=0 - The preconditioning is suppressed.  
\$MOS2=±1 - The positive definite preconditioner [187] based on a diagonal approximation of the Hessian matrix is used.  
\$MOS2=±2 - The polynomial preconditioner [213] based on a decomposition of the normal equation is used.



If  $\$MOS2 > 0$ , a complete Gill-Murray decomposition is used. If  $\$MOS2 < 0$ , an incomplete Gill-Murray decomposition is used. The default value is  $\$MOS2 = 1$ .

Macrovariable  $\$MOS3$  specifies residual smoothing of the conjugate gradient method.

- $\$MOS3 = 0$  - The residual smoothing is suppressed.
- $\$MOS3 = 1$  - A simple one-dimensional residual smoothing is used.

The default value is  $\$MOS3 = 1$ .

If  $\$TYPE = 'G'$ , only the default specifications  $\$DECOMP = 'Z'$  and  $\$NUMBER = '3'$  are possible, which correspond to the trust region conjugate gradient (TRCG) method. Macrovariable  $\$MOS2$  specifies a way for computing the projection step.

- $\$MOS2 = 1$  - The projection step is computed by using the orthogonal projection matrix determined from a range-space basis.
- $\$MOS2 = 2$  - The projection step is computed by using the augmented system determined from a range-space basis.

The default value is  $\$MOS2 = 1$ .

Possible specifications (type-decomposition-number) for recursive quadratic programming methods for sparse equality constrained nonlinear programming problems are these:

- L-K-1,
- L-K-3, L-Z-3, L-G-3,
- L-K-4, L-G-4,
- L-K-5,
- L-K-6,
- G-Z-3.

The default choice is L-K-3. The choices L-K-1, L-G-3, and L-G-4 can be used only if  $\$CLASS = 'MN'$  or  $\$CLASS = 'VM'$  and  $\$HESF = 'S'$  (if a sparsity pattern is available).

### 3.32 Primal-dual interior point methods for sparse nonlinear programming problems

Primal-dual interior point methods for sparse nonlinear programming problems [177] are specified by the statement  $\$FORM = 'SI'$ . These methods, which are intended for large problems, belong to the following classes:

- $\$CLASS = 'VM'$  - Primal-dual interior point variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates.
- $\$CLASS = 'VL'$  - Primal-dual interior point variable metric methods with limited memory based on compact representations of variable metric updates. The number of VM steps is specified by the macrovariable  $\$MF$  (the default value is  $\$MF = 5$ ). Variable metric methods with limited memory use several small-size matrices which are updated in every iteration in such a way that their product approximates the Lagrangian function Hessian matrix as precisely as possible [31].
- $\$CLASS = 'MN'$  - Primal-dual interior point modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically. The sparsity pattern is required.
- $\$CLASS = 'TN'$  - Primal-dual interior point truncated Newton methods. These methods differ from modified Newton methods in that the directional derivatives are determined by the numerical differentiation instead of the sparse Hessian matrix multiplication. The sparsity pattern is not required.

The default value is `$CLASS='MN'`.

If `$CLASS='VM'` and `$HESF='D'`, dense variable metric updates are used. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The DFP method [62], [83] is used.
- `$MET=3` - The Hoshino method [119] is used.
- `$MET=4` - The safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

If `$CLASS='VM'` and `$HESF='S'`, the individual variable metric updates are specified by using the macrovariable `$UPDATE`:

- `$UPDATE='M'` - The simple Marwil projection update [203].
- `$UPDATE='B'` - The partitioned variable metric updates from the Broyden family [102]. The partitioned variable metric updates can only be used if `$MODEL='AF'` or `$MODEL='AQ'` or `$MODEL='AP'`.

The default value is `$UPDATE='B'` if `$MODEL='AF'` or `$MODEL='AQ'` or `$MODEL='AP'` and `$UPDATE='M'` in the opposite case. If `$UPDATE='B'`, the particular variable metric method is specified by using macrovariables `$MET`, `$MET1`, `$MET5`. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The partitioned BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The partitioned DFP method [62], [83] is used.
- `$MET=3` - The partitioned Hoshino method [119] is used.
- `$MET=4` - The partitioned safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

Macrovariable `$MET1` determines scaling of variable metric updates [224].

- `$MET1=1` - No scaling is used.
- `$MET1=2` - The initial scaling [251] is used.
- `$MET1=3` - The controlled scaling [157] is used.
- `$MET1=4` - The interval scaling [180] is used.
- `$MET1=5` - The scaling in each iteration is used.

The default value is `$MET1=3`.

Macrovariable `$MET5` determines subjects of variable metric updates.

- `$MET5=0` - Updates concerns approximating functions.
- `$MET5=2` - Updates concerns approximating functions multiplied by the signs of the Lagrange multipliers.
- `$MET5=3` - Updates concerns terms of the Lagrangian function.

The default value is `$MET5=1`.

If `$CLASS='VL'`, two variable metric updates with limited memory, belonging to the Broyden family, can be used. These updates are specified by using the macrovariable `$MET`.

- `$MET=1` - The BFGS method [22], [78], [97], [247] is used.
- `$MET=4` - The safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

Primal-dual interior point methods for sparse nonlinear programming problems are realized in the way which is specified by using the macrovariable `$TYPE`:

`$TYPE='L'` - Line search methods based on various merit functions or filter structures.  
`$TYPE='G'` - Trust region methods. These methods use two direction determination subproblems [70], [129]. The vertical subproblem, solved by using the dog-leg method, serves for a sufficient decrease of constraint violations. The horizontal subproblem, solved by a special realization of the conjugate gradient method, serves for minimization of a quadratic approximation of a particular merit function.

If `$TYPE='L'`, various penalty functions or filter structures can be used for step-size selection. The corresponding choice is determined by the macrovariable `$MERIT`:

`$MERIT='P'` - A penalty function is used.  
`$MERIT='M'` - The Markov filter [275], [170] is used.  
`$MERIT='F'` - The Fletcher-Leyffer filter [82], [275] is used.  
`$MERIT='B'` - The barrier filter [275] is used.

If `$MERIT='P'`, then macrovariable `$MEP` determines the particular merit function:

`$MEP=0` - No merit function is used. In this case, the line search option `$KTERS=6` is implicitly assumed.  
`$MEP=1` - The augmented logarithmic barrier function is used.  
`$MEP=2` - The augmented Lagrangian function is used.

The default value is `$MEP=0`.

If `$TYPE='G'`, only the default specification `$MERIT='P'` is possible. In this case, macrovariable `$MEP` determines the particular merit function:

`$MEP=1` - The augmented logarithmic barrier function is used.  
`$MEP=2` - The simplified augmented logarithmic barrier function is used.

The default value is `$MEP=1`.

Other important specifications can be determined by using macrovariables `$MEP2`, `$MEP3`, `$MEP4`, `$MEP5` and `$MOS5`. Macrovariable `$MEP2` determines a strategy for computation of primal and dual stepsizes.

`$MEP2=0` - Individual components of primal and dual stepsizes are handled separately.  
`$MEP2=±1` - Primal and dual stepsizes are not greater than their maximum values.  
`$MEP2=±2` - Primal and dual stepsizes are not greater than their maximum values, but the dual stepsize is not greater than the primal stepsize.

If `$MEP2>0`, the basic stepsize does not depend on the primal stepsize. If `$MEP2<0`, the basic stepsize is not greater than the primal stepsize (this is implicitly assumed when `$MEP=1`). The default value is `$MEP2=0`.

Macrovariable `$MEP3` determines a strategy for computation of the barrier parameter.

`$MEP3=1` - The Shanno-Vanderbei formula [274] is used.  
`$MEP3=2` - The El Bakry formula [75] is used.

The default value is `$MEP3=1`.

Macrovariable `$MEP4` determines a regularization strategy if gradients of constraint functions are approximately linearly dependent.

`$MEP4=0` - Regularization is not used.  
`$MEP4=1` - Regularization described in [167] is used.

The default value is \$MEP4=0.

Macrovariable \$MEP5 determines a restart strategy.

- \$MEP5=0 - The restart is suppressed.
- \$MEP5=1 - The uniform descent is assured.

The default value is \$MEP5=0.

Macrovariable \$MOS5 determines an approach to the KKT-equations formulation.

- \$MOS5=1 - The primal formulation is used
- \$MOS5=2 - The primal-dual formulation is used.

The default value is \$MOS5=2.

Macrovariables \$MEP4 and \$MEP5 can be used only if \$TYPE='L'. Moreover the value \$MEP5=1 is significant only if \$MERIT='P' and \$MEP=0.

The direction vector can be computed in the way specified by using the macrovariable \$DECOMP:

- \$DECOMP='I' - The direction vector is determined as a solution of the indefinite Karush-Kuhn-Tucker system [187].

If \$TYPE='L', two realizations are possible, which are specified by the macrovariable \$NUMBER:

- \$NUMBER=1 - An exact sparse Bunch-Parlett (BP) decomposition [74] of the indefinite Karush-Kuhn-Tucker system is used. This choice can be used only if \$CLASS='MN' or \$CLASS='VM' and \$HESF='S' (if a sparsity pattern is available).
- \$NUMBER=3 - An inexact preconditioned conjugate gradient (PCG) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. A realization of the inexact preconditioned conjugate gradient method depends on specifications given by the macrovariables \$MOS1, \$MOS2 and \$MOS3.

The default value is \$NUMBER='3'.

Macrovariable \$MOS1 specifies the precision control and the choice of the penalty parameter.

- \$MOS1=0 - The precision control is suppressed.
- \$MOS1=1 - A precision guaranteeing descent direction is used together with the basic choice of the penalty parameter.
- \$MOS1=2 - A precision guaranteeing descent direction is used together with an extended choice of the penalty parameter.

The default value is \$MOS1=0.

Macrovariable \$MOS2 specifies a preconditioning technique.

- \$MOS2=0 - Preconditioning is suppressed.
- \$MOS2=±1 - The indefinite preconditioner [177] based on a diagonal approximation of the Hessian matrix is used in the normal equation form.
- \$MOS2=2 - The indefinite preconditioner [177] based on a diagonal approximation of the Hessian matrix is used in the augmented system form.
- \$MOS2=±3 - The indefinite preconditioner [167] based on the Gill-Murray decomposition of the approximation of the Hessian matrix is used.

If \$MOS2>0, a complete Gill-Murray decomposition is used. If \$MOS2<0, an incomplete Gill-Murray decomposition is used. The default value is \$MOS2=1.

Macrovariable \$MOS3 specifies residual smoothing of the conjugate gradient method.

- \$MOS3=0 - The residual smoothing is suppressed.
- \$MOS3=0 - A simple one-dimensional residual smoothing is used.

The default value is \$MOS3=0.

If \$TYPE='G', only one realization is possible, which is specified by the macrovariable \$NUMBER:

- \$NUMBER=3 - Trust region method that use two direction determination subproblems [28], [29]. The vertical subproblem, solved by using the dog-leg method, serves for a sufficient decrease of constraint violations. The horizontal subproblem, solved by a special realization of the conjugate gradient method, serves for minimization of a quadratic approximation of a particular merit function. A realization of the inexact preconditioned conjugate gradient method depends on specifications given by the macrovariables \$MOS1, \$MOS2, \$MOS3 and \$MOS4.

Macrovariable \$MOS1 specifies the choice of the penalty parameter.

- \$MOS1=1 - A constant penalty parameter is used.
- \$MOS1=2 - Penalty parameter is determined from the quadratic model.

The default value is \$MOS1=1.

Macrovariable \$MOS2 specifies a preconditioning technique.

- \$MOS2=1 - The preconditioner is computed by using the orthogonal projection matrix determined from a range-space basis.

Macrovariable \$MOS3 determines a decision in the case when a negative curvature is detected.

- \$MOS3=1 - Additional conjugate gradient iterations are performed.
- \$MOS3=2 - A boundary step is determined.

The default value is \$MOS3=1.

Macrovariable \$MOS4 determines a norm for the trust region definition.

- \$MOS4=0 - The trust region is defined by slack variables.
- \$MOS4=1 - The trust region is defined by the Chebyshev  $l_\infty$  norm.
- \$MOS4=2 - The trust region is defined by the Euclidean  $l_2$  norm.

The default value is \$MOS4=2.

Possible specifications (type-decomposition-number) for interior point methods for sparse equality and inequality constrained nonlinear programming problems are these:

L-I-1,  
L-I-3,  
G-I-3.

The default choice is L-I-3. The choice L-I-1 can be used only if \$CLASS='MN' or \$CLASS='VM' and \$HESF='S' (if a sparsity pattern is available).

Primal-dual interior point methods can be used for sparse nonlinear programming problems with various objective functions. If we set \$MODEL='AA' (sum of absolute values) or \$MODEL='AM' (minimax) then an extended nonlinear programming problem containing extra variables is defined and solved.

### 3.33 Nonsmooth equation methods for sparse nonlinear programming problems

Nonsmooth equation methods for sparse nonlinear programming problems [193] are specified by the statement \$FORM='SF'. These methods, which are intended for large problems, belong to the following classes:

- `$CLASS='VM'` - Nonsmooth equation variable metric methods. An approximation of the Lagrangian function Hessian matrix is updated in each iteration by using variable metric updates.
- `$CLASS='VL'` - Nonsmooth equation variable metric methods with limited memory based on compact representations of variable metric updates. The number of VM steps is specified by the macrovariable `$MF` (the default value is `$MF=5`). Variable metric methods with limited memory use several small-size matrices which are updated in every iteration in such a way that their product approximates the Lagrangian function Hessian matrix as precisely as possible [31].
- `$CLASS='MN'` - Nonsmooth equation modified Newton methods. The Lagrangian function Hessian matrix is computed in each iteration either analytically or numerically.
- `$CLASS='TN'` - Nonsmooth equation truncated Newton methods. These methods differ from modified Newton methods in that the directional derivatives are determined by the numerical differentiation instead of the sparse Hessian matrix multiplication (sparsity pattern is not required)

The default value is `$CLASS='MN'`.

If `$CLASS='VM'` and `$HESF='D'`, dense variable metric updates are used. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The DFP method [62], [83] is used.
- `$MET=3` - The Hoshino method [119] is used.
- `$MET=4` - The safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

If `$CLASS='VM'` and `$HESF='S'`, the individual variable metric updates are specified by using the macrovariable `$UPDATE`:

- `$UPDATE='M'` - The simple Marwil projection update [203].
- `$UPDATE='B'` - The partitioned variable metric updates from the Broyden family [102]. The partitioned variable metric updates can only be used if `$MODEL='AF'` or `$MODEL='AQ'` or `$MODEL='AP'`.

The default value is `$UPDATE='B'` if `$MODEL='AF'` or `$MODEL='AQ'` or `$MODEL='AP'` and `$UPDATE='M'` in the opposite case. If `$UPDATE='B'`, the particular variable metric method is specified by using macrovariables `$MET`, `$MET1`, `$MET5`. Macrovariable `$MET` determines the variable metric update.

- `$MET=1` - The partitioned BFGS method [22], [78], [97], [247] is used.
- `$MET=2` - The partitioned DFP method [62], [83] is used.
- `$MET=3` - The partitioned Hoshino method [119] is used.
- `$MET=4` - The partitioned safeguarded rank-one method [153] is used.

The default value is `$MET=1`.

Macrovariable `$MET1` determines scaling of variable metric updates [224].

- `$MET1=1` - No scaling is used.
- `$MET1=2` - The initial scaling [251] is used.
- `$MET1=3` - The controlled scaling [157] is used.
- `$MET1=4` - The interval scaling [180] is used.
- `$MET1=5` - The scaling in each iteration is used.

The default value is `$MET1=3`.

Macrovariable `$MET5` determines subjects of variable metric updates.

- \$MET5=0 - Updates concerns approximating functions.
- \$MET5=2 - Updates concerns approximating functions multiplied by the signs of the Lagrange multipliers.
- \$MET5=3 - Updates concerns terms of the Lagrangian function.

The default value is \$MET5=1.

If \$CLASS='VL', two variable metric updates with limited memory, belonging to the Broyden family, can be used. These updates are specified by using the macrovariable \$MET.

- \$MET=1 - The BFGS method [22], [78], [97], [247] is used.
- \$MET=4 - The safeguarded rank-one method [153] is used.

The default value is \$MET=1.

Nonsmooth equation methods for sparse nonlinear programming problems are realized in the way which are specified by using the macrovariable \$TYPE:

- \$TYPE='L' - Line search methods based on various merit functions or filter structures.

If \$TYPE='L', various penalty functions or filter structures can be used for step-size selection. The corresponding choice is determined by the macrovariable \$MERIT:

- \$MERIT='P' - A penalty function is used.
- \$MERIT='M' - The Markov filter [275], [170] is used.
- \$MERIT='F' - The Fletcher-Leyffer filter [82], [275] is used.

If \$MERIT='P', then macrovariable \$MEP determines the particular merit function:

- \$MEP=0 - No merit function is used. In this case, the line search option \$KTERS=6 is implicitly assumed.
- \$MEP=2 - The augmented Lagrangian function is used together with the residual function.

The default value is \$MEP=2.

Other important specifications can be determined by using macrovariables \$MEP3 and \$MEP4. If \$MERIT='P', then macrovariable \$MEP3 determines a strategy for the use of the residual function.

- \$MEP3=0 - The augmented Lagrangian function is used as the merit function (the residual function is not used).
- \$MEP3=1 - The combination of the augmented Lagrangian function and the residual function is used as the merit function.
- \$MEP3=1 - The residual function is used as the merit function (with the exception of restarted iterations).

The default value is \$MEP3=0.

Macrovariable \$MEP4 determines a regularization strategy if gradients of constraint functions are approximately linearly dependent.

- \$MEP4=0 - Regularization is not used.
- \$MEP4=1 - Regularization described in [167] is used.

The default value is \$MEP4=0.

The direction vector can be computed in the way, which is specified by using the macrovariable \$DECOMP:

- \$DECOMP='I' - The direction vector is determined as a solution of the indefinite Karush-Kuhn-Tucker system [187].

Two realizations are possible, which are specified by the macrovariable \$NUMBER:

- \$NUMBER=1 - An exact sparse Bunch-Parlett (BP) decomposition [74] of the indefinite Karush-Kuhn-Tucker system is used. This choice can be used only if \$CLASS='MN' or \$CLASS='VM' and \$HESF='S' (if a sparsity pattern is available).
- \$NUMBER=3 - An inexact preconditioned conjugate gradient (PCG) method for the indefinite Karush-Kuhn-Tucker system is applied which uses a special determination of the required precision. The particular realization of the inexact preconditioned conjugate gradient method depends on specifications given by the macrovariables \$MOS1, \$MOS2 and \$MOS3.

The default value is \$NUMBER='3'.

Macrovariable \$MOS1 specifies the precision control and the choice of the penalty parameter.

- \$MOS1=0 - The precision control is suppressed.
- \$MOS1=1 - A precision guaranteeing descent direction is used together with the basic choice of the penalty parameter.
- \$MOS1=2 - A precision guaranteeing descent direction is used together with an extended choice of the penalty parameter.

The default value is \$MOS1=0.

Macrovariable \$MOS2 specifies a preconditioning technique.

- \$MOS2=0 - Preconditioning is suppressed.
- \$MOS2=±1 - The indefinite preconditioner [177] based on a diagonal approximation of the Hessian matrix is used in the normal equation form.
- \$MOS2=2 - The indefinite preconditioner [177] based on a diagonal approximation of the Hessian matrix is used in the augmented system form.
- \$MOS2=±3 - The indefinite preconditioner [167] based on the Gill-Murray decomposition of the approximation of the Hessian matrix is used.

If \$MOS2>0, a complete Gill-Murray decomposition is used. If \$MOS2<0, an incomplete Gill-Murray decomposition is used. The default value is \$MOS2=1.

Macrovariable \$MOS3 specifies residual smoothing of the conjugate gradient method.

- \$MOS3=0 - The residual smoothing is suppressed.
- \$MOS3=0 - A simple one-dimensional residual smoothing is used.

The default value is \$MOS3=0.

Possible specifications (type-decomposition-number) for nonsmooth equation methods for sparse equality and inequality constrained nonlinear programming problems are these:

L-F-1,  
L-F-3.

The default choice is L-F-3. The choice L-F-1 can be used only if \$CLASS='MN' or \$CLASS='VM' and \$HESF='S' (if a sparsity pattern is available).

Nonsmooth equation methods can be used for sparse nonlinear programming problems with various objective functions. If we set \$MODEL='AA' (sum of absolute values) or \$MODEL='AM' (minimax) then an extended nonlinear programming problem containing extra variables is defined and solved.

### 3.34 Methods for initial value problems for ordinary differential equations

Methods for initial value problems for ordinary differential equations are specified by using the macrovariable \$SOLVER. The UFO system contains five types of integration methods:



- `$$SOLVER='DP5'` - The Dormand and Prince method of the fifth order with a stepsize control for nonstiff problems.
- `$$SOLVER='DP8'` - The Dormand and Prince method of the eighth order with a stepsize control for nonstiff problems.
- `$$SOLVER='EX1'` - The extrapolation method with a stepsize control, based on the midpoint rule, for nonstiff problems.
- `$$SOLVER='RD5'` - The Radau method of the fifth order with a stepsize control for stiff problems.
- `$$SOLVER='RS4'` - The Rosenbrock method of the fourth order with a stepsize control for stiff problems.

The default value is `$$SOLVER='DP8'`. These methods, described in [105], use a stepsize control based on a local truncation error.

A solution to the initial value problem for ordinary differential equations can be stored for subsequent processing. The extent of the data stored is determined by using the macrovariable `$MED`.

- `$MED=0` - No data are stored.
- `$MED=1` - The data in all solution steps are stored.
- `$MED=2` - The data in equidistant mesh points are stored. The number of mesh points is specified by using the statement `$NA=number_of_mesh_points` in this case.

### 3.35 Methods for direction determination

Optimization methods, contained in the UFO system, are usually implemented in such a way that they use the same modules for direction determination. These modules, realized with different kinds of matrix decomposition, are distinguished by using the macrovariables `$TYPE` and `$NUMBER`. The meaning of the specification `$TYPE` has been explained above. Now we will explain the specification `$NUMBER`.

If `$TYPE='L'`, then line search methods are supposed. In this case, relatively simple procedures are used for direction determination. There are five possibilities:

- `$NUMBER=1` - Direct methods for solving linear systems based on various matrix decompositions. The Gill-Murray decomposition [94] of the Hessian matrix is applied if `$DECOMP='M'` and `$MOS2=0` or if `$DECOMP='G'`. The Schnabel-Eskow decomposition [245] of the Hessian matrix is used if `$DECOMP='M'` and `$MOS2=1` or if `$DECOMP='S'`. The Choleski decomposition of the Hessian matrix is utilized if `$DECOMP='R'` or `$DECOMP='C'`. The Bunch-Parlett decomposition [27] of the Hessian matrix is applied if `$DECOMP='B'`. The inverse matrix is used if `$DECOMP='T'`. The orthogonal QR decomposition [271] of the Jacobian matrix is utilized if `$DECOMP='A'` or `$DECOMP='Q'`. The complete LU decomposition [63] of the Jacobian matrix is applied if `$DECOMP='E'`. Moreover, symbolic decomposition is always determined before the iterative process in the sparse case, so that only numerical computations with known factors are carried out in the subsequent iterations.
- `$NUMBER=2` - An alternative possibility to the previous case. The direct solution is combined with a conjugate gradient direction if the Hessian matrix is indefinite. This possibility can be advantageously used in connection with the modified Newton method.

- `$NUMBER=3` - Inexact iterative methods. The conjugate gradient method [64] for solving linear systems with the Hessian matrix is applied if `$DECOMP='M'`. The CGLS method [226] for solving linear least squares problems with the Jacobian matrix is used if `$DECOMP='A'`. The smoothed CGS method [268] for solving linear systems with the Jacobian matrix is utilized if `$DECOMP='E'`. The precision control is specified by the macrovariable `$MOS`. If `$DECOMP='M'` and `$HESF='S'`, the conjugate gradient method can be preconditioned by using the incomplete Gill-Murray decomposition. This possibility is specified by the macrovariable `$MOS2`. If `$DECOMP='E'` and `$JACA='S'`, the smoothed CGS method can be preconditioned by using either the incomplete LU decomposition or the SSOR preconditioner. This possibility is specified by the macrovariable `$MOS2`.
- `$NUMBER=4` - Inexact iterative methods. The LSQR method [226] for solving linear least squares problems with the Jacobian matrix is applied if `$DECOMP='A'`. The GMRES method [243] for solving linear systems with the Jacobian matrix is used if `$DECOMP='E'`. The precision control is specified by the macrovariable `$MOS`.
- `$NUMBER=5` - Inexact iterative methods. The smoothed BICGSTAB method [273] for solving linear systems with the sparse Jacobian matrix is used if `$DECOMP='E'`. The precision control is specified by the macrovariable `$MOS`.

Macrovariable `$MOS` used in iterative methods has the following meaning.

- `$MOS=1` - A simple strategy is used for precision control.  
`$MOS=2` - The geometric decreasing strategy is used for precision control.  
`$MOS=3` - The harmonic decreasing strategy is used for precision control.

The default value is `$MOS=3`.

If `$DECOMP='M'`, the macrovariable `$MOS2` used in iterative methods has the following meaning.

- `$MOS2=0` - Preconditioning is suppressed.  
`$MOS2=±1` - The incomplete Gill-Murray decomposition (full implementation) is used.  
`$MOS2=±2` - The ILUT type decomposition (the first type) is used.  
`$MOS2=±3` - The ILUT type decomposition (the second type) is used.  
`$MOS2=±4` - The ILUT type decomposition (the third type) is used.  
`$MOS2=±5` - The incomplete Gill-Murray decomposition (simplified implementation) is used.  
`$MOS2=±6` - The SSOR preconditioner is used.

If `$MOS2<0`, only preconditioning is applied. If `$MOS2>0`, preliminary solution obtained by the preconditioner is tested. If it is not sufficiently accurate, preconditioned iterations follow. The default value is `$MOS2=-1`.

If `$DECOMP='E'`, the macrovariable `$MOS2` used in iterative methods has the following meaning.

- `$MOS2=0` - Preconditioning is suppressed.  
`$MOS2=±1` - The incomplete LU decomposition is used.  
`$MOS2=±2` - The SSOR preconditioner is used.

If `$MOS2<0`, only preconditioning is applied. If `$MOS2>0`, preliminary solution obtained by the preconditioner is tested. If it is not sufficiently accurate, preconditioned iterations follow. The default value is `$MOS2=-1`.

If the line search method is used then a descent property of the determined direction is tested. If

$$-s^T g \geq \varepsilon_0 \|s\| \|g\|$$

where  $s^T g$  is the directional derivative,  $s$  is the direction, and  $g$  is the objective function gradient, then the direction is accepted. In the opposite case the optimization method is restarted. The value  $\varepsilon_0$  is specified

by using the macrovariable \$EPS0.

If \$TYPE='G', then trust region methods are supposed. The initial trust region radius can be specified by the statement \$XDEL=trust\_region\_radius, but the default automatically derived value is recommended. The trust region methods can be internally scaled. This way is very advantageous for nonlinear regression problems containing exponentials. The trust region scaling is specified by the macrovariable \$MOS1.

\$MOS1=1           - No scaling is performed.  
\$MOS1=2           - The scaling coefficients are derived from the normal equation matrix diagonal elements [162].

The default value is \$MOS1=1.

There are six trust region methods:

\$NUMBER=1       - Dog-leg methods based on various matrix decompositions. The Gill-Murray decomposition [94] of the Hessian matrix is applied if \$DECOMP='M' and \$MOS2=0 or if \$DECOMP='G'. The Schnabel-Eskow decomposition [245] is used if \$DECOMP='M' and \$MOS2=1 or if \$DECOMP='S'. The Choleski decomposition of the Hessian matrix is utilized if \$DECOMP='R' or \$DECOMP='C'. The Bunch-Parlett decomposition [27] of the Hessian matrix is applied if \$DECOMP='B'. The orthogonal QR decomposition [271] of the Jacobian matrix is utilized if \$DECOMP='A' or \$DECOMP='Q'. The complete LU decomposition [63] of the Jacobian matrix is applied if \$DECOMP='E'. Moreover, symbolic decomposition is always determined before the iterative process in the sparse case, so that only numerical computations with known factors are carried out in the subsequent iterations. The individual dog-leg methods are specified by the macrovariable \$MOS. If \$MOS=1, the single dog-leg method [231] is used. If \$MOS=2, the double dog-leg method [68] is used. If \$MOS=3, the optimum dog-leg method [33] is used. The default value is \$MOS=2.

\$NUMBER=2       - An alternative possibility to the previous case. The multiple dog-leg methods (combinations of dog-leg methods and conjugate gradient methods) [161] are supposed. The number of dog-leg steps is specified by the statement \$MOS=number\_of\_steps.

\$NUMBER=3       - Iterative trust region methods. The conjugate gradient trust region method [260] with the Hessian matrix is applied if \$DECOMP='M'. The CGLS trust region method [158] with the Jacobian matrix is used if \$DECOMP='A'. The smoothed CGS trust region method [183] with the Jacobian matrix is utilized if \$DECOMP='E'. The precision control is specified by the macrovariable \$MOS in the same way as in line search methods. If \$DECOMP='M' and \$HESF='S', the conjugate gradient method can be preconditioned by using the incomplete Gill-Murray decomposition. This possibility is specified by macrovariable \$MOS2 in the same way as in line search methods. If \$DECOMP='E' and \$JACA='S', the smoothed CGS method can be preconditioned by using either the incomplete LU decomposition or the SSOR preconditioner. This possibility is specified by macrovariable \$MOS2 in the same way as in line search methods.

\$NUMBER=4       - Iterative trust region methods. The combined Lanczos and CG trust region method [161] with the Hessian matrix is applied if \$DECOMP='M'. The LSQR trust region method [158] with the Jacobian matrix is used if \$DECOMP='A'. The GMRES trust region method [183] with the Jacobian matrix is utilized if \$DECOMP='E'. The precision control is specified by the macrovariable \$MOS as in the previous case. If \$DECOMP='M' and \$HESF='S', the CG iterations can be preconditioned by using the incomplete Gill-Murray decomposition. This possibility is specified by the macrovariable \$MOS2 in the same way as in line search methods.

- \$NUMBER=5 - Iterative trust region methods. The combined CG and Lanczos trust region method [161] with the Hessian matrix is applied if \$DECOMP='M'. The smoothed BICGSTAB trust region method [183] with the Jacobian matrix is utilized if \$DECOMP='E'. The precision is specified by the macrovariable \$MOS as in the previous case. If \$DECOMP='M' and \$HESF='S', the CG iterations can be preconditioned by using the incomplete Gill-Murray decomposition. This possibility is specified by the macrovariable \$MOS2 in the same way as in line search methods.
- \$NUMBER=6 - Iterative trust region method using the Lanczos process to obtain an approximation to the optimal locally constrained step [101]. The number of Lanczos steps is specified by the macrovariable \$MOS3. The CG iterations value is \$MOS3=100. If \$DECOMP='M' and \$HESF='S', the method can be preconditioned by using the incomplete Gill-Murray decomposition. This possibility is specified by macrovariable \$MOS2 in the same way as in line search methods.
- \$NUMBER=7 - An optimum locally constrained trust region method [210]. The Gill-Murray decomposition [94] of the Hessian matrix is applied if \$DECOMP='M' and \$MOS2=0. The Schnabel-Eskow decomposition [245] of the Hessian matrix is used if \$DECOMP='M' and \$MOS2=1 or if \$DECOMP='S'. The special augmented Jacobian matrix is used if \$DECOMP='A'.
- \$NUMBER=8 - The sequential subspace method [107]. The quadratic function is minimized over a subspace which is adjusted in successive iterations to ensure convergence to an optimum. The subspace contains one step of SQP method [17]. This iterate is obtained by solving a linear system whose preconditioner is specified by the macrovariable \$MOS2. If \$MOS2=1, then the diagonal preconditioning is used, if \$MOS2=2, then the incomplete Gill-Murray decomposition is used, and if \$MOS2=3, then the SSOR preconditioning is used.
- \$NUMBER=9 - The trust-region subproblem is recasted as a parametrized eigenvalue problem [242]. An optimal value for the parameter is computed and the optimal solution of the trust-region subproblem is found from the eigenvectors associated with two of the smallest eigenvalues of the parameterized eigenvalue problem corresponding to the optimal parameter. The eigenpairs are computed using ARPACK Implicitly Restarted Lanczos Method [131]. The number of Lanczos steps is determined by the macrovariable \$MOS3. The default value is \$MOS3=100.

If \$TYPE='T', only specifications \$NUMBER=1, \$NUMBER=2 and \$NUMBER=7 can be used. These specifications have the same meaning as in the case \$TYPE='G', but the implementation is simpler. If \$NUMBER=7, the simplified optimum locally constrained trust region method [162] is used. If \$TYPE='M', only specification \$NUMBER=1 can be used. In this case a modified Marquardt method proposed by Fletcher [77] is applied.

### 3.36 Methods for stepsize selection

Stepsize selection is a very important part of optimization methods. The UFO system contains two types of stepsize selection procedures: line search methods and trust region methods. Line search methods are realized in two modifications specified by the macrovariable \$SEARCH:

- \$SEARCH='B' - Basic line search methods based on various interpolation and extrapolation formulas.
- \$SEARCH='M' - Mixed line search methods which control the maximum stepsize like the trust region methods.

The choice of individual line search procedures is influenced by the order of directional derivatives being used. This order can be specified by the macrovariable \$KDS. The value of the macrovariable \$KDS is usually derived internally from the order of analytically supplied partial derivatives. If this order is zero, then always \$KDS=0. In the opposite case, the value of the macrovariable \$KDS can be specified by

the user. If \$KDS=0, only the function values are used during the line search. If \$KDS=1, the function values and the first directional derivatives are used. If \$KDS=2 then, in addition, the Hessian matrices or their approximations are computed during the line search (this case is very useful for a line search implementation of modified Gauss-Newton methods).

The particular interpolation and extrapolation rule is specified by the macrovariable \$MES. If \$KDS=0, we have the following possibilities:

- \$MES=1 - The uniformly increasing extrapolation or bisection interpolation is used.
- \$MES=2 - Two point quadratic extrapolation or interpolation is used.
- \$MES=3 - Three point quadratic extrapolation or interpolation is used.
- \$MES=4 - Three point cubic extrapolation or interpolation is used.
- \$MES=5 - Special extrapolation or interpolation is used based on the special form of the objective function.

If \$KDS=1 or \$KDS=2, the following possibilities, based on the first directional derivatives, can be used:

- \$MES=1 - The uniformly increasing extrapolation or bisection interpolation is used.
- \$MES=2 - Quadratic extrapolation or interpolation (with one directional derivative) is used.
- \$MES=3 - Quadratic extrapolation or interpolation (with two directional derivatives) is used.
- \$MES=4 - Cubic extrapolation or interpolation [62] is used.
- \$MES=5 - Conic extrapolation or interpolation [13] is used.

More detailed specifications concerning the line search selection can be chosen by using macrovariables \$MES1, \$MES2, \$MES3:

- \$MES1=1 - Constant extrapolation is used.
- \$MES1=2 - Extrapolation specified by the macrovariable \$MES is used.
- \$MES1=3 - Extrapolation is suppressed.
- \$MES2=1 - Standard line search termination criterion is used.
- \$MES2=2 - Special termination criterion for nonconvex functions is used.
- \$MES2=3 - Line search is terminated after at least two function evaluations.
- \$MES3=1 - Safeguard against rounding errors is suppressed.
- \$MES3=2 - The first level of safeguard is used.
- \$MES3=3 - The second level of safeguard is used.

Another useful specification for the line search selection is a termination criterion which is determined by using the macrovariable \$KTERS:

- \$KTERS<0 - The nonmonotone line search procedure proposed in [103] is used. The absolute value of the macrovariable \$KTERS, which cannot be greater than 10, gives the number of nonmonotone steps.
- \$KTERS=1 - Perfect stepsize. The relative precision of the stepsize parameter is given by the value \$EPS3.
- \$KTERS=2 - The Goldstein stepsize [99]. The termination precision is given by the value \$EPS1.
- \$KTERS=3 - The Curry-Altman stepsize [44] (Wolfe conditions). The termination precision is given by the values \$EPS1 and \$EPS2.
- \$KTERS=4 - The extended Curry-Altman stepsize [80] (strict Wolfe conditions). The termination precision is given by the values \$EPS1 and \$EPS2.
- \$KTERS=5 - The Armijo stepsize [9]. The termination is given by the value \$EPS1.
- \$KTERS=6 - The first stepsize. The stepsize selection is terminated after the first function evaluation.

The last useful specification for the line search methods is the initial stepsize choice which is determined by the macrovariable \$INITS. The initial stepsize is usually computed by the rule

$$\alpha = \min(c_1, -c_2(\Delta F/s^T g))$$

where  $s^T g$  is the initial directional derivative and  $\Delta F = F - F_{min}$  or  $\Delta F = F_{old} - F$  if the value of the macrovariable \$INITS is positive or negative respectively. The absolute value of the macrovariable \$INITS determines coefficients  $c_1$  and  $c_2$

- \$INITS=±1 - Values  $c_1 = 1$  and  $c_2 = 0$  are used.
- \$INITS=±2 - Values  $c_1 = 1$  and  $c_2 = 4$  are used.
- \$INITS=±3 - Values  $c_1 = 1$  and  $c_2 = 2$  are used.
- \$INITS=±4 - Values  $c_1 = 0$  and  $c_2 = 2$  are used.

Trust region methods are also realized in two modifications specified by the macrovariable \$SEARCH:

- \$SEARCH='B' - The basic trust region methods with stepsize control based on the comparison of both the actual and the predicted function decreases.
- \$SEARCH='M' - Mixed trust region methods which use interpolation formulas for stepsize reduction like the line search methods [223].

Trust region methods are also influenced by using the macrovariable \$KTERS. If \$KTERS<0, then nonmonotone trust region procedure proposed in [66] is used. The absolute value of the macrovariable \$KTERS, which cannot be greater than 10, gives the number of nonmonotone steps.

### 3.37 Methods for numerical differentiation

The UFO system computes derivatives of the model function (approximating functions, constraint functions) numerically whenever they are not given analytically. This is made possible by the macroprocessor which generates a corresponding part of the UFO source program. The main problem of a numerical differentiation is a difference determination which has to be chosen in such a way that the total influence of both the cancellation and the roundoff error is as small as possible. There are three possibilities in the UFO system which are distinguished by using the macrovariable \$MCG:

- \$MCG=0 - The simple difference determination described in [69] is used.
- \$MCG=1 - The optimum difference determination proposed in [96] is used.
- \$MCG=2 - The optimum difference determination proposed in [261] is used.

The default option is \$MCG=2. The above possibilities are used for a computation of the model function first order derivatives. The others (second order derivatives or derivatives of the approximating functions and constraint functions) are always computed with the simple difference determination.

### 3.38 Methods for objective function evaluation in the case of dynamical systems optimization

If either \$MODEL='DF' or \$MODEL='DQ', the objective function is computed from the solution of an initial value problem for ordinary differential equations. The initial value problem is solved and the integral criterion is evaluated by using integration methods specified by the macrovariable \$SOLVER as is described above. If the partial derivatives of all the functions used are given analytically, the gradient of the objective function is computed by integration methods. There are two possibilities specified by the macrovariable \$SYSTEM:

- \$SYSTEM='F' - Forward integration using an augmented system of ordinary differential equations.
- \$SYSTEM='B' - Backward integration using the adjoint system of ordinary differential equations.

The default value is `$SYSTEM='F'`. In the case of modified Gauss-Newton methods (`$CLASS='GN'`), an approximation of the Hessian matrix is also computed by using forward integration of an augmented system.

### 3.39 Global optimization methods

Global optimization methods are used if `$EXTREM='G'` is specified. The global optimization methods use local optimization methods for finding local minima. Therefore the particular local optimization method has to be chosen by using the macrovariables `$CLASS`, `$TYPE` and others. Individual global optimization methods are specified by using the macrovariables `$GCLASS` and `$GTYPE`. The UFO system contains four classes of global optimization methods:

- `$GCLASS=1` - Random search methods. These methods are simple and robust, but less efficient.
- `$GCLASS=2` - Continuation methods. These methods use some penalty functions which are adjusted after reaching an arbitrary local minimum so that another local minimum is found.
- `$GCLASS=3` - Clustering methods. These methods are based on randomly generated sample points which are processed by using clustering algorithms to determine basins of attraction (clusters) of the individual minima. The basins of attraction (clusters) obtained are not searched repeatedly.
- `$GCLASS=4` - Multi-level methods. Modern stochastic methods which involve a combination of sampling and local search techniques. These methods combine strong theoretical properties with an attractive computational behavior. These methods are simpler but more efficient than the clustering methods.

If `$GCLASS=1`, we can choose four types of global optimization methods:

- `$GTYPE=1` - Single-start methods. Random points, uniformly distributed in a given region, are generated and a local minimization method is started from the point with the lowest function value.
- `$GTYPE=2` - Multi-start methods. Random points, uniformly distributed in a given region, are generated and local minimization is started from every point. The local minima obtained are compared and selected.
- `$GTYPE=3` - Modified multi-start methods. Random points, distributed in a given region uniformly, are generated and local minimization is started whenever a point is found which has a lower function value than that reached up to date.
- `$GTYPE=4` - Bayesian reduced multi-start methods [14]. Random samples of points are repeatedly generated. Every random sample is reduced and local minimization is started from all points belonging to the reduced sample. Obtained local minima are compared and selected. This process is repeated while the Bayesian termination criterion is not satisfied.

If `$GCLASS=2`, then we can choose three types of global optimization methods:

- `$GTYPE=1` - Tunneling function methods [132]. These methods consist of two phases: a local minimization phase and a tunneling phase. The starting point for the second phase is the local minimum. At the end of the tunneling phase a new point is found which has a function value equal or lower than the starting point.
- `$GTYPE=2` - Combined tunneling function and random search methods. In this case a random search is used in the tunneling phase if the minimization of a tunneling function has failed to find a new starting point.

`$GTYPE=3` - Filled function methods [88], [89]. The idea of filled function methods is based on a filled function. This function has a maximum in the point of a known minimum of the objective function. On the other hand, this function does not have minimizers or saddle points in any basin of a higher minimizer of the objective function, but it does have a minimizer or a saddle point in a basin of a lower minimizer of the objective function.

If `$GCLASS=3`, we can choose two types of global optimization methods:

`$GTYPE=1` - Density clustering method [15]. Density clustering refers to a class of clustering techniques by using nonparametric probability density estimates to form clusters. All unclustered points from a reduced sample, which are within the threshold distance from the seed point, are added to the cluster.

`$GTYPE=2` - Single linkage clustering method [15]. In this case, the next two clusters to be merged are those for which the distance between the nearest points is the smallest. When this distance becomes larger than the threshold distance, the procedure is stopped. Starting with each point in a separate cluster, the points at distances smaller than the threshold distance are linked. A cluster is recognized as a set of points linked together.

If `$GCLASS=4`, we can choose three types of global optimization methods:

`$GTYPE=1` - Multi-level single linkage method [241]. In this case, the function values of the sample points are used in a very simple manner to obtain a very powerful method. The local search procedure is applied to every sample point, except if there is another sample point within the critical distance which has a smaller function value. Clusters can be constructed by associating a point with a local minimum, if there exists a chain of points linking it to that minimum. This is done so that the distance between each successive pair is, at most, equal to the critical distance and the function value is decreasing along the chain. A point in this way could be assigned to more than one minimum.

`$GTYPE=2` - Multi-level mode analysis method [241]. This method is a generalization of the mode analysis method. The region is partitioned into cells. After the sample reduction, it is determined which cells contain enough points to be "full". For each full cell the function value of the cell is defined to be equal to the smallest function value of any of the sample points in the cell. Finally, for every full cell, local minimization is applied except if a cell has a neighboring cell which is full and has a smaller function value.

`$GTYPE=3` - Modified multi-level single linkage method. This is a multi level single linkage method with some modifications which are described in [241].

The number of points randomly generated in the given region can be specified by using the macrovariable `$MNRND`. The default value is usually  $100+20*Nf$ . Since it depends on the number of variables and for  $Nf > 20$  is too large, we recommend to use global optimization methods up to 20 variables only. If we use clustering or multi level single linkage methods (`$GCLASS=3` or `$GCLASS=4`), we can specify additional parameters.

`$MNLMIN` - Maximum considered number of local minima. The default value is  $50+20*Nf$ .

`$GAMA` - Reduction of random sample (typically 0.1D0 - 0.2D0). A greater value of `GAMA` usually leads to a greater number of local minima, but it requires a greater amount of work.

`$SIGMA` - Parameter of cluster or single linkage termination (typically 1 - 8).



## 4 Input possibilities in the UFO system

The UFO system has many input possibilities including interactive dialogues. These input possibilities can be divided into three basic groups which are batch mode, text dialogue mode and graphic dialogue mode. Batch and dialogue modes can be combined. The basic means for the batch and combined modes is the UFO control language.

### 4.1 The UFO control language

The form of the UFO source program can be determined by using the statements of the UFO control language. The UFO control language is based on the batch editing language (BEL) [263] described in Appendix B. The UFO control language contains four types of instructions:

1. Standard Fortran 77 instructions which can be written in the free format.
2. Fortran 77 instructions containing macrovariables. These instructions get a final form after the first pass of the UFO preprocessor.
3. Substitutions and directives. These macroinstructions control the UFO preprocessor execution.
4. Special substitutions. These macroinstructions are special tools of the UFO control language that realize the most useful sets of single instructions.

Standard Fortran 77 instructions used in the UFO control language have some extensions and limitations. The main extension is the free format. The instructions need not have a limited length, they can be written everywhere in the input file and if they are written in the same line, the character ';' is used to separate the instructions. The continuation of an instruction is specified by character '& '. The main limitation concerns the application of instructions in the UFO source program. Therefore, statement numbers greater than 9999 cannot be used, comments can be introduced by character '\*' only and the only continuation character can be '& '. At the same time, it is recommended to use identifiers beginning with character 'W' which are not used in the UFO system.

Macrovariables used in the UFO system begin with character '\$' and are supposed to be of the type character. Their values are always in the form of a string of characters which can be sometimes interpreted as an integer or a real or a logical constant. The chief significance of the macrovariables is their use in substituting their values for their names in the Fortran 77 statements. In this case we place the macrovariable (beginning with '\$') in the text, but if it is followed by a letter or a digit we have to use brackets. For example if we write

```
$FLOAT W(100)
```

or

```
CALL UD$HESF$TYPE$DECOMP$NUMBER
```

or

```
X(1)=1.0$(P)0
```

and if the values of \$FLOAT, \$HESF, \$TYPE, \$DECOMP, \$NUMBER and \$P are 'REAL\*8' (this is default), 'D', 'L', 'G', 'I' and 'D' (this is default), we get REAL\*8 W(100) or CALL UDDLGI or X(1)=1.0D0 respectively, after the UFO preprocessor application. The values of macrovariables can be defined and changed by assignments or by special directives as will be shown later.

Substitutions and directives are very important for the UFO control language since they make the substitutions of texts, definitions and changes of macrovariables, branching, loops, etc., possible. We briefly describe the most useful of them. A more detailed description is given in Appendix B.

1. Assignment: The assignment of a string of characters for a macrovariable is specified by the macroinstruction \$MACRO='value'. For example, in order to obtain the result given above, we have to set \$HESF='D', \$TYPE='L', \$DECOMP='G', \$NUMBER=1 (the integers do not need to be substituted as strings).

2. Text insertion: If we write

```
$SET(MACRO)   or   $ADD(MACRO)
    text                text
$ENDSET       $ENDADD
```

then a given text (that can contain a large number of Fortran 77 statements) is inserted into the macrovariable \$MACRO. The macroinstruction \$SET is used for the definition of a new macrovariable. The macroinstruction \$ADD appends a new text into the old macrovariable so that it can be used repeatedly.

3. Logical substitutions: The macrovariables \$INT, \$REAL, \$LOG and \$DEF have logical values. If we write \$INT(MACRO) (or \$REAL(MACRO) or \$LOG(MACRO)), the resulting value is either .TRUE., if the value of the macrovariable \$MACRO is an integer constant (or a real constant or a logical constant), or .FALSE. in the opposite case. If we write \$DEF(MACRO), the value of \$DEF is either .TRUE., if the macrovariable \$MACRO was previously defined (by the substitution \$MACRO='value' or by using macroinstructions \$SET and \$ADD), or .FALSE. in the opposite case. This possibility can be used for branching. If we use the directive \$ERASE(MACRO), the previously defined macrovariable \$MACRO becomes undefined (so that \$DEF(MACRO)=.FALSE.).
4. List of items macrovariables: Values of macrovariables can be lists of items, i.e. they can have a more complicated form \$MACRO='item\_1 \item\_2\...\item\_n' where every item corresponds to one value. The list of items macrovariables use pointers which point out the current items. The current item can be obtained by the substitution \$DATA(MACRO) which also moves the pointer to the next item. The directive \$RESTORE(MACRO) returns the pointer to the first item.
5. Branching: This possibility is very similar to the branching in the Fortran 77 language:

```
$IF(condition)
    statements
$ELSEIF(condition)
    statements
$ELSE
    statements
$ENDIF
```

Conditions can be logical constants .TRUE., .FALSE., or logical macrovariables \$INT(MACRO), \$REAL(MACRO), \$LOG(MACRO), \$DEF(MACRO), or they can have a form of comparisons MACRO=MACRO1, MACRO='value' etc. (besides the relation =, we can also use other relations < or > or <= or >= or <>). Branching is used in the UFO preprocessor stage and has an influence on the form of the UFO source program.

6. Loops: The basic looping directives have the following form (similarly as in the Fortran 77 or Pascal languages):

```
$DO(MACRO=INDEX1,INDEX2,INDEX3)
    statements
$ENDDO
or
$REPEAT
```

```
statements
$UNTIL(condition)
```

For example if we set \$NF=2, \$NC=3 and write

```
$DO(I=1,NF,1)
  $DO(J=1,NC,1)
    CALL $SETCG($I,$J,$I.0D0+$J.0D0)
  $ENDDO
$ENDDO
```

then the UFO preprocessor generates the sequence

```
$SETCG(1,1,1.0D0+1.0D0)
$SETCG(1,2,1.0D0+2.0D0)
$SETCG(1,3,1.0D0+3.0D0)
$SETCG(2,1,2.0D0+1.0D0)
$SETCG(2,2,2.0D0+2.0D0)
$SETCG(2,3,2.0D0+3.0D0)
```

Similarly, if we set \$FLOAT='REAL\*8', \$N=20, \$MACRO='X(\$N)\G(\$N)\H(\$N,\$N)\.END.', and write

```
$REPEAT
  $I='$DATA(MACRO)'
  $IF(I<>'.END.') $FLOAT $I
$UNTIL(I='.END.')
```

then the UFO preprocessor generates the sequence

```
REAL*8 X(20)
REAL*8 G(20)
REAL*8 H(20,20)
```

7. File substitutions: Suppose we have a file with a name file\_name.extension. Then we can include it into the UFO source program by using the macroinstructions

```
$INCLUDE('file_name.extension')
```

or

```
$SUBST('file_name.extension')
```

The main difference between these possibilities is that the directive \$INCLUDE includes a text without change (it has to be a regular Fortran 77 text with a fixed format) while the directive \$SUBST substitutes a text executed consecutively by the UFO preprocessor (so that it can contain the macrovariables and macroinstructions and can be written in the free format). Moreover, the directives \$SUBST can be nested. This possibility is widely used for the UFO source program generation by using nested templates. If the included file has the name file\_name.I, we can use a simpler form without extension. For example, the file UZLINS.I can be substituted by using the macroinstruction \$SUBST('UZLINS').

8. Special substitutions: Besides macroinstructions of the batch editing language BEL, the UFO control language contains special substitutions which realize sets of instructions and are useful for controlling the UFO preprocessor:

\$BATCH	- Switch to the batch mode.
\$DIALOGUE	- Switch to the default dialogue mode (text or graphic).
\$TDIALOGUE	- Switch to the text dialogue mode.
\$GDIALOGUE	- Switch to the graphic dialogue mode.
\$GLOBAL	- Global declarations.
\$INITIATION	- Initiation of the global variables.
\$INPUT	- User supplied input.
\$OUTPUT	- User supplied output.
\$METHOD	- Generation of the optimization method.
\$MODERASE	- Cancellation of the current model and the current system settings.
\$METERASE	- Cancellation of the current method.
\$RUNERASE	- Cancellation of the current method and the current system settings.
\$VARERASE	- Clearing the common variables.
\$TSTART	- Start of the time measurement.
\$TSTOP	- Termination of the time measurement and print of the measured time.
\$END	- End of the optimization block.
\$STANDARD	- Standard optimization block: The macroinstruction \$STANDARD substitutes the sequence of macroinstructions \$GLOBAL, \$INITIATION, \$INPUT, \$TSTART, \$METHOD, \$TSTOP, \$OUTPUT.

9. Comments: Every line beginning with \$REM is a comment line, which is ignored by the macroprocessor.

Moreover \$UYTES1, \$UYTES2, \$UYTES3, \$UOTES4, \$UKMAI1, \$UKMCI1, \$UKMCI2 are simplified substitutions of subroutines UYTES1, UYTES2, UYTES3, UOTES4, UKMAI1, UKMCI1, UKMCI2 respectively and \$SETAG, \$SETCG are simplified calling statements (Sections 2.6 and 2.14).

We have described basic possibilities of the UFO control language that are sufficient for preparing the batch input file. More details are given in subsequent chapters and especially in Appendix B. Usually, the macrovariable \$STANDARD is used if one optimization problem is solved by one optimization method. The following three examples show possibilities how to use various methods for solving various optimization problems. The first example demonstrates the use of the UFO control language for solving one optimization problem by using two different optimization methods: the robust but less efficient pattern-search method (\$CLASS='HM', \$TYPE='P') and the superlinearly convergent variable metric method with numerical computation of gradients (\$CLASS='VM'). The input template has the form:

```

$REM +-----+
$REM +           MODEL DESCRIPTION           +
$REM +-----+
$SET(INPUT)
  X(1)=-1.2D0; X(2)= 1.0D0
$ENDSET
$SET(FMODEL)
  FF=1.0D2*(X(1)**2-X(2))**2+(X(1)-1.0D0)**2
$ENDSET

```

```

$NF=2
$REM : output specifications
$LOUT=2
$MOUT=1
$NOUT=1

$REM +-----+
$REM +           START OF THE PROGRAM GENERATION           +
$REM +-----+
$BATCH; $REM : the batch mode
$GLOBAL; $REM : global declarations
$TSTART; $REM : start of time measurement

$REM +-----+
$REM +           THE FIRST METHOD                             +
$REM +-----+
$REM : parameters of the first method
$XDEL='1.0$P-2'
$TOLB='1.0$P-2'
$MIT=800
$MFV=800
$CLASS='HM'
$TYPE='P'
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation
$METERASE; $REM : clearing parameters of the first method

$REM +-----+
$REM +           THE SECOND METHOD                             +
$REM +-----+
$REM : parameters of the second method
$HESF='D'
$TOLX='1.0$P-16'
$TOLF='1.0$P-16'
$TOLG='1.0$P-8'
$TOLB='1.0$P-16'
$MIT=800
$MFV=800
$CLASS='VM'
$TYPE='L'
$DECOMP='I'
$NUMBER=1
$METHOD; $REM : method specification and generation

$TSTOP; $REM : print of the CPU time
$END; $REM : end of the program generation

```

The text output file follows:

```

CLASS = HM - PN1   UPDATE = N   MODEL = FF   HESF = N   NF =       2
  0 NIT=   53 NFV=  222 NDC=    0 FV BOUND F= 0.5266202783E-02
FF = 0.5266202783D-02
X = 0.9322729460D+00 0.8717390863D+00
CLASS = VM - LI1   UPDATE = B   MODEL = FF   HESF = D   NF =       2
  0 NIT=   12 NFV=   42 NFG=    0 FV BOUND F= 0.1936407754E-20 G=0.607D-09
FF = 0.1936407754D-20
X = 0.1000000000D+01 0.1000000000D+01
TIME= 0:00:00.00

```

The second example demonstrates the use of the UFO control language for solving three similar optimization problems by the same optimization method. All problems concern nonlinear approximation (sum of values and sum of squares) with the same number of variables. The input template has the form:

```

$REM +-----+
$REM +          GLOBAL PARAMETERS          +
$REM +-----+
$NF=100
$NA=500
$MA=2000
$M=9000
$REM : output specifications
$LOUT=2
$MOUT=1
$NOUT=0

$REM +-----+
$REM +          METHOD SELECTION          +
$REM +-----+
$REM : parameters of the method
$TOLX='1.0$P-10'
$TOLF='1.0$P-15'
$TOLG='1.0$P-5'
$JACA='S'
$HESF='S'
$MIT=800
$MFV=1200
$CLASS='VM'
$TYPE='L'
$DECOMP='M'
$NUMBER=3
$UPDATE='B'
$MOS2=0
$NEXT=1

$REM +-----+
$REM +          START OF THE PROGRAM GENERATION          +
$REM +-----+
$BATCH; $REM : the batch mode $GLOBAL; $REM : global declarations

```

```

$REM +-----+
$REM +           THE FIRST MODEL           +
$REM +-----+
$MODEL='AF'
$SET(INPUT)
  CALL EIUB14(NF,NA,MA,X,IAG,JAG,FMIN,XMAX,NEXT,IEXT,IERR)
$ENDSET
$SET(FMODELA)
  CALL EAFU14(NF,KA,X,FA,NEXT)
$ENDSET
$SET(GMODELA)
  CALL EAGU14(NF,KA,X,GA,NEXT)
$ENDSET
$COLLECTION='Y'; $REM : cycle for testing
$NEXT=3; $REM : number of steps in the cycle
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation
$MODERASE; $REM : clearing parameters of the first model

$REM +-----+
$REM +           THE SECOND MODEL          +
$REM +-----+
$MODEL='AQ'
$SET(INPUT)
  CALL EIUB15(NF,NA,MA,X,IAG,JAG,FMIN,XMAX,NEXT,IEXT,IERR)
$ENDSET
$SET(FMODELA)
  CALL EAFU15(NF,KA,X,FA,NEXT)
$ENDSET
$SET(GMODELA)
  CALL EAGU15(NF,KA,X,GA,NEXT)
$ENDSET
$NEXT=1; $REM : THE PROBLEM NUMBER
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
$TSTART; $REM : start of time measurement
$METHOD; $REM : method specification and generation
$TSTOP; $REM : print of the CPU time
$MODERASE; $REM : clearing parameters of the second model

$REM +-----+
$REM +           THE THIRD MODEL           +
$REM +-----+
$SET(INPUT)
  CALL EIUB18(NF,NA,MA,X,IAG,JAG,FMIN,XMAX,NEXT,IEXT,IERR)
$ENDSET
$SET(FMODELA)
  CALL EAFU18(NF,KA,X,FA,NEXT)
$ENDSET

```

```

$SET(GMODELA)
  CALL EAGU18(NF,KA,X,GA,NEXT)
$ENDSET
$COLLECTION='Y'; $REM : cycle for testing
$NEXT=5; $REM : number of steps in the cycle
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation

$END; $REM : end of the program generation

```

The text output file follows:

```

CLASS = VM - LM3  UPDATE = B  MODEL = AF  HESF = S  NF = 100
  1 NIT= 303 NFV= 364 NFG= 364 GRAD TOL F= 0.3916196246E-14 G=0.229D-05
  2 NIT= 104 NFV= 168 NFG= 168 GRAD TOL F= 10.69009184 G=0.993D-06
  3 NIT= 35 NFV= 38 NFG= 38 GRAD TOL F= 0.5708941253E-10 G=0.567D-05
TOTAL  NIT= 442 NFV= 570 NFG= 570 NDC= 0 * 3
      NCG= 9078 NLS= 125 NSC= 0 NUP= 25779
      NRS= 3 NMX= 0 NAD= 0 NRM= 0

TIME= 0:00:00.09
CLASS = VM - LM3  UPDATE = B  MODEL = AQ  HESF = S  NF = 100
  1 NIT= 306 NFV= 389 NFG= 389 GRAD TOL F= 0.5452508290E-14 G=0.161D-05
TIME= 0:00:00.08
CLASS = VM - LM3  UPDATE = B  MODEL = AQ  HESF = S  NF = 100
  1 NIT= 38 NFV= 40 NFG= 40 GRAD TOL F= 0.3359765546E-10 G=0.268D-05
  2 NIT= 34 NFV= 36 NFG= 36 GRAD TOL F= 0.4552663103E-12 G=0.919D-06
  3 NIT= 4 NFV= 5 NFG= 5 GRAD TOL F= 0.8693979881E-14 G=0.375D-07
  4 NIT= 12 NFV= 16 NFG= 16 GRAD TOL F= 7.086281648 G=0.655D-06
  5 NIT= 69 NFV= 132 NFG= 132 GRAD TOL F= 320.3198802 G=0.940D-05
TOTAL  NIT= 157 NFV= 229 NFG= 229 NDC= 0 * 5
      NCG= 2517 NLS= 67 NSC= 0 NUP= 8284
      NRS= 5 NMX= 0 NAD= 0 NRM= 0

```

The third example demonstrates the use of the UFO control language for solving one optimization problem by using two different models: general objective function and sum of squares. Three optimization methods: the robust but less efficient heuristic pattern-search method (\$CLASS='HM', \$TYPE='P'), the variable metric method with numerical computation of gradients (\$CLASS='VM') and the modified Gauss-Newton method with numerical computation of Jacobian matrices (\$CLASS='GN', \$UPDATE='F') are used. The input template has the form:

```

$REM +-----+
$REM +          GLOBAL PARAMETERS          +
$REM +-----+
$REM : declaration of the user variables
$FLOAT W,WA,WB,WC,WM
$REM : output specifications
$LOUT=2
$MOUT=1
$NOUT=1

```



```

$REM +-----+
$REM +          START OF THE PROGRAM GENERATION          +
$REM +-----+
$BATCH; $REM : the batch mode
$GLOBAL; $REM : global declarations

$REM +-----+
$REM +          THE FIRST MODEL                          +
$REM +-----+
$SET(INPUT)
  X(1)=ONE ; X(2)=TWO ; X(3)=ONE
  X(4)=ONE ; X(5)=ONE ; X(6)=ONE
  XMAX=TEN
  FMIN=ZERO
$ENDSET
$SET(FMODEL)
  FF=ZERO
  DO 2 KA=1,20
    W=$DBLE(KA)/TEN
    WA=EXP(-W*X(1))
    WB=EXP(-W*X(2))
    WC=EXP(-W*X(3))
    WM=EXP(-W)-FIVE*EXP(-TEN*W)+THREE*EXP(-FOUR*W)
    FF=FF+(X(4)*WA-X(5)*WB+X(6)*WC-WM)**2
  2 CONTINUE
  FF=HALF*FF
$ENDSET
$NF=6

$REM +-----+
$REM +          THE FIRST METHOD                          +
$REM +-----+
$XDEL='1.0$P-2'
$TOLB='1.0$P-1'
$MIT=800
$MFV=8000
$CLASS='HM'
$TYPE='P'
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation
$METERASE; $REM : clearing parameters of the first method

$REM +-----+
$REM +          THE SECOND METHOD                          +
$REM +-----+
$TOLX='1.0$P-8'
$TOLF='1.0$P-8'
$TOLG='1.0$P-2'
$TOLB='1.0$P-2'
$MIT=800
$MFV=8000

```

```

$HESF='D'
$CLASS='VM'
$TYPE='L'
$DECOMP='I'
$NUMBER=1
$METHOD; $REM : method specification and generation
$METERASE; $REM : clearing parameters of the second method
$MODERASE; $REM : clearing parameters of the first model

$REM +-----+
$REM +           THE SECOND MODEL           +
$REM +-----+
$SET(INPUT)
  DO 3 KA=1,NA
    W=$DBLE(KA)/TEN
    AM(KA)=EXP(-W)-FIVE*EXP(-TEN*W)+THREE*EXP(-FOUR*W)
  3 CONTINUE
$ENDSET
$SET(FMODEL)
  W=$DBLE(KA)/TEN
  WA=EXP(-W*X(1))
  WB=EXP(-W*X(2))
  WC=EXP(-W*X(3))
  FA=X(4)*WA-X(5)*WB+X(6)*WC
$ENDSET
$NA=20
$NAL=0
$KBA=1
$MODEL='AQ'

$REM +-----+
$REM +           THE THIRD METHOD           +
$REM +-----+
$TOLX='1.0$P-16'
$TOLF='1.0$P-16'
$TOLG='1.0$P-8'
$TOLB='1.0$P-16'
$MIT=800
$MFV=8000
$CLASS='GN'
$TYPE='G'
$DECOMP='M'
$NUMBER=7
$UPDATE='F'
$JACA='D'
$HESF='D'
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation

$END; $REM : end of the program generation

```

The text output file follows:

```

CLASS = HM - PN1   UPDATE = N   MODEL = FF   HESF = N   NF =      6
      0 NIT=   18 NRV=  176 NDC=    0 FV BOUND F= 0.9758203329E-01
FF = 0.9758203329D-01
X  = 0.1577350877D+01 0.3818216079D+01 0.1711759499D+01 0.1324516197D+01
      0.1282806727D+01 0.1275838294D+01
CLASS = VM - LI1   UPDATE = B   MODEL = FF   HESF = D   NF =      6
      0 NIT=    9 NRV=   72 NFG=    0 GRAD TOL F= 0.5384948810E-01 G=0.740D-02
FF = 0.5384948810D-01
X  = 0.2184135468D+01 0.5676842761D+01 0.1972958227D+01 0.1753256141D+01
      0.2957973274D+01 0.1724924127D+01
CLASS = GN - GM7   UPDATE = F   MODEL = AQ   HESF = D   NF =      6
      0 NIT=   14 NRV=  105 NFG=    0 FV BOUND F= 0.3963563826E-30 G=0.171D-14
F  = 0.3963563826D-30
X  = 0.4000000000D+01 0.1000000000D+02 0.1000000000D+01 0.3000000000D+01
      0.5000000000D+01 0.1000000000D+01

```

## 4.2 The batch mode

A switch to the batch mode is realized by using the special substitution \$BATCH. If we want to process either the batch mode or the mixed mode we have to prepare a batch input file written in the UFO control language. This input file prescribes the structure of the UFO source program. If a macrovariable is used, it has to be the one defined previously. Therefore definitions of macrovariables usually lie at the beginning of the input file. Many macrovariables serve for defining a given optimization problem. The most important among them are the macrovariable \$INPUT which determines initial input values (user supplied input) and macrovariables which define problem functions, specifically the model (or objective) function, approximating functions for nonlinear approximation, constraint functions for nonlinear programming, state functions, initial functions and the terminal function for optimization of dynamical systems. These functions are specified by using special macrovariables whose names consist of three parts. The first part can contain letters F, G, D, H or their combinations:

F        - Function value.  
G        - Gradient with respect to basic variables.  
D        - Gradient with respect to state variables.  
H        - Hessian matrix with respect to basic variables.  
FG       - Function value and gradient with respect to basic variables.  
FD       - Function value and gradient with respect to state variables.  
GD       - Gradient with respect to basic variables and gradient with respect to state variables.  
FGD      - Function value, gradient with respect to basic variables and gradient with respect to state variables.  
FGH      - Function value, gradient with respect to basic variables and Hessian matrix with respect to basic variables.

The second part always has the form MODEL. The third part can contain letters F, A, C, E, Y and also an additional letter S:

F        - The model function or the terminal function.  
A        - The selected approximating function.  
AS       - All approximating functions.  
C        - The selected constraint function.  
CS       - All constraint functions.  
E        - The selected state function.  
ES       - All state functions.

Y - The selected initial function.  
 YS - All initial functions.

The following combinations are possible:

\$FMODEL	\$FMODEL	\$FMODEL	\$FMODEL	\$FMODEL
	LA	LC	LE	LY
	LA	LC	LE	LY
\$GMODEL	\$GMODEL	\$GMODEL	\$GMODEL	\$GMODEL
	LA	LC	LE	LY
	LA	LC	LE	LY
\$DMODEL	\$DMODEL		\$DMODEL	
	LA		LE	
	LA		LE	
\$HMODEL	\$HMODEL	\$HMODEL		
	LA	LC		
	LA	LC		
\$FGMODEL	\$FGMODEL	\$FGMODEL	\$FGMODEL	\$FGMODEL
	LA	LC	LE	LY
	LA	LC	LE	LY
\$FDMODEL	\$FDMODEL		\$FDMODEL	
	LA		LE	
	LA		LE	
\$GDMODEL	\$GDMODEL		\$GDMODEL	
	LA		LE	
	LA		LE	
\$FGDMODEL	\$FGDMODEL		\$FGDMODEL	
	LA		LE	
	LA		LE	
\$FGHMODEL	\$FGHMODEL	\$FGHMODEL		
	LA	LC		
	LA	LC		

The choice of a suitable way for problem function definitions is ambiguous and problem dependent. We can only give several remarks:

1. The basic and most general way is the use of different macrovariables for different quantities (values, gradients, Hessian matrices) together with an independent evaluation of individual functions (the last letter is different from S). This way saves the computer storage and frequently also the computational time.
2. Sometimes, evaluations of gradients require function values. In this case, it can be advantageous to compute values and gradients simultaneously. A similar consideration also holds for Hessian matrices.
3. Even if simultaneous evaluations of all approximating (constraint, state, initial) functions increase storage requirements, it can be advantageous if there are complicated computations common for all such functions, and also if a problem has a low dimension or a sparse structure. It is frequently advantageous for the evaluation of state and initial functions when the dynamical systems are optimized.
4. If the gradients of approximating (constraint, state, initial) functions are computed simultaneously (the last letter is equal to S), then also function values have to be computed simultaneously. Similarly if the Hessian matrices are computed simultaneously, then also function values and gradients have to be computed simultaneously.

A simple example of a batch input file was shown in Section 1.2. We repeat it here with some explanations:

```

$REM : model specifications
$SET(INPUT)
  X(1)=-1.2D0; X(2)= 1.0D0
$ENDSET
$SET(FMODEL)
  FF=1.0D2*(X(1)**2-X(2))**2+(X(1)-1.0D0)**2
$ENDSET

```

```

$NF=2
$REM : the default method is used
$REM : print specifications
$MOUT=1
$NOUT=1
$REM : the batch mode is used
$BATCH
$REM : the standard form of the source program is used
$STANDARD

```

By using the macrovariable \$INPUT, we specify the initial values of variables  $x_1 = -1.2$  and  $x_2 = 1.0$ . By using the macrovariable \$FMODEL, we specify the model function value (the model function gradient is not specified, it will be computed numerically). The macrovariable \$NF defines the number of variables and \$MOUT, \$NOUT are print specifications. The macroinstruction \$BATCH switches the mode to the batch mode. The macroinstruction \$STANDARD defines the standard form of the UFO source program. Descriptions of more complicated problems are shown in Chapter 7.

In the above example, a direct definition of a model function value is used. We can also use indirect specifications by means of the Fortran 77 subroutines or the files prepared beforehand. Suppose that the model function value is defined by using the subroutine EFFU01 or is specified in the file FVAL.FOR. Then we can write:

```

$SET(FMODEL)
  CALL EFFU01(NF,X,FF,NEXT)
$ENDSET

```

or

```

$SET(FMODEL)
  $INCLUDE('FVAL.FOR')
$ENDSET

```

or

```

$SET(FMODEL)
  $SUBST('FVAL.FOR')
$ENDSET

```

The last possibility is useful if the model function value specification is written in a free format or it contains the BEL macroinstructions.

If we need to utilize user supplied subroutines, we can include them into the UFO source program by using the macrovariable \$SUBROUTINES:

```

$SET(SUBROUTINES)
  user supplied subroutines
$ENDSET

```

In this case, some exceptions laid on the text of user supplied subroutines forced by the UFO preprocessor have to be satisfied. All comments have to begin with character '\*', the continuation line has to begin with character '&', character '\$' has to be replaced by '\$\$' and character ';' does not have to be present.

The batch input file should also contain optimization method selection. Fortunately, this selection is not critical since the optimization method can be chosen automatically by using knowledge bases contained in the UFO system templates. Here we will only demonstrate some possibilities. The greatest influence on the optimization method selection have the following macrovariables:

```

$CLASS          - Class of optimization methods (heuristic, conjugate gradient, variable metric, variable metric with limited storage, modified Newton, truncated Newton, Gauss-Newton, quasi-Newton, quasi-Newton with limited storage, proximal bundle, bundle-Newton).
$TYPE           - Type of optimization methods (line search, trust region, SQP filter).
$DECOMP         - Type of matrix decomposition (original matrix, Choleski decomposition, inversion).

```

\$NUMBER - Individual methods for direction determination (various direct, various iterative).  
\$UPDATE - Type of variable metric or quasi-Newton update.

A more detailed description of these choices together with other choices (\$MET, \$MET1, \$MET2, \$MET3, \$MES, \$MES1, \$MES2, \$MES3, \$MOS, \$MOS1, \$MOS2, \$MOS3) is given in Section 3.

### 4.3 The text dialogue mode

A switch to the text dialogue mode is realized by using the special substitution \$TDIALOGUE. This is equivalent to the substitution \$DIALOGUE in the UNIX version of the UFO system. If this is the case, a sequence of questions appear on the screen in the text form. Each question, which is placed in its own frame, consists of the macrovariable description usually followed by the list of its possible values. The name of a macrovariable together with its default value is written on the top of the frame. We have two possibilities for an answer. First, the required value can be entered from the keyboard. Secondly, we can press ENTER to choose the default value. After the assignment of a value to the macrovariable, a new question immediately appears on the screen until the last one is exhausted. The dialogue mode can be terminated by entering character '!' from the keyboard. We demonstrate four questions as an example:

\_\_\_\_\_ ? INPUT ( ) ? \_\_\_\_\_

USER SUPPLIED INPUT:

HERE THE STARTING POINT, BOUNDS FOR VARIABLES,  
TYPES OF CONSTRAINTS, THE STRUCTURE OF SPARSE PROBLEM,  
AND OTHER INPUT DATA HAVE TO BE SPECIFIED.

Here a user supplied input is expected. This is a text which should be entered from the keyboard.

\_\_\_\_\_ ? MODEL (FF) ? \_\_\_\_\_

TYPE OF OBJECTIVE FUNCTION

FF - GENERAL FUNCTION

FL - LINEAR FUNCTION

FQ - QUADRATIC FUNCTION

AF - SUM OF FUNCTIONS

AQ - SUM OF SQUARES

AP - SUM OF POWERS

AM - MINIMAX

DF - DIFFERENTIAL SYSTEM WITH GENERAL INTEGRAL CRITERION

DQ - DIFFERENTIAL SYSTEM WITH INTEGRAL OF SQUARES

NO - MODEL IS NOT SPECIFIED

Here an optimization model, i.e. a type of the objective function, is chosen. We have 10 possibilities, FF, FL, FQ, AF, AQ, AP, AM, DF, DQ, NO. The default value of the macrovariable \$MODEL corresponding to the general objective function is FF. By pressing ENTER, the default value FF is accepted.

\_\_\_\_\_ ? NF (0) ? \_\_\_\_\_

NUMBER OF VARIABLES

Here the number of variables is expected. This is a positive integer. No default value is offered, i.e. we have to set a value. If this value is not a positive integer, the answer is ignored and the same question appears on the screen.

```
_____ ? FMIN (-1.0D 60) ? _____  
LOWER BOUND FOR FUNCTION VALUE
```

Here a real constant is expected. By pressing ENTER the default value -1.0D 60 is accepted.

More details concerning a text dialogue mode are given in Appendix A, where a complete text dialogue concerning unconstrained minimization of the Rosenbrock function is shown.

#### 4.4 The graphic dialogue mode

The graphic dialogue mode can be used only on PC computers under the MS DOS system. This possibility is not allowed on the UNIX workstations. A switch to the graphic dialogue mode is realized by using the special substitution \$GDIALOGUE. This is an equivalent to the substitution \$DIALOGUE in the PC version of the UFO system. If this is the case, a sequence of screens follows. Each screen realizes one question which is in fact the same as that in the text dialogue mode. Nevertheless, the graph dialogue mode has several advantages over the text one:

1. Information is better arranged on the screen.
2. The window for typing answers is in fact a simple editor. Therefore the text can be easily corrected and a movement controlled by arrows is possible.
3. Application of the special UFO editor is possible for realizing more complicated answers. The UFO editor works with multiple windows so that an answer can be set up from several sources. Therefore a convenient utility of the batch mode can also be used in the dialogue mode.

To compare text and graphic dialogue modes, we again demonstrate the above four questions:

```
USER SUPPLIED INPUT:  
  
HERE THE STARTING POINT, BOUNDS FOR VARIABLES,  
TYPES OF CONSTRAINTS, THE STRUCTURE OF SPARSE PROBLEM,  
AND OTHER INPUT DATA HAVE TO BE SPECIFIED.  
  
Type string for INPUT:  
  
E - using the UFO EDITOR (ALT+5 returns to dialogue)  
! - end of dialogue
```

Here a user supplied input is expected. This is a text which should be written into the window displayed on the screen (followed by pressing ENTER). If this text is more complicated, we can use the UFO editor by typing character 'E' and pressing ENTER. The return from the UFO editor to the graphic dialogue is realized by pressing <alt-5> (Section 1.3). The dialogue mode can be terminated by typing character '!' and pressing ENTER.

**TYPE OF OBJECTIVE FUNCTION**  
FF - GENERAL FUNCTION  
FL - LINEAR FUNCTION  
FQ - QUADRATIC FUNCTION  
AF - SUM OF FUNCTIONS  
AQ - SUM OF SQUARES  
AP - SUM OF POWERS  
AM - MINIMAX  
DF - DIFFERENTIAL SYSTEM WITH INTEGRAL CRITERION  
DQ - DIFFERENTIAL SYSTEM WITH INTEGRAL OF SQUARES  
NO - MODEL IS NOT SPECIFIED

Type your choice for MODEL: **FF**

or press ENTER for default

! - end of dialogue

Here an optimization model, i.e. a type of the objective function, is chosen. We have 10 possibilities, FF, FL, FQ, AF, AQ, AP, AM, DF, DQ, NO. The default value of the macrovariable \$MODEL, corresponding to the general objective function, is FF. By pressing ENTER, the default value FF is accepted. If we want to choose a different possibility, it has to be written into the two-character window, followed by pressing ENTER. The dialogue mode can be terminated by typing character '!' and pressing ENTER.

**NUMBER OF VARIABLES**

Type integer for NF:

Positive value is required - no default

! - end of dialogue

Here the number of variables is expected. This is a positive integer. No default value is offered, i.e. we have to enter a value. If this value is not a positive integer, the answer is ignored and another answer is expected. The dialogue mode can be terminated by typing character '!' and pressing ENTER.



**LOWER BOUND FOR FUNCTION VALUE**

Type real for FMIN: -1.0D 60

or press ENTER for default

! - end of dialogue

Here a real constant is expected. By pressing ENTER the default value -1.0D 60 is accepted. If we want to choose a different value, it has to be written into the twenty-character window, followed by pressing ENTER. The dialogue mode can be terminated by typing character '!' and pressing ENTER.

## 5 Output possibilities in the UFO system

The UFO system has many output possibilities including graphic pictures. These output possibilities can be divided into five basic groups.

### 5.1 Basic screen output

The basic screen output can be used only if  $\$GRAPH='N'$  and  $\$DISPLAY='N'$ . In this case, individual rows corresponding to the iterations and the final results are printed on the screen consequently. A print level of the screen output is determined by using the macrovariables  $\$MOUT$  and  $\$NOUT$ . The macrovariable  $\$MOUT$  can have the following values:

- $\$MOUT= 0$  - Screen output is suppressed.
- $\$MOUT=\pm 1$  - Standard output. The final results appear on the screen.
- $\$MOUT=\pm 2$  - Extended output. Additional information from every iteration appears on the screen.
- $\$MOUT=\pm 3$  - Extended output. Additional final results of linear or quadratic programming subproblems appear on the screen.
- $\$MOUT=\pm 4$  - Extended output. Additional information from every iteration of linear or quadratic programming subproblems appears on the screen.

If  $\$MOUT>0$ , printed results have the standard form while if  $\$MOUT<0$ , additional information containing various method specifications and optimization options is printed.

The macrovariable  $\$NOUT$  can have the following values:

- $\$NOUT= 0$  - Short final results (scalar variables) appear on the screen.
- $\$NOUT= 1$  - Extended final results (vectors) appear on the screen.

The basic screen output can be copied into file `P.OUT` (text file output) if  $\$KOUT=0$  and  $\$LOUT>0$  (see Section 5.4. In this case, macrovariable  $\$LOUT$  can have the following values:

- $\$LOUT= 0$  - Basic screen output is not copied into text file `P.OUT`.
- $\$LOUT= 1$  - Basic screen output is copied into text file `P.OUT`. Standard line of the final results is printed.
- $\$LOUT= 2$  - Basic screen output is copied into text file `P.OUT`. Modified line of the final results, containing the termination criterion, is printed.

Typical basic screen outputs are shown in Section 7. If we use values  $\$LOUT=2$ ,  $\$MOUT=-2$ ,  $\$NOUT=2$  in the problem specification introduced in Section 7.1, then text file `P.OUT` contains the following information.

```
CLASS = VM - LG1   UPDATE = B   MODEL = FF   HESF = D   NF =      5
  NIT=   0 NFV=   1 NFG=   1 F=  1.866666667   G=0.667D-01
  NIT=   1 NFV=   4 NFG=   4 F=  1.550000000   G=0.150D+00
  NIT=   2 NFV=   7 NFG=   7 F=  1.200000000   G=0.200D+00
  NIT=   3 NFV=   9 NFG=   9 F=  1.000000000   G=0.000D+00
0 NIT=   3 NFV=   9 NFG=   9 GRAD TOL F=  1.000000000   G=0.000D+00
FF = -0.1000000000D+01
X  =  0.1000000000D+01  0.2000000000D+01  0.3000000000D+01  0.4000000000D+01
      0.5000000000D+01
TIME= 0:00:00.00

OPTIONS:
```

```

MET   = 8   MET1  = 3   MET2  = 2   MET3  = 1   MET4  = 3   MET5  = 0
MOT   = 2   MOT1  = 0   MOT2  = 0   MOT3  = 0   MOT4  = 0   MOT5  = 0
MES   = 4   MES1  = 2   MES2  = 2   MES3  = 2   MES4  = 0   MES5  = 0
MOS   = 0   MOS1  = 0   MOS2  = 0   MOS3  = 0   MOS4  = 0   MOS5  = 0
MEP   = 0   MEP1  = 0   MEP2  = 0   MEP3  = 0   MEP4  = 0   MEP5  = 0
MED   = 0   MED1  = 0   MED2  = 0   MED3  = 0   MED4  = 0   MED5  = 0
MEG   = 0   MEG1  = 0   MEX   = 0   MEX1  = 0   MEQ   = 0   MEQ1  = 0
MFP   = 0   MFP1  = 0   MLP   = 0   MLP1  = 0   MQP   = 0   MQP1  = 0
KTERS = 3   INITD = 1   INITS = 2   INITH = 1   IREM  = 0   IADD  = 0

MIT   =    500   MIC   =    0   MRED  =    10   MF    =    0
MFV   =   1000   MCG   =    0   IRES1 =   999   MA    =    0
MFG   =  10000   MSTP  =    0   IRES2 =    0   MB    =    0

TOLX = 0.100D-11   EPS0 = 0.100D-05   ETA0 = 0.100D-14   ALF1 = 0.100D-09
TOLF = 0.100D-13   EPS1 = 0.100D-03   ETA1 = 0.100D-12   ALF2 = 0.100D+11
TOLG = 0.100D-05   EPS2 = 0.900D+00   ETA2 = 0.100D-14   ALF3 = 0.000D+00
TOLB = -0.100D+61  EPS3 = 0.000D+00   ETA3 = 0.000D+00   BET1 = 0.000D+00
TOLC = 0.000D+00   EPS4 = 0.000D+00   ETA4 = 0.000D+00   BET2 = 0.000D+00
TOLA = 0.000D+00   EPS5 = 0.000D+00   ETA5 = 0.000D+00   BET3 = 0.000D+00
TOLR = 0.000D+00   EPS6 = 0.000D+00   ETA6 = 0.000D+00   GAM1 = 0.000D+00
XDEL = 0.000D+00   EPS7 = 0.000D+00   ETA7 = 0.000D+00   GAM2 = 0.000D+00
XMAX = 0.100D+04   EPS8 = 0.500D+00   ETA8 = 0.000D+00   GAM3 = 0.000D+00
RPF1 = 0.000D+00   EPS9 = 0.100D-07   ETA9 = 0.100D+61   DEL1 = 0.000D+00
RPF2 = 0.000D+00   FMIN = -0.100D+61  FMAX = 0.100D+21   DEL1 = 0.000D+00
RPF3 = 0.000D+00                                     DEL3 = 0.000D+00

```

The first line contains specifications of the method used (\$CLASS='VM', \$TYPE='L', \$DECOMP='G', \$NUMBER=1, \$UPDATE='B') and the problem characteristics (\$MODEL='FF', \$HESF='D', \$NF=5). Then information concerning individual iterations and the final results are printed (0 corresponds to \$NEXT=0 and GRAD TOL is the cause of termination). Since \$MOUT<0, the options (parameters of the method) follows.

## 5.2 Extended screen output

If we want to use an extended screen output, we have to set \$DISPLAY='Y' (the default value is \$DISPLAY='N'). This type of screen output consists of text pages which correspond to individual iterations and the final results. The final results are divided into several groups which can be displayed successively. We can change the displayed group by typing particular characters from the keyboard.

Change of the displayed group of the final results:

F - (function) : Value of the objective function and statistics.  
V - (variables) : Values of variables if NF>0 (with their bounds if KBF>0).  
A - (approximation) : Values of approximating functions if NA>0 (with their prescribed values if KBA>0). Values of selected components of a solution of the set of ordinary differential equations at the prescribed mesh points if NE>0.  
C - (constraints) : Values of constraint functions if NC>0 (with their bounds if KBC>0).  
D - (data) : Data which specify the problem solved (sizes of problem and additional specifications).  
O - (options) : Options which specify the method used.

Exit:

Q - (quit) : Exit from the extended screen output.

After typing each character we must use ENTER.

Besides these possibilities we can stop every iteration for scanning the iterative process. It is specified if we set `$SCAN='Y'` (the default value is `$SCAN='N'`). If `$SCAN='N'`, the output of iterations is suppressed. Scanning of the iterative process can be terminated by using character `'!` from the keyboard.

### 5.3 Graphic screen output

The graphic screen output can be used only on PC computers under the MS DOS system. This possibility is not allowed on the UNIX workstations. If we want to use a graphic screen output, we have to set `$GRAPH='Y'` (the default value is `$GRAPH='N'`). In this case, both iterations and the final results appear in the graphic mode. In general, the graphic screen output is a sequence of screens which can be examined successively in a required order. A change of the screen is carried out by using the menu given on the top of this screen. We have three possibilities. First, the character displayed as a capital at the menu item can immediately be typed from the keyboard. Secondly, we can use characters  $\rightarrow$  and  $\leftarrow$  (or keys `+` and `-` in Windows 2000 versions), which realize movement in the top menu. The underlined menu item is then selected by pressing ENTER. Finally, we can apply a mouse click to the menu item when DOS graphics is used. In the subsequent graphic screen output description, we focus our attention to the first possibility without a loss of generality.

The graphic form of the final results can be specified in detail by using macrovariables `$PATH` ('N'- no, 'Y'- yes, 'E'- extended), `$MAP` ('N'- no, 'Y'- yes, 'E'- extended), `$HIL` ('N'- no, 'Y'- yes) and `$ISO` ('N'- no, 'Y'- yes). The final results are divided into several groups which can be displayed successively. We can change the displayed group by typing particular characters from the keyboard.

Change of the displayed group of the final results:

F - (function) : Value of the objective function and statistics.  
V - (variables) : Values of variables if `NF>0` (with their bounds if `KBF>0`).  
A - (approximation) : Values of approximating functions if `NA>0` (with their prescribed values if `KBA>0`). Values of selected components of a solution of the set of ordinary differential equations at the prescribed mesh points if `NE>0`.  
C - (constraints) : Values of constraint functions if `NC>0` (with their bounds if `KBC>0`).  
D - (data) : Data which specify the problem solved (sizes of problem and additional specifications).  
O - (options) : Options which specify the method used.  
T - (path) : Values of the objective function and selected variables (we can change these variables during the graphic output, if we have specified `$PATH='E'`) in the last NPA iterations (only if `$PATH='Y'` or `$PATH='E'`).

Exit:

Q - (quit) : Exit from the graphic output.

X - (exit) : Exit from the UFO system.

Besides these possibilities we can stop every iteration for scanning the iterative process. It is specified if we set `$SCAN='Y'` (the default value is `$SCAN='N'`). In every iteration, we can choose one of the possibilities F, V, A, C, D, O as in the case above. If we have chosen either V (variables) or A (approximation) or C (constraints), the intermediate results can be displayed graphically by typing G (graph) from the keyboard. In all these cases we can execute a single iteration by typing SPACE merely. We can also execute all iterations until the  $k$ -th one by typing J (jump) and entering the number  $k$ . Finally, by typing U (automatic), all remaining iterations are executed without scanning.

Besides text representations in the graphic mode, which are essentially like the ones in the extended screen output (with the choice `$DISPLAY='Y'`), we can chose several types of graphic data representation.

a) Graphic picture:

If we have chosen either V (variables) or A (approximation) or C (constraints), the results can be displayed graphically by typing G (graph) from the keyboard. A graphic picture appears on the screen in this case. It contains either values of variables with indices I,  $1 \leq I \leq NF$ , or values of the approximating functions with indices KA,  $1 \leq KA \leq NA$ , or values of the constraint functions with indices KC,  $1 \leq KC \leq NC$ . If we have chosen A (approximation) in the case of  $NE > 0$ , the graphic picture contains a component (with the index VAR) of a solution of the set of ordinary differential equations at the mesh points AT(KA),  $1 \leq KA \leq NA$ . We have to define the index VAR from the keyboard in this case. The graphic picture can be changed by typing the particular characters from the keyboard.

Change of representation:

V - (values) : Values are drawn.  
O - (ordinates) : Values and ordinates from zero axis are drawn.  
C - (curve) : Values are connected by a curve.  
M - (mixed) : Curve and ordinates are drawn.

Change of graph (if either  $KBF > 0$  or  $KBA > 0$  or  $KBC > 0$ ):

F - (functions) : Either values of variables X(I),  $1 \leq I \leq NF$ , or values of the approximating functions AF(KA),  $1 \leq KA \leq NA$ , or values of the constraint functions CF(KC),  $1 \leq KC \leq NC$ , are demonstrated.  
A - (approximation) : Either values of variables X(I) together with their bounds XL(I) and XU(I),  $1 \leq I \leq NF$ , or values of the approximating functions AF(KA) together with their prescribed values AM(KA),  $1 \leq KA \leq NA$ , or values of the constraint functions CF(KC) together with their bounds CL(KC) and CU(KC),  $1 \leq KC \leq NC$ , are demonstrated.  
D - (differences) : Either the differences between variables and their bounds or the differences between the approximating functions and their prescribed values or the differences between the constraint functions and their bounds are demonstrated.

Continuation (if either  $NF > 200$  or  $NA > 200$  or  $NC > 200$ ):

P - (previous) : Previous set of at most 200 values is drawn.  
N - (next) : Next set of at most 200 values is drawn.

Choice of the next displayed iteration (only if  $\$SCAN='Y'$ ):

J - (jump) : The iterative process is stopped at the  $k$ -th iteration. Number  $k$  is read from the keyboard.  
U - (automatic) : All remaining iterations are executed without scanning.

New graph or return:

W - (new) : This possibility can be used only if  $NE > 0$ . Then a new component (with a new index VAR) of a solution of the set of ordinary differential equations is drawn. We have to define a new index VAR from the keyboard in this case.  
Q - (quit) : Return to the displayed group of final results.

If we have chosen F (function) as a group of final results, we can use additional graphic representations.

b) Two-dimensional orbit:

If  $NE > 1$ , we can draw an orbit of two components of a solution of the set of ordinary differential equations by typing G (graph) from the keyboard. We have to define an index VAR for every selected component of a solution (according to the text appeared on the screen). The two-dimensional orbit can be changed by typing particular characters from the keyboard.

Change of the orbit:

V - (values) : Values are drawn.  
C - (curves) : Values are connected by a curve.

New orbit or return:

W - (new) : New components of a solution of the set of ordinary differential equations are drawn. We have to define new two indices from the keyboard in this case.  
Q - (quit) : Return to the displayed group of final results.

c) Three-dimensional orbit:

If  $NE > 2$ , then we can draw an orbit of three components of a solution of the set of ordinary differential equations by typing I (picture) from the keyboard. We have to define an index VAR for every selected component of a solution (according to the text appeared on the screen). The three-dimensional orbit can be changed by typing particular characters from the keyboard.

Change of the orbit:

V - (values) : Values are drawn.  
C - (curves) : Values are connected by a curve.  
O - (rotate) : Rotation of values or curves about a vertical axis by a subsequently entered angle Dfi.  
T - (tilt) : Tilting rotated values or curves by a subsequently entered angle Dtheta.  
A - (axes) : Drawing a picture with rotated and tilted axes.  
S - (scale) : Scaling of rotated and tilted values or curves to make full use of the screen.

New orbit or return:

W - (new) : New components of a solution of the set of ordinary differential equations are drawn. We have to define new three indices from the keyboard in this case.  
Q - (quit) : Return to the displayed group of final results.

d) Colored map of the objective function:

If we have specified either  $\$MAP='Y'$  or  $\$MAP='E'$  (default value is  $\$MAP='N'$ ), we can draw a colored map of the objective function by typing M (map) from the keyboard. This picture can be changed by typing particular characters from the keyboard.

Change of the map:

L - (linear) : Linear scale of the colored map.  
G - (logarithmic) : Logarithmic scale of the colored map.  
R - (refinement) : Refinement of the colored map.  
B - (back) : Back refinement of the colored map.  
N - (inverse) : Colored map of the objective function negation.

Another type of picture, new map or return:

H - (hills) : Drawing an objective function surface with respect to visibility (only if  $\$HIL='Y'$  is specified).  
S - (isolines) : Drawing contours of the objective function (only if  $\$ISO='Y'$  is specified).  
W - (new) : Selection of new variables and drawing a new colored map.  
Q - (quit) : Return to the displayed group of final results.

If we set  $\$MAP='Y'$ , one picture for two variables is drawn. If we set  $\$MAP='E'$ , three pictures for all combinations of two from three variables are drawn. In both cases we have to define, from the keyboard, an index VAR and bounds XL(VAR), XU(VAR) for every variable used (according to the text appeared

on the screen). Note that the choice \$MAP='E' excludes the choices \$HIL='Y' and \$ISO='Y' so that the other pictures cannot be used.

e) Objective function surface:

If we have specified \$HIL='Y' (default value is \$HIL='N'), we can draw an objective function surface with respect to visibility by typing H (hills) from the keyboard. This picture can be changed by typing particular characters from the keyboard.

Change of the surface:

L - (linear) :	Linear scale of the surface.
G - (logarithmic) :	Logarithmic scale of the surface.
R - (refinement) :	Refinement of the surface.
B - (back) :	Back refinement of the surface.
O - (rotate) :	Rotation of the surface about a vertical axis by a subsequently entered angle Dfi.
T - (tilt) :	Tilting the rotated surface by a subsequently entered angle Dtheta.
F - (face) :	Facing the rotated surface (drawing the rotated surface without tilting).
N - (inverse) :	Surface of the objective function negation.

Another type of picture, new surface or return:

M - (map) :	Drawing a colored map of the objective function (only if \$MAP='Y' is specified).
S - (isolines) :	Drawing contours of the objective function (only if \$ISO='Y' is specified).
W - (new) :	Selection of new variables and drawing new surface.
Q - (quit) :	Return to the displayed group of final results.

Before drawing the objective function surface we have to define, from the keyboard, an index VAR and bounds XL(VAR), XU(VAR) for every variable used (according to the text appeared on the screen).

f) Objective function contours:

If we have specified \$ISO='Y' (default value is \$ISO='N'), we can draw an objective function contours by typing S (isolines) from the keyboard. This picture can be changed by typing particular characters from the keyboard.

Change of contours:

L - (linear) :	Linear scale of contours.
G - (logarithmic) :	Logarithmic scale of contours.
R - (refinement) :	Refinement of contours.
B - (back) :	Back refinement of contours.
O - (color) :	Coloring of contours and used levels.
N - (inverse) :	Inverse coloring of contours and used levels.

Another type of picture, new contours or return:

M - (map) :	Drawing a colored map of the objective function (only if \$MAP='Y' is specified).
H - (hills) :	Drawing an objective function surface with respect to visibility (only if \$HIL='Y' is specified).
W - (new) :	Selection of new variables and drawing a new surface.
Q - (quit) :	Return to the displayed group of final results.

g) Graphic path of the objective function and selected variables:

If we have chosen T (path), we can display the values of the objective function as a function graph by typing G (graph) or draw the objective function contours with the path in the last NPA iterations by typing S (isolines). The graph can be changed in the same way as in a).

Change of contours:

L - (linear) : Linear scale of contours.  
G - (logarithmic) : Logarithmic scale of contours.  
R - (refinement) : Refinement of contours.  
B - (back) : Back refinement of contours.  
Z - (zoom) : Zoom of the path for the number of last iterations entered.

Another type of picture, new contours or return:

W - (new) : Selection of new variables and drawing new contours (only if we have specified \$PATH='E').  
Q - (quit) : Return to the displayed group of final results.

Before drawing the objective function contours we have to define, from the keyboard, an index VAR and bounds XL(VAR), XU(VAR) for every variable used (according to the text appeared on the screen).

## 5.4 Text file output

The UFO system contains a great number of text file output procedures which are controlled by using the macrovariables \$KOUT, \$KOUT1, \$KOUT2, \$KOUT3 and \$LOUT. These text file output procedures are useful especially for debugging new optimization methods. The UFO system works with the output file P.OUT. The Fortran number of this output file defines the common variable IWR. The macrovariables \$KOUT, \$KOUT1, \$KOUT2, \$KOUT3 determine what is printed and the macrovariable \$LOUT has an influence on the extent of the print.

The macrovariable \$KOUT can have the following values:

\$KOUT= 0 - Text file output is suppressed (the file P.OUT is empty) .  
\$KOUT= ± 1 - Standard output. The heading and the final results are printed together with selected information in each accepted iteration.  
\$KOUT= ± 2 - Extended output. Additional information, obtained from stepsize selection, is printed.  
\$KOUT= ± 3 - Extended output. Additional information, obtained from direction determination and variable metric update, is printed.  
\$KOUT= ± 4 - Extended output. Additional information, obtained from linear constraint addition and deletion, is printed.  
\$KOUT= ± 5 - Extended output. Additional information, obtained from numerical differentiation, is printed.

If \$KOUT>0, the standard heading is printed while if \$KOUT<0, the extended heading, containing problem specifications and optimization options, is printed.

The selection of iterations accepted for print is controlled by the contents of the macrovariables \$KOUT1, \$KOUT2, \$KOUT3. If \$KOUT1≤\$KOUT2, only the iterations whose numbers are between \$KOUT1 and \$KOUT2 are assumed, but the \$KOUT3–1 ones are always omitted (\$KOUT1 is a lower bound, \$KOUT2 is an upper bound and \$KOUT3 is a step). Similarly, if \$KOUT1>\$KOUT2, only the iterations whose numbers are smaller than \$KOUT2 or greater than \$KOUT1 are assumed, but the \$KOUT3–1 ones are always omitted. If \$KOUT3=0, no iterations are assumed.

While the macrovariable \$KOUT specifies what information is printed, the macrovariable \$LOUT specifies how much information is printed:

\$LOUT= 0 - Basic output. The basic information (one row if \$KOUT=1) is printed in each accepted iteration.  
\$LOUT=± 1 - Extended output. Additional scalars, together with the vector of variables, are printed.  
\$LOUT=± 2 - Extended output. Additional vectors (usually gradients) are printed.



\$LOUT= $\pm$  3 - Extended output. Additional matrices (usually Hessian matrices) are printed.  
 \$LOUT= $\pm$  4 - The most extended output. All useful data are printed.

If \$LOUT $>$ 0, the basic part of the information is printed. If \$LOUT $<$ 0, a more extensive part of the information is printed.

The macrovariable \$LOUT has an additional significance. If \$KOUT=0 and \$LOUT $>$ 0, a copy of the basic screen output is performed (see Section 5.1). If \$KOUT=0 and \$LOUT $<$ 0, paper saving print for line printers is assumed. In the last case, only several rows are printed for every solution. This type of output is useful for simultaneous tests of optimization methods.

To show a typical text file output, we use values \$KOUT=5, \$LOUT=0 in the problem specification introduced in Section 7.1. In this case, text file P.OUT contains the following information.

UNCONSTRAINED MAXIMIZATION USING UFO SYSTEM

```
-----
OPTIMIZATION SUBROUTINE : U1FDU1
DIRECTION DETERMINATION : UDDL1G1
STEP SIZE DETERMINATION : US1L01
FUNCTION DETERMINATION : UF1F01
GRADIENT DETERMINATION :
H MATRIX DETERMINATION :
VARIABLE METRIC UPDATE : UUDBG1
```

PROBLEM

```
-----
NF=   5           KDF=  1 KSF=  1 KCF=  2 KBF=  2 ISNF=  1 NORMF=  0
NA=   0 NAL=  0 MAL=  0 KDA= -1 KSA=  0 KCA=  0 KBA=  0 ISNA=  0 NORMA=  0
NC=   0 NCL=  0 MCL=  0 KDC= -1 KSC=  0 KCC=  0 KBC=  0 ISNC=  0 NORMC=  0

NIT=   0 NFV=   1 NFG=   1 F=0.187D+01 G=0.667D-01 D=0.000D+00
```

-----  
 DIRECTION DETERMINATION USING UDDL1G1

```
0 ALF=0.000D+00 SIG=0.000D+00
END OF DIRECTION DETERMINATION : G.M. POS
G = 0.11547D+00 S = 0.11547D+00 P = -0.13333D-01
STEP SIZE SELECTION USING US1L01 : P/(G*S) = -.100D+01
R=0.000D+00 S=0.115D+00 F=0.186666666667D+01 P=-.133D-01 B=0.150D+02
0 0 R=0.100D+01 D=0.115D+00 F=0.185288395062D+01 P=-.142D-01 INIT-1
1 -1 R=0.400D+01 D=0.462D+00 F=0.180590617284D+01 P=-.171D-01 BISECT.
1 -2 R=0.150D+02 D=0.173D+01 F=0.155000000000D+01 P=-.300D-01 BISECT.
R=0.150D+02 D=0.173D+01 F=0.155000000000D+01
```

END OF STEP SIZE SELECTION : R BOUND

VARIABLE METRIC UPDATE USING UUDBG1

```
GAM=0.000D+00 RHO=0.000D+00 PAR=0.000D+00 BET=0.000D+00
A =0.000D+00 B =-.250D+00 C =0.000D+00
END OF VARIABLE METRIC UPDATE : B - NEG.
```

CONSTRAINT ADDITION USING UYADS1

```
NEW= -3 N= 2
END OF CONSTRAINT ADDITION : N = 2 IER = 0
```

NIT= 1 NFV= 4 NFG= 4 F=0.155D+01 G=0.150D+00 D=0.333D+00  
UNIT G.M. POS R BOUND NORMAL

-----  
DIRECTION DETERMINATION USING UDDLG1

0 ALF=0.000D+00 SIG=0.000D+00  
END OF DIRECTION DETERMINATION : G.M. POS  
G = 0.21213D+00 S = 0.21213D+00 P = -0.45000D-01  
STEPWISE SELECTION USING US1L01 : P/(G\*S) = -.100D+01  
R=0.000D+00 S=0.212D+00 F=0.155000000000D+01 P=-.450D-01 B=0.667D+01  
0 0 R=0.100D+01 D=0.212D+00 F=0.150387500000D+01 P=-.473D-01 INIT-1  
1 -1 R=0.400D+01 D=0.849D+00 F=0.135200000000D+01 P=-.540D-01 BISECT.  
1 -2 R=0.667D+01 D=0.141D+01 F=0.120000000000D+01 P=-.600D-01 CUBIC  
R=0.667D+01 D=0.141D+01 F=0.120000000000D+01

END OF STEPWISE SELECTION : R BOUND

VARIABLE METRIC UPDATE USING UDBG1

GAM=0.000D+00 RHO=0.000D+00 PAR=0.000D+00 BET=0.000D+00

A =0.000D+00 B =-.100D+00 C =0.000D+00

END OF VARIABLE METRIC UPDATE : B - NEG.

CONSTRAINT ADDITION USING UYADS1

NEW= -4 N= 1

END OF CONSTRAINT ADDITION : N = 1 IER = 0

NIT= 2 NFV= 7 NFG= 7 F=0.120D+01 G=0.200D+00 D=0.250D+00

UNIT G.M. POS R BOUND UPDATE

-----  
DIRECTION DETERMINATION USING UDDLG1

0 ALF=0.000D+00 SIG=0.000D+00  
END OF DIRECTION DETERMINATION : G.M. POS  
G = 0.20000D+00 S = 0.20000D+00 P = -0.40000D-01  
STEPWISE SELECTION USING US1L01 : P/(G\*S) = -.100D+01  
R=0.000D+00 S=0.200D+00 F=0.120000000000D+01 P=-.400D-01 B=0.500D+01  
0 0 R=0.100D+01 D=0.200D+00 F=0.116000000000D+01 P=-.400D-01 INIT-1  
1 -1 R=0.500D+01 D=0.100D+01 F=0.100000000000D+01 P=-.400D-01 CUBIC  
R=0.500D+01 D=0.100D+01 F=0.100000000000D+01

END OF STEPWISE SELECTION : R BOUND

VARIABLE METRIC UPDATE USING UDBG1

GAM=0.000D+00 RHO=0.000D+00 PAR=0.000D+00 BET=0.000D+00

A =0.000D+00 B =0.000D+00 C =0.000D+00

END OF VARIABLE METRIC UPDATE : B - NEG.

CONSTRAINT ADDITION USING UYADS1

NEW= -5 N= 0

END OF CONSTRAINT ADDITION : N = 0 IER = 0

NIT= 3 NFV= 9 NFG= 9 F=0.100D+01 G=0.000D+00 D=0.200D+00

UNIT G.M. POS R BOUND UPDATE

-----  
FINAL RESULTS

FF= -0.1000000000D+01  
X = 0.1000000000D+01 0.2000000000D+01 0.3000000000D+01 0.4000000000D+01  
0.5000000000D+01

TERMINATION: 4 GRAD TOL F=0.100D+01 G=0.000D+00 D=0.200D+00

#### STATISTICS

-----

NIT	=	3	NDEC	=	0				
NFV	=	9	NAV	=	0	NCV	=	0	NRES = 3
NFG	=	9	NAG	=	0	NCG	=	0	NREM = 0
NFH	=	0	NAH	=	0	NCH	=	0	NADD = 0

Here the optimization subroutines used are listed on the top followed by problem specifications. After the line containing the starting function value and gradient norm (zero iteration), results of individual iterations follow. Since \$KOUT=5, information concerning direction determination, step-size selection, variable metric update and constraint handling is printed. The last two lines describing every iteration contain the current number of iterations NIT, function evaluations NFV, gradient evaluations NFG, the function value F, the gradient norm G, the step-size D and the information concerning the status of the current iteration (UNIT, G.M. POS, R BOUND, UPDATE). After results of individual iterations, the final results are printed. They contain the cause of termination ITERM=4 (GRAD TOL) corresponding to the attainment of the required gradient norm. Furthermore, F is the objective function value, G is the maximum absolute value of gradient elements and D is the maximum relative change of variables. The statistics contains the final number of iterations NIT, (objective, particular, constraint) function evaluations (NFV, NAV, NCV), (objective, particular, constraint) gradient evaluations (NFG, NAG, NCG), (objective, particular, constraint) Hessian matrix evaluations (NFH, NAH, NCH), the number of decompositions NDEC, the number of restarts NRES, and the numbers of constraint deletions NREM and additions NADD.

## 5.5 User supplied output

The UFO system allows utilizing both the user supplied output subroutines and the post-processing subroutines. These subroutines can be inserted in the UFO source program by using the macrovariable \$OUTPUT:

```
$SET(OUTPUT)
    Calling the user supplied output subroutines.
    Calling the post-processing subroutines.
$ENDSET
```

The parameters of the user supplied output subroutines and the post-processing subroutines must satisfy the UFO conventions. For example, the vector of variables, the model function value and the model function gradient must be denoted by X, FF and GF, respectively (see Chapter 2).

## 5.6 Storing final results

If we set \$OUTPUTDATA='Y', the final values of variables X(I),  $1 \leq I \leq NF$ , are stored in file P.DAT. Similarly, if we set \$INPUTDATA='Y', the values of variables X(I),  $1 \leq I \leq NF$ , from file P.DAT are used as input data for a new optimization process.

## 5.7 Other output files

The UFO system uses two other output files P.DIM and P.SIF which contain additional information about the problem solved. File P.DIM shows us the problem dimension. It contains the numbers of variables, approximating functions, constraints and also numbers of nonzero elements in sparse structures. For example, if we apply the UFO system to the input file TSIF21.UFO, then the file P.DIM contains the following text:

```

PROBLEM: NEXT =          0
NUMBER OF VARIABLES:      NF =      450
NUMBER OF CONSTRAINTS:    NC =      360
NUMBER OF NONZERO ELEMENTS: MC =    1576
NUMBER OF NONZERO ELEMENTS: MHC =     55
NUMBER OF NONZERO ELEMENTS: MCH =   4736
NUMBER OF NONZERO ELEMENTS: M   =   2779

```

File P.SIF contains information concerning SIF files of the CUTE collection (Section 6.4). This file is generated by the SIF decoder. For example, if we apply the UFO system to the input file TSIF21.UFO, the file P.SIF contains the following text:

```

Problem name: BRITGAS
The objective function uses      1 nonlinear group

There are      360 nonlinear equality constraints

There are      426 variables bounded only from below
There are      24 variables bounded from below and above

```

## 5.8 Error messages

If we use the specification \$MOUT>0 (basic screen output), then nonstandard terminations are indicated. The message consists of the three parts: the name of a critical subroutine, the number of a message, and an explanation text. For example, if the number of iterations is exceeded, we obtain the following message:

```

0 NIT= 500 NRV=2545 NFG= 0 NDC=1556 NCG= 0 F= .122D+06 G= .112D+05
UYFUT1: (-2) MAXIMUM NUMBER OF ITERATIONS

```

Error messages are very useful especially in case the problem dimension is invalid. For example, if the number of nonzero elements in the Jacobian matrix is specified incorrectly, then we obtain the message:

```

0 NIT= 0 NRV= 0 NFG= 0 NDC= 0 NCG= 0 F= .000D+00 G= .000D+00
UZLMIN: (78) LACK OF SPACE : MA TOO SMALL
ACTUAL VALUE:      298 - DECLARED VALUE:      250

```

Here UZLMIN is the subroutine where the error was detected, 78 is the error number and MA TOO SMALL is the explanation. In this case, additional information (ACTUAL VALUE and DECLARED VALUE) is given.

The following table presents all UFO error messages (error numbers and explanations):

```

-1 - MAXIMUM NUMBER OF FUNCTION EVALUATIONS : MFV TOO SMALL
-2 - MAXIMUM NUMBER OF ITERATIONS : MIT TOO SMALL
-3 - MAXIMUM NUMBER OF CYCLES : MIC TOO SMALL
 1 - BAD DECOMPOSITION
 2 - BAD INTERVAL IN THE OLC DIRECTION DETERMINATION
 3 - MAXIMUM NUMBER OF STEPS IN THE OLC DIRECTION DETERMINATION
 4 - BREAKDOWN IN THE ITERATIVE METHOD
 5 - BREAKDOWN IN THE ITERATIVE METHOD
 6 - MAXIMUM NUMBER OF REDUCTIONS
 7 - NEGATIVE DIRECTIONAL DERIVATIVE
 8 - BAD INTERVAL FOR INTERPOLATION
 9 - BAD PREDICTION IN THE TRUST REGION METHOD
10 - RESTART
11 - FEASIBLE SOLUTION DOES NOT EXIST

```

12 - BOUNDED SOLUTION DOES NOT EXIST  
13 - FEASIBLE SOLUTION DOES NOT EXIST  
14 - FEASIBLE TRUST REGION DOES NOT EXIST  
15 - INVALID SITUATION IN CONSTRAINT HANDLING  
16 - INVALID SITUATION IN CONSTRAINT HANDLING  
17 - LACK OF SPACE IN CONSTRAINT HANDLING : MMAX TOO SMALL  
18 - LACK OF SPACE IN CONSTRAINT HANDLING : MMAX TOO SMALL  
19 - BAD INPUT DATA  
20 - BAD INPUT DATA  
21 - UXSGFM: NAU IS DECLARED TOO SMALL  
22 - UXSGFM: NZ IS DECLARED TOO SMALL  
23 - UXSGFM: JACOBIAN MATRIX IS TOO UNSTABLE  
24 - UXSGFM: JACOBIAN MATRIX IS SINGULAR  
25 - UXSGFM: NZ IS TOO SMALL FOR THE FACTOR  
26 - UXSGFM: NAU IS TOO SMALL FOR THE FACTOR  
27 - UXSGFM: NZ IS TOO SMALL FOR DATA MANIPULATIONS AFTER FACTORIZATION  
28 - UXSGFM: COLUMN SCHEME FOR THE FACTOR IS NOT CREATED: LACK OF SPACE  
29 - BAD INPUT DATA  
30 - BAD INPUT DATA  
31 - UXSGUM: NAU IS DECLARED TOO SMALL  
32 - UXSGUM: JACOBIAN MATRIX IS SINGULAR  
33 - UXSGUM: FACTOR IS BADLY CONDITIONED  
34 - UXSGUM: LITTLE SPACE FOR L-UPDATES  
35 - UXSGUM: JACOBIAN MATRIX SINGULARITY IS FACED  
36 - UNBOUNDEDNESS IS FACED  
37 - UKLTS3: ROWS ARE NOT SPECIFIED  
38 - UKLTS3: COLUMNS ARE NOT SPECIFIED  
39 - UKLTS3: TYPE IS NOT SPECIFIED  
40 - UKLTS3: TYPC IS NOT DEFINED  
41 - LACK OF SPACE FOR THE CHOLESKI FACTOR : MMAX TOO SMALL  
42 - LACK OF SPACE FOR A SYMBOLIC FACTORIZATION : MMAX TOO SMALL  
43 - LACK OF SPACE FOR THE FILL-IN  
44 - LACK OF SPACE FOR NUMERICAL DIFFERENTIATION : M TOO SMALL  
45 - STRUCTURAL SINGULARITY DURING INCOMPLETE LU FACTORIZATION  
46 - INVALID STRUCTURE FOR INCOMPLETE LU FACTORIZATION  
47 - LACK OF SPACE IN NUMERICAL DIFFERENTIATION : NVAR TOO SMALL  
48 - LACK OF SPACE IN THE INCOMPLETE DECOMPOSITION : MMAX TOO SMALL  
49 - LACK OF SPACE IN THE SCHUR COMPLEMENT : MMAX TOO SMALL  
50 - LACK OF SPACE FOR THE FACTOR  
51 - INSUFFICIENT STORAGE FOR NONZERO SUBSCRIPTS  
52 - LACK OF SPACE IN THE FRONTAL SCHEME  
53 - ERROR IN THE FRONTAL SCHEME  
54 - LACK OF SPACE IN THE FRONTAL SCHEME  
55 - ERROR IN THE FRONTAL SCHEME  
56 - LACK OF SPACE IN THE INTEGER FIELD  
57 - LACK OF SPACE IN THE REAL FIELD  
58 - ZERO INDEX  
59 - DIMENSION ERROR  
60 - LACK OF SPACE IN THE WORKING FIELD  
61 - INVALID MATRIX ORDER  
62 - NUMBER OF NONZEROS SMALLER THAN ZERO  
63 - INVALID ENTRIES IN THE INPUT MATRIX

64 - INCONSISTENT MEMORY  
65 - LACK OF SPACE IN THE INTEGER FIELD : MMAX TOO SMALL  
66 - LACK OF SPACE IN THE REAL FIELD : MMAX TOO SMALL  
67 - INVALID LU FACTORS  
68 - MAXIMUM INTEGER TOO SMALL  
69 - INVALID INPUTS  
70 - ZERO PIVOT WHEN DEFINITENESS IS DECLARED  
71 - CHANGE IN SIGN OF PIVOT ENCOUNTERED  
72 - SINGULARITY DETECTED  
73 - NONZERO ELEMENT IGNORED  
74 - PIVOT HAS DIFFERENT SIGN FROM THE PREVIOUS ONE  
75 - LACK OF SPACE : M TOO SMALL  
76 - LACK OF SPACE : MAH TOO SMALL  
77 - LACK OF SPACE : MCH TOO SMALL  
78 - LACK OF SPACE : MA TOO SMALL  
79 - LACK OF SPACE : MC TOO SMALL  
80 - LACK OF SPACE : NF TOO SMALL  
81 - LACK OF SPACE : NA TOO SMALL  
82 - LACK OF SPACE : NC TOO SMALL  
83 - SIMPLE BOUNDS ARE NOT PERMITTED  
84 - INEQUALITY CONSTRAINTS ARE NOT PERMITTED  
85 - TOO MANY DENSE ROWS : ND TOO SMALL  
86 - LACK OF SPACE : MHA TOO SMALL  
87 - LACK OF SPACE : MHC TOO SMALL  
88 - LINEAR DEPENDENCE OF ACTIVE CONSTRAINTS  
89 - INFEASIBLE SOLUTION  
90 - MAXIMUM NUMBER OF SIMPLEX ITERATIONS : MIS TOO SMALL  
91 - DIFFERENTIAL EQUATION IS UNSTABLE  
92 - MAXIMUM NUMBER OF INTEGRATION STEPS EXCEEDED  
93 - TOO SMALL INTEGRATION STEP  
94 - DIFFERENTIAL EQUATION IS STIFF  
95 - SINGULAR JACOBIAN IN IMPLICIT INTEGRATION METHOD  
96 - LACK OF SPACE IN DIFFERENTIAL EQUATION SOLVER  
97 - LACK OF SPACE : MMAX TOO SMALL  
98 - MAXIMUM NUMBER OF MINOR CYCLES : MIQ TOO SMALL  
99 - MAXIMUM NUMBER OF MAJOR CYCLES : MAQ TOO SMALL  
100 - LACK OF SPACE : MF TOO SMALL  
101 - LARGE LAGRANGIAN FUNCTION RETURN  
102 - INVALID TERMINATION: EPS9 TOO SMALL  
103 - FEASIBILITY RESTORATION PHASE IS EXPECTED

## 6 Special tools of the UFO system

The UFO system contains special tools that facilitate the user's activity. There are tools for checking the correctness of optimization problems and for testing optimization methods.

### 6.1 Checking external subroutines

The values, gradients, Hessian matrices of the model function or the approximating functions or the constraint functions are specified by using the macrovariables \$FMODEL, \$GMODEL, \$HMODEL or \$FMODELA, \$GMODELA, \$HMODELA or \$FMODEL, \$GMODEL, \$HMODEL, respectively. Sometimes the correctness of these models needs to be checked up. If this is the case, then both the analytical and the numerical differentiation can be compared. The checking of optimization problems can be specified by using the macrovariable \$TEST. If \$TEST='N', no checking is performed. If \$TEST='Y', both the analytical and the numerical differentiation are executed before optimization is started (at the initial starting point) and the derivatives obtained are printed. Only the derivatives that are analytically specified (the first, the second) are checked. If \$TEST='A', the checking is performed after the optimization is finished (at the final optimum point). Finally, if \$TEST='O', only checking is performed and optimization is not started. The output of checking an optimization problem has the following form:

```
STANDARD TEST OF EXTERNAL SUBROUTINES
-----
PROBLEM NO      1

PROBLEM
-----
NF = 2           KDF = 2  KSF = 1  KCF = 2  NORMF = 0
NA = 0  NAL = 0  MAL = 0  KDA = -1  KSA = 0  KCA = 0  NORMA = 0
NC = 3  NCL = 0  MCL = 0  KDC = 1  KSC = 0  KCC = 2  NORMC = 0

PARAMETERS
-----
X      =  -.2000000000D+01   .1000000000D+01

DERIVATIVES
-----
FF A =  .9090000000D+03
GF N =  -.2405999822D+04  -.6000004263D+03
GF A =  -.2406000000D+04  -.6000000000D+03
HF N =  .4402000148D+04   .8000000070D+03   .2000000002D+03
HF A =  .4402000000D+04   .8000000000D+03   .2000000000D+03

FC A =  -.1000000000D+01
GC N =  .1000000000D+01   .2000000032D+01
GC A =  .1000000000D+01   .2000000000D+01

FC A =  .5000000000D+01
GC N =  -.4000000070D+01   .9999999930D+00
GC A =  -.4000000000D+01   .1000000000D+01

FC A =  .5000000000D+01
GC N =  -.4000000070D+01   .2000000042D+01
GC A =  -.4000000000D+01   .2000000000D+01
```

Here the letter 'N' indicates a numerical differentiation and the letter 'A' indicates an analytical differentiation.

## 6.2 Testing optimization methods

The UFO system contains a great number of subroutines (collections of test problems) which serve for testing optimization methods. All of these subroutines begin with letter 'E' (external). The input subroutines have the second letter 'I' and the third letter 'U' or 'L' or 'N' for unconstrained or linearly constrained or nonlinearly constrained problems, respectively. The model specification subroutines have the second letter 'F' or 'A' or 'C' or 'E' or 'Y' for a model function or approximating functions or constraint functions or state functions or initial functions, respectively, and the third letter 'F' or 'G' or 'H' for values or gradients or Hessian matrices. The fourth letter is always 'U' or 'D' or 'S' or 'B' for universal or dense or sparse or partitioned problems, respectively. The last two digits specify individual test problems collections. When we want to carry out a test of the method selected, we define macrovariable \$COLLECTION and set \$NEXT=number\_of\_test\_problems. The following specifications can be used:

```
$COLLECTION='N' - test is suppressed.
$COLLECTION='Y' - test is performed and basic output is printed.
$COLLECTION='E' - test is performed and extended output is printed.
$COLLECTION='P' - test is performed, basic output is printed and data for performance profiles are prepared.
```

The default value is \$COLLECTION='N'.

Tests corresponding to individual test problems collections are realized by using the following input files (asterisks denote possible additional letters):

```
TEST01*.UFO - tests for unconstrained optimization (25 dense problems from [42], [153]). External subroutines EIUD01, EFFU01, EFGU01, EFHD01 are used.
TEST02*.UFO - tests for the sum of squares minimization (30 dense problems from [209]). External subroutines EIUD02, EAFU02, EAGU02, EAHD02 are used.
TEST03*.UFO - tests for linearly constrained optimization (15 dense problems from [117]). External subroutines EILD03, EFFU03, EFGU03 are used.
TEST04*.UFO - tests for medium-size linear programming (6 dense problems). External subroutine EILD04 is used.
TEST05*.UFO - tests for medium-size quadratic programming (5 dense problems). External subroutine EILD05 is used.
TEST06*.UFO - tests for minimax (25 dense problems from [192]). External subroutines EIUD06, EAFU06, EAGU06, EAHD06 are used.
TEST07*.UFO - tests for inequality constrained nonlinear programming (34 dense problems from [117]). External subroutines EIND07, EFFU07, EFGU07, ECFU07, ECGU07 are used.
TEST08*.UFO - tests for equality constrained nonlinear programming (31 dense problems from [117]). External subroutines EIND08, EFFU08, EFGU08, ECFU08, ECGU08 are used.
TEST09*.UFO - tests for unconstrained global optimization (13 problems from [308]). External subroutines EIUD09, EFFU09, EFGU09 are used.
TEST10*.UFO - tests for unconstrained optimization (15 sparse problems from [184]). External subroutines EIUS10, EFFU10, EFGU10, EFHS10 are used.
TEST11*.UFO - tests for large-scale linear programming (18 sparse problems). External subroutine EILS11 is used.
TEST12*.UFO - tests for large-scale quadratic programming (11 sparse problems). External subroutine EILS12 is used.
TEST13*.UFO - tests for linearly constrained optimization (6 sparse problems). External subroutines EILS13, EFFU13, EFGU13 are used.
```



TEST14\*.UFO - tests for the sum of functions minimization (22 sparse problems from [184]). External subroutines EIUB14, EAFU14, EAGU14 are used.

TEST15\*.UFO - tests for the sum of squares minimization (24 sparse problems from [184]). External subroutines EIUB15, EAFU15, EAGU15 are used.

TEST16\*.UFO - tests for nonlinear equations solutions (32 dense problems). External subroutines EIUD16, EAFU16, EAGU16 are used.

TEST17\*.UFO - tests for nonlinear equations solutions (30 dense problems). External subroutines EIUD17, EAFU17, EAGU17 are used.

TEST18\*.UFO - tests for nonlinear equations (32 sparse problems from [184]). External subroutines EIUB18, EAFU18, EAGU18 are used.

TEST19\*.UFO - tests for nonsmooth unconstrained optimization (25 dense problems from [192]). External subroutines EIUD19, EFFU19, EFGU19, EFHD19 are used.

TEST20\*.UFO - tests for equality constrained sparse nonlinear programming (18 sparse problems from [192]). External subroutines EIUB20, EIUS20, EIND20, EINS20, EFFU20, EFGU20, EAFU20, EAGU20, ECFU20, ECGU20 are used.

TEST21\*.UFO - tests for inequality constrained sparse nonlinear programming (1 dense problem). External subroutines EIUS21, EINS21, EFFU21, EFGU21, ECFU21, ECGU21 are used.

TEST22\*.UFO - tests for linearly constrained minimax optimization (15 dense problems from [192]). External subroutines EIUD22, EAFU22, EAGU22, EAHD22 are used.

TEST23\*.UFO - extended tests for unconstrained optimization (74 dense problems from [184]). External subroutines EIUD23, EFFU23, EFGU23 are used.

TEST24\*.UFO - extended tests for the sum of squares minimization (115 dense problems from [42], [153], [184], [209]). External subroutines EIUD24, EAFU24, EAGU24 are used.

TEST25\*.UFO - extended tests for the sum of functions minimization (74 sparse problems from [184]). External subroutines EIUB25, EAFU25, EAGU25 are used.

TEST26\*.UFO - extended tests for the sum of squares minimization (52 sparse problems from [184]). External subroutines EIUB26, EAFU26, EAGU26 are used.

TEST27\*.UFO - extended tests for the sum of squares minimization (82 dense problems from [184]). External subroutines EIUD27, EAFU27, EAGU27 are used.

TEST28\*.UFO - extended tests for unconstrained optimization (92 dense problems from [42], [153], [184], [209]). External subroutines EIUD28, EFFU28, EFGU28 are used.

TEST29\*.UFO - tests for nonsmooth unconstrained optimization (31 dense problems). External subroutines EIUD29, EFFU29, EFGU29, EFBU29 are used.

TEST30\*.UFO - tests for optimization of dynamical systems (4 dense problems). External subroutines EIUD30, EEFU30, EEGU30, EYFU30, EYGU30 are used.

TEST31\*.UFO - tests for differential equations (6 dense problems). External subroutines EIUD31, EEFU31 are used.

TEST32\*.UFO - tests for the sum of squares minimization (6 dense problems from [155]). External subroutines EIUD32, EAFU32, EAGU32 are used.

TEST33\*.UFO - tests for the sum of squares minimization (6 dense problems from [155]). External subroutines EIUD33, EAFU33, EAGU33 are used.

TEST34\*.UFO - tests for large-scale linear programming (18 sparse problems). External subroutine EINS20 is used.

TEST35\*.UFO - tests for large-scale quadratic programming (8 sparse problems). External subroutine EIQS35 is used.

TEST36\*.UFO - extended tests for nonlinear equations solutions (94 dense problems). External subroutines EIUD36, EAFU36, EAGU36 are used.

TEST37\*.UFO - extended tests for nonlinear equations solutions (64 dense problems). External subroutines EIUD37, EAFU37, EAGU37 are used.

In these input files, all necessary macrovariables are defined and the external subroutines are called. The external subroutines with the last two digits 01, . . . , 37 are briefly described in the text files E01.TXT, . . . , E37.TXT. The external subroutines with the last two digits 01, . . . , 22 contain original test problems. The remaining external subroutines are their various combinations.

To demonstrate the use of the test input file we perform a test of the sum of squares minimization by using a hybrid method realized as a trust region method. The test input file TEST02.UFO has the following form:

```

$REM : model specifications
$SET(INPUT)
  CALL EIUD02(NF,NA,NAL,X,FMIN,XMAX,NEXT,IEXT,IERR)
  IF(NEXT.EQ.10) XMAX=1.0$P 1
  IF(IERR.NE. 0) GO TO $$ENDTEST
$ENDSET
$SET(FMODELA)
  CALL EAFU02(NF,KA,X,FA,NEXT)
$ENDSET
$NF=12
$NA=400
$MODEL='AQ'
$REM : method specifications
$CLASS='GN'
$TYPE='G'
$DECOMP='M'
$NUMBER=7
$UPDATE='F'
$TOLX='1.0$P-16'
$TOLF='1.0$P-14'
$TOLB='1.0$P-16'
$TOLG='1.0$P-6'
$REM : print specifications
$MOUT=1
$REM : the cycle for testing 30 test problems is generated
$COLLECTION='YES'
$NEXT=30
$REM : the batch mode is used
$BATCH
$REM : the standard form of the source program is used
$STANDARD

```

The result (screen output) obtained has the following form (each row corresponds to one test problem and the last row is the summary):

CLASS = GN - GM7	UPDATE = F	MODEL = AQ	HESF = D	NF =	2
1 NIT= 12 NFV= 41 NFG= 0	FV BOUND	F= 0.2465190329E-31	G=0.222D-15		
2 NIT= 20 NFV= 69 NFG= 0	GRAD TOL	F= 24.49212684	G=0.667D-06		
3 NIT= 33 NFV= 102 NFG= 0	FV BOUND	F= 0.2035629458E-22	G=0.581D-06		
4 NIT= 14 NFV= 47 NFG= 0	FV BOUND	F= 0.000000000	G=0.000D+00		
5 NIT= 6 NFV= 21 NFG= 0	GRAD TOL	F= 0.1422753549E-15	G=0.807D-07		
6 NIT= 11 NFV= 41 NFG= 0	GRAD TOL	F= 62.18109118	G=0.927D-06		
7 NIT= 7 NFV= 32 NFG= 0	FV BOUND	F= 0.2748725380E-26	G=0.728D-12		
8 NIT= 5 NFV= 24 NFG= 0	GRAD TOL	F= 0.4107438653E-02	G=0.293D-08		
9 NIT= 1 NFV= 8 NFG= 0	GRAD TOL	F= 0.5639663969E-08	G=0.177D-07		
10 NIT= 235 NFV= 954 NFG= 0	STEP TOL	F= 43.97292759	G=0.178D-03		
11 NIT= 80 NFV= 329 NFG= 0	FV BOUND	F= 0.7813056212E-24	G=0.350D-08		
12 NIT= 12 NFV= 53 NFG= 0	FV BOUND	F= 0.3624914786E-20	G=0.107D-09		
13 NIT= 10 NFV= 55 NFG= 0	GRAD TOL	F= 0.1686647891E-09	G=0.247D-06		

14	NIT=	40	NFV=	213	NFG=	0	GRAD TOL	F=	0.7422398347E-15	G=	0.538D-06
15	NIT=	11	NFV=	62	NFG=	0	GRAD TOL	F=	0.1537528024E-03	G=	0.511D-06
16	NIT=	24	NFV=	134	NFG=	0	FV TOL	F=	42911.10081	G=	0.108D-03
17	NIT=	22	NFV=	139	NFG=	0	GRAD TOL	F=	0.2732447349E-04	G=	0.127D-06
18	NIT=	17	NFV=	128	NFG=	0	GRAD TOL	F=	0.1023149745E-15	G=	0.273D-07
19	NIT=	13	NFV=	169	NFG=	0	GRAD TOL	F=	0.2192388901E-01	G=	0.379D-07
20	NIT=	9	NFV=	130	NFG=	0	GRAD TOL	F=	0.2361205389E-09	G=	0.138D-06
21	NIT=	12	NFV=	171	NFG=	0	FV BOUND	F=	0.1479114197E-30	G=	0.222D-15
22	NIT=	10	NFV=	143	NFG=	0	GRAD TOL	F=	0.5059943643E-09	G=	0.247D-06
23	NIT=	19	NFV=	264	NFG=	0	GRAD TOL	F=	0.4393003951E-04	G=	0.361D-06
24	NIT=	17	NFV=	244	NFG=	0	GRAD TOL	F=	0.3081274513E-03	G=	0.820D-06
25	NIT=	10	NFV=	143	NFG=	0	FV BOUND	F=	0.1248755103E-25	G=	0.190D-11
26	NIT=	9	NFV=	133	NFG=	0	GRAD TOL	F=	0.1376154725E-06	G=	0.491D-07
27	NIT=	6	NFV=	91	NFG=	0	FV BOUND	F=	0.9458284017E-18	G=	0.142D-08
28	NIT=	7	NFV=	104	NFG=	0	GRAD TOL	F=	0.2107337939E-10	G=	0.379D-06
29	NIT=	2	NFV=	39	NFG=	0	GRAD TOL	F=	0.7987932563E-13	G=	0.203D-06
30	NIT=	5	NFV=	78	NFG=	0	FV BOUND	F=	0.2322757795E-26	G=	0.181D-12
TOTAL	NIT=	679	NFV=	4161	NFG=	0	NDC=	1549	*	29	
	NCG=	0	NLS=	0	NSC=	0	NUP=	670			
	NRS=	0	NMX=	0	NAD=	0	NRM=	0			

### 6.3 Computation of performance profiles

Performance profiles introduced in [72] are a very suitable way for comparing optimization method, since they take into account results corresponding to individual problems. The performance profile  $\rho_m(\tau)$  is defined by the formula

$$\rho_m(\tau) = \frac{\text{number of problems where } \log_2(\tau_{p,m}) \leq \tau}{\text{total number of problems}}$$

with  $\tau \geq 0$ , where  $\tau_{p,m}$  is the performance ratio between the measured quantity (number of iterations, number of function evaluations, CPU time) to solve problem  $p$  by method  $m$  over the lowest one required. The ratio  $\tau_{p,m}$  is set to infinity (or some large number) if method  $m$  fails to solve problem  $p$ . The value of  $\rho_m(\tau)$  at  $\tau = 0$  gives the percentage of test problems for which the method  $m$  is the best and the value for  $\tau$  large enough is the percentage of test problems that method  $m$  can solve. The relative efficiency and reliability of each method can be directly seen from the performance profiles: the higher is the particular curve the better is the corresponding method.

Data for performance profiles are prepared if we set `$COLLECTION='P'`. In this case, values of NIT (numbers of iterations), NFV (numbers of function evaluations), NFG (numbers of gradient evaluations), NCG (numbers of inner iterations), NTM (CPU time in hundredths of seconds) are saved for individual problems in file `P.PER`. We can specify a name of the method tested by the statement `$NAME='name_of_the_method'`. Before tests are carried out, the file `P.PER` has to be deleted, since results of individual tests are always appended to `P.PER`. Deletion of the file `P.PER` can be specified in the input template by using the statement `$DELETION='Y'`. Performance profiles are computed if it is specified by the macrovariable `$PROFILE`, which can have the following values:

<code>\$PROFILE='NO'</code>	- performance profiles are not computed immediately.
<code>\$PROFILE='NIT'</code>	- performance profiles for NIT are computed immediately.
<code>\$PROFILE='NFV'</code>	- performance profiles for NFV are computed immediately.
<code>\$PROFILE='NFG'</code>	- performance profiles for NFG are computed immediately.
<code>\$PROFILE='NCG'</code>	- performance profiles for NCG are computed immediately.
<code>\$PROFILE='NTM'</code>	- performance profiles for NTM are computed immediately.

The default value is \$PROFILE='NO'. If performance profiles are computed immediately, we can give an upper bound of the parameter  $\tau$  by the statement \$BOUND='upper\_bound' (e.g., \$BOUND='5.0D0'). The default value is \$BOUND='3.0D0'. If macrovariable \$PROFILE has one of the values NIT, NFV, NFG, NCG, NTM, performance profiles for this quantity are computed immediately and corresponding results are written into file P.PRO. More advantageous is to set \$PROFILE='NO' and compute performance profiles from the data given in P.PER prepared by the UFO system (if \$COLLECTION='P'). In this case, we type

```
PROFILE quantity upper_bound
```

where parameter quantity if one of the values NIT, NFV, NFG, NCG, NTM and upper\_bound is a real number (upper bound for  $\tau$ ). Then procedure PROFILE is called, which generates file P.PRO from data contained in P.PER.

The following example demonstrates the computation of performance profiles for three variable metric methods: BFGS without scaling (BFGS - NS), BFGS with preliminary scaling (BFGS - PS) and BFGS with controlled scaling (BFGS - CS). The input template has the form:

```
$REM +-----+
$REM +           MODEL DESCRIPTION           +
$REM +-----+
$SET(INPUT)
  CALL EIUN28(NF,X,FMIN,XMAX,NEXT,IERR)
  IF (NEXT.EQ.24) XMAX=5.0$P 0
  IF(IERR.NE.0) GO TO $$ENDTEST
$ENDSET
$SET(FGMODEL)
  CALL EFBU28(NF,X,FF,GF,NEXT)
$ENDSET
$NF=200
$COLLECTION='P'; $REM : computation of performance profiles
$NEXT=92; $REM : number of test problems
$PROFILE='NTM'; $REM : performance profiles for CPU times
$BOUND='6.0$P 0'; $REM : upper bound for performance ratio
$DELETION='Y'; $REM : preliminary deletion of P.PER

$REM +-----+
$REM +           START OF THE PROGRAM GENERATION           +
$REM +-----+
$BATCH; $REM : the batch mode
$GLOBAL; $REM : global declarations

$REM +-----+
$REM +           THE FIRST METHOD           +
$REM +-----+
$REM : parameters of the first method
$CLASS='VM'
$MET=1
$MET1=1
$MET2=2
$MIT=3000
$MFV=4000
$NAME='BFGS - NS'; $REM : name of the first method
$INITIATION; $REM : model specification and generation
$INPUT; $REM : the user input
```

```
$METHOD; $REM : method specification and generation
$RUNERASE; $REM : clearing parameters of the first run
```

```
$REM +-----+
$REM +           THE SECOND METHOD           +
$REM +-----+
$REM : parameters of the second method
$CLASS='VM'
$MET=1
$MET1=2
$MET2=2
$MIT=3000
$MFV=4000
$NAME='BFGS - PS'; $REM : name of the second method
$INITIATION; $REM : run specification
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation
$RUNERASE; $REM : clearing parameters of the second run
```

```
$REM +-----+
$REM +           THE THIRD METHOD           +
$REM +-----+
$REM : parameters of the third method
$CLASS='VM'
$MET=1
$MET1=3
$MET2=2
$MIT=3000
$MFV=4000
$NAME='BFGS - CS'; $REM : name of the third method
$INITIATION; $REM : run specification
$INPUT; $REM : the user input
$METHOD; $REM : method specification and generation
```

```
$END; $REM : end of the program generation
```

Here \$COLLECTION='P' specifies computation of performance profiles, \$NEXT=92 determines the number of test problems, \$PROFILE='NTM' specifies that CPU times will be considered, \$BOUND='6.0\$P 0' gives the upper bound for performance ratio and \$DELETION='Y' means that file P.PER will be deleted before the computation. Other specifications are explained in Section 4. Using the above template, the UFO system creates file P.PRO which has the form:

```
PERFORMANCE PROFILES:  NTM  BOUND =  6.0000  STEP =  0.2000

METHOD    1  BFGS - NS  EFFICIENCY =  0.6069

0.3804    0.4130    0.4565    0.5000    0.5217    0.5652    0.5761    0.6196
0.7174    0.7174    0.7174    0.7717    0.8043    0.8261    0.8478    0.9022
0.9130    0.9239    0.9457    0.9457    0.9565    0.9565    0.9674    0.9783
0.9783    0.9783    0.9891    0.9891    0.9891    0.9891    1.0000
```

METHOD	2	BFGS - PS	EFFICIENCY =	0.7320				
	0.4783	0.5000	0.5543	0.6196	0.6522	0.7935	0.8043	0.8261
	0.8913	0.8913	0.9130	0.9239	0.9348	0.9348	0.9348	0.9565
	0.9565	0.9565	0.9674	0.9674	0.9674	0.9674	0.9674	0.9674
	0.9674	0.9674	0.9674	0.9674	0.9674	0.9674	0.9674	

METHOD	3	BFGS - CS	EFFICIENCY =	0.8806				
	0.6739	0.7065	0.7826	0.8043	0.8587	0.9457	0.9565	0.9565
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

## 6.4 Interface to the CUTE collection

The CUTE collection [19] is the most famous set of problems for testing optimization methods. These problems are written in the special so-called SIF format. Therefore a SIF decoder is necessary. Such a decoder is part of the CUTE collection, but this version can only be used for a relatively small set of optimization codes (e.g. for the LANCELOT [41] code). Since the UFO system has a special nature, the original SIF decoder had to be modified. This modification consists in replacing the subroutine `SDLANC` by the subroutine `SDUFO` and in preparing new interface subroutines `SIUBXX`, `SIUDXX`, `SIUSXX`, `SINBXX`, `SINDXX`, `SINSXX`, `SFFUXX`, `SFGUXX`, `SFFGUX`, `SCFUXX`, `SCGUXX`, `SCFGUX`, `SAFUXX`, `SAGUXX`, `SAFGUX` (instead of `USETUP`, `UFN`, `UGR`, `CSETUP`, `CFN`, `CGR` etc.).

The CUTE collection is not distributed with the UFO system. The SIF files together with SIF decoder subroutines can be obtained from their authors (they are also available on the INTERNET address <http://www.cse.clrc.ac.uk/nag/cuter/>). The special interface subroutines listed above are exceptions. They are contained in the library `CUTELIB.LIB`.

If we want to use a SIF file for testing the UFO system methods, it suffices to write the macroinstruction `$SIF='SIF_file_name'` in the input batch file. For example, if we want to use problem `DTOC3` for testing recursive quadratic programming methods for sparse equality constrained nonlinear programming problems, then the problem specification (input batch file) has the following form.

```
$SIF='DTOC3'
$FORM='SE'
$JACC='S'
$BATCH
$STANDARD
```

Here `DTOC3` is a name of the SIF file and `SE` is the form of recursive quadratic programming methods (Section 3.31) and `$JACC='S'` means that the sparse Jacobian matrix is considered. The problem solution (basic screen output) has this form.

```
CLASS = MN - LK3   UPDATE = N   MODEL = FF   HESF = S   NF =   1499
  NIC=  0 NIT=  0 NfV=   1 NfG=   8 F=0.000D+00 C=0.150D+02 G=0.000D+00
  NIC=  0 NIT=  1 NfV=   2 NfG=  16 F=0.235D+03 C=0.355D-14 G=0.739D-14
  0 NIC=  0 NIT=  2 NfV=   2 NfG=  16 F=0.235D+03 C=0.355D-14 G=0.739D-14
```

The form of additional output files `P.DIM` and `P.SIF` is shown in Section 5.7.

If the sparsity pattern contains a relatively great number of nonzero elements, then default dimensions (e.g. `$M`, `$MA`, `$MAH`, `$MHA`, `$MC`, `$MCH`, `$MHC`) might be too small and therefore they must be specified in the input batch file.

## 7 Application of the UFO system (examples)

Before the solution of a given problem, the input file containing the problem description and other specifications for the macroprocessor must usually be prepared. This input file can contain only the macroinstruction \$STANDARD (input file STANDARD.UFO). Then a full dialogue is processed. However, a more advantageous possibility is to prepare an input file containing the problem description while a method selection is left to the dialogue. Moreover, since a method selection can be made automatically by using knowledge bases coded in UFO templates, the batch mode is recommended.

When writing input file instructions, we have to observe some conventions. Since the UFO source program contains a great number of common variables, we recommend using variables beginning with the letter 'W' for the problem description to avoid their double use. Real variables of this type should be declared at the beginning of the UFO source program by the statement \$FLOAT (for example \$FLOAT W,W1,W2). Simple integers I,J,K,L need not be declared. We recommend using statement numbers smaller than 10000 for the problem description to avoid their double use.

The basic implementation of the UFO system is in a double precision arithmetic. Therefore usually \$FLOAT='REAL\*8' and \$P='D'. We recommend writing real constants always in the form of \$P or D specification (for example 1.0\$P 2, 4.0\$P-1 or 1.0D 2, 4.0D-1) since the conversions from a single precision, which depend on a compiler, can be incorrect. Instead of constants 0.0D0, 0.5D0, 1.0D0, 2.0D0, 3.0D0, 4.0D0, 5.0D0, 1.0D1, we can use the common variables ZERO, HALF, ONE, TWO, THREE, FOUR, FIVE, TEN, which contain corresponding values.

In the following text, we demonstrate the application of the UFO system to 20 typical problems. Every example consists of the problem description, the problem specification (input file), comments to the problem specification and the problem solution (basic screen output) obtained on a PC computer. All input files contain the necessary data and can be used in the batch mode. These input files are included into the UFO system as the demo-files PROB01.UFO,...,PROB34.UFO.

### 7.1 Optimization with simple bounds

a) Problem description:

Suppose we have to find a local maximum of the objective function

$$F(x) = \frac{1}{n!} \left( \prod_{i=1}^n x_i \right) - 2$$

with simple bounds  $0 \leq x_i \leq i$  for  $1 \leq i \leq n$ , where  $n = 5$ . The starting point is  $x_i = 2$  for  $1 \leq i \leq n$ .

b) Problem specification (input file):

```
$REM +-----+
$REM + DENSE BOX CONSTRAINED MAXIMIZATION +
$REM +-----+
$FLOAT W
$SET(INPUT)
  DO 1 I=1,NF
  X(I)=2.0D0; XL(I)=0.0D0; XU(I)=DBLE(I); IX(I)=3
1 CONTINUE
$ENDSET
$SET(FGMODEL)
  W=1.0D0
  DO 2 I=1,NF
  W=W*X(I)/DBLE(I)
2 CONTINUE
  FF=W-2.0D0
```

```

DO 3 I=1,NF
  GF(I)=W/X(I)
3 CONTINUE
$ENDSET
$IEXT=1
$NF=5
$NX=5
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values and the simple bounds for variables. By using the macrovariable \$FGMODEL we specify analytically the value and the gradient of the model function. Because we look for a maximum, we set \$IEXT=1.

d) Problem solution (basic screen output):

```

CLASS = VM - LG1   UPDATE = B   MODEL = FF   HESF = D   NF =      5
  NIT=   0  NFV=   1  NFG=   1  F=  1.866666667   G=0.667D-01
  NIT=   1  NFV=   4  NFG=   4  F=  1.550000000   G=0.150D+00
  NIT=   2  NFV=   7  NFG=   7  F=  1.200000000   G=0.200D+00
  NIT=   3  NFV=   9  NFG=   9  F=  1.000000000   G=0.000D+00
  0 NIT=   3  NFV=   9  NFG=   9  GRAD TOL F=  1.000000000   G=0.000D+00
FF = -0.1000000000D+01
X  =  0.1000000000D+01  0.2000000000D+01  0.3000000000D+01  0.4000000000D+01
      0.5000000000D+01
TIME= 0:00:00.00

```

## 7.2 Unconstrained least squares optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \frac{1}{2} \sum_{i=1}^m (x_4 e^{-x_1 t_i} - x_5 e^{-x_2 t_i} + x_6 e^{-x_3 t_i} - y_i)^2,$$

where  $m = 20$ ,  $t_i = i/10$  and  $y_i = e^{-t_i} - 5e^{-10t_i} + 3e^{-4t_i}$  for  $1 \leq i \leq m$ . The starting point is  $x_1 = 1$ ,  $x_2 = 2$ ,  $x_3 = 1$ ,  $x_4 = 1$ ,  $x_5 = 1$ ,  $x_6 = 1$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE UNCONSTRAINED LEAST SQUARES +
$REM +-----+
$FLOAT W,WA,WB,WC
$SET(INPUT)
  X(1)=1.0D0; X(2)=2.0D0; X(3)=1.0D0
  X(4)=1.0D0; X(5)=1.0D0; X(6)=1.0D0
  DO 1 KA=1,NA
    W=0.1D0*DBLE(KA)
    AM(KA)=EXP(-W)-5.0D0*EXP(-1.0D1*W)+3.0D0*EXP(-4.0D0*W)
  1 CONTINUE
$ENDSET

```



```

$SET(FMODELA)
W=0.1DO*DBLE(KA)
WA=EXP(-W*X(1))
WB=EXP(-W*X(2))
WC=EXP(-W*X(3))
FA=X(4)*WA-X(5)*WB+X(6)*WC
$ENDSET
$NF=6
$NA=20
$MODEL='AQ'
$KBA=1
$MOS1=1
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the vector AM containing values  $y_i, 1 \leq i \leq m$ . By using the macrovariable \$FMODELA we specify analytically the values of the approximating function. The gradients of the approximating functions are computed numerically. For the sum of squares minimization we set \$MODEL='AQ'. The specification \$KBA=1 indicates that the vector AM is used.

d) Problem solution (basic screen output):

```

CLASS = GN - GM7  UPDATE = N  MODEL = AQ  HESF = D  NF = 6
NIT= 0 NFV= 7 NFG= 0 F= 0.4652437783 G=0.675D+00
NIT= 1 NFV= 14 NFG= 0 F= 0.1755707488 G=0.172D+00
NIT= 2 NFV= 22 NFG= 0 F= 0.1310718542 G=0.103D+00
NIT= 3 NFV= 30 NFG= 0 F= 0.1014547191 G=0.987D-01
NIT= 4 NFV= 37 NFG= 0 F= 0.8325110963E-01 G=0.315D+00
NIT= 5 NFV= 44 NFG= 0 F= 0.5149743622E-01 G=0.225D+00
NIT= 6 NFV= 52 NFG= 0 F= 0.1248663022E-01 G=0.964D-01
NIT= 7 NFV= 59 NFG= 0 F= 0.4692732603E-02 G=0.479D-01
NIT= 8 NFV= 66 NFG= 0 F= 0.2350712826E-02 G=0.292D-01
NIT= 9 NFV= 73 NFG= 0 F= 0.1276233060E-02 G=0.420D-01
NIT= 10 NFV= 80 NFG= 0 F= 0.7512378805E-03 G=0.482D-01
NIT= 11 NFV= 87 NFG= 0 F= 0.3065852206E-03 G=0.395D-01
NIT= 12 NFV= 95 NFG= 0 F= 0.3583746062E-04 G=0.409D-02
NIT= 13 NFV= 103 NFG= 0 F= 0.2339698105E-04 G=0.374D-03
NIT= 14 NFV= 110 NFG= 0 F= 0.1221490232E-04 G=0.272D-02
NIT= 15 NFV= 117 NFG= 0 F= 0.7308108434E-05 G=0.287D-02
NIT= 16 NFV= 124 NFG= 0 F= 0.3876069070E-05 G=0.202D-02
NIT= 17 NFV= 131 NFG= 0 F= 0.2023584146E-05 G=0.190D-02
NIT= 18 NFV= 138 NFG= 0 F= 0.1029329293E-05 G=0.193D-02
NIT= 19 NFV= 145 NFG= 0 F= 0.6760021848E-06 G=0.196D-02
NIT= 20 NFV= 152 NFG= 0 F= 0.5022053254E-09 G=0.553D-04
NIT= 21 NFV= 159 NFG= 0 F= 0.4846290353E-19 G=0.610D-09
0 NIT= 21 NFV= 159 NFG= 0 FV BOUND F= 0.4846290353E-19 G=0.610D-09
F = 0.4846290353D-19
X = 0.1000000000D+01 0.1000000000D+02 0.4000000000D+01 0.1000000000D+01
    0.5000000000D+01 0.3000000000D+01
TIME= 0:00:00.00

```

### 7.3 Unconstrained least powers optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \frac{1}{4} \sum_{i=1}^m (x_4 e^{-x_1 t_i} - x_5 e^{-x_2 t_i} + x_6 e^{-x_3 t_i} - y_i)^4,$$

where  $m = 20$ ,  $t_i = i/10$  and  $y_i = e^{-t_i} - 5e^{-10t_i} + 3e^{-4t_i}$  for  $1 \leq i \leq m$ . The starting point is  $x_1 = 1$ ,  $x_2 = 2$ ,  $x_3 = 1$ ,  $x_4 = 1$ ,  $x_5 = 1$ ,  $x_6 = 1$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE UNCONSTRAINED LEAST POWERS +
$REM +-----+
$FLOAT W,WA,WB,WC
$SET(INPUT)
  X(1)=1.0D0; X(2)=2.0D0; X(3)=1.0D0
  X(4)=1.0D0; X(5)=1.0D0; X(6)=1.0D0
  DO 1 KA=1,NA
    W=0.1D0*DBLE(KA)
    AM(KA)=EXP(-W)-5.0D0*EXP(-1.0D1*W)+3.0D0*EXP(-4.0D0*W)
  1 CONTINUE
$ENDSET
$SET(FMODEL)
  W=0.1D0*DBLE(KA)
  WA=EXP(-W*X(1))
  WB=EXP(-W*X(2))
  WC=EXP(-W*X(3))
  FA=X(4)*WA-X(5)*WB+X(6)*WC
$ENDSET
$NF=6
$NA=20
$MODEL='AP'
$REXP='4.0D0'
$KBA=1
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the vector AM containing values  $y_i$ ,  $1 \leq i \leq m$ . By using the macrovariable \$FMODEL we specify analytically the values of the approximating function. The gradients of the approximating functions are computed numerically. For the sum of fourth powers minimization we set \$MODEL='AP' and \$REXP='4.0D0' (macrovariable \$REXP defines a corresponding power). The specification \$KBA=1 indicates that the vector AM is used.

d) Problem solution (basic screen output):

```

CLASS = GN - GM7   UPDATE = N   MODEL = AP   HESF = D   NF =      6
  NIT=    0 NfV=    7 NfG=    0 F= 0.3430916991E-01 G=0.194D+00
  NIT=    1 NfV=   14 NfG=    0 F= 0.1419887081E-01 G=0.665D-01
  NIT=    2 NfV=   21 NfG=    0 F= 0.4944922455E-02 G=0.111D-01

```

```

NIT=   3 NFV=  28 NFG=   0 F= 0.3678302435E-02 G=0.124D-02
NIT=   4 NFV=  35 NFG=   0 F= 0.2424810434E-02 G=0.624D-02
NIT=   5 NFV=  42 NFG=   0 F= 0.9409619052E-03 G=0.294D-02
NIT=   6 NFV=  50 NFG=   0 F= 0.6512073068E-03 G=0.111D-02
NIT=   7 NFV=  57 NFG=   0 F= 0.4137478404E-03 G=0.615D-03
NIT=   8 NFV=  64 NFG=   0 F= 0.2198851619E-03 G=0.505D-03
NIT=   9 NFV=  72 NFG=   0 F= 0.1728982452E-03 G=0.122D-03
NIT=  10 NFV=  79 NFG=   0 F= 0.1213304154E-03 G=0.272D-03
NIT=  11 NFV=  86 NFG=   0 F= 0.6351299654E-04 G=0.679D-03
NIT=  12 NFV=  93 NFG=   0 F= 0.1567466646E-04 G=0.685D-04
NIT=  13 NFV= 100 NFG=   0 F= 0.4367175375E-05 G=0.121D-03
NIT=  14 NFV= 107 NFG=   0 F= 0.9045737965E-06 G=0.307D-04
NIT=  15 NFV= 114 NFG=   0 F= 0.2000673033E-06 G=0.119D-04
NIT=  16 NFV= 121 NFG=   0 F= 0.4446122812E-07 G=0.463D-05
NIT=  17 NFV= 128 NFG=   0 F= 0.9700186794E-08 G=0.167D-05
NIT=  18 NFV= 135 NFG=   0 F= 0.2066473108E-08 G=0.561D-06
0 NIT=  18 NFV= 135 NFG=   0 GRAD TOL F= 0.2066473108E-08 G=0.561D-06
F = 0.2066473108D-08
X = 0.1014994274D+01 0.9976207912D+01 0.3907664874D+01 0.1011751654D+01
    0.4777298750D+01 0.2851199655D+01
TIME= 0:00:00.00

```

## 7.4 Unconstrained minimax optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max_{1 \leq i \leq m} \left| \frac{x_1 + t_i x_2}{1 + t_i x_3 + t_i^2 x_4 + t_i^3 x_5} - y_i \right|,$$

where  $m = 21$ ,  $t_i = (i-1)/10 - 1$  and  $y_i = e^{t_i}$  for  $1 \leq i \leq m$ . The starting point is  $x_1 = 0.5$ ,  $x_2 = 0$ ,  $x_3 = 0$ ,  $x_4 = 0$ ,  $x_5 = 0$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE UNCONSTRAINED MINIMAX OPTIMIZATION +
$REM +-----+
$FLOAT W
$SET(INPUT)
  X(1)=0.5D0; X(2)=0.0D0; X(3)=0.0D0; X(4)=0.0D0; X(5)=0.0D0
$ENDSET
$SET(FMODELA)
  W=0.1D0*DBLE(KA-1)-1.0D0
  FA=(X(1)+W*X(2))/(1.0D0+W*(X(3)+W*(X(4)+W*X(5))))-EXP(W)
$ENDSET
$MODEL='AM'
$NF=5
$NA=21
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables. By using the macrovariable \$FMODELA we specify analytically the values of the approximating functions. The gradients of the approximating functions are computed numerically. For minimax approximation we set \$MODEL='AM'.

d) Problem solution (basic screen output):

```

CLASS = VM - LQ1   UPDATE = B   MODEL = AM   HESF = D   NF =      5
  NIT=   0 NFV=   6 NFG=   0 F=  2.218281828   G=0.100+121
  NIT=   1 NFV=  13 NFG=   0 F=  0.4253776661   G=0.783D+00
  NIT=   2 NFV=  19 NFG=   0 F=  0.8271278131E-01 G=0.223D+00
  NIT=   3 NFV=  25 NFG=   0 F=  0.1256835525E-01 G=0.114D+00
  NIT=   4 NFV=  31 NFG=   0 F=  0.6693249048E-02 G=0.290D-01
  NIT=   5 NFV=  37 NFG=   0 F=  0.6121578248E-02 G=0.140D-01
  NIT=   6 NFV=  43 NFG=   0 F=  0.1771793241E-02 G=0.550D-01
  NIT=   7 NFV=  49 NFG=   0 F=  0.2044251553E-03 G=0.208D-01
  NIT=   8 NFV=  55 NFG=   0 F=  0.1225222546E-03 G=0.833D-04
  NIT=   9 NFV=  61 NFG=   0 F=  0.1223712525E-03 G=0.240D-07
0 NIT=   9 NFV=  61 NFG=   0 GRAD TOL F= 0.1223712525E-03 G=0.240D-07
F = 0.1223712525D-03
X = 0.9998776287D+00 0.2535884404D+00 -0.7466075717D+00 0.2452015019D+00
    -0.3749029100D-01
TIME= 0:00:00.00

```

## 7.5 Unconstrained nonsmooth optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = -x_1 + 2 * (x_1^2 + x_2^2 - 1) + \frac{7}{4} |x_1^2 + x_2^2 - 1|.$$

The starting point is  $x_1 = -1$ ,  $x_2 = -1$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE UNCONSTRAINED NONSMOOTH OPTIMIZATION +
$REM +-----+
$FLOAT W
$SET(INPUT)
  X(1)=-1.0D0
  X(2)=-1.0D0
$ENDSET
$SET(FGMODEL F)
  W=X(1)**2+X(2)**2-1.0D0
  FF=-X(1)+2.0D0*W+1.75D0*ABS(W)
  W=SIGN(3.5D0,W)+4.0D0
  GF(1)=W*X(1)-1.0D0
  GF(2)=W*X(2)
$ENDSET
$NF=2
$KSF=3
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables. By using the macrovariable \$FGMODEL we specify analytically the value and the gradient of the objective function. Since the objective function is nonsmooth, we set \$KSF=3.

d) Problem solution (basic screen output):

```

CLASS = BM - L11    UPDATE = N    MODEL = FF    HESF = N    NF =      2
  NIT=   0 NFV=   1 NFG=   1 F=  4.750000000    G=0.100+121
  NIT=   1 NFV=   3 NFG=   3 F=-0.378888889    G=0.850D+01
  NIT=   2 NFV=   4 NFG=   4 F=-0.6061514369   G=0.933D+00
  NIT=   3 NFV=   5 NFG=   5 F=  9.250000000    G=0.802D+00
  NIT=   4 NFV=   6 NFG=   6 F=-0.7284826639   G=0.802D+00
  NIT=   5 NFV=   7 NFG=   7 F=  0.8708184712   G=0.348D+00
  NIT=   6 NFV=   8 NFG=   8 F=-0.8275709577   G=0.722D+00
  NIT=   7 NFV=   9 NFG=   9 F=-0.8436035754   G=0.162D+00
  NIT=   8 NFV=  10 NFG=  10 F=-0.9986081259   G=0.984D-01
  NIT=   9 NFV=  11 NFG=  11 F=-0.9980886265   G=0.114D+00
  NIT=  10 NFV=  12 NFG=  12 F=-0.9992851884   G=0.501D+00
  NIT=  11 NFV=  13 NFG=  13 F=-0.9999999867   G=0.530D-01
  NIT=  12 NFV=  14 NFG=  14 F=-0.9999922205   G=0.454D-05
  NIT=  13 NFV=  15 NFG=  15 F= -1.000000000    G=0.986D-07
  0 NIT=  13 NFV=  15 NFG=  15 GRAD TOL F= -1.000000000    G=0.986D-07
FF = -0.1000000000D+01
X   =  0.1000000000D+01  0.0000000000D+00
TIME= 0:00:00.00

```

## 7.6 Optimization with linear constraints

a) Problem specification:

Suppose we have to find a local minimum of the objective function

$$F(x) = (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6$$

over the set given by the linear constraints

$$x_1 + x_2 + x_3 + 4x_4 = 7,$$

$$x_3 + 5x_5 = 6.$$

The starting point is  $x_1 = 10$ ,  $x_2 = 7$ ,  $x_3 = 2$ ,  $x_4 = -3$ ,  $x_5 = 0.8$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE MINIMIZATION WITH LINEAR CONSTRAINTS +
$REM +-----+
$SET(INPUT)
X(1)= 1.0D1; X(2)=7.0D0; X(3)= 2.0D0
X(4)=-3.0D0; X(5)=0.8D0
IC(1)=5; CL(1)=7.0D0
CG(1)=1.0D0; CG(2)=1.0D0; CG(3)=1.0D0
CG(4)=4.0D0; CG(5)=0.0D0
IC(2)=5; CL(2)=6.0D0

```

```

CG(6)=0.0D0; CG(7)=0.0D0; CG(8)=1.0D0
CG(9)=0.0D0; CG(10)=5.0D0
$ENDSET
$SET(FMODEL)
FF=(X(1)-X(2))**2+(X(3)-1.0D0)**2+(X(4)-1.0D0)**4+(X(5)-1.0D0)**6
$ENDSET
$SET(GMODEL)
GF(1)= 2.0D0*(X(1)-X(2))
GF(2)=-2.0D0*(X(1)-X(2))
GF(3)= 2.0D0*(X(3)-1.0D0)
GF(4)= 4.0D0*(X(4)-1.0D0)**3
GF(5)= 6.0D0*(X(5)-1.0D0)**5
$ENDSET
$NF=5
$NC=2
$NCL=2
$FMIN='0.0D0'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the types and values of the linear constraints. By using the macrovariable \$FMODEL we specify analytically the value of the model function. By using the macrovariable \$GMODEL we specify analytically the gradient of the model function. The specification \$FMIN='0.0D0' is used, since the objective function value cannot be smaller than zero.

d) Problem solution (basic screen output):

```

CLASS = VM - LG1   UPDATE = B   MODEL = FF   HESF = D   NF =      5
  NIT=   0 NFV=   1 NFG=   1 F=  266.0000640   G=0.853D+02
  NIT=   1 NFV=   2 NFG=   2 F=  23.36590202   G=0.911D+01
  NIT=   2 NFV=   3 NFG=   3 F=   9.082029409   G=0.573D+01
  NIT=   3 NFV=   4 NFG=   4 F=   1.180947507   G=0.213D+01
  NIT=   4 NFV=   5 NFG=   5 F=  0.3278298455   G=0.713D+00
  NIT=   5 NFV=   6 NFG=   6 F=  0.1969059589   G=0.356D+00
  NIT=   6 NFV=   7 NFG=   7 F=  0.6972244149E-01 G=0.176D+00
  NIT=   7 NFV=   8 NFG=   8 F=  0.1791441524E-01 G=0.115D+00
  NIT=   8 NFV=   9 NFG=   9 F=  0.5643511948E-02 G=0.887D-01
  NIT=   9 NFV=  10 NFG=  10 F=  0.2190093826E-02 G=0.438D-01
  NIT=  10 NFV=  11 NFG=  11 F=  0.8383567168E-03 G=0.639D-02
  NIT=  11 NFV=  12 NFG=  12 F=  0.1933645225E-03 G=0.101D-01
  NIT=  12 NFV=  13 NFG=  13 F=  0.5495677142E-04 G=0.137D-02
  NIT=  13 NFV=  14 NFG=  14 F=  0.1463468059E-04 G=0.117D-02
  NIT=  14 NFV=  15 NFG=  15 F=  0.4872666765E-05 G=0.220D-02
  NIT=  15 NFV=  16 NFG=  16 F=  0.1203915690E-05 G=0.118D-02
  NIT=  16 NFV=  17 NFG=  17 F=  0.7914796148E-06 G=0.108D-02
  NIT=  17 NFV=  18 NFG=  18 F=  0.2070004312E-06 G=0.164D-03
  NIT=  18 NFV=  19 NFG=  19 F=  0.6014332855E-07 G=0.162D-04
  NIT=  19 NFV=  20 NFG=  20 F=  0.1821407821E-07 G=0.116D-03
  NIT=  20 NFV=  21 NFG=  21 F=  0.9713555354E-08 G=0.201D-03
  NIT=  21 NFV=  22 NFG=  22 F=  0.2625654615E-08 G=0.406D-04
  NIT=  22 NFV=  23 NFG=  23 F=  0.1884926025E-08 G=0.508D-04

```

```

NIT= 23 NFV= 24 NFG= 24 F= 0.8035144110E-09 G=0.107D-04
NIT= 24 NFV= 25 NFG= 25 F= 0.2551768226E-09 G=0.238D-05
NIT= 25 NFV= 26 NFG= 26 F= 0.7613012252E-10 G=0.776D-05
NIT= 26 NFV= 27 NFG= 27 F= 0.2444365765E-10 G=0.706D-05
NIT= 27 NFV= 28 NFG= 28 F= 0.9517068620E-11 G=0.249D-05
NIT= 28 NFV= 29 NFG= 29 F= 0.3624829699E-11 G=0.832D-06
0 NIT= 28 NFV= 29 NFG= 29 GRAD TOL F= 0.3624829699E-11 G=0.832D-06
FF = 0.3624829699D-11
X = 0.1002725618D+01 0.1002725912D+01 0.1000000296D+01 0.9986370434D+00
0.9999999409D+00
TIME= 0:00:00.00

```

## 7.7 Least squares optimization with linear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max_{1 \leq i \leq 163} \left( \frac{1}{15} + \frac{2}{15} \sum_{j=1}^7 \cos(2\pi x_j \sin \vartheta_i) \right),$$

where  $\vartheta_i = \pi(8.5 + i0.5)/180$  for  $1 \leq i \leq 163$ , over set given by the simple constraints  $x_1 \geq 0.4$ ,  $x_7 = 3.5$  and by the linear constraints

$$\begin{aligned}
-x_1 + x_2 &\geq 0.4, \\
-x_2 + x_3 &\geq 0.4, \\
-x_3 + x_4 &\geq 0.4, \\
-x_4 + x_5 &\geq 0.4, \\
-x_5 + x_6 &\geq 0.4, \\
-x_6 + x_7 &\geq 0.4, \\
-x_4 + x_6 &= 1.0.
\end{aligned}$$

The starting point is  $x_1 = 0.5$ ,  $x_2 = 1.0$ ,  $x_3 = 1.5$ ,  $x_4 = 2.0$ ,  $x_5 = 2.5$ ,  $x_6 = 3.0$ ,  $x_7 = 3.5$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE LEAST SQUARES WITH LINEAR CONSTRAINTS +
$REM +-----+
$FLOAT W(163),WA,WPI
$SET(INPUT)
  WPI=3.1415926535897932D0
  DO 1 I=1,NF
    X(I)=DBLE(I)*0.5D0; IX(I)=0
1 CONTINUE
  XL(1)=0.4D0; IX(1)=1; IX(7)=5
  DO 2 I=1,NA
    W(I)=2.0D0*WPI*SIN(WPI*(8.5D0+DBLE(I)*0.5D0)/1.8D2)
2 CONTINUE
  DO 3 I=1,6
    CL(I)=0.4D0; IC(I)=1
3 CONTINUE

```

```

CL(7)=1.0D0; CU(7)=1.0D0; IC(7)=5
DO 4 I=1,NF*NC
  CG(I)=0.0D0
4 CONTINUE
K=0
DO 5 I=1,6
  CG(K+I)=-1.0D0; CG(K+I+1)=1.0D0
  K=K+NF
5 CONTINUE
CG(46)=-1.0D0; CG(48)= 1.0D0
$ENDSET
$SET(FMODELA)
WA=0.0D0
DO 6 I=1,NF
  WA=WA+COS(W(KA)*X(I))
6 CONTINUE
FA=(1.0D0+2.0D0*WA)/1.5D1
$ENDSET
$MODEL='AQ'
$NF=7
$NX=7
$NA=163
$NC=7
$NCL=7
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, types and values of the simple bounds and types and values of the linear constraints. By using the macrovariable \$FMODELA we specify analytically the values of the approximating function. The gradients of the approximating functions are computed numerically. For the sum of squares minimization we set \$MODEL='AQ'.

d) Problem solution (basic screen output):

CLASS =	GN -	GM7	UPDATE =	N	MODEL =	AQ	HESF =	D	NF =	7
NIT=	0	NFV=	7	NFG=	0	F=	0.4638565295	G=	0.114D+01	
NIT=	1	NFV=	14	NFG=	0	F=	0.4461088576	G=	0.327D+00	
NIT=	2	NFV=	21	NFG=	0	F=	0.4348755804	G=	0.383D+00	
NIT=	3	NFV=	28	NFG=	0	F=	0.4117773441	G=	0.557D+00	
NIT=	4	NFV=	35	NFG=	0	F=	0.3640897673	G=	0.831D+00	
NIT=	5	NFV=	42	NFG=	0	F=	0.2855278593	G=	0.118D+01	
NIT=	6	NFV=	49	NFG=	0	F=	0.2459276580	G=	0.145D+01	
NIT=	7	NFV=	56	NFG=	0	F=	0.2428262783	G=	0.805D+00	
NIT=	8	NFV=	63	NFG=	0	F=	0.2346115275	G=	0.263D-01	
NIT=	9	NFV=	70	NFG=	0	F=	0.2345064740	G=	0.623D-02	
NIT=	10	NFV=	77	NFG=	0	F=	0.2345039665	G=	0.752D-03	
NIT=	11	NFV=	84	NFG=	0	F=	0.2345039533	G=	0.837D-04	
NIT=	12	NFV=	91	NFG=	0	F=	0.2345039531	G=	0.966D-05	
NIT=	13	NFV=	98	NFG=	0	F=	0.2345039531	G=	0.111D-05	
NIT=	14	NFV=	105	NFG=	0	F=	0.2345039531	G=	0.128D-06	
0 NIT=	14	NFV=	105	NFG=	0	GRAD TOL	F= 0.2345039531	G=	0.128D-06	



```

F = 0.2345039531D+00
X = 0.4000000000D+00 0.8399686447D+00 0.1239968645D+01 0.1761071079D+01
    0.2161071079D+01 0.2761071079D+01 0.3500000000D+01
TIME= 0:00:00.00

```

## 7.8 Minimax optimization with linear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max(-\exp(x_1 - x_2), \sinh(x_1 - 1) - 1, -\log(x_2) - 1)$$

over the set given by the simple constraint  $x_2 \geq 1/100$  and by the linear constraint

$$\frac{5}{100} x_1 - x_2 + \frac{1}{2} \geq 0.$$

The starting point is  $x_1 = -1$ ,  $x_2 = 1/100$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE MINIMAX OPTIMIZATION WITH LINEAR CONSTRAINTS +
$REM +-----+
$SET(INPUT)
  X(1)=-1.0D 0;          IX(1)=0
  X(2)= 1.0D-2; XL(2)= 1.0D-2; IX(2)=1
                    CL(1)=-5.0D-1; IC(1)=1
  CG(1)=5.0D-2; CG(2)=-1.0D 0
$ENDSET
$SET(FMODELA)
  IF (KA.EQ.1) FA=-EXP(X(1)-X(2))
  IF (KA.EQ.2) FA= SINH(X(1)-1.0D0)-1.0D0
  IF (KA.EQ.3) FA=-LOG(X(2))-1.0D0
$ENDSET
$MODEL='AM'
$IEXT=-1
$NF=2
$NX=2
$NA=3
$NC=1
$NCL=1
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, types and values of the simple bounds and types and values of the linear constraints. By using the macrovariable \$FMODELA we specify analytically the values of the approximating functions. The gradients of the approximating functions are computed numerically. For minimax approximation we set \$MODEL='AM' and \$IEXT=-1.

d) Problem solution (basic screen output):

```

CLASS = VM - LQ1   UPDATE = B   MODEL = AM   HESF = D   NF =      2
  NIT=    0 NFV=    3 NFG=    0 F=  3.605170186   G=0.100+121
  NIT=    1 NFV=    6 NFG=    0 F=  1.978554385   G=0.363D+00
  NIT=    2 NFV=    9 NFG=    0 F=  0.6817245960   G=0.487D+00
  NIT=    3 NFV=   12 NFG=    0 F=-0.2766371994   G=0.595D+00
  NIT=    4 NFV=   15 NFG=    0 F=-0.3838143331   G=0.344D+00
  NIT=    5 NFV=   18 NFG=    0 F=-0.4394007715   G=0.281D-01
  NIT=    6 NFV=   21 NFG=    0 F=-0.4488931954   G=0.124D-02
  NIT=    7 NFV=   24 NFG=    0 F=-0.4489107860   G=0.215D-05
  0 NIT=    8 NFV=   24 NFG=    0 GRAD TOL F=-0.4489107860   G=0.737D-11
F   = -0.4489107860D+00
X    =  0.1526434615D+01  0.5763217308D+00
TIME= 0:00:00.00

```

## 7.9 Nonsmooth optimization with linear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max \left( -\sqrt{(x_1 - x_3)^2 + (x_2 - x_4)^2}, -\sqrt{(x_3 - x_5)^2 + (x_4 - x_6)^2}, -\sqrt{(x_5 - x_1)^2 + (x_6 - x_2)^2} \right)$$

over the set given by the linear constraints

$$x_1 \cos(2\pi i/5) + x_2 \sin(2\pi i/5) \leq 1, \quad 0 \leq j \leq 4,$$

$$x_3 \cos(2\pi i/5) + x_4 \sin(2\pi i/5) \leq 1, \quad 0 \leq j \leq 4,$$

$$x_5 \cos(2\pi i/5) + x_6 \sin(2\pi i/5) \leq 1, \quad 0 \leq j \leq 4.$$

The starting point is  $x_1 = -1$ ,  $x_2 = 0$ ,  $x_3 = 0$ ,  $x_4 = -1$ ,  $x_5 = 1$ ,  $x_6 = 1$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE NONSMOOTH OPTIMIZATION WITH LINEAR CONSTRAINTS +
$REM +-----+
$FLOAT W1,W2,W3,WA,WPI
$SET(INPUT)
  WPI=3.1415926535897932D0
  X(1)=-1.0D0; X(2)= 0.0D0; X(3)= 0.0D0
  X(4)=-1.0D0; X(5)= 1.0D0; X(6)= 1.0D0
  DO 1 I=1,NC
    CU(I)=1.0D0; IC(I)=2
1 CONTINUE
  DO 2 I=1,NF*NC
    CG(I)=0.0D0
2 CONTINUE
  K=1
  DO 4 I=1,3
    L=2*(I-1)

```

```

DO 3 J=1,5
  CG(K+L)=SIN(2.0D0*WPI*DBLE(J-1)/5.0D0)
  CG(K+L+1)=COS(2.0D0*WPI*DBLE(J-1)/5.0D0)
  K=K+NF
3 CONTINUE
4 CONTINUE
$ENDSET
$SET(FMODEL)
  W1=-SQRT((X(1)-X(3))**2+(X(2)-X(4))**2)
  W2=-SQRT((X(3)-X(5))**2+(X(4)-X(6))**2)
  W3=-SQRT((X(5)-X(1))**2+(X(6)-X(2))**2)
  FF=MAX(W1,W2,W3)
$ENDSET
$NF=6
$NX=6
$NC=15
$NCL=15
$KSF=3
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the types and values of the linear constraints. By using the macrovariable \$FMODEL we specify analytically the value of the objective function. The subgradients of the objective function is computed numerically. Since the objective function is nonsmooth, we set \$KSF=3.

d) Problem solution (basic screen output):

```

CLASS = BM - L11  UPDATE = N  MODEL = FF  HESF = N  NF = 6
  NIT= 0 NFV= 7 NFG= 0 F= -1.414213562 G=0.100+121
  NIT= 1 NFV= 14 NFG= 0 F= -1.723937907 G=0.447D+00
  NIT= 2 NFV= 21 NFG= 0 F= -1.769198225 G=0.800D-01
  NIT= 3 NFV= 28 NFG= 0 F= -1.819486899 G=0.106D+00
  NIT= 4 NFV= 35 NFG= 0 F= -1.831224488 G=0.807D-01
  NIT= 5 NFV= 42 NFG= 0 F= -1.833047757 G=0.802D-01
  NIT= 6 NFV= 49 NFG= 0 F= -1.725011873 G=0.801D-01
  NIT= 7 NFV= 56 NFG= 0 F= -1.843442148 G=0.791D-01
  NIT= 8 NFV= 63 NFG= 0 F= -1.852803685 G=0.609D-01
  NIT= 9 NFV= 70 NFG= 0 F= -1.827414255 G=0.609D-01
  NIT= 10 NFV= 77 NFG= 0 F= -1.857698751 G=0.324D-01
  NIT= 11 NFV= 84 NFG= 0 F= -1.857554478 G=0.310D-01
  NIT= 12 NFV= 91 NFG= 0 F= -1.859265289 G=0.134D+00
  NIT= 13 NFV= 98 NFG= 0 F= -1.859548882 G=0.173D-01
  NIT= 14 NFV= 105 NFG= 0 F= -1.859596171 G=0.135D-01
  NIT= 15 NFV= 112 NFG= 0 F= -1.859614932 G=0.166D-03
  NIT= 16 NFV= 119 NFG= 0 F= -1.859617277 G=0.980D-05
  NIT= 17 NFV= 126 NFG= 0 F= -1.859615967 G=0.206D-04
  NIT= 18 NFV= 133 NFG= 0 F= -1.859618401 G=0.158D-04
  NIT= 19 NFV= 140 NFG= 0 F= -1.859618685 G=0.573D-05
  NIT= 20 NFV= 147 NFG= 0 F= -1.859618696 G=0.220D-07
0 NIT= 20 NFV= 147 NFG= 0 GRAD TOL F= -1.859618696 G=0.220D-07

```

```

FF = -0.1859618696D+01
X   = -0.9723036844D+00  0.2436249348D+00  0.5321594418D+00 -0.8494315113D+00
      0.7265425280D+00  0.1000000000D+01
TIME= 0:00:00.00

```

## 7.10 Optimization with nonlinear constraints (nonlinear programming)

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = x_1 x_3$$

over the set given by the simple bounds  $x_1 \geq 0$ ,  $x_3 \geq 0$ ,  $x_5 \geq 1$ ,  $x_7 \geq 1$  and by the nonlinear constraints

$$(x_4 - x_6)^2 + (x_5 - x_7)^2 \geq 4,$$

$$\frac{x_3 x_4 - x_2 x_5}{\sqrt{x_2^2 + x_3^2}} \geq 1,$$

$$\frac{x_3 x_6 - x_2 x_7}{\sqrt{x_2^2 + x_3^2}} \geq 1,$$

$$\frac{x_1 x_3 + (x_2 - x_1) x_5 - x_3 x_4}{\sqrt{(x_2 - x_1)^2 + x_3^2}} \geq 1,$$

$$\frac{x_1 x_3 + (x_2 - x_1) x_7 - x_3 x_6}{\sqrt{(x_2 - x_1)^2 + x_3^2}} \geq 1.$$

The starting point is  $x_1 = 3.0$ ,  $x_2 = 0.0$ ,  $x_3 = 2.0$ ,  $x_4 = -1.5$ ,  $x_5 = 1.5$ ,  $x_6 = 5.0$ ,  $x_7 = 0.0$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE MINIMIZATION WITH NONLINEAR CONSTRAINTS +
$REM +-----+
$FLOAT W
$SET(INPUT)
  X(1)= 3.0D0; XL(1)= 0.0D0; IX(1)= 1
  X(2)= 0.0D0
  X(3)= 2.0D0; XL(3)= 0.0D0; IX(3)= 1
  X(4)=-1.5D0
  X(5)= 1.5D0; XL(5)= 1.0D0; IX(5)= 1
  X(6)= 5.0D0
  X(7)= 0.0D0; XL(7)= 1.0D0; IX(7)= 1
  CL(1)=4.0D0; IC(1)= 1
  CL(2)=1.0D0; IC(2)= 1
  CL(3)=1.0D0; IC(3)= 1
  CL(4)=1.0D0; IC(4)= 1
  CL(5)=1.0D0; IC(5)= 1
$ENDSET
$SET(FMODEL)
  FF=X(1)*X(3)
$ENDSET

```

```

$SET(FMODEL C)
IF(KC.LE.0)THEN
ELSEIF(KC.EQ.1)THEN
    FC=(X(4)-X(6))**2+(X(5)-X(7))**2
ELSEIF(KC.EQ.2)THEN
    W=SQRT(X(2)**2+X(3)**2)
    FC=(X(3)*X(4)-X(2)*X(5))/W
ELSEIF(KC.EQ.3)THEN
    W=SQRT(X(2)**2+X(3)**2)
    FC=(X(3)*X(6)-X(2)*X(7))/W
ELSEIF(KC.EQ.4)THEN
    W=SQRT((X(2)-X(1))**2+X(3)**2)
    FC=(X(1)*X(3)+(X(2)-X(1))*X(5)-X(3)*X(4))/W
ELSEIF(KC.EQ.5)THEN
    W=SQRT((X(2)-X(1))**2+X(3)**2)
    FC=(X(1)*X(3)+(X(2)-X(1))*X(7)-X(3)*X(6))/W
ENDIF
$ENDSET
$IADF=1
$IADC=1
$NF=7
$NX=7
$NC=5
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of the variables, types and values of the simple bounds and types and values of the nonlinear constraints. By using the macrovariable \$FMODEL F we specify the value of the model function analytically. The gradient of the model function is computed by automatic differentiation, since \$IADF=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used.

d) Problem solution (basic screen output):

```

CLASS = VM - LQ1    UPDATE = B    MODEL = FF    HESF = D    NF =      7
NIC=  0 NIT=  0 NFV=   1 NFG=   1 F=0.600D+01 C=0.294D+01 G=0.000D+00
NIC=  0 NIT=  1 NFV=   3 NFG=   3 F=0.340D+02 C=0.961D+00 G=0.267D+01
NIC=  0 NIT=  2 NFV=   5 NFG=   5 F=0.292D+02 C=0.540D-01 G=0.139D+01
NIC=  0 NIT=  3 NFV=   7 NFG=   7 F=0.247D+02 C=0.156D-01 G=0.105D+01
NIC=  0 NIT=  4 NFV=   9 NFG=   9 F=0.236D+02 C=0.356D-01 G=0.691D+00
NIC=  0 NIT=  5 NFV=  11 NFG=  11 F=0.235D+02 C=0.277D-01 G=0.797D+00
NIC=  0 NIT=  6 NFV=  13 NFG=  13 F=0.233D+02 C=0.171D-02 G=0.127D+00
NIC=  0 NIT=  7 NFV=  15 NFG=  15 F=0.233D+02 C=0.122D-03 G=0.107D+00
NIC=  0 NIT=  8 NFV=  17 NFG=  17 F=0.233D+02 C=0.345D-03 G=0.182D+00
NIC=  0 NIT=  9 NFV=  19 NFG=  19 F=0.233D+02 C=0.122D-04 G=0.101D+00
NIC=  0 NIT= 10 NFV=  21 NFG=  21 F=0.233D+02 C=0.188D-04 G=0.308D-01
NIC=  0 NIT= 11 NFV=  23 NFG=  23 F=0.233D+02 C=0.604D-08 G=0.457D-03
NIC=  0 NIT= 12 NFV=  25 NFG=  25 F=0.233D+02 C=0.701D-10 G=0.707D-04
NIC=  0 NIT= 13 NFV=  27 NFG=  27 F=0.233D+02 C=0.611D-13 G=0.171D-05
0 NIC=  0 NIT= 14 NFV=  27 NFG=  27 F=0.233D+02 C=0.611D-13 G=0.228D-09

```

```

FF = 0.2331370850D+02
X   = 0.4828427125D+01 -0.4592416296D-10 0.4828427125D+01 0.1000000000D+01
      0.2414213562D+01 0.2414213562D+01 0.1000000000D+01
TIME= 0:00:00.00

```

## 7.11 Least squares optimization with nonlinear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = (x_1 - x_4)^2 + (x_2 - x_5)^2 + (x_3 - x_6)^2$$

over the set given by the simple bound  $4 \leq x_6 \leq 8$  and by the nonlinear constraints

$$x_1^2 + x_2^2 + x_3^2 \leq 5,$$

$$x_5^2 + (x_4 - 3)^2 \leq 1.$$

The starting point is  $x_1 = 1$ ,  $x_2 = 1$ ,  $x_3 = 1$ ,  $x_4 = 3$ ,  $x_5 = 0$ ,  $x_6 = 0.5$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE LEAST SQUARES WITH NONLINEAR CONSTRAINTS +
$REM +-----+

$SET(INPUT)
  X(1)=1.0D0; X(2)=1.0D0; X(3)=1.0D0
  X(4)=3.0D0; X(5)=0.0D0; X(6)=0.5D0
  IX(6)=3; XL(6)=4.0D0; XU(6)=8.0D0
  IC(1)=2; CU(1)=5.0D0
  IC(2)=2; CU(2)=1.0D0
$ENDSET

$SET(FMODEL A)
  IF(KA.EQ.1)THEN
    FA=X(1)-X(4)
  ELSEIF(KA.EQ.2)THEN
    FA=X(2)-X(5)
  ELSE
    FA=X(3)-X(6)
  ENDIF
$ENDSET

$SET(FMODEL C)
  IF(KC.EQ.1)THEN
    FC=X(1)**2+X(2)**2+X(3)**2
  ELSE
    FC=X(5)**2+(X(4)-3.0D0)**2
  ENDIF
$ENDSET

$IADA=1
$IADC=1
$NF=6
$NX=6
$NA=3

```

```

$NC=2
$MODEL='AQ'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, types and values of the simple bounds and types and values of the nonlinear constraints. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum of squares minimization we set \$MODEL='AQ'.

d) Problem solution (basic screen output):

```

CLASS = VM - LQ1   UPDATE = B   MODEL = AQ   HESF = D   NF =      6
  NIC=  0 NIT=  0 NFV=   1 NFG=   1 F=0.700D+01 C=0.000D+00 G=0.000D+00
  NIC=  0 NIT=  1 NFV=   4 NFG=   4 F=0.363D+01 C=0.125D+01 G=0.200D+01
  NIC=  0 NIT=  2 NFV=   6 NFG=   6 F=0.266D+01 C=0.158D+00 G=0.111D+01
  NIC=  0 NIT=  3 NFV=   8 NFG=   8 F=0.254D+01 C=0.315D-01 G=0.357D+00
  NIC=  0 NIT=  4 NFV=  10 NFG=  10 F=0.250D+01 C=0.320D-02 G=0.130D+00
  NIC=  0 NIT=  5 NFV=  12 NFG=  12 F=0.250D+01 C=0.230D-02 G=0.533D-01
  NIC=  0 NIT=  6 NFV=  14 NFG=  14 F=0.250D+01 C=0.798D-05 G=0.529D-02
  NIC=  0 NIT=  7 NFV=  16 NFG=  16 F=0.250D+01 C=0.876D-07 G=0.431D-03
  NIC=  0 NIT=  8 NFV=  18 NFG=  18 F=0.250D+01 C=0.934D-10 G=0.200D-04
0 NIC=  0 NIT=  9 NFV=  18 NFG=  18 F=0.250D+01 C=0.934D-10 G=0.864D-07
F =  0.2500000000D+01
X =  0.1000000020D+01 -0.6092964516D-08  0.1999999990D+01  0.2000000000D+01
      0.4014414082D-07  0.4000000000D+01
TIME=  0:00:00.02

```

## 7.12 Minimax optimization with nonlinear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max (x_1 - 14, x_1^2 - 4x_1 + x_2^2 - 4x_2 + x_3^2 - 4x_3, x_1^2 - 4x_1 - 8)$$

over the set given by the nonlinear constraints

$$6x_2 + 4x_3 - x_1^3 \geq 3,$$

$$8x_1 + 14x_2 + 7x_3 = 56,$$

$$6x_1^2 + x_2^2 + x_3^2 = 25,$$

The starting point is  $x_1 = 2$ ,  $x_2 = 3$ ,  $x_3 = 2$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE MINIMAX OPTIMIZATION WITH NONLINEAR CONSTRAINTS +
$REM +-----+

```

```

$SET(INPUT)
X(1)=2.0D0; X(2)=3.0D0; X(3)=2.0D0
CL(1)=3.0D0; IC(1)=1
CL(2)=5.6D1; IC(2)=5
CL(3)=2.5D1; IC(3)=5
$ENDSET
$SET(FMODELA)
IF(KA.EQ.1)THEN
  FA=X(1)-1.4D1
ELSEIF(KA.EQ.2) THEN
  FA=X(1)**2-4.0D0*X(1)+X(2)**2-4.0D0*X(2)+X(3)**2-4.0D0*X(3)
ELSE
  FA=X(1)**2-4.0D0*X(1)-8.0D0
ENDIF
$ENDSET
$SET(FMODEL C)
IF(KC.EQ.1)THEN
  FC=6.0D0*X(2)+4.0D0*X(3)-X(1)**3
ELSEIF(KC.EQ.2) THEN
  FC=8.0D0*X(1)+1.4D1*X(2)+7.0D0*X(3)
ELSE
  FC=X(1)**2+X(2)**2+X(3)**2
ENDIF
$ENDSET
$MODEL='AM'
$IEXT=-1
$IADA=1
$IADC=1
$NF=3
$NA=3
$NC=3
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and types and values of the nonlinear constraints. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For minimax approximation we set \$MODEL='AM' and \$IEXT=-1.

d) Problem solution (basic screen output):

```

CLASS = VM - LQ1   UPDATE = B   MODEL = AM   HESF = D   NF =      4
NIC=  0 NIT=  0 NFV=   1 NFG=   1 F=0.000D+00 C=0.160D+02 G=0.000D+00
NIC=  0 NIT=  1 NFV=   2 NFG=   2 F=0.276D+03 C=0.280D+03 G=0.140D+02
NIC=  0 NIT=  2 NFV=   3 NFG=   3 F=0.745D+02 C=0.664D+02 G=0.701D+01
NIC=  0 NIT=  3 NFV=   4 NFG=   4 F=0.936D+01 C=0.136D+02 G=0.140D+01
NIC=  0 NIT=  4 NFV=   5 NFG=   5 F=0.217D+01 C=0.236D+01 G=0.526D+00
NIC=  0 NIT=  5 NFV=   6 NFG=   6 F=-.374D+01 C=0.853D-01 G=0.168D+00

```



```

      NIC=  0 NIT=   6 NFV=   7 NFG=   7 F=-.393D+01 C=0.125D-03 G=0.418D-02
      NIC=  0 NIT=   7 NFV=   8 NFG=   8 F=-.393D+01 C=0.262D-09 G=0.514D-05
0  NIC=  0 NIT=   8 NFV=   8 NFG=   8 F=-.393D+01 C=0.262D-09 G=0.107D-10
F  = -0.3934510577D+01
X   =  0.2548204707D+01  0.4023431118D+00  0.4283079826D+01 -0.3934510577D+01
TIME=  0:00:00.00

```

### 7.13 Global optimization

a) Problem description:

Suppose we have to find the global minimum of the objective function

$$F(x) = (x_1 - 3)^2(x_1 + 5)^2 + (x_2 - 2)^2(x_2 + 3)^2 - x_1^2x_2^2$$

over the set given by the inequalities  $-12 \leq x_1 \leq 10$  and  $-12 \leq x_2 \leq 10$ . The starting point is  $x_1 = 0$ ,  $x_2 = 0$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + DENSE UNCONSTRAINED GLOBAL MINIMIZATION +
$REM +-----+
$SET(INPUT)
  XL(1)=-1.2D1; XU(1)=1.0D1
  XL(2)=-1.2D1; XU(2)=1.0D1
$ENDSET
$SET(FMODEL)
  FF=((X(1)-3.0D0)*(X(1)+5.0D0))**2+((X(2)-2.0D0)*(X(2)+3.0D0))**2&
-(X(1)*X(2))**2
$ENDSET
$NF=2
$MOUT=1
$EXTREM='G'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the bounds defining the investigated region. By using the macrovariable \$FMODEL we specify analytically the value of the model function. The gradient of the model function is computed numerically. Since we require to find the global minimum we set \$EXTREM='G'.

d) Problem solution (basic screen output):

```

CLASS = VM - LI1  UPDATE = B  MODEL = FF  HESF = D  NF = 2
      0 NIT=   64 NFV=   709 NEX=   4 F=-0.806D+03

EXTREM  1 :
F = -0.8060772623D+03
X = -0.7329989920D+01 -0.6447506446D+01

```

```

EXTREM  2 :
F = -0.3072281498D+03
X = -0.6228926454D+01  0.4363683102D+01

EXTREM  3 :
F = -0.1504539067D+03
X =  0.3836710556D+01 -0.4317610574D+01

EXTREM  4 :
F = -0.5795091449D+02
X =  0.3368245192D+01  0.2827173202D+01
TIME= 0:00:00.00

```

## 7.14 Large-scale nonlinear equations

a) Problem description:

Suppose we have to solve the system of the nonlinear equations

$$\begin{aligned}
 f_i^A(x) &= (3 - 2x_i)x_i - x_{i+1} + 1 = 0 & , & \quad i = 1 \\
 f_i^A(x) &= (3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1 = 0 & , & \quad 2 \leq i \leq n - 1 \\
 f_i^A(x) &= (3 - 2x_i)x_i - x_{i-1} + 1 = 0 & , & \quad i = n
 \end{aligned}$$

where  $n = 100$ . The starting point is  $x_i = -1$  for  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + LARGE-SCALE NONLINEAR EQUATIONS +
$REM +-----+
$SET(INPUT)
  DO 1 I=1,NF
    X(I)=-1.0D0
1 CONTINUE
$ENDSET
$SET(FMODELA)
  I=KA
  FA=(3.0D0-2.0D0*X(I))*X(I)+1.0D0
  IF (I.GT. 1) FA=FA-X(I-1)
  IF (I.LT.NA) FA=FA-X(I+1)
$ENDSET
$NF=100
$NA=100
$MODEL='NE'
$JACA='N'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables. By using the macrovariable \$FMODELA we specify analytically the values of functions in the nonlinear equations. For solving nonlinear equations we set \$MODEL='NE'. Since the Jacobian matrix is not defined, we set \$JACA='N'.

d) Problem solution (basic screen output):

```

CLASS = TN - GE3   UPDATE = N   MODEL = AQ   HESF = N   NF =   100
  NIT=   0 NFV=    1 F=  205.0000000
  NIT=   1 NFV=    5 F=   5.266293104
  NIT=   2 NFV=    9 F=  0.1672522970E-01
  NIT=   3 NFV=   13 F=  0.1523614858E-05
  NIT=   4 NFV=   19 F=  0.9562244762E-11
  NIT=   5 NFV=   27 F=  0.3893958431E-18
  0 NIT=   5 NFV=   27 NDC=    0 FV BOUND F=  0.3893958431E-18
TIME= 0:00:00.00

```

## 7.15 Large scale unconstrained optimization (sparse Hessian matrix)

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^n ((3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1)^2, \quad x_{n+1} = x_0 = 0$$

where  $n = 100$ . The starting point is  $x_i = -1$  for  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE UNCONSTRAINED MINIMIZATION +
$REM +-----+
$FLOAT W,WF(0:100)
INTEGER IAD_WF(0:100)
$SET(INPUT)
  DO 1 I=1,NF
    X(I)=-1.0D0
    J=2*(I-1)+1
    IH(I)=J
    JH(J)=I
    JH(J+1)=I+1
1 CONTINUE
  IH(NF+1)=2*NF
$ENDSET
$SET(FMODEL)
  WF(0)=0.0D0
  DO 2 J=1,NF
    IF(J.EQ.1)THEN
      W=(3.0D0-2.0D0*X(J))*X(J)+1.0D0-X(J+1)
    ELSEIF(J.EQ.NF)THEN
      W=(3.0D0-2.0D0*X(J))*X(J)+1.0D0-X(J-1)
    ELSE
      W=(3.0D0-2.0D0*X(J))*X(J)+1.0D0-X(J-1)-X(J+1)
    ENDIF
    WF(J)=WF(J-1)+W*W
2 CONTINUE
  FF=WF(NF)
$ENDSET

```

```

$IADF=1
$NF=100
$HESF='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the sparsity pattern of the Hessian matrix. The sparse Hessian matrix, indicated by the statement \$HESF='S', is tridiagonal. By using the macrovariable \$FMODEL we specify the value of the model function analytically. The gradient of the model function is computed by automatic differentiation, since \$IADF=1. Since values of the objective function are counted in the cycle, we have to use real array WF(0:100) and the corresponding integer array IAD\_WF(0:100) (see Section 2.19) Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used.

d) Problem solution (basic screen output):

```

CLASS = MN - GM3   UPDATE = N   MODEL = FF   HESF = S   NF =   100
  NIT=   0 NFV=   4 NFG=   4 F= 410.0000000   G=0.380D+02
  NIT=   1 NFV=   8 NFG=   8 F= 51.26265782   G=0.123D+02
  NIT=   2 NFV=  12 NFG=  12 F= 6.162928967   G=0.694D+01
  NIT=   3 NFV=  16 NFG=  16 F= 0.2405291876   G=0.141D+01
  NIT=   4 NFV=  20 NFG=  20 F= 0.7598786964E-03 G=0.122D+00
  NIT=   5 NFV=  24 NFG=  24 F= 0.2442230554E-05 G=0.636D-02
  NIT=   6 NFV=  28 NFG=  28 F= 0.3898601055E-07 G=0.822D-03
  NIT=   7 NFV=  32 NFG=  32 F= 0.1062176777E-09 G=0.481D-04
  NIT=   8 NFV=  36 NFG=  36 F= 0.5393508095E-12 G=0.401D-05
  NIT=   9 NFV=  40 NFG=  40 F= 0.3871096527E-14 G=0.271D-06
  0 NIT=   9 NFV=  40 NFG=  40 GRAD TOL F= 0.3871096527E-14 G=0.271D-06
TIME= 0:00:00.00

```

## 7.16 Large-scale unconstrained optimization (sparse Jacobian matrix)

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^n f_i^A(x)$$

where  $n=100$  and

$$\begin{aligned}
 f_i^A(x) &= ((3 - 2x_i)x_i - x_{i+1} + 1)^2, & i = 1 \\
 f_i^A(x) &= ((3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1)^2, & 2 \leq i \leq n - 1 \\
 f_i^A(x) &= ((3 - 2x_i)x_i - x_{i-1} + 1)^2, & i = n
 \end{aligned}$$

The starting point is  $x_i = -1$  for  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE PARTIALLY SEPARABLE UNCONSTRAINED MINIMIZATION +
$REM +-----+

```

```

$FLOAT WA
$SET(INPUT)
  DO 1 I=1,NF
    X(I)=-1.0D0
1 CONTINUE
  L=1
  DO 2 I=1,NA
    IAG(I)=L
    IF (I.GT.1) THEN
      JAG(L)=I-1
      L=L+1
    ENDIF
    JAG(L)=I
    L=L+1
    IF (I.LT.NA) THEN
      JAG(L)=I+1
      L=L+1
    ENDIF
2 CONTINUE
  IAG(NA+1)=L
$ENDSET
$SET(FMODELA)
  IF(KA.EQ.1)THEN
    WA=(3.0D0-2.0D0*X(KA))*X(KA)+1.0D0-X(KA+1)
  ELSEIF(KA.EQ.NA)THEN
    WA=(3.0D0-2.0D0*X(KA))*X(KA)+1.0D0-X(KA-1)
  ELSE
    WA=(3.0D0-2.0D0*X(KA))*X(KA)+1.0D0-X(KA-1)-X(KA+1)
  ENDIF
  FA=WA*WA
$ENDSET
$IADA=1
$NF=100
$NA=100
$MODEL='AF'
$JACA='S'
$FMIN='0.0D0'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the sparsity pattern of the Jacobian matrix. The sparse Jacobian matrix, indicated by the statement \$JACA='S', is tridiagonal. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum of values minimization we set \$MODEL='AF'.

d) Problem solution (basic screen output):

```

CLASS = VM - LM3  UPDATE = B  MODEL = AF  HESF = S  NF = 100
  NIT= 0  NFV= 1  NFG= 1  F= 410.000000  G=0.380D+02
  NIT= 1  NFV= 2  NFG= 2  F= 56.19847311  G=0.887D+01

```

```

NIT=   2 NFV=   3 NFG=   3 F= 33.21669961 G=0.563D+01
NIT=   3 NFV=   5 NFG=   5 F= 8.034550334 G=0.672D+01
NIT=   4 NFV=   7 NFG=   7 F= 2.498266945 G=0.753D+01
NIT=   5 NFV=   9 NFG=   9 F= 1.126771725 G=0.617D+01
NIT=   6 NFV=  10 NFG=  10 F= 0.2724480160E-01 G=0.730D+00
NIT=   7 NFV=  11 NFG=  11 F= 0.1670056108E-03 G=0.647D-01
NIT=   8 NFV=  12 NFG=  12 F= 0.1033932683E-05 G=0.365D-02
NIT=   9 NFV=  13 NFG=  13 F= 0.1781676012E-07 G=0.834D-03
NIT=  10 NFV=  14 NFG=  14 F= 0.2260632676E-09 G=0.981D-04
NIT=  11 NFV=  15 NFG=  15 F= 0.2888717918E-11 G=0.995D-05
NIT=  12 NFV=  16 NFG=  16 F= 0.1281762413E-12 G=0.216D-05
NIT=  13 NFV=  17 NFG=  17 F= 0.2194981848E-15 G=0.782D-07
0 NIT=  13 NFV=  17 NFG=  17 GRAD TOL F= 0.2194981848E-15 G=0.782D-07
TIME= 0:00:00.00

```

## 7.17 Large scale unconstrained least squares optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \frac{1}{2} \sum_{i=1}^{2(n-1)} (f_i^A(x))^2$$

where  $n = 100$  and

$$\begin{aligned} f_i^A(x) &= x_1 - 1 && , \quad i = 1, \\ f_i^A(x) &= 10(x_i^2 - x_{i+1}), && , \quad k > 1, \quad \text{mod}(k, 2) = 0, \\ f_i^A(x) &= 2 \exp(-(x_i - x_{i+1})^2) + \exp(-2(x_{i+1} - x_{i+2})^2), && , \quad k > 1, \quad \text{mod}(k, 2) = 1, \end{aligned}$$

The starting point is  $x_i = -1.2$  for  $\text{mod}(i, 2) = 1$  and  $x_i = 1.0$  for  $\text{mod}(i, 2) = 0$ , where  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE UNCONSTRAINED LEAST SQUARES +
$REM +-----+
$FLOAT W1,W2
$SET(INPUT)
DO 1 I=1,NF
  IF(MOD(I,2).EQ.1) THEN
    X(I)=-1.2D0
  ELSE
    X(I)= 1.0D0
  ENDIF
1 CONTINUE
NA=2*(NF-1)
MA=2
IAG(1)=1
IAG(2)=2
JAG(1)=1
DO 2 KA=1,NA-1
  I=(KA+1)/2

```

```

      IF (MOD(KA,2).EQ.1) THEN
        JAG(MA)=I
        MA=MA+1
        JAG(MA)=I+1
        MA=MA+1
      ELSE
        JAG(MA)=I
        MA=MA+1
        JAG(MA)=I+1
        MA=MA+1
        JAG(MA)=I+2
        MA=MA+1
      ENDIF
      IAG(KA+2)=MA
2 CONTINUE
      MA=MA-1
$ENDSET
$SET(FMODEL)
      I=KA/2
      IF(KA.EQ.1)THEN
        FA=X(KA)-1.0D0
      ELSEIF(MOD(KA,2).EQ.0)THEN
        FA=1.0D1*(X(I)**2-X(I+1))
      ELSE
        FA=2.0D0*EXP(-(X(I)-X(I+1))**2)+EXP(-2.0D0*(X(I+1)-X(I+2))**2)
      ENDIF
$ENDSET
$IADA=1
$NF=100
$NA=198
$MODEL='AQ'
$JACA='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables and the sparsity pattern of the Jacobian matrix. The sparse Jacobian matrix, indicated by the statement \$JACA='S', is tridiagonal. By using the macrovariable \$FMODEL we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum-of-squares minimization we set \$MODEL='AQ'.

d) Problem solution (basic screen output):

```

CLASS = GN - GM3   UPDATE = M   MODEL = AQ   HESF = S   NF =   100
  NIT=   0 NFV=   1 NFG=   1 F= 12344.43235   G=0.396D+03
  NIT=   1 NFV=   3 NFG=   3 F= 1211.833625   G=0.149D+03
  NIT=   2 NFV=   5 NFG=   5 F= 556.3758005   G=0.663D+02
  NIT=   3 NFV=   7 NFG=   7 F= 466.7731338   G=0.324D+02
  NIT=   4 NFV=   9 NFG=   9 F= 447.6914295   G=0.127D+02
  NIT=   5 NFV=  11 NFG=  11 F= 443.4171082   G=0.480D+01
  NIT=   6 NFV=  13 NFG=  13 F= 442.2501148   G=0.182D+02

```

```

NIT=    7 NFV=   15 NFG=   15 F= 439.5246294 G=0.283D+01
NIT=    8 NFV=   18 NFG=   18 F= 438.8538272 G=0.285D+01
NIT=    9 NFV=   20 NFG=   20 F= 438.3971433 G=0.325D+01
NIT=   10 NFV=   22 NFG=   22 F= 438.1103835 G=0.101D+01
-----
NIT=   31 NFV=   56 NFG=   56 F= 436.9702388 G=0.948D-03
NIT=   32 NFV=   57 NFG=   57 F= 436.9702388 G=0.336D-03
NIT=   33 NFV=   59 NFG=   59 F= 436.9702388 G=0.129D-03
NIT=   34 NFV=   60 NFG=   60 F= 436.9702388 G=0.232D-03
NIT=   35 NFV=   62 NFG=   62 F= 436.9702388 G=0.578D-04
NIT=   36 NFV=   63 NFG=   63 F= 436.9702388 G=0.338D-04
NIT=   37 NFV=   65 NFG=   65 F= 436.9702388 G=0.135D-04
NIT=   38 NFV=   67 NFG=   67 F= 436.9702388 G=0.511D-05
NIT=   39 NFV=   69 NFG=   69 F= 436.9702388 G=0.124D-05
NIT=   40 NFV=   71 NFG=   71 F= 436.9702388 G=0.490D-06
0 NIT=   40 NFV=   71 NFG=   71 GRAD TOL F= 436.9702388 G=0.490D-06
TIME= 0:00:00.03

```

## 7.18 Large-scale unconstrained $l_1$ optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^n |f_i^A(x)|$$

where  $n = 100$  and

$$\begin{aligned} f_i^A(x) &= x_1 - 1 && , \quad i = 1, \\ f_i^A(x) &= 10(x_i^2 - x_{i+1}), && , \quad k > 1, \quad \text{mod}(k, 2) = 0, \\ f_i^A(x) &= 2 \exp(-(x_i - x_{i+1})^2) + \exp(-2(x_{i+1} - x_{i+2})^2), && , \quad k > 1, \quad \text{mod}(k, 2) = 1, \end{aligned}$$

The starting point is  $x_i = -1.2$  for  $\text{mod}(i, 2) = 1$  and  $x_i = 1.0$  for  $\text{mod}(i, 2) = 0$ , where  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE UNCONSTRAINED L-1 OPTIMIZATION +
$REM +-----+
$FLOAT W1,W2
$SET(INPUT)
DO 1 I=1,NF
  IF(MOD(I,2).EQ.1) THEN
    X(I)=-1.2D0
  ELSE
    X(I)= 1.0D0
  ENDIF
1 CONTINUE
NA=2*(NF-1)
MA=2
IAG(1)=1
IAG(2)=2
JAG(1)=1

```



```

DO 2 KA=1,NA-1
  I=(KA+1)/2
  IF (MOD(KA,2).EQ.1) THEN
    JAG(MA)=I
    MA=MA+1
    JAG(MA)=I+1
    MA=MA+1
  ELSE
    JAG(MA)=I
    MA=MA+1
    JAG(MA)=I+1
    MA=MA+1
    JAG(MA)=I+2
    MA=MA+1
  ENDIF
  IAG(KA+2)=MA
2 CONTINUE
  MA=MA-1
$ENDSET
$SET(FMODELA)
  I=KA/2
  IF(KA.EQ.1)THEN
    FA=X(KA)-1.0D0
  ELSEIF(MOD(KA,2).EQ.0)THEN
    FA=1.0D1*(X(I)**2-X(I+1))
  ELSE
    FA=2.0D0*EXP(-(X(I)-X(I+1))**2)+EXP(-2.0D0*(X(I+1)-X(I+2))**2)
  ENDIF
$ENDSET
$IADA=1
$NF=100
$NA=198
$MODEL='AA'
$JACA='S'
$XMAX='1.0D1'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the Jacobian matrix and the maximum step-size \$XMAX. The sparse Jacobian matrix, indicated by the statement \$JACA='S', is tridiagonal. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum-of-absolute-values minimization we set \$MODEL='AA'.

d) Problem solution (basic screen output):

```

CLASS = MN - LG1   UPDATE = N   MODEL = AA   HESF = S   NF =   100
  NIT=   0 NFV=    6 NFG=    6 F= 1157.710087   G=0.287D+02
  NIT=   1 NFV=   12 NFG=   12 F= 309.1650110   G=0.921D+01
  NIT=   2 NFV=   21 NFG=   21 F= 302.5945304   G=0.127D+02

```

```

NIT=   3 NFV=   30 NFG=   30 F= 299.5018688      G=0.165D+02
NIT=   4 NFV=   40 NFG=   40 F= 297.5686249      G=0.164D+02
NIT=   5 NFV=   51 NFG=   51 F= 296.7110526      G=0.162D+02
NIT=   6 NFV=   58 NFG=   58 F= 291.6634238      G=0.139D+02
NIT=   7 NFV=   68 NFG=   68 F= 289.7015897      G=0.139D+02
NIT=   8 NFV=   78 NFG=   78 F= 286.1805123      G=0.133D+02
NIT=   9 NFV=   88 NFG=   88 F= 285.7754331      G=0.131D+02
NIT=  10 NFV=   99 NFG=   99 F= 285.6742808      G=0.131D+02
-----
NIT=  51 NFV=  420 NFG=  420 F= 293.4040912      G=0.620D+01
NIT=  52 NFV=  427 NFG=  427 F= 293.4040800      G=0.303D+01
NIT=  53 NFV=  433 NFG=  433 F= 293.4040781      G=0.359D+00
NIT=  54 NFV=  439 NFG=  439 F= 293.4040781      G=0.408D-02
NIT=  55 NFV=  445 NFG=  445 F= 293.4040781      G=0.538D-01
NIT=  56 NFV=  451 NFG=  451 F= 293.4040781      G=0.927D-04
NIT=  57 NFV=  457 NFG=  457 F= 293.3718245      G=0.679D+01
NIT=  58 NFV=  464 NFG=  464 F= 293.3718226      G=0.295D+01
NIT=  59 NFV=  470 NFG=  470 F= 293.3718225      G=0.139D+00
NIT=  60 NFV=  476 NFG=  476 F= 293.3718225      G=0.108D-03
NIT=  61 NFV=  482 NFG=  482 F= 293.3718225      G=0.124D-04
NIT=  62 NFV=  497 NFG=  497 F= 293.3718225      G=0.246D-07
0 NIT=  62 NFV=  497 NFG=  497 GRAD TOL F= 293.3677360      G=0.246D-07
TIME= 0:00:00.47

```

## 7.19 Large-scale unconstrained $l_\infty$ (minimax) optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max_{1 \leq i \leq n} |f_i^A(x)|$$

where  $n = 100$  and

$$\begin{aligned}
 f_i^A(x) &= x_1 - 1 && , \quad i = 1, \\
 f_i^A(x) &= 10(x_i^2 - x_{i+1}), && , \quad k > 1, \quad \text{mod}(k, 2) = 0, \\
 f_i^A(x) &= 2 \exp(-(x_i - x_{i+1})^2) + \exp(-2(x_{i+1} - x_{i+2})^2), && , \quad k > 1, \quad \text{mod}(k, 2) = 1,
 \end{aligned}$$

The starting point is  $x_i = -1.2$  for  $\text{mod}(i, 2) = 1$  and  $x_i = 1.0$  for  $\text{mod}(i, 2) = 0$ , where  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE UNCONSTRAINED L1-INFINITY (MINIMAX) OPTIMIZATION      +
$REM +-----+
$FLOAT W1,W2
$SET(INPUT)
DO 1 I=1,NF
  IF(MOD(I,2).EQ.1) THEN
    X(I)=-1.2D0
  ELSE
    X(I)= 1.0D0
  ENDIF

```

```

1 CONTINUE
  NA=2*(NF-1)
  MA=2
  IAG(1)=1
  IAG(2)=2
  JAG(1)=1
  DO 2 KA=1,NA-1
    I=(KA+1)/2
    IF (MOD(KA,2).EQ.1) THEN
      JAG(MA)=I
      MA=MA+1
      JAG(MA)=I+1
      MA=MA+1
    ELSE
      JAG(MA)=I
      MA=MA+1
      JAG(MA)=I+1
      MA=MA+1
      JAG(MA)=I+2
      MA=MA+1
    ENDIF
    IAG(KA+2)=MA
2 CONTINUE
  MA=MA-1
$ENDSET
$SET(FMODELA)
  I=KA/2
  IF(KA.EQ.1)THEN
    FA=X(KA)-1.0D0
  ELSEIF(MOD(KA,2).EQ.0)THEN
    FA=1.0D1*(X(I)**2-X(I+1))
  ELSE
    FA=2.0D0*EXP(-(X(I)-X(I+1))**2)+EXP(-2.0D0*(X(I+1)-X(I+2))**2)
  ENDIF
$ENDSET
$IADA=1
$NF=100
$NA=198
$MODEL='AM'
$JACA='S'
$XMAX='5.0D0'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the Jacobian matrix and the maximum step-size \$XMAX. The sparse Jacobian matrix, indicated by the statement \$JACA='S', is tridiagonal. Therefore we set \$MA=300. Since we use the sparse Hessian matrix, we set \$HESF='S' and specify the number of its nonzero elements \$M=3000 (this is a reasonable upper bound). By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the

automatic differentiation is used. For the sum-of-absolute-values minimization we set \$MODEL='AA'.

d) Problem solution (basic screen output):

```

CLASS = VM - LG1   UPDATE = B   MODEL = AM   HESF = S   NF =   100
  NIT=   0 NFV=   1 NFG=   1 F= -1972.482481   G=0.506D-02
  NIT=   1 NFV=   2 NFG=   2 F=  2.251130699   G=0.183D+01
  NIT=   2 NFV=   3 NFG=   3 F=-0.7786205801E-01 G=0.676D-01
  NIT=   3 NFV=   4 NFG=   4 F=  0.5616156203   G=0.336D-01
  NIT=   4 NFV=   5 NFG=   5 F=  2.357525149   G=0.265D-01
  NIT=   5 NFV=   6 NFG=   6 F=  2.395877538   G=0.222D-01
  NIT=   6 NFV=   7 NFG=   7 F=  2.447059519   G=0.653D-02
  NIT=   7 NFV=   8 NFG=   8 F=  2.529085464   G=0.917D+00
  NIT=   8 NFV=  11 NFG=  11 F=  2.494000970   G=0.627D+00
  NIT=   9 NFV=  12 NFG=  12 F=  2.488049570   G=0.635D-01
  NIT=  10 NFV=  13 NFG=  13 F=  2.481679082   G=0.243D-01
-----
  NIT=  51 NFV= 128 NFG= 128 F=  2.354874985   G=0.228D-03
  NIT=  52 NFV= 129 NFG= 129 F=  2.354874985   G=0.350D-04
  NIT=  53 NFV= 130 NFG= 130 F=  2.354874985   G=0.990D-07
  NIT=  54 NFV= 132 NFG= 132 F=  2.354874054   G=0.975D+00
  NIT=  55 NFV= 135 NFG= 135 F=  2.354874043   G=0.560D+00
  NIT=  56 NFV= 136 NFG= 136 F=  2.354874042   G=0.260D-01
  NIT=  57 NFV= 137 NFG= 137 F=  2.354874041   G=0.598D-02
  NIT=  58 NFV= 138 NFG= 138 F=  2.354874041   G=0.502D-03
  NIT=  59 NFV= 139 NFG= 139 F=  2.354874041   G=0.459D-05
  NIT=  60 NFV= 140 NFG= 140 F=  2.354874041   G=0.230D-08
  0 NIT=  60 NFV= 140 NFG= 140 GRAD TOL F=  2.354873722   G=0.230D-08
TIME= 0:00:00.45

```

## 7.20 Large-scale unconstrained nonsmooth optimization

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^{n-1} \max(x_i^2 + (x_{i+1} - 1)^2 + x_{i+1} - 1, -x_i^2 - (x_{i+1} - 1)^2 + x_{i+1} - 1),$$

where  $n = 100$ . The starting point is  $x_i = -1.5$  for  $\text{mod}(i, 2) = 1$  and  $x_i = 2.0$  for  $\text{mod}(i, 2) = 0$ , where  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + LARGE SCALE UNCONSTRAINED NONSMOOTH OPTIMIZATION +
$REM +-----+
$FLOAT W1,W2
$SET(INPUT)
DO 1 I=1,NF
  IF(MOD(I,2).EQ.1) THEN
    X(I)=-1.5DO
  ELSE
    X(I)= 2.0DO

```

```

        ENDIF
1 CONTINUE
$ENDSET
$SET(FGMODEL F)
    FF=0.0D0
    GF(1)=0.0D0
    DO 2 I=1,NF-1
        W1= X(I)*X(I)+(X(I+1)-1.0D0)*(X(I+1)-1.0D0)+X(I+1)-1.0D0
        W2=-X(I)*X(I)-(X(I+1)-1.0D0)*(X(I+1)-1.0D0)+X(I+1)+1.0D0
        IF (W1.GE.W2) THEN
            FF=FF+W1
            GF(I)=GF(I)+2.0D0*X(I)
            GF(I+1)= 2.0D0*(X(I+1)-1.0D0)+1.0D0
        ELSE
            FF=FF+W2
            GF(I)=GF(I)-2.0D0*X(I)
            GF(I+1)=-2.0D0*(X(I+1)-1.0D0)+1.0D0
        END IF
    2 CONTINUE
$ENDSET
$NF=100
$HESF='N'
$KSF=3
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables. By using the macrovariable \$FGMODEL F we specify analytically the value and the gradient of the objective function. For nonsmooth optimization we set \$KSF=3. Since the pattern of the Hessian matrix is not given, we set \$HESF='N'.

d) Problem solution (basic screen output):

```

CLASS = BL - LI1    UPDATE = N    MODEL = FF    HESF = N    NF =    100
NIT=    0 NFV=    1 NFG=    1 F= 592.2500000    G=0.700D+01
NIT=    1 NFV=    2 NFG=    2 F= 592.2500000    G=0.700D+01
NIT=    2 NFV=    3 NFG=    3 F= 61.35842142    G=0.300D+01
NIT=    3 NFV=    4 NFG=    4 F= 61.35842142    G=0.300D+01
NIT=    4 NFV=    5 NFG=    5 F= 8.817561091    G=0.300D+01
NIT=    5 NFV=    6 NFG=    6 F= 4.412558758    G=0.238D+01
NIT=    6 NFV=    7 NFG=    7 F= 3.624278568    G=0.243D+01
NIT=    7 NFV=    8 NFG=    8 F= 2.725409817    G=0.253D+01
NIT=    8 NFV=    9 NFG=    9 F= 0.8653137918    G=0.288D+01
NIT=    9 NFV=   10 NFG=   10 F= 0.8653137918    G=0.300D+01
NIT=   10 NFV=   11 NFG=   11 F= 0.8653137918    G=0.224D+01
-----
NIT=  211 NFV=  212 NFG=  212 F= 0.7717072027E-06 G=0.300D+01
NIT=  212 NFV=  213 NFG=  213 F= 0.7717072027E-06 G=0.300D+01
NIT=  213 NFV=  214 NFG=  214 F= 0.7717072027E-06 G=0.300D+01
NIT=  214 NFV=  215 NFG=  215 F= 0.7717072027E-06 G=0.300D+01
NIT=  215 NFV=  216 NFG=  216 F= 0.7717072027E-06 G=0.300D+01
NIT=  216 NFV=  217 NFG=  217 F= 0.7717072027E-06 G=0.300D+01

```

```

      NIT= 217 NFV= 218 NFG= 218 F= 0.7717072027E-06 G=0.300D+01
      NIT= 218 NFV= 219 NFG= 219 F= 0.7717072027E-06 G=0.300D+01
0     NIT= 219 NFV= 219 NFG= 219 GRAD TOL F= 0.7717072027E-06 G=0.434D-06
TIME= 0:00:00.02

```

## 7.21 Sparse linear programming

a) Problem description:

Suppose we have to find the global maximum of the linear function

$$F(x) = \sum_{i=1}^n (-1)^i x_i$$

with simple bounds  $-20 \leq x_i \leq 20$ ,  $1 \leq i \leq n$ , and linear constraints

$$-x_i + x_{i+1} - x_{i+2} = i, \quad 1 \leq i \leq n_C$$

where  $n = 20$  and  $n_C = 18$ . The starting point is not given. The maximum value of the linear objective function is  $F = 7.0$

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE LINEAR PROGRAMMING +
$REM +-----+
$SET(INPUT)
  DO 1 I=1,NF
    IX(I)=3; XL(I)=-2.0D1; XU(I)=2.0D1
    GF(I)=DBLE((-1)**I)
1 CONTINUE
  DO 2 KC=1,NC
    IC(KC)=5; CL(KC)=DBLE(KC)
    $SETCG(KC,KC,-1.0D0)
    $SETCG(KC,KC+1, 1.0D0)
    $SETCG(KC,KC+2,-1.0D0)
2 CONTINUE
$ENDSET
$IEXT=1
$NF=20
$NX=20
$NC=18
$NCL=18
$MODEL='FL'
$JACC='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the bounds for variables and the sparsity pattern with numerical values of the constraint Jacobian matrix. We use the procedure UKMCI1. The sparse Jacobian matrix, indicated by the statement \$JACC='S', is tridiagonal. The option \$MODEL='FL' indicates the linear programming problem.

d) Problem solution (basic screen output):

```

CLASS = LP - LN1   UPDATE = N   MODEL = FL   HESF = N   NF =      20
NUMITR=   1 INEW=   20 IOLD=   15 KINP=   0 IU=   48 F=0.980D+04
NUMITR=   2 INEW=   19 IOLD=   20 KINP=   0 IU=   49 F=0.208D+04
NUMITR=   3 INEW=    0 IOLD=   20 KINP=   0 IU=   49 F=0.000D+00
NUMITR=   3 NEL=    3 NREF=    1 KINP=   0 IU=   49 F=0.000D+00 ITERL=  1
NUMITR=   1 INEW=   15 IOLD=   19 KINP=   0 IU=   49 F=0.900D+01
NUMITR=   2 INEW=   20 IOLD=   18 KINP=   0 IU=   48 F=0.700D+01
NUMITR=   3 INEW=    0 IOLD=   18 KINP=   0 IU=   48 F=0.700D+01
NUMITR=   3 NEL=    3 NREF=    1 KINP=   0 IU=   48 F=0.700D+01 ITERL=  2
  0 NIT=   6 NFV=   2 NFG=    0 OPTIMUM F=  7.00000   C=0.0D+00 G=0.0D+00
FF = -0.7000000000D+01
X   = -0.2000000000D+01  0.0000000000D+00  0.1000000000D+01 -0.1000000000D+01
      -0.5000000000D+01 -0.8000000000D+01 -0.8000000000D+01 -0.6000000000D+01
      -0.5000000000D+01 -0.7000000000D+01 -0.1100000000D+02 -0.1400000000D+02
      -0.1400000000D+02 -0.1200000000D+02 -0.1100000000D+02 -0.1300000000D+02
      -0.1700000000D+02 -0.2000000000D+02 -0.2000000000D+02 -0.1800000000D+02
TIME=  0:00:00.00

```

## 7.22 Sparse quadratic programming

a) Problem description:

Suppose we have to find the global minimum of the quadratic function

$$F(x) = \frac{1}{2}(2x_1^2 + x_2^2 + 2x_3^2 + x_4^2 - 2x_1x_3 + 2x_3x_4) - x_1 - 3x_2 + x_3 - x_4$$

with simple bounds  $x_i \geq 0$  for  $1 \leq i \leq 4$  and linear constraints

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &\leq 5, \\ 3x_1 + x_2 + 2x_3 - x_4 &\leq 4, \\ 2x_2 + 8x_3 &\geq 3. \end{aligned}$$

The starting point is  $x_i = 1/2$  for  $1 \leq i \leq 4$ . The minimum value of the quadratic objective function is  $F = -4.681818$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE QUADRATIC PROGRAMMING +
$REM +-----+
$SET(INPUT)
  DO 1 I=1,NF
  X(I)= 0.5D0; XL(I)=0.0D0; IX(I)=1
1 CONTINUE
  GF(1)=-1.0D0; GF(2)=-3.0D0; GF(3)= 1.0D0; GF(4)=-1.0D0
  IH(1)= 1; IH(2)= 3; IH(3)= 4; IH(4)= 6; IH( 5)= 7
  JH(1)= 1; JH(2)= 3; JH(3)= 2; JH(4)= 3; JH( 5)= 4; JH(6)= 4
  HF(1)= 2.0D0; HF(2)=-1.0D0; HF(3)= 1.0D0
  HF(4)= 2.0D0; HF(5)= 1.0D0; HF(6)= 1.0D0
  IC(1)=2; IC(2)=2; IC(3)=1
  CU(1)= 5.0D0; CU(2)= 4.0D0; CL(3)= 3.0D0

```

```

ICG(1)=1; ICG(2)=5; ICG(3)=9; ICG(4)=11
JCG(1)=1; JCG(2)=2; JCG(3)=3; JCG(4)=4; JCG( 5)=1
JCG(6)=2; JCG(7)=3; JCG(8)=4; JCG(9)=2; JCG(10)=3
CG(1)= 1.0D0; CG(2)= 2.0D0; CG(3)= 1.0D0; CG(4)= 1.0D0; CG( 5)= 3.0D0
CG(6)= 1.0D0; CG(7)= 2.0D0; CG(8)=-1.0D0; CG(9)= 2.0D0; CG(10)= 8.0D0
$ENDSET
$NF=4
$NX=4
$NC=3
$NCL=3
$MODEL='FQ'
$JACC='S'
$HESF='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the bounds for variables, the sparsity pattern with numerical values of the model Hessian matrix, and the sparsity pattern with numerical values of the constraint Jacobian matrix. The sparse Hessian matrix is indicated by the statement \$HESF='S'. The sparse Jacobian matrix is indicated by the statement \$JACC='S'. The option \$MODEL='FQ' indicates the quadratic programming problem.

d) Problem solution (basic screen output):

```

CLASS = QP - LN2   UPDATE = N   MODEL = FQ   HESF = S   NF =      4
MODE =  1  NRED =  0   N =  4   IOLD =  0   INEW =  0
MODE =  1  NRED =  1   N =  3   IOLD =  0   INEW =  3   ADDITION
MODE =  1  NRED =  2   N =  2   IOLD =  0   INEW = -3   ADDITION
MODE =  1  NRED =  3   N =  2   IOLD =  0   INEW =  0
MODE =  1  NRED =  4   N =  2   IOLD =  0   INEW =  0
MODE =  1  NRED =  4   N =  3   IOLD =  1   INEW =  0   DELETION
MODE =  1  NRED =  5   N =  2   IOLD =  0   INEW =  1   ADDITION
MODE =  1  NRED =  6   N =  2   IOLD =  0   INEW =  0
MODE =  1  NRED =  7   N =  2   IOLD =  0   INEW =  0
  0 NIT=  7  NFV=  2  NFG=  0  OPTIMUM  F= -4.68182   C=0.0D+00 G=0.0D+00
FF = -0.4681818182D+01
X =  0.2727272727D+00  0.2090909091D+01  0.0000000000D+00  0.5454545455D+00
TIME= 0:00:00.00

```

## 7.23 Large-scale optimization with linear constraints

a) Problem description:

The problem we have solved is in fact the Hock and Schittkowski problem number 119 (see [44]) which has 16 variables and 8 linear constraints.

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE MINIMIZATION WITH LINEAR CONSTRAINTS +
$REM +-----+
$FLOAT WI,WJ

```



```

$SET(INPUT)
  DO 1 I=1,NF
    X(I)=10.0D0; XL(I)=0.0D0; XU(I)=5.0D0; IX(I)=3
1 CONTINUE
  IH( 1)= 1; IH( 2)= 6; IH( 3)=10; IH( 4)=15; IH( 5)=19
  IH( 6)=24; IH( 7)=27; IH( 8)=30; IH( 9)=33; IH(10)=36
  IH(11)=38; IH(12)=40; IH(13)=42; IH(14)=44; IH(15)=45
  IH(16)=46; IH(17)=47;
  JH( 1)= 1; JH( 2)= 4; JH( 3)= 7; JH( 4)= 8; JH( 5)=16
  JH( 6)= 2; JH( 7)= 3; JH( 8)= 7; JH( 9)=10;
  JH(10)= 3; JH(11)= 7; JH(12)= 9; JH(13)=10; JH(14)=14
  JH(15)= 4; JH(16)= 7; JH(17)=11; JH(18)=15;
  JH(19)= 5; JH(20)= 6; JH(21)=10; JH(22)=12; JH(23)=16
  JH(24)= 6; JH(25)= 8; JH(26)=15;
  JH(27)= 7; JH(28)=11; JH(29)=13;
  JH(30)= 8; JH(31)=10; JH(32)=15;
  JH(33)= 9; JH(34)=12; JH(35)=16;
  JH(36)=10; JH(37)=14;
  JH(38)=11; JH(39)=13;
  JH(40)=12; JH(41)=14;
  JH(42)=13; JH(43)=14;
  JH(44)=14;
  JH(45)=15;
  JH(46)=16;
  DO 2 I=1,NC
    IC(I)=5
2 CONTINUE
  CL(1)= 2.5D0; CL(2)= 1.1D0; CL(3)=-3.1D0; CL(4)=-3.5D0
  CL(5)= 1.3D0; CL(6)= 2.1D0; CL(7)= 2.3D0; CL(8)=-1.5D0
  $SETCG(1, 1, 0.22D0); $SETCG(1, 2, 0.20D0); $SETCG(1, 3, 0.19D0)
  $SETCG(1, 4, 0.25D0); $SETCG(1, 5, 0.15D0); $SETCG(1, 6, 0.11D0)
  $SETCG(1, 7, 0.12D0); $SETCG(1, 8, 0.13D0); $SETCG(1, 9, 1.00D0)
  $SETCG(2, 1,-1.46D0); $SETCG(2, 3,-1.30D0); $SETCG(2, 4, 1.82D0)
  $SETCG(2, 5,-1.15D0); $SETCG(2, 7, 0.80D0); $SETCG(2,10, 1.00D0)
  $SETCG(3, 1, 1.29D0); $SETCG(3, 2,-0.89D0); $SETCG(3, 5,-1.16D0)
  $SETCG(3, 6,-0.96D0); $SETCG(3, 8,-0.49D0); $SETCG(3,11, 1.00D0)
  $SETCG(4, 1,-1.10D0); $SETCG(4, 2,-1.06D0); $SETCG(4, 3, 0.95D0)
  $SETCG(4, 4,-0.54D0); $SETCG(4, 6,-1.78D0); $SETCG(4, 7,-0.41D0)
  $SETCG(4,12, 1.00D0); $SETCG(5, 4,-1.43D0); $SETCG(5, 5, 1.51D0)
  $SETCG(5, 6, 0.59D0); $SETCG(5, 7,-0.33D0); $SETCG(5, 8,-0.43D0)
  $SETCG(5,13, 1.00D0); $SETCG(6, 2,-1.72D0); $SETCG(6, 3,-0.33D0)
  $SETCG(6, 5, 1.62D0); $SETCG(6, 6, 1.24D0); $SETCG(6, 7, 0.21D0)
  $SETCG(6, 8,-0.26D0); $SETCG(6,14, 1.00D0); $SETCG(7, 1, 1.12D0)
  $SETCG(7, 4, 0.31D0); $SETCG(7, 7, 1.12D0); $SETCG(7, 9,-0.36D0)
  $SETCG(7,15, 1.00D0); $SETCG(8, 2, 0.45D0); $SETCG(8, 3, 0.26D0)
  $SETCG(8, 4,-1.10D0); $SETCG(8, 5, 0.58D0); $SETCG(8, 7,-1.03D0)
  $SETCG(8, 8, 0.10D0); $SETCG(8,16, 1.00D0)
$ENDSET
$SET(FGMODEL F)
  FF=0.0D0
  DO 3 I=1,NF
    GF(I)=0.0D0

```

```

3 CONTINUE
DO 5 I=1,NF
  WI=X(I)*(X(I)+1.0D0)+1.0D0
  K1=IH(I)
  K2=IH(I+1)-1
DO 4 K=K1,K2
  J=JH(K)
  WJ=X(J)*(X(J)+1.0D0)+1.0D0
  FF=FF+WI*WJ
  GF(I)=GF(I)+(2.0D0*X(I)+1.0D0)*WJ
  GF(J)=GF(J)+WI*(2.0D0*X(J)+1.0D0)
4 CONTINUE
5 CONTINUE
$ENDSET
$NF=16
$NX=16
$NC=8
$NCL=8
$JACC='S'
$HESF='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the bounds for variables, the sparsity pattern with numerical values of the model Hessian matrix, and the sparsity pattern with numerical values of the constraint Jacobian matrix. We use the procedure UKMCI1. The sparse Hessian matrix is indicated by the statement \$HESF='S'. The sparse Jacobian matrix is indicated by the statement \$JACC='S'. The option \$MODEL='FF' indicates a general objective function. By using the macrovariable \$FGMODEL we specify analytically the value and the gradient of the model function.

d) Problem solution (basic screen output):

```

CLASS = VM - LM3   UPDATE = M   MODEL = FF   HESF = S   NF =      16
  NIT=   0  NFV=   2  NFG=   2  F=  4528.573861   G=0.100D+01
  NIT=   1  NFV=   3  NFG=   3  F=   325.0955478   G=0.000D+00
  NIT=   2  NFV=   4  NFG=   4  F=   252.2712344   G=0.000D+00
  NIT=   3  NFV=   5  NFG=   5  F=   246.0828245   G=0.185D+02
  NIT=   4  NFV=   6  NFG=   6  F=   245.0236003   G=0.887D+01
  NIT=   5  NFV=   7  NFG=   7  F=   244.9100700   G=0.161D+01
  NIT=   6  NFV=   8  NFG=   8  F=   244.8997060   G=0.351D-01
  NIT=   7  NFV=   9  NFG=   9  F=   244.8996978   G=0.130D-01
  NIT=   8  NFV=  10  NFG=  10  F=   244.8996975   G=0.124D-02
  NIT=   9  NFV=  11  NFG=  11  F=   244.8996975   G=0.159D-03
  NIT=  10  NFV=  12  NFG=  12  F=   244.8996975   G=0.322D-06
0 NIT=  10  NFV=  12  NFG=  12  GRAD TOL F=  244.8996975   G=0.322D-06
FF =  0.2448996975D+03
X  =  0.3984735060D-01  0.7919831554D+00  0.2028703315D+00  0.8443579157D+00
      0.1269906451D+01  0.9347387094D+00  0.1681961969D+01  0.1553008761D+00
      0.1567870334D+01  0.0000000000D+00  0.0000000000D+00  0.0000000000D+00
      0.6602040658D+00  0.0000000000D+00  0.6742559281D+00  0.0000000000D+00
TIME= 0:00:00.00

```

## 7.24 Large-scale optimization with nonlinear equality constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^n (f_i^A(x))^2$$

where  $n = 100$  and

$$\begin{aligned} f_i^A(x) &= (3 - 2x_i)x_i - x_{i+1} + 1 & , \quad i = 1 \\ f_i^A(x) &= (3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1 & , \quad 2 \leq i \leq n - 1 \\ f_i^A(x) &= (3 - 2x_i)x_i - x_{i-1} + 1 & , \quad i = n \end{aligned}$$

over the set given by the nonlinear equality constraints

$$8x_i(x_i^2 - x_{i-1}) - 2(1 - x_i) + 4(x_i - x_{i+1}^2) + x_{i-1}^2 - x_{i-2} + x_{i+1} - x_{i+2}^2 = 0, \quad 3 \leq i \leq n - 2.$$

The starting point is  $x_i = -1$ ,  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE MINIMIZATION WITH NONLINEAR EQUALITY CONSTRAINTS +
$REM +-----+
$FLOAT WA,WB
$SET(INPUT)
  DO 1 I=1,NF
  X(I)=-1.0D0
1 CONTINUE
  M=0
  IH(1)=1
  DO 2 I=1,NF
  M=M+1
  JH(M)=I
  IF (I.LE.NF-1) THEN
  M=M+1
  JH(M)=I+1
  ENDIF
  IF (I.LE.NF-2) THEN
  M=M+1
  JH(M)=I+2
  ENDIF
  IH(I+1)=M+1
2 CONTINUE
  MC=0
  ICG(1)=1
  DO 3 I=3,NF-2
  MC=MC+1
  JCG(MC)=I-2
  MC=MC+1
  JCG(MC)=I-1
  MC=MC+1

```

```

JCG(MC)=I
MC=MC+1
JCG(MC)=I+1
MC=MC+1
JCG(MC)=I+2
ICG(I-1)=MC+1
3 CONTINUE
DO 4 KC=1,NC
  IC(KC)=5
  CL(KC)=0.0DO
4 CONTINUE
$ENDSET
$SET(FMODEL F)
  FF=0.0DO
  DO 5 J=1,NF
    WA=(3.0DO-2.0DO*X(J))*X(J)+1.0DO
    IF (J.GT. 1) WA=WA-X(J-1)
    IF (J.LT.NF) WA=WA-X(J+1)
    FF=FF+WA**2
5 CONTINUE
$ENDSET
$SET(GMODEL F)
  DO 6 J=1,NF
    GF(J)=0.0DO
6 CONTINUE
  DO 7 J=1,NF
    WA=(3.0DO-2.0DO*X(J))*X(J)+1.0DO
    IF (J.GT. 1) WA=WA-X(J-1)
    IF (J.LT.NF) WA=WA-X(J+1)
    WB=2.0DO*WA
    GF(J)=GF(J)+WB*(3.0DO-4.0DO*X(J))
    IF (J.GT. 1) GF(J-1)=GF(J-1)-WB
    IF (J.LT.NF) GF(J+1)=GF(J+1)-WB
7 CONTINUE
$ENDSET
$SET(FMODEL C)
  K=KC+2
  FC=8.0DO*X(K)*(X(K)**2-X(K-1))-2.0DO*(1.0DO-X(K))&
+4.0DO*(X(K)-X(K+1)**2)+X(K-1)**2-X(K-2)+X(K+1)-X(K+2)**2
$ENDSET
$SET(GMODEL C)
  K=KC+2
  GC(K-2)=-1.0DO
  GC(K-1)=-8.0DO*X(K)+2.0DO*X(K-1)
  GC(K)=2.4D1*X(K)**2-8.0DO*X(K-1)+6.0DO
  GC(K+1)=-8.0DO*X(K+1)+1.0DO
  GC(K+2)=-2.0DO*X(K+2)
$ENDSET
$NF=100
$NC=96
$NCE=NC
$JACC='S'

```

```

$HESF='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Hessian matrix, the sparsity pattern of the constraint Jacobian matrix, and the constraint specifications. The sparse Hessian matrix, indicated by the statement \$HESF='S', is tridiagonal. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. The statement \$NCE=NC specifies that all constraints are equalities (thus the special method for equality constraints can be used). By using the macrovariable \$FMODEL we specify analytically the value of the model function. By using the macrovariable \$GMODEL we specify analytically the gradient of the model function. By using the macrovariable \$FMODEL C we specify analytically the values of the constraint functions. By using the macrovariable \$GMODEL C we specify analytically the gradients of the constraint functions.

d) Problem solution (basic screen output):

```

CLASS = MN - LK3   UPDATE = N   MODEL = FF   HESF = S   NF =   100
  NIC=  0 NIT=  0 NFV=   1 NFG=  10 F=0.410D+03 C=0.280D+02 G=0.380D+02
  NIC=  0 NIT=  1 NFV=   2 NFG=  20 F=0.374D+04 C=0.880D+01 G=0.126D+02
  NIC=  0 NIT=  2 NFV=   3 NFG=  30 F=0.587D+03 C=0.285D+01 G=0.489D+01
  NIC=  0 NIT=  3 NFV=   4 NFG=  40 F=0.269D+03 C=0.127D+01 G=0.640D+01
  NIC=  0 NIT=  4 NFV=   5 NFG=  50 F=0.966D+02 C=0.851D+00 G=0.890D+01
  NIC=  0 NIT=  5 NFV=   9 NFG=  60 F=-.557D+02 C=0.731D+00 G=0.776D+01
  NIC=  0 NIT=  6 NFV=  11 NFG=  70 F=0.295D+01 C=0.311D+00 G=0.385D+01
  NIC=  0 NIT=  7 NFV=  12 NFG=  80 F=0.533D+01 C=0.490D-01 G=0.850D+00
  NIC=  0 NIT=  8 NFV=  13 NFG=  90 F=0.530D+01 C=0.252D-02 G=0.189D+00
  NIC=  0 NIT=  9 NFV=  14 NFG= 100 F=0.529D+01 C=0.927D-03 G=0.514D-01
  NIC=  0 NIT= 10 NFV=  15 NFG= 110 F=0.529D+01 C=0.190D-03 G=0.111D-01
  NIC=  0 NIT= 11 NFV=  16 NFG= 120 F=0.529D+01 C=0.238D-04 G=0.117D-02
  NIC=  0 NIT= 12 NFV=  17 NFG= 130 F=0.529D+01 C=0.447D-06 G=0.220D-04
  NIC=  0 NIT= 13 NFV=  18 NFG= 140 F=0.529D+01 C=0.168D-09 G=0.824D-08
  0 NIC=  0 NIT= 14 NFV=  18 NFG= 140 F=0.529D+01 C=0.168D-09 G=0.824D-08
TIME= 0:00:00.00

```

## 7.25 Large scale least squares optimization with nonlinear equality constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^n (f_i^A(x))^2$$

where  $n = 100$  and

$$\begin{aligned} f_i^A(x) &= (3 - 2x_i)x_i - x_{i+1} + 1 & , \quad i = 1 \\ f_i^A(x) &= (3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1 & , \quad 2 \leq i \leq n - 1 \\ f_i^A(x) &= (3 - 2x_i)x_i - x_{i-1} + 1 & , \quad i = n \end{aligned}$$

over the set given by the nonlinear equality constraints

$$8x_i(x_i^2 - x_{i-1}) - 2(1 - x_i) + 4(x_i - x_{i+1}^2) + x_{i-1}^2 - x_{i-2} + x_{i+1} - x_{i+2}^2 = 0, \quad 3 \leq i \leq n - 2.$$

The starting point is  $x_i = -1$ ,  $1 \leq i \leq n$ .

b) Problem specification (input file):

```
$REM +-----+
$REM + SPARSE LEAST SQUARES WITH NONLINEAR EQUALITY CONSTRAINTS +
$REM +-----+
$SET(INPUT)
  DO 1 I=1,NF
    X(I)=-1.0DO
1 CONTINUE
  MA=0
  IAG(1)=1
  DO 2 I=1,NA
    IF (I.GT.1) THEN
      MA=MA+1
      JAG(MA)=I-1
    ENDIF
    MA=MA+1
    JAG(MA)=I
    IF (I.LT.NF) THEN
      MA=MA+1
      JAG(MA)=I+1
    ENDIF
    IAG(I+1)=MA+1
2 CONTINUE
  MC=0
  ICG(1)=1
  DO 3 I=3,NF-2
    MC=MC+1
    JCG(MC)=I-2
    MC=MC+1
    JCG(MC)=I-1
    MC=MC+1
    JCG(MC)=I
    MC=MC+1
    JCG(MC)=I+1
    MC=MC+1
    JCG(MC)=I+2
    ICG(I-1)=MC+1
3 CONTINUE
  DO 4 KC=1,NC
    IC(KC)=5
    CL(KC)=0.0DO
4 CONTINUE
$ENDSET
$SET(FMODELA)
  IF(KA.EQ.1)THEN
    FA=(3.0DO-2.0DO*X(KA))*X(KA)+1.0DO-X(KA+1)
  ELSEIF(KA.EQ.NF)THEN
    FA=(3.0DO-2.0DO*X(KA))*X(KA)+1.0DO-X(KA-1)
  ELSE
    FA=(3.0DO-2.0DO*X(KA))*X(KA)+1.0DO-X(KA-1)-X(KA+1)
```

```

ENDIF
$ENDSET
$SET(FMODEL C)
K=KC+2
FC=8.0D0*X(K)*(X(K)**2-X(K-1))-2.0D0*(1.0D0-X(K))&
+4.0D0*(X(K)-X(K+1)**2)+X(K-1)**2-X(K-2)+X(K+1)-X(K+2)**2
$ENDSET
$IADA=1
$IADC=1
$NF=100
$NA=100
$NC=96
$NCE=NC
$MODEL='AQ'
$JACA='S'
$JACC='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Jacobian, matrix the sparsity pattern of the constraint Jacobian matrix, and the constraint specifications. The sparse Jacobian matrix, indicated by the statement \$JACA='S' is tridiagonal. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. The statement \$NCE=NC specifies that all constraints are equalities (thus the special method for equality constraints can be used). By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum-of-squares minimization we set \$MODEL='AQ'.

d) Problem solution (basic screen output):

```

CLASS = MN - LK3   UPDATE = N   MODEL = AQ   HESF = S   NF = 100
NIC= 0 NIT= 0 NFV= 19 NFG= 19 F=0.205D+03 C=0.280D+02 G=0.190D+02
NIC= 0 NIT= 1 NFV= 39 NFG= 39 F=0.363D+04 C=0.880D+01 G=0.631D+01
NIC= 0 NIT= 2 NFV= 59 NFG= 59 F=0.484D+03 C=0.285D+01 G=0.244D+01
NIC= 0 NIT= 3 NFV= 79 NFG= 79 F=0.169D+03 C=0.127D+01 G=0.320D+01
NIC= 0 NIT= 4 NFV= 99 NFG= 99 F=0.641D+02 C=0.851D+00 G=0.445D+01
NIC= 0 NIT= 5 NFV= 121 NFG= 121 F=-.181D+02 C=0.593D+00 G=0.330D+01
NIC= 0 NIT= 6 NFV= 141 NFG= 141 F=0.465D+01 C=0.307D+00 G=0.765D+00
NIC= 0 NIT= 7 NFV= 161 NFG= 161 F=0.267D+01 C=0.157D-01 G=0.231D+00
NIC= 0 NIT= 8 NFV= 181 NFG= 181 F=0.265D+01 C=0.278D-02 G=0.889D-01
NIC= 0 NIT= 9 NFV= 201 NFG= 201 F=0.265D+01 C=0.107D-02 G=0.257D-01
NIC= 0 NIT= 10 NFV= 221 NFG= 221 F=0.265D+01 C=0.198D-03 G=0.516D-02
NIC= 0 NIT= 11 NFV= 241 NFG= 241 F=0.265D+01 C=0.214D-04 G=0.534D-03
NIC= 0 NIT= 12 NFV= 261 NFG= 261 F=0.265D+01 C=0.373D-06 G=0.918D-05
NIC= 0 NIT= 13 NFV= 281 NFG= 281 F=0.265D+01 C=0.117D-09 G=0.287D-08
0 NIC= 0 NIT= 14 NFV= 281 NFG= 281 F=0.265D+01 C=0.117D-09 G=0.287D-08
TIME= 0:00:00.24

```

## 7.26 Large-scale nonlinear programming (sparse Hessian matrix)

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^{n-3} (x_i^2 + x_{i+1}^2 + 2x_{i+2}^2 + x_{i+3}^2 - 5x_i - 5x_{i+1} - 21x_{i+2} + 7x_{i+3}),$$

where  $n = 100$ , over the set given by the nonlinear constraints

$$\begin{aligned} x_{j-1}^2 + x_j^2 + x_{j+1}^2 + x_{j+2}^2 + x_{j-1} - x_j + x_{j+1} - x_{j+2} &\leq 8, & \text{mod}(k, 3) = 0 \\ x_{j-1}^2 + 2x_j^2 + x_{j+1}^2 + 2x_{j+2}^2 - x_{j-1} - x_{j+2} &\leq 10, & \text{mod}(k, 3) = 1 \\ 2x_{j-1}^2 + x_j^2 + x_{j+1}^2 + 2x_{j-1} - x_j - x_{j+2} &\leq 5, & \text{mod}(k, 3) = 2 \end{aligned}$$

where  $j = 2(\text{div}(k-1, 3) + 1)$ ,  $1 \leq k \leq 3(n-2)/2$  ( $\text{div}(k, l)$  is the integer division and  $\text{mod}(k, l)$  is the remainder after integer division). The starting point is  $x_i = 0$ ,  $1 \leq i \leq n$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE OPTIMIZATION WITH NONLINEAR CONSTRAINTS +
$REM +-----+
$SET(INPUT)
  DO 1 I=1,NF
  X(I)=0.0DO
1 CONTINUE
  DO 2 I=1,NF
  IH(I)=I
  JH(I)=I
2 CONTINUE
  IH(NF+1)=NF+1
  MC=0
  KC=0
  DO 5 J=1,NC/3
  CU(KC+1)=8.0DO
  CU(KC+2)=1.0D1
  CU(KC+3)=5.0DO
  I=2*(J-1)
  DO 4 K=1,3
  IC(KC+K)=2
  ICG(KC+K)=MC+1
  DO 3 L=1,4
  JCG(MC+L)=I+L
3 CONTINUE
  MC=MC+4
4 CONTINUE
  KC=KC+3
5 CONTINUE
  ICG(NC+1)=MC+1
$ENDSET
$SET(FMODEL)
  FF=0.0DO
  DO 11 I=1,NF-3

```



```

      FF=FF+X(I)**2+X(I+1)**2+2.0D0*X(I+2)**2+X(I+3)**2&
-5.0D0*X(I)-5.0D0*X(I+1)-2.1D1*X(I+2)+7.0D0*X(I+3)
11 CONTINUE
$ENDSET
$SET(GMODEL F)
      DO 12 I=1,NF
      GF(I)=0.0D0
12 CONTINUE
      DO 13 I=1,NF-3
      GF(I)=GF(I)+2.0D0*X(I)-5.0D0
      GF(I+1)=GF(I+1)+2.0D0*X(I+1)-5.0D0
      GF(I+2)=GF(I+2)+4.0D0*X(I+2)-2.1D1
      GF(I+3)=GF(I+3)+2.0D0*X(I+3)+7.0D0
13 CONTINUE
$ENDSET
$SET(FMODEL C)
      J=2*((KC-1)/3+1)
      L=MOD(KC,3)
      GO TO (21,22,23), L+1
21 FC=X(J-1)**2+X(J)**2+X(J+1)**2+X(J+2)**2+X(J-1)-X(J)+X(J+1)-X(J+2)
      GO TO 24
22 FC=X(J-1)**2+2.0D0*X(J)**2+X(J+1)**2+2.0D0*X(J+2)**2-X(J-1)-X(J+2)
      GO TO 24
23 FC=2.0D0*X(J-1)**2+X(J)**2+X(J+1)**2+2.0D0*X(J-1)-X(J)-X(J+2)
24 CONTINUE
$ENDSET
$SET(GMODEL C)
      J=2*((KC-1)/3+1)
      L=MOD(KC,3)
      GO TO (25,26,27), L+1
25 GC(J-1)=2.0D0*X(J-1)+1.0D0
      GC(J)=2.0D0*X(J)-1.0D0
      GC(J+1)=2.0D0*X(J+1)+1.0D0
      GC(J+2)=2.0D0*X(J+2)-1.0D0
      GO TO 28
26 GC(J-1)=2.0D0*X(J-1)-1.0D0
      GC(J)=4.0D0*X(J)
      GC(J+1)=2.0D0*X(J+1)
      GC(J+2)=4.0D0*X(J+2)-1.0D0
      GO TO 28
27 GC(J-1)=4.0D0*X(J-1)+2.0D0
      GC(J)=2.0D0*X(J)-1.0D0
      GC(J+1)=2.0D0*X(J+1)
      GC(J+2)=-1.0D0
28 CONTINUE
$ENDSET
$NF=100
$NC=147
$JACC='S'
$HESF='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Hessian matrix, the sparsity pattern of the constraint Jacobian matrix, and the constraint specifications. The sparse Hessian matrix, indicated by the statement \$HESF='S', is diagonal. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. By using the macrovariable \$FMOD-ELF we specify analytically the value of the model function. By using the macrovariable \$GMODEL F we specify analytically the gradient of the model function. By using the macrovariable \$FMODELC we specify analytically the values of the constraint functions. By using the macrovariable \$GMODEL C we specify analytically the gradients of the constraint functions.

d) Problem solution (basic screen output):

```

CLASS = MN - LI3    UPDATE = N    MODEL = FF    HESF = S    NF =    100
  NIC=  0 NIT=  0 NfV=   1 NfG=   7 F=0.000D+00 C=0.000D+00 G=0.307D+02
  NIC=  0 NIT=  1 NfV=   2 NfG=  14 F=-.273D+04 C=0.177D+02 G=0.580D+01
  NIC=  0 NIT=  2 NfV=   3 NfG=  21 F=-.238D+04 C=0.107D+02 G=0.402D+01
  NIC=  0 NIT=  3 NfV=   4 NfG=  28 F=-.203D+04 C=0.227D+01 G=0.199D+02
  NIC=  0 NIT=  4 NfV=   5 NfG=  35 F=-.208D+04 C=0.227D+02 G=0.175D+02
  NIC=  0 NIT=  5 NfV=   6 NfG=  42 F=-.207D+04 C=0.573D+01 G=0.206D+02
  NIC=  0 NIT=  6 NfV=   7 NfG=  49 F=-.207D+04 C=0.179D+01 G=0.105D+01
  NIC=  0 NIT=  7 NfV=   8 NfG=  56 F=-.207D+04 C=0.613D+00 G=0.131D+01
  NIC=  0 NIT=  8 NfV=   9 NfG=  63 F=-.207D+04 C=0.189D+00 G=0.169D+01
  NIC=  0 NIT=  9 NfV=  10 NfG=  70 F=-.207D+04 C=0.150D-01 G=0.912D-01
  NIC=  0 NIT= 10 NfV=  11 NfG=  77 F=-.207D+04 C=0.307D-03 G=0.859D-03
  NIC=  0 NIT= 11 NfV=  12 NfG=  84 F=-.207D+04 C=0.178D-07 G=0.123D-05
  NIC=  0 NIT= 12 NfV=  13 NfG=  91 F=-.207D+04 C=0.000D+00 G=0.629D-07
  NIC=  0 NIT= 13 NfV=  14 NfG=  98 F=-.207D+04 C=0.000D+00 G=0.314D-08
  NIC=  0 NIT= 14 NfV=  15 NfG= 105 F=-.207D+04 C=0.888D-15 G=0.157D-09
  0 NIC=  0 NIT= 15 NfV=  15 NfG= 105 F=-.207D+04 C=0.888D-15 G=0.157D-09
TIME=  0:00:00.02

```

## 7.27 Large-scale nonlinear programming (sparse Jacobian matrix)

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^{n-3} f_i^A(x),$$

where

$$f_i^A(x) = (x_i^2 + x_{i+1}^2 + 2x_{i+2}^2 + x_{i+3}^2 - 5x_i - 5x_{i+1} - 21x_{i+2} + 7x_{i+3}), \quad 1 \leq i \leq n-3,$$

and  $n = 100$ , over the set given by the nonlinear constraints

$$\begin{aligned} x_{j-1}^2 + x_j^2 + x_{j+1}^2 + x_{j+2}^2 + x_{j-1} - x_j + x_{j+1} - x_{j+2} &\leq 8, & \text{mod}(k, 3) &= 0 \\ x_{j-1}^2 + 2x_j^2 + x_{j+1}^2 + 2x_{j+2}^2 - x_{j-1} - x_{j+2} &\leq 10, & \text{mod}(k, 3) &= 1 \\ 2x_{j-1}^2 + x_j^2 + x_{j+1}^2 + 2x_{j-1} - x_j - x_{j+2} &\leq 5, & \text{mod}(k, 3) &= 2 \end{aligned}$$

where  $j = 2(\text{div}(k-1, 3) + 1)$ ,  $1 \leq k \leq 3(n-2)/2$  ( $\text{div}(k, l)$  is the integer division and  $\text{mod}(k, l)$  is the remainder after integer division). The starting point is  $x_i = 0$ ,  $1 \leq i \leq n$ .

b) Problem specification (input file):

```
$REM +-----+
$REM + PARTIALLY SEPARABLE OPTIMIZATION WITH NONLINEAR CONSTRAINTS +
$REM +-----+
$SET(INPUT)
  DO 1 I=1,NF
  X(I)=0.0DO
1 CONTINUE
  MA=0
  IAG(1)=1
  DO 8 KA=1,NA
  DO 9 K=1,4
  JAG(MA+K)=KA+K-1
9 CONTINUE
  MA=MA+4
  IAG(KA+1)=MA+1
8 CONTINUE
  MC=0
  KC=0
  DO 5 J=1,NC/3
  CU(KC+1)=8.0DO
  CU(KC+2)=1.0D1
  CU(KC+3)=5.0DO
  I=2*(J-1)
  DO 4 K=1,3
  IC(KC+K)=2
  ICG(KC+K)=MC+1
  DO 3 L=1,4
  JCG(MC+L)=I+L
3 CONTINUE
  MC=MC+4
4 CONTINUE
  KC=KC+3
5 CONTINUE
  ICG(NC+1)=MC+1
$ENDSET
$SET(FMODELA)
  FA=X(KA)**2+X(KA+1)**2+2.0DO*X(KA+2)**2+X(KA+3)**2&
-5.0DO*X(KA)-5.0DO*X(KA+1)-2.1D1*X(KA+2)+7.0DO*X(KA+3)
$ENDSET
$SET(FMODEL C)
  J=2*((KC-1)/3+1)
  L=MOD(KC,3)
  GO TO (21,22,23), L+1
21 FC=X(J-1)**2+X(J)**2+X(J+1)**2+X(J+2)**2+X(J-1)-X(J)+X(J+1)-X(J+2)
  GO TO 24
22 FC=X(J-1)**2+2.0DO*X(J)**2+X(J+1)**2+2.0DO*X(J+2)**2-X(J-1)-X(J+2)
  GO TO 24
23 FC=2.0DO*X(J-1)**2+X(J)**2+X(J+1)**2+2.0DO*X(J-1)-X(J)-X(J+2)
24 CONTINUE
$ENDSET
```

```

$IADA=1
$IADC=1
$NF=100
$NA=97
$NC=147
$MODEL='AF'
$JACA='S'
$JACC='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Jacobian matrix, the sparsity pattern of the constraint Jacobian matrix and the constraint specifications. The sparse objective Jacobian matrix is indicated by the statement \$JACA='S'. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum of values minimization we set \$MODEL='AF'.

d) Problem solution (basic screen output):

```

CLASS = MN - LI3   UPDATE = N   MODEL = AF   HESF = S   NF =   100
  NIC=  0 NIT=  0 NFV=   8 NFG=   8 F=0.000D+00 C=0.000D+00 G=0.304D+02
  NIC=  0 NIT=  1 NFV=  17 NFG=  17 F=-.271D+04 C=0.202D+02 G=0.595D+01
  NIC=  0 NIT=  2 NFV=  26 NFG=  26 F=-.247D+04 C=0.899D+01 G=0.876D+01
  NIC=  0 NIT=  3 NFV=  35 NFG=  35 F=-.218D+04 C=0.239D+01 G=0.224D+02
  NIC=  0 NIT=  4 NFV=  44 NFG=  44 F=-.231D+04 C=0.308D+02 G=0.250D+01
  NIC=  0 NIT=  5 NFV=  53 NFG=  53 F=-.227D+04 C=0.115D+02 G=0.185D+02
  NIC=  0 NIT=  6 NFV=  62 NFG=  62 F=-.228D+04 C=0.296D+01 G=0.497D+02
  NIC=  0 NIT=  7 NFV=  71 NFG=  71 F=-.229D+04 C=0.192D+00 G=0.342D+01
  NIC=  0 NIT=  8 NFV=  80 NFG=  80 F=-.229D+04 C=0.229D-01 G=0.185D+00
  NIC=  0 NIT=  9 NFV=  89 NFG=  89 F=-.229D+04 C=0.240D-02 G=0.147D-02
  NIC=  0 NIT= 10 NFV=  98 NFG=  98 F=-.229D+04 C=0.606D-06 G=0.237D-06
  NIC=  0 NIT= 11 NFV= 107 NFG= 107 F=-.229D+04 C=0.000D+00 G=0.524D-08
  NIC=  0 NIT= 12 NFV= 116 NFG= 116 F=-.229D+04 C=0.000D+00 G=0.262D-09
  NIC=  0 NIT= 13 NFV= 125 NFG= 125 F=-.229D+04 C=0.000D+00 G=0.131D-10
  0 NIC=  0 NIT= 14 NFV= 125 NFG= 125 F=-.229D+04 C=0.000D+00 G=0.131D-10
TIME= 0:00:00.17

```

## 7.28 Large scale least squares optimization with nonlinear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \frac{1}{2} \sum_{i=1}^{n-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2),$$

where  $n = 100$ , with constraints

$$3x_{i+1}^3 + 2x_{i+2} - 5 + \sin(x_{i+1} - x_{i+2}) \sin(x_{i+1} + x_{i+2}) + 4x_{i+1} - x_i \exp(x_i - x_{i+1}) - 3 \geq 2$$

for  $1 \leq i \leq n - 2$ . The starting point is  $x_i = -1.2, \text{mod}(i, 2) = 1, x_i = 1.0, \text{mod}(i, 2) = 0$ . The minimum value of the objective function is  $F = 0.0$  (This problem is equivalent to the previous one).

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE LEAST SQUARES WITH NONLINEAR CONSTRAINTS +
$REM +-----+
$FLOAT W1,W2,W3
$SET(INPUT)
  DO 1 I=1,NF
    IF(MOD(I,2).EQ.1) THEN
      X(I)=-1.2DO
    ELSE
      X(I)=1.0DO
    ENDIF
1 CONTINUE
  DO 2 I=1,NF-1
    J=3*(I-1)+1
    JAG(J)=I
    JAG(J+1)=I+1
    JAG(J+2)=I
    K=2*(I-1)+1
    IAG(K)=J
    IAG(K+1)=J+2
2 CONTINUE
  IAG(K+2)=J+3
  NA=2*(NF-1)
  MA=3*(NF-1)
  NC=NF-2
  MC=0
  ICG(1)=1
  DO 3 I=2,NF-1
    MC=MC+1
    JCG(MC)=I-1
    MC=MC+1
    JCG(MC)=I
    MC=MC+1
    JCG(MC)=I+1
    ICG(I)=MC+1
3 CONTINUE
  CALL UXVINS(NC,1,IC)
  CALL UXVSET(NC,2.0DO,CL)
$ENDSET
$SET(FMODELA)
  I=(KA+1)/2
  IF(MOD(KA,2).EQ.1)THEN
    FA=1.0D1*(X(I)**2-X(I+1))
  ELSE
    FA=X(I)-1.0DO
  ENDIF
$ENDSET

```

```

$SET(FMODEL C)
K=KC+1
FC=3.0DO*X(K)**3+2.0DO*X(K+1)-5.0DO+SIN(X(K)-X(K+1))*SIN(X(K)&
+X(K+1))+4.0DO*X(K)-X(K-1)*EXP(X(K-1)-X(K))-3.0DO
$ENDSET
$IADA=1
$IADC=1
$NF=100
$NA=198
$NC=98
$MODEL='AQ'
$JACA='S'
$JACC='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Jacobian matrix, the sparsity pattern of the constraint Jacobian matrix and the constraint specifications. The sparse objective Jacobian matrix is indicated by the statement \$JACA='S'. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the sum-of-squares minimization we set \$MODEL='AQ'.

d) Problem solution (basic screen output):

```

CLASS = MN - LI3    UPDATE = N    MODEL = AQ    HESF = S    NF =    100
  NIC=  0 NIT=  0 NFV=   11 NFG=   11 F=0.126D+05 C=0.268D+02 G=0.396D+03
  NIC=  0 NIT=  1 NFV=   23 NFG=   23 F=0.162D+05 C=0.125D+02 G=0.181D+03
  NIC=  0 NIT=  2 NFV=   35 NFG=   35 F=-.136D+05 C=0.594D+01 G=0.170D+04
  NIC=  0 NIT=  3 NFV=   47 NFG=   47 F=-.694D+03 C=0.379D+00 G=0.196D+03
  NIC=  0 NIT=  4 NFV=   59 NFG=   59 F=0.107D+03 C=0.142D-01 G=0.192D+02
  NIC=  0 NIT=  5 NFV=   71 NFG=   71 F=0.108D+03 C=0.331D-01 G=0.473D+01
  NIC=  0 NIT=  6 NFV=   83 NFG=   83 F=0.108D+03 C=0.000D+00 G=0.134D+02
  NIC=  0 NIT=  7 NFV=   95 NFG=   95 F=0.107D+03 C=0.138D-03 G=0.144D+02
  NIC=  0 NIT=  8 NFV=  107 NFG=  107 F=0.107D+03 C=0.113D-05 G=0.664D+01
  NIC=  0 NIT=  9 NFV=  119 NFG=  119 F=0.106D+03 C=0.431D-09 G=0.889D+01
  NIC=  0 NIT= 10 NFV=  131 NFG=  131 F=0.106D+03 C=0.000D+00 G=0.127D+02
  NIC=  0 NIT= 11 NFV=  143 NFG=  143 F=0.106D+03 C=0.116D-03 G=0.619D+00
  NIC=  0 NIT= 12 NFV=  155 NFG=  155 F=0.106D+03 C=0.000D+00 G=0.717D+01
  NIC=  0 NIT= 13 NFV=  167 NFG=  167 F=0.106D+03 C=0.733D-05 G=0.268D+00
  NIC=  0 NIT= 14 NFV=  179 NFG=  179 F=0.106D+03 C=0.000D+00 G=0.835D-01
  NIC=  0 NIT= 15 NFV=  191 NFG=  191 F=0.106D+03 C=0.433D-07 G=0.296D-02
  NIC=  0 NIT= 16 NFV=  203 NFG=  203 F=0.106D+03 C=0.178D-14 G=0.154D-04
  NIC=  0 NIT= 17 NFV=  215 NFG=  215 F=0.106D+03 C=0.178D-14 G=0.130D-11
  0 NIC=  0 NIT= 18 NFV=  215 NFG=  215 F=0.106D+03 C=0.178D-14 G=0.130D-11
TIME= 0:00:00.22

```

## 7.29 Large-scale $l_1$ optimization with nonlinear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \sum_{i=1}^{n-1} (10|x_i^2 - x_{i+1}| + |x_i - 1|),$$

where  $n = 100$ , with constraints

$$3x_{i+1}^3 + 2x_{i+2} - 5 + \sin(x_{i+1} - x_{i+2}) \sin(x_{i+1} + x_{i+2}) + 4x_{i+1} - x_i \exp(x_i - x_{i+1}) - 3 \geq 2$$

for  $1 \leq i \leq n - 2$ . The starting point is  $x_i = -1.2$ ,  $\text{mod}(i, 2) = 1$ ,  $x_i = 1.0$ ,  $\text{mod}(i, 2) = 0$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE L-1 OPTIMIZATION WITH NONLINEAR CONSTRAINTS +
$REM +-----+
$FLOAT W1,W2,W3
$SET(INPUT)
  DO 1 I=1,NF
    IF (MOD(I,2).EQ.1) THEN
      X(I)=-1.2DO
    ELSE
      X(I)=1.0DO
    ENDIF
1 CONTINUE
  DO 2 I=1,NF-1
    J=3*(I-1)+1
    JAG(J)=I
    JAG(J+1)=I+1
    JAG(J+2)=I
    K=2*(I-1)+1
    IAG(K)=J
    IAG(K+1)=J+2
2 CONTINUE
  IAG(K+2)=J+3
  NA=2*(NF-1)
  MA=3*(NF-1)
  NC=NF-2
  MC=0
  ICG(1)=1
  DO 3 I=2,NF-1
    MC=MC+1
    JCG(MC)=I-1
    MC=MC+1
    JCG(MC)=I
    MC=MC+1
    JCG(MC)=I+1
    ICG(I)=MC+1
3 CONTINUE
  CALL UXVINS(NC,1,IC)

```

```

CALL UXVSET(NC,2.0D0,CL)
$ENDSET
$SET(FMODELA)
I=(KA+1)/2
IF(MOD(KA,2).EQ.1)THEN
  FA=1.0D1*(X(I)**2-X(I+1))
ELSE
  FA=X(I)-1.0D0
ENDIF
$ENDSET
$SET(FMODEL C)
K=KC+1
FC=3.0D0*X(K)**3+2.0D0*X(K+1)-5.0D0+SIN(X(K)-X(K+1))*SIN(X(K)&
+X(K+1))+4.0D0*X(K)-X(K-1)*EXP(X(K-1)-X(K))-3.0D0
$ENDSET
$IADA=1
$IADC=1
$NF=100
$NA=198
$NC=98
$M=10000
$MODEL='AA'
$JACA='S'
$JACC='S'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Jacobian matrix, the sparsity pattern of the constraint Jacobian matrix and the constraint specifications. The sparse objective Jacobian matrix is indicated by the statement \$JACA='S'. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the  $l_1$  (sum of absolute values) minimization we set \$MODEL='AA'.

d) Problem solution (basic screen output):

```

CLASS = MN - LI3    UPDATE = N    MODEL = AA    HESF = S    NF =    298
NIC=  0 NIT=  0 NFV=   1 NFG=   7 F=0.302D+03 C=0.268D+02 G=0.280D+01
NIC=  0 NIT=  1 NFV=   2 NFG=  14 F=0.908D+04 C=0.110D+02 G=0.302D+03
NIC=  0 NIT=  2 NFV=   3 NFG=  21 F=0.955D+04 C=0.149D+02 G=0.503D+03
NIC=  0 NIT=  3 NFV=   4 NFG=  28 F=0.332D+04 C=0.136D+02 G=0.274D+03
NIC=  0 NIT=  4 NFV=   5 NFG=  35 F=0.204D+03 C=0.783D+02 G=0.157D+04
NIC=  0 NIT=  5 NFV=   6 NFG=  42 F=0.288D+04 C=0.262D+02 G=0.116D+04
NIC=  0 NIT=  6 NFV=   7 NFG=  49 F=0.120D+03 C=0.160D+02 G=0.207D+03
NIC=  0 NIT=  7 NFV=  11 NFG=  56 F=0.151D+03 C=0.141D+02 G=0.181D+03
NIC=  0 NIT=  8 NFV=  15 NFG=  63 F=0.164D+03 C=0.123D+02 G=0.158D+03
NIC=  0 NIT=  9 NFV=  16 NFG=  70 F=0.175D+03 C=0.994D+00 G=0.127D+03
NIC=  0 NIT= 10 NFV=  17 NFG=  77 F=0.156D+03 C=0.186D+01 G=0.143D+02

```



```

NIC= 0 NIT= 11 NFV= 18 NFG= 84 F=0.156D+03 C=0.350D+00 G=0.632D+01
NIC= 0 NIT= 12 NFV= 19 NFG= 91 F=0.155D+03 C=0.173D+01 G=0.830D+01
NIC= 0 NIT= 13 NFV= 21 NFG= 98 F=0.325D+03 C=0.210D+03 G=0.626D+04
NIC= 0 NIT= 14 NFV= 22 NFG= 105 F=0.175D+03 C=0.412D+03 G=0.312D+03
NIC= 0 NIT= 15 NFV= 23 NFG= 112 F=0.155D+03 C=0.304D+02 G=0.530D+01
NIC= 0 NIT= 16 NFV= 24 NFG= 119 F=0.155D+03 C=0.132D-01 G=0.167D+01
NIC= 0 NIT= 17 NFV= 25 NFG= 126 F=0.155D+03 C=0.176D+01 G=0.652D+00
NIC= 0 NIT= 18 NFV= 26 NFG= 133 F=0.155D+03 C=0.951D+00 G=0.115D-01
NIC= 0 NIT= 19 NFV= 27 NFG= 140 F=0.155D+03 C=0.258D-03 G=0.289D-04
NIC= 0 NIT= 20 NFV= 28 NFG= 147 F=0.155D+03 C=0.899D-07 G=0.110D-07
NIC= 0 NIT= 21 NFV= 29 NFG= 154 F=0.155D+03 C=0.888D-15 G=0.449D-09
0 NIC= 0 NIT= 22 NFV= 29 NFG= 154 F=0.155D+03 C=0.888D-15 G=0.449D-09
TIME= 0:00:00.42

```

### 7.30 Large-scale $l_\infty$ (minimax) optimization with nonlinear constraints

a) Problem description:

Suppose we have to find a local minimum of the objective function

$$F(x) = \max_{1 \leq i \leq n-1} (\max(10|x_i^2 - x_{i+1}|, |x_i - 1|)),$$

where  $n = 100$ , with constraints

$$3x_{i+1}^3 + 2x_{i+2} - 5 + \sin(x_{i+1} - x_{i+2}) \sin(x_{i+1} + x_{i+2}) + 4x_{i+1} - x_i \exp(x_i - x_{i+1}) - 3 \geq 2$$

for  $1 \leq i \leq n - 2$ . The starting point is  $x_i = -1.2$ ,  $\text{mod}(i, 2) = 1$ ,  $x_i = 1.0$ ,  $\text{mod}(i, 2) = 0$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SPARSE MINIMAX OPTIMIZATION WITH NONLINEAR CONSTRAINTS +
$REM +-----+
$FLOAT W1,W2,W3
$SET(INPUT)
DO 1 I=1,NF
  IF(MOD(I,2).EQ.1) THEN
    X(I)=-1.2D0
  ELSE
    X(I)=1.0D0
  ENDIF
1 CONTINUE
DO 2 I=1,NF-1
  J=3*(I-1)+1
  JAG(J)=I
  JAG(J+1)=I+1
  JAG(J+2)=I
  K=2*(I-1)+1
  IAG(K)=J
  IAG(K+1)=J+2
2 CONTINUE
IAG(K+2)=J+3
NA=2*(NF-1)
MA=3*(NF-1)

```

```

NC=NF-2
MC=0
ICG(1)=1
DO 3 I=2,NF-1
    MC=MC+1
    JCG(MC)=I-1
    MC=MC+1
    JCG(MC)=I
    MC=MC+1
    JCG(MC)=I+1
    ICG(I)=MC+1
3 CONTINUE
CALL UXVINS(NC,1,IC)
CALL UXVSET(NC,2.0D0,CL)
$ENDSET
$SET(FMODELA)
I=(KA+1)/2
IF(MOD(KA,2).EQ.1)THEN
    FA=1.0D1*(X(I)**2-X(I+1))
ELSE
    FA=X(I)-1.0D0
ENDIF
$ENDSET
$SET(FMODEL C)
K=KC+1
FC=3.0D0*X(K)**3+2.0D0*X(K+1)-5.0D0+SIN(X(K)-X(K+1))*SIN(X(K)&
+X(K+1))+4.0D0*X(K)-X(K-1)*EXP(X(K-1)-X(K))-3.0D0
$ENDSET
$IADA=1
$IADC=1
$NF=100
$NA=198
$NC=98
$M=10000
$MODEL='AM'
$JACA='S'
$JACC='S'
$EPS6='1.0D-2'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of variables, the sparsity pattern of the objective Jacobian matrix, the sparsity pattern of the constraint Jacobian matrix and the constraint specifications. The sparse objective Jacobian matrix is indicated by the statement \$JACA='S'. The sparse constraint Jacobian matrix is indicated by the statement \$JACC='S'. By using the macrovariable \$FMODELA we specify values of the approximating function analytically. The gradients of the approximating function are computed by automatic differentiation, since \$IADA=1. By using the macrovariable \$FMODEL C we specify the values of the constraint functions analytically. The gradients of the constraint functions are computed by automatic differentiation, since \$IADC=1. Note that statements IF-THEN and ELSEIF-THEN have not to contain blanks if the automatic differentiation is used. For the  $l_\infty$  minimization (minimax) we set \$MODEL='AM'.

d) Problem solution (basic screen output):

```

CLASS = MN - LI3    UPDATE = N    MODEL = AM    HESF = S    NF =    101
  NIC=  0 NIT=  0 NFV=   1 NFG=   7 F=0.302D+03 C=0.268D+02 G=0.386D+02
  NIC=  0 NIT=  1 NFV=   2 NFG=  14 F=0.924D+04 C=0.118D+02 G=0.475D+03
  NIC=  0 NIT=  2 NFV=   3 NFG=  21 F=0.386D+04 C=0.157D+02 G=0.794D+03
  NIC=  0 NIT=  3 NFV=   4 NFG=  28 F=-.193D+05 C=0.201D+02 G=0.262D+03
  NIC=  0 NIT=  4 NFV=   5 NFG=  35 F=-.387D+03 C=0.711D+00 G=0.633D+02
  NIC=  0 NIT=  5 NFV=   6 NFG=  42 F=-.443D+02 C=0.280D+01 G=0.171D+02
  NIC=  0 NIT=  6 NFV=   7 NFG=  49 F=0.861D+01 C=0.330D+01 G=0.194D+01
  NIC=  0 NIT=  7 NFV=   8 NFG=  56 F=0.220D+01 C=0.291D+01 G=0.357D+00
  NIC=  0 NIT=  8 NFV=   9 NFG=  63 F=0.116D+01 C=0.182D+01 G=0.395D-01
  NIC=  0 NIT=  9 NFV=  10 NFG=  70 F=0.224D+01 C=0.143D+01 G=0.167D+01
  NIC=  0 NIT= 10 NFV=  11 NFG=  77 F=0.695D+00 C=0.831D+01 G=0.262D+01
-----
  NIC=  0 NIT= 61 NFV=  303 NFG=  434 F=0.192D+01 C=0.907D-02 G=0.344D-04
  NIC=  0 NIT= 62 NFV=  310 NFG=  441 F=0.192D+01 C=0.893D-02 G=0.338D-04
  NIC=  0 NIT= 63 NFV=  315 NFG=  448 F=0.192D+01 C=0.832D-02 G=0.312D-04
  NIC=  0 NIT= 64 NFV=  316 NFG=  455 F=0.192D+01 C=0.000D+00 G=0.308D-07
  NIC=  0 NIT= 65 NFV=  323 NFG=  462 F=0.192D+01 C=0.000D+00 G=0.303D-07
  NIC=  0 NIT= 66 NFV=  329 NFG=  469 F=0.192D+01 C=0.000D+00 G=0.288D-07
  NIC=  0 NIT= 67 NFV=  331 NFG=  476 F=0.192D+01 C=0.000D+00 G=0.143D-07
  NIC=  0 NIT= 68 NFV=  338 NFG=  483 F=0.192D+01 C=0.000D+00 G=0.141D-07
  NIC=  0 NIT= 69 NFV=  345 NFG=  490 F=0.192D+01 C=0.000D+00 G=0.427D-07
  0 NIC=  0 NIT= 70 NFV=  345 NFG=  490 F=0.192D+01 C=0.000D+00 G=0.427D-07
TIME= 0:00:01.80

```

### 7.31 Optimization of dynamical systems - general integral criterion

a) Problem description:

Suppose we have to find a minimum of the objective function

$$F(x) = \frac{1}{2} \int_0^T (y_1^2(t) + y_2^2(t)) dt + \frac{1}{2} (y_1^2(T) + y_2^2(T))$$

where  $T = 1.5$  and where

$$\frac{dy_1(t)}{dt} = y_2(t), \quad y_1(0) = x_1$$

$$\frac{dy_2(t)}{dt} = (1 - y_1^2(t))y_2(t) - y_1(t), \quad y_2(0) = 1$$

b) Problem specification (input file):

```

$REM +-----+
$REM + OPTIMIZATION OF DYNAMICAL SYSTEMS - GENERAL CRITERION          +
$REM +-----+
$SET(INPUT)
  X(1)=0.0D0
  TA=0.0D0
  TAMAX=1.5D0
$ENDSET

```

```

$SET(FMODEL F)
  FF=0.5D0*(YA(1)**2+YA(2)**2)
$ENDSET
$SET(DMODEL F)
  DF(1)=YA(1)
  DF(2)=YA(2)
$ENDSET
$SET(FMODEL A)
  FA=HALF*(YA(1)**2+YA(2)**2)
$ENDSET
$SET(DMODEL A)
  DA(1)=YA(1)
  DA(2)=YA(2)
$ENDSET
$SET(FMODEL E)
  GO TO (1,2) KE
1 FE=YA(2)
  GO TO 3
2 FE=-YA(1)+(ONE-YA(1)**2)*YA(2)
3 CONTINUE
$ENDSET
$SET(DMODEL E)
  GO TO (4,5) KE
4 DE(1)=ZERO
  DE(2)=ONE
  GO TO 6
5 DE(1)=-ONE-TWO*YA(1)*YA(2)
  DE(2)=ONE-YA(1)**2
6 CONTINUE
$ENDSET
$SET(FMODEL Y)
  GO TO (7,8) KE
7 FE=X(1)
  GO TO 9
8 FE=ONE
9 CONTINUE
$ENDSET
$SET(GMODEL Y)
  GO TO (10,11) KE
10 GE(1)=ONE
  GO TO 12
11 GE(1)=ZERO
12 CONTINUE
$ENDSET
$NF=1
$NE=2
$MODEL='DF'
$TOLR='1.0D-9'
$TOLA='1.0D-9'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial value of the variable  $x_1$  as well as the initial and terminal times 0 and T, respectively. By using the macrovariables \$FMODELA and \$DMODELA we specify the subintegral function and by using the macrovariables \$FMODEL and \$DMODEL we specify the terminal function. The right hand sides of the differential equations are specified by using the macrovariables \$FMODELE and \$DMODELE, while the initial values and their derivatives are given by using the macrovariables \$FMODELY and \$GMODELY. The option \$MODEL='DF' indicates a general integral criterion.

d) Problem solution (basic screen output):

```

CLASS = VM - LI1   UPDATE = B   MODEL = DF   HESF = D   NF =      1
  NIT=    0 NFV=    1 NFG=    0 F=  2.763393900   G=0.242D+01
  NIT=    1 NFV=    3 NFG=    0 F=  1.974913643   G=0.513D+00
  NIT=    2 NFV=    4 NFG=    0 F=  1.944408382   G=0.398D-01
  NIT=    3 NFV=    5 NFG=    0 F=  1.944233193   G=0.478D-03
  NIT=    4 NFV=    6 NFG=    0 F=  1.944233168   G=0.338D-06
  0 NIT=    4 NFV=    6 NFG=    0 GRAD TOL F=  1.944233168   G=0.338D-06
FF =  0.7671653593D+00
X   =  0.6169839176D+00
TIME= 0:00:00.00

```

### 7.32 Optimization of dynamical systems - special integral criterion

a) Problem description:

Suppose we have to find a minimum of the objective function

$$F(x) = \frac{1}{2} \int_0^T (y_1(t) - 1/(1+t))^2 dt$$

where  $T = 1$  and where

$$\frac{dy_1(t)}{dt} = -x_1 y_1(t), \quad y_1(0) = x_2$$

b) Problem specification (input file):

```

$REM +-----+
$REM + OPTIMIZATION OF DYNAMICAL SYSTEMS - QUADRATIC CRITERION      +
$REM +-----+
$SET(INPUT)
  X(1)=2.0D0
  X(2)=0.0D0
  TA=0.0D0
  TAMAX=1.0D0
$ENDSET
$SET(FMODELE)
  FE=-X(1)*YA(1)**2
  YE=1.0D0/(1.0D0+TA)
  WE=1.0D0
$ENDSET
$SET(GDMODELE)
  GE(1)=-YA(1)**2
  GE(2)= 0.0D0

```

```

DE(1)=-2.0D0*X(1)*YA(1)
$ENDSET
$SET(FMODELY)
FE=X(2)
$ENDSET
$SET(GMODELY)
GE(1)=0.0D0
GE(2)=1.0D0
$ENDSET
$MODELA='Y'
$NF=2
$NE=1
$MODEL='DQ'
$TOLR='1.0D-9'
$TOLA='1.0D-9'
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of the variables  $x_1$  and  $x_2$  as well as the initial and terminal times 0 and T, respectively. The right hand side of the differential equation is specified by using the macrovariables \$FMODELE and \$GDMODELE, while the initial values and their derivatives are given by using the macrovariables \$FMODELY and \$GMODELY. The option \$MODEL='DQ' together with \$MODELA='Y' indicate a special integral criterion.

d) Problem solution (basic screen output):

```

CLASS = GN - GM7  UPDATE = N  MODEL = DQ  HESF = D  NF = 2
NIT= 0  NFV= 1  NFG= 1  F= 0.2500000000  G=0.693D+00
NIT= 1  NFV= 3  NFG= 2  F= 0.3379696559E-01  G=0.114D+00
NIT= 2  NFV= 5  NFG= 3  F= 0.1598937577E-02  G=0.613D-02
NIT= 3  NFV= 7  NFG= 4  F= 0.1195750953E-04  G=0.225D-02
NIT= 4  NFV= 9  NFG= 5  F= 0.1909017677E-08  G=0.300D-04
NIT= 5  NFV= 11  NFG= 6  F= 0.2793082948E-15  G=0.200D-08
0 NIT= 5  NFV= 11  NFG= 6  GRAD TOL  F= 0.2793082948E-15  G=0.200D-08
F = 0.2793082948D-15
X = 0.9999999725D+00  0.9999999990D+00
TIME= 0:00:00.00

```

### 7.33 Non-stiff initial value problem for ordinary differential equations

a) Problem description:

Suppose we have to find a solution of a three-body problem

$$\frac{dy_1(t)}{dt} = y_3(t), \quad y_1(0) = 0.9$$

$$\frac{dy_2(t)}{dt} = y_4(t), \quad y_2(0) = 0$$

$$\frac{dy_3(t)}{dt} = y_1(t) + 2y_4(t) - \mu' \frac{y_1(t)+\mu}{D_1(t)} - \mu \frac{y_1(t)-\mu'}{D_2(t)}, \quad y_3(0) = 0$$

$$\frac{dy_4(t)}{dt} = y_2(t) - 2y_3(t) - \mu' \frac{y_2(t)}{D_1(t)} - \mu \frac{y_2(t)}{D_2(t)}, \quad y_4(0) = -2$$

with  $\mu = 0.012$ ,  $\mu' = 1 - \mu$ ,

$$D1 = ((y_1(t) + \mu)^2 + y_2(t)^2)^{3/2},$$

$$D2 = ((y_1(t) - \mu')^2 + y_2(t)^2)^{3/2},$$

in the interval  $0 \leq t \leq T$ , where  $T = 18$ .

b) Problem specification (input file):

```

$REM +-----+
$REM + SOLUTION OF NONSTIFF ORDINARY DIFFERENTIAL EQUATIONS +
$REM +-----+
$FLOAT W,W1,W2
$SET(INPUT)
  TA=0.0D0
  YA(1)= 0.9D0
  YA(2)= 0.0D0
  YA(3)= 0.0D0
  YA(4)=-2.0D0
  TAMAX= 1.8D1
$ENDSET
$SET(FMODELE)
  W=1.2D-2
  W1=(YA(1)+W)**2+YA(2)**2
  W1=W1*SQRT(W1)
  W2=(YA(1)+W-1.D0)**2+YA(2)**2
  W2=W2*SQRT(W2)
  GO TO (1,2,3,4) KE
1 FE=YA(3)
  GO TO 5
2 FE=YA(4)
  GO TO 5
3 FE=YA(1)+2*YA(4)+(W-1.D0)*(YA(1)+W)/W1-W*(YA(1)+W-1.D0)/W2
  GO TO 5
4 FE=YA(2)-2*YA(3)+(W-1.D0)*YA(2)/W1-W*YA(2)/W2
5 CONTINUE
$ENDSET
$NA=19
$NE=4
$MODEL='DE'
$MED=2
$BATCH
$STANDARD

```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of the variables  $y_1$ ,  $y_2$ ,  $y_3$ ,  $y_4$  as well as the initial and terminal times 0 and T, respectively. The right hand sides of the differential equations are specified by using the macrovariable \$FMODELE. The option \$MODEL='DE' indicates integration of a system of ordinary differential equations. Macrovariable \$NA denotes the number of time points (nodes) in which the results are printed.

d) Problem solution (basic screen output):

```

CLASS = DE - DP8   UPDATE = N   MODEL = NO   HESF = N   NF =      0
  0 NSTP=   129  NACC=    99  NREJ=    29  NEV =  6296  NEG =      0
  1 AT=  0.000000000D+00
    AY=  0.900000000D+00  0.000000000D+00  0.000000000D+00 -0.200000000D+01
  2 AT=  0.100000000D+01
    AY= -0.472474065D+00 -0.831552587D+00 -0.169925866D+01  0.933246989D+00
  3 AT=  0.200000000D+01
    AY= -0.522790604D+00  0.777581853D+00  0.164553511D+01  0.102899578D+01
  4 AT=  0.300000000D+01
    AY=  0.860391993D+00 -0.437352811D-02 -0.262361582D-01 -0.198574759D+01
  5 AT=  0.400000000D+01
    AY= -0.545402744D+00 -0.728281933D+00 -0.158624165D+01  0.112488972D+01
  6 AT=  0.500000000D+01
    AY= -0.361844730D+00  0.857723938D+00  0.179231945D+01  0.743627310D+00
  7 AT=  0.600000000D+01
    AY=  0.900662074D+00 -0.136996194D+00 -0.190925673D+00 -0.195779058D+01
  8 AT=  0.700000000D+01
    AY= -0.560957109D+00 -0.823081310D+00 -0.162128701D+01  0.106071764D+01
  9 AT=  0.800000000D+01
    AY= -0.539311035D+00  0.804263515D+00  0.165764912D+01  0.100452367D+01
 10 AT=  0.900000000D+01
    AY=  0.852371731D+00  0.255993603D-01 -0.165348326D-01 -0.198405038D+01
 11 AT=  0.100000000D+02
    AY= -0.552637953D+00 -0.670747471D+00 -0.153518712D+01  0.120708594D+01
 12 AT=  0.110000000D+02
    AY= -0.236500690D+00  0.877334516D+00  0.186864292D+01  0.531341150D+00
 13 AT=  0.120000000D+02
    AY=  0.910238785D+00 -0.267656880D+00 -0.360816991D+00 -0.191426158D+01
 14 AT=  0.130000000D+02
    AY= -0.609930562D+00 -0.895015297D+00 -0.163634493D+01  0.104639816D+01
 15 AT=  0.140000000D+02
    AY= -0.713359518D+00  0.770343601D+00  0.150308593D+01  0.122606580D+01
 16 AT=  0.150000000D+02
    AY=  0.822862768D+00  0.266689821D+00  0.396871774D+00 -0.191636216D+01
 17 AT=  0.160000000D+02
    AY= -0.414633818D+00 -0.671205422D+00 -0.169537478D+01  0.101918630D+01
 18 AT=  0.170000000D+02
    AY= -0.172307075D+00  0.856357518D+00  0.188430339D+01  0.497722001D+00
 19 AT=  0.180000000D+02
    AY=  0.101278207D+01 -0.321748683D+00 -0.191710973D-01 -0.194769624D+01
TIME= 0:00:00.02

```

### 7.34 Stiff initial value problem for ordinary differential equations

a) Problem description:

Suppose we have to find a solution of the Van der Pol equation

$$\frac{dy_1(t)}{dt} = y_2(t), \quad y_1(0) = 2$$

$$\frac{dy_2(t)}{dt} = (1 - y_1^2(t))y_2(t) - y_1(t), \quad y_2(0) = 0$$



in the interval  $0 \leq t \leq T$  where  $T = 20$ .

b) Problem specification (input file):

```
$REM +-----+
$REM + SOLUTION OF STIFF ORDINARY DIFFERENTIAL EQUATIONS +
$REM +-----+
$FLOAT W1,W2,W3,W4
$SET(INPUT)
  TA=0.0D0
  YA(1)=2.0D0
  YA(2)=-6.6D-1
  TAMAX=2.0D0
$ENDSET
$SET(FMODELE)
  W1=1.0D-6
  GO TO (1,2) KE
1 FE=YA(2)
  GO TO 3
2 FE=((1.0D0-YA(1)**2)*YA(2)-YA(1))/W1
3 CONTINUE
$ENDSET
$SET(DMODELE)
  W1=1.0D-6
  GO TO (4,5) KE
4 DE(1)=0.0D0
  DE(2)=1.0D0
  GO TO 6
5 DE(1)=(-2.0D0*YA(1)*YA(2)-1.0D0)/W1
  DE(2)=(1.0D0-YA(1)**2)/W1
6 CONTINUE
$ENDSET
$NA=301
$NE=2
$MODEL='DE'
$ODE='STIFF'
$TOLR='1.0D-4'
$TOLA='1.0D-8'
$MED=1
$BATCH
$STANDARD
```

c) Comments on the problem specification:

By using the macrovariable \$INPUT we specify the initial values of the variables  $y_1$  and  $y_2$  as well as the initial and terminal times 0 and T, respectively. The right hand sides of the differential equations are specified by using the macrovariable \$FMODELE. The option \$MODEL='DE' indicates integration of a system of ordinary differential equations. Macrovariable \$NA denotes the number of time points (nodes) in which the results are printed.

d) Problem solution (basic screen output):

```
CLASS = DE - RD5   UPDATE = N   MODEL = NO   HESF = N   NF =      0
  0 NSTP=   345  NACC=   290  NREJ=   38  NEV =  5532  NEG =   374
  1 AT=  0.000000000D+00
    AY=  0.200000000D+01 -0.660000000D+00
  2 AT=  0.100000000D-06
    AY=  0.199999993D+01 -0.661727883D+00
  3 AT=  0.200000000D-06
    AY=  0.199999987D+01 -0.663007940D+00
  4 AT=  0.656175225D-06
    AY=  0.199999956D+01 -0.665734985D+00
  5 AT=  0.126092836D-05
    AY=  0.199999916D+01 -0.666514522D+00
  6 AT=  0.218147783D-05
    AY=  0.199999855D+01 -0.666657189D+00
  7 AT=  0.375735133D-05
    AY=  0.199999750D+01 -0.666667678D+00
  8 AT=  0.804409324D-05
    AY=  0.199999464D+01 -0.666669509D+00
  9 AT=  0.292949166D-04
    AY=  0.199998047D+01 -0.666677397D+00
 10 AT=  0.199301503D-03
    AY=  0.199986713D+01 -0.666740383D+00
-----
281 AT=  0.161429359D+01
    AY=  0.200007205D+01 -0.666537135D+00
282 AT=  0.161429463D+01
    AY=  0.200007136D+01 -0.666622471D+00
283 AT=  0.161429652D+01
    AY=  0.200007010D+01 -0.666627481D+00
284 AT=  0.161430239D+01
    AY=  0.200006618D+01 -0.666629772D+00
285 AT=  0.161433549D+01
    AY=  0.200004412D+01 -0.666642040D+00
286 AT=  0.161460030D+01
    AY=  0.199986757D+01 -0.666740131D+00
287 AT=  0.161671877D+01
    AY=  0.199845427D+01 -0.667526465D+00
288 AT=  0.163366649D+01
    AY=  0.198708726D+01 -0.673925875D+00
289 AT=  0.173214063D+01
    AY=  0.191874076D+01 -0.715528198D+00
290 AT=  0.193035891D+01
    AY=  0.176619569D+01 -0.833326803D+00
291 AT=  0.200000000D+01
    AY=  0.170616796D+01 -0.892808294D+00
TIME= 0:00:00.03
```

## 8 Model examples for demonstration of graphic output

Here we introduce several problem specifications (input files) which demonstrate the application of the graphic screen output. The graphic screen output can be used only on PC computers under the MS DOS system. This possibility is not allowed on the UNIX workstations.

The input files are included into the UFO system as demo-files PROC01.UFO,...,PROC08.UFO. Corresponding graphic pictures are included in the Appendix C. The data recommended for graphic pictures are introduced in lines which begin by the directive \$REM.

### 8.1 Nonlinear regression

```
$SET(INPUT)
  LDIM=5
  X(1)=7.0D20
  X(2)=1.0D4
  X(3)=2.2D0
  X(4)=1.01D0
  X(5)=7.0D17
  X(6)=7.0D3
  X(7)=1.6D0
  X(8)=1.01D0
  X(9)=1.0D16
  X(10)=4.0D3
  X(11)=1.5D0
  X(12)=1.01D0
  X(13)=2.0D15
  X(14)=4.0D3
  X(15)=1.3D0
  X(16)=1.01D0
  X(17)=1.0D16
  X(18)=5.0D2
  X(19)=1.2D0
  X(20)=1.01D0
  BETA=5.95D0
  CALL BIUD01(NF,LDIM,NA,X,XL,XU,IX,AT,AM)
$ENDSET
$SET(FMODELA)
  CALL BAFU01(NF,LDIM,KA,NA,X,AT,FA,BETA)
$ENDSET
$SET(GMODELA)
  CALL BAGU01(NF,LDIM,KA,NA,X,AT,GA,BETA)
$ENDSET
$NF=30
$NA=500
$MIT=100
$MODEL='AQ'
$CLASS='GN'
$TYPE='G'
$DECOMP='M'
$NUMBER=7
$UPDATE='F'
$TOLX='1.0$P-16'
```

```

$TOLF='1.0$P-16'
$TOLB='1.0$P-16'
$TOLG='1.0$P-6'
$KBA=1
$KBF=2
$GRAPH='Y'
$SCAN='Y'
$BATCH
$ADD(REAL,'\BETA\AT($NA)')
$ADD(SUBROUTINES)
  SUBROUTINE BIUDO1(N,L,NA,X,XL,XU,IX,AT,AM)
    INTEGER N,L,NA,IX(N),I,K
    REAL*8 X(N),XL(N),XU(N),AT(NA),AM(NA)
    N=4*L
    K=0
    DO 1 I=1,L
      X(K+1)=LOG(X(K+1))
      XL(K+1)=LOG(1.0D+0)
      XU(K+1)=LOG(1.0D+40)
      IX(K+1)=3
      X(K+2)=LOG(X(K+2))
      XL(K+2)=LOG(1.0D+0)
      XU(K+2)=LOG(1.0D+10)
      IX(K+2)=3
      XL(K+3)=1.0D-2
      XU(K+3)=1.0D+2
      IX(K+3)=3
      XL(K+4)=1.00001D0
      XU(K+4)=1.00000D1
      IX(K+4)=3
      K=K+4
1 CONTINUE
    OPEN (11,FILE='PROCO1.DAT',STATUS='OLD')
    NA=0
2 NA=NA+1
    READ (11,'(2D14.6)',ERR=3) AT(NA),AM(NA)
    GO TO 2
3 NA=NA-1
    RETURN
  END
  SUBROUTINE BAFU01(N,L,KA,NA,X,AT,FA,BETA)
    INTEGER N,L,KA,NA
    REAL*8 X(N),AT(NA),FA,Q(8),QD(8)
    REAL*8 ARG,POM,BK,B6INT,BETA
    INTEGER J,K
    COMMON /BCOM/ Q,QD
    DATA BK /8.617385D-5/
    FA=0.0D 0
    K=0
    DO 1 J=1,L
      ARG=X(K+3)/(BK*AT(KA))
      IF (KA.EQ.1) THEN

```

```

Q(J)=B6INT(AT(KA),ARG)
FA=FA+EXP(X(K+1)+X(K+2)-ARG)
ELSE
POM=X(K+4)-1.0DO
FA=FA+EXP(X(K+1)+X(K+2)-ARG)*
& (1.0DO+(POM/BETA)*EXP(X(K+1)))*(B6INT(AT(KA),ARG)-
& Q(J))**(-X(K+4)/POM)
ENDIF
K=K+4
1 CONTINUE
RETURN
END
SUBROUTINE BAGU01(N,L,KA,NA,X,AT,GA,BETA)
INTEGER N,L,KA,NA
REAL*8 X(N),AT(NA),GA(N)
REAL*8 FAC,ARG,POM,POW,BK,B6INT,B6INTD,A,B,C,D,E,F,G
REAL*8 Q(8),QD(8),QQ,QQD,BETA
INTEGER J,K
COMMON /BCOM/ Q,QD
DATA BK /8.617385D-5/
K=0
DO 1 J=1,L
FAC=1.0DO/(BK*AT(KA))
ARG=FAC*X(K+3)
IF (KA.EQ.1) THEN
Q(J)=B6INT(AT(KA),ARG)
QD(J)=FAC*B6INTD(AT(KA),ARG)
QQ=0.0DO
QQD=0.0DO
ELSE
QQ=B6INT(AT(KA),ARG)-Q(J)
QQD=FAC*B6INTD(AT(KA),ARG)-QD(J)
ENDIF
POM=X(K+4)-1.0DO
POW=-X(K+4)/POM
A=EXP(X(K+1)+X(K+2)-ARG)
B=EXP(X(K+1))
G=B*QQ
C=(1.0DO+(POM/BETA)*G)
D=C**POW
E=POW*D/C
F=POM*POM
GA(K+1)=A*(D+E*(POM/BETA)*G)
GA(K+2)=A*D
GA(K+3)=A*(-FAC*D+E*(POM/BETA)*B*QQD)
GA(K+4)=A*D*(LOG(C)/F+POW*G/(C*BETA))
K=K+4
1 CONTINUE
RETURN
END

```

```

FUNCTION B6INT(T,X)
REAL*8 T,X,B6INT
REAL*8 A1,A2,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6
DATA A1,A2,A3,A4,A5,A6 /41.0D+0, 590.0D+0, 3648.0D+0,
&          9432.0D+0, 8028.0D+0, 720.0D+0/
DATA B1,B2,B3,B4,B5,B6 /42.0D+0, 630.0D+0, 4200.0D+0,
&          12600.0D+0, 15120.0D+0, 5040.0D+0/
B6INT=(1.0D0-(A6+X*(A5+X*(A4+X*(A3+X*(A2+X*(A1+X)))))))/
&      (B6+X*(B5+X*(B4+X*(B3+X*(B2+X*(B1+X))))))*EXP(-X)*T
RETURN
END
FUNCTION B6INTD(T,X)
REAL*8 T,X,B6INTD
REAL*8 A1,A2,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6
REAL*8 C1,C2,C3,C4,C5,D1,D2,D3,D4,D5,DIS,DEN,DISD,DEND
DATA A1,A2,A3,A4,A5,A6 /41.0D+0, 590.0D+0, 3648.0D+0,
&          9432.0D+0, 8028.0D+0, 720.0D+0/
DATA B1,B2,B3,B4,B5,B6 /42.0D+0, 630.0D+0, 4200.0D+0,
&          12600.0D+0, 15120.0D+0, 5040.0D+0/
DATA C1,C2,C3,C4,C5 /205.0D+0, 2360.0D+0, 10944.0D+0,
&          18863.0D+0, 8028.0D+0/
DATA D1,D2,D3,D4,D5 /210.0D+0, 2520.0D+0, 12600.0D+0,
&          25200.0D+0, 15120.0D+0/
DIS=A6+X*(A5+X*(A4+X*(A3+X*(A2+X*(A1+X))))
DEN=B6+X*(B5+X*(B4+X*(B3+X*(B2+X*(B1+X))))
DISD=C5+X*(C4+X*(C3+X*(C2+X*(C1+6.0D0*X)))
DEND=D5+X*(D4+X*(D3+X*(D2+X*(D1+6.0D0*X)))
B6INTD=((DIS-DISD+DEND*DIS/DEN)/DEN-1.0D0)*EXP(-X)*T
RETURN
END

```

```

$ENDADD
$STANDARD

```

## 8.2 Nonlinear minimax optimization

```

$FLOAT W
$SET(INPUT)
  X(1)=0.5D0 ; X(2)=0.0D0 ; X(3)=0.0D0
  X(4)=0.0D0 ; X(5)=0.0D0
$ENDSET
$SET(FMODELA)
  W=0.1D0*DBLE(KA-1)-1.0D0
  FA=(X(1)+W*X(2))/(1.0D0+W*(X(3)+W*(X(4)+W*X(5))))-EXP(W)
$ENDSET
$MODEL='AM'
$NF=5
$NA=21
$NAL=0
$GRAPH='Y'
$MAP='Y'
$HIL='Y'
$ISO='Y'

```

```
$PATH='E'  
$BATCH  
$STANDARD
```

```
$REM VAR=1, XL=-5, XU=5  
$REM VAR=3, XL=-5, XU=5
```

### 8.3 Transformer network design

```
$SET(INPUT)  
  NEXT=4  
  CALL EIUD06(NF,NA,NAL,X,FMIN,XMAX,NEXT,IEXT,IERR)  
$ENDSET  
$SET(FMODELA)  
  CALL EAFU06(NF,KA,X,FA,NEXT)  
$ENDSET  
$SET(GMODELA)  
  CALL EAGU06(NF,KA,X,GA,NEXT)  
$ENDSET  
$NF=6  
$NA=11  
$NAL=0  
$MOUT=1  
$MODEL='AM'  
$GRAPH='Y'  
$MAP='Y'  
$HIL='Y'  
$ISO='Y'  
$PATH='E'  
$BATCH  
$STANDARD  
  
$REM VAR=1, XL=-5, XU=5  
$REM VAR=3, XL=-5, XU=5
```

### 8.4 Global optimization

```
$SET(INPUT)  
  NEXT=4  
  CALL EIUD09(NF,XL,XU,NEXT,IERR)  
$ENDSET  
$SET(FMODEL)  
  CALL EFFU09(NF,X,FF,NEXT)  
$ENDSET  
$NF=4  
$MOUT=1  
$GCLASS=1  
$GRAPH='Y'  
$MAP='Y'  
$HIL='Y'  
$ISO='Y'  
$EXTREM='G'
```

```
$BATCH
$STANDARD
```

```
$REM VAR=1, XL=-3.8, XU=3.8
$REM VAR=2, XL=-3.8, XU=3.8
```

## 8.5 Nonsmooth optimization

```
$SET(INPUT)
  NEXT=15
  CALL EIUD19(NF,X,FMIN,XMAX,NEXT,IEXT,IERR)
  MA=NF+3
$ENDSET
$SET(FMODEL)
  CALL EFFU19(NF,X,FF,NEXT)
$ENDSET
$SET(GMODEL)
  CALL EFGU19(NF,X,GF,NEXT)
$ENDSET
$KSF=3
$NF=30
$MOUT=-1
$MODEL='FF'
$GRAPH='Y'
$MAP='Y'
$HIL='Y'
$ISO='Y'
$PATH='Y'
$BATCH
$STANDARD

$REM VAR=1, XL=-5, XU=5
$REM VAR=4, XL=-5, XU=5
```

## 8.6 The Rosenbrock function

```
$SET(INPUT)
  X(1)=-1.2D0
  X(2)= 1.0D0
$ENDSET
$SET(FMODEL)
  FF=1.0D2*(X(1)**2-X(2))**2+(X(1)-1.0D0)**2
$ENDSET
$NF=2
$GRAPH='Y'
$MAP='Y'
$ISO='Y'
$PATH='Y'
$BATCH
$STANDARD
```



## 8.7 Ordinary differential equations

```
$FLOAT W1,W2,W3,W4
$SET(INPUT)
  TA=0.0D0
  YA(1)=0.994D0
  YA(2)=0.0D0
  YA(3)=0.0D0
  YA(4)=-2.00158510637908252240537862224D0
  TAMAX=17.0652165601579625588917206249D0
$ENDSET
$SET(FMODELE)
  W1=0.012277471D0
  W2=1.0D0-W1
  W3=(YA(1)+W1)**2+YA(2)**2
  W3=W3*SQRT(W3)
  W4=(YA(1)-W2)**2+YA(2)**2
  W4=W4*SQRT(W4)
  GO TO (1,2,3,4) KE
1 FE=YA(3)
  GO TO 5
2 FE=YA(4)
  GO TO 5
3 FE=YA(1)+2*YA(4)-W2*(YA(1)+W1)/W3-W1*(YA(1)-W2)/W4
  GO TO 5
4 FE=YA(2)-2*YA(3)-W2*YA(2)/W3-W1*YA(2)/W4
5 CONTINUE
$ENDSET
$NE=4
$NA=2000
$MODEL='DE'
$SOLVER='DP5'
$TOLR='1.0$P-9'
$TOLA='1.0$P-9'
$MED=1
$GRAPH='Y'
$BATCH
$STANDARD
```

## 8.8 The Lorenz attractor

```
$FLOAT W1,W2,W3
$SET(INPUT)
  W1=10.0D0
  W2=28.0D0
  W3=8.0D0/3.0D0
  TA=0.0D0
  YA(1)=-8.0D0
  YA(2)= 8.0D0
  YA(3)=W2-1.0D0
  TAMAX=50.0D0
$ENDSET
```

```
$SET(FMODELE)
  GO TO (1,2,3) KE
1 FE=-W1*YA(1)+W1*YA(2)
  GO TO 4
2 FE=-YA(1)*YA(3)+W2*YA(1)-YA(2)
  GO TO 4
3 FE=YA(1)*YA(2)-W3*YA(3)
4 CONTINUE
$ENDSET
$NE=3
$NA=2000
$MODEL='D'
$SOLVER='DP8'
$TOLR='1.0$P-9'
$TOLA='1.0$P-9'
$MED=1
$GRAPH='Y'
$BATCH
$STANDARD
```

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## A Demonstration of the text dialogue mode

Suppose that the model function has the form

$$f^F(x) = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2$$

(the Rosenbrock function) and the starting point is  $x_1 = -1.2$  and  $x_2 = 1.0$ . If we type the statement **UFOGO** (without batch input file specification), then the following questions (which we supplement together with answers) appear on the screen.

UFO PREPROCESSOR V.3.1.

\_\_\_\_\_ ? INPUT ( ) ? \_\_\_\_\_

USER SUPPLIED INPUT:

HERE THE STARTING POINT, BOUNDS FOR VARIABLES,  
TYPES OF CONSTRAINTS, THE STRUCTURE OF SPARSE PROBLEM,  
AND OTHER INPUT DATA HAVE TO BE SPECIFIED.

X(1) = -1.2D0; X(2) = 1.0D0

\_\_\_\_\_ ? GRAPH (N) ? \_\_\_\_\_

SPECIFICATION OF GRAPHICAL OUTPUT

N - GRAPHICAL OUTPUT SUPPRESSED  
Y - GRAPHICAL OUTPUT REQUIRED

\_\_\_\_\_ ? DISPLAY (N) ? \_\_\_\_\_

SPECIFICATION OF EXTENDED SCREEN OUTPUT

N - EXTENDED SCREEN OUTPUT SUPPRESSED  
Y - EXTENDED SCREEN OUTPUT REQUIRED

\_\_\_\_\_ ? MODEL (FF) ? \_\_\_\_\_

TYPE OF OBJECTIVE FUNCTION

FF - GENERAL FUNCTION  
FL - LINEAR FUNCTION  
FQ - QUADRATIC FUNCTION  
AF - SUM OF FUNCTIONS  
AQ - SUM OF SQUARES  
AP - SUM OF POWERS  
AM - MINIMAX  
DF - DIFFERENTIAL SYSTEM WITH GENERAL INTEGRAL CRITERION  
DQ - DIFFERENTIAL SYSTEM WITH INTEGRAL OF SQUARES  
DE - DIFFERENTIAL EQUATIONS  
NE - NONLINEAR EQUATIONS  
NO - MODEL IS NOT SPECIFIED

NUMBER OF VARIABLES ? NF (0) ?

2

TYPE OF EXTREMUM ? IEXT (0) ?  
0 - MINIMUM  
1 - MAXIMUM

MODEL OF OBJECTIVE FUNCTION ? FMODEL (\*) ?  
FF = <FORTRAN\_EXPRESSION>

$$FF = 1.0D2*(X(1)**2 - X(2))**2 + (X(1) - 1.0D0)**2$$

MODEL OF GRADIENT OF OBJECTIVE FUNCTION ? GMODEL (\*) ?  
GF(1) = <FORTRAN\_EXPRESSION>  
GF(2) = <FORTRAN\_EXPRESSION>  
.  
.  
GF(NF) = <FORTRAN\_EXPRESSION>

MODEL OF HESSIAN MATRIX ? HMODEL (\*) ?  
HF(1) = <FORTRAN\_EXPRESSION>  
HF(2) = <FORTRAN\_EXPRESSION>  
.  
.  
HF(M) = <FORTRAN\_EXPRESSION>

COMPLEXITY OF THE OBJECTIVE FUNCTION ? KCF (2) ?  
1 - EASY COMPUTED FUNCTION  
2 - REASONABLE BUT NOT EASY COMPUTED FUNCTION  
3 - EXTREMELY COMPLICATED FUNCTION

? KSF (1) ?

SMOOTHNESS OF THE OBJECTIVE FUNCTION:

- 1 - SMOOTH AND WELL-CONDITIONED FUNCTION
- 2 - SMOOTH BUT ILL-CONDITIONED FUNCTION
- 3 - NONSMOOTH FUNCTION

? HESF (D) ?

TYPE OF HESSIAN MATRIX:

- D - DENSE
- S - SPARSE WITH KNOWN (GENERAL) STRUCTURE
- N - HESSIAN MATRIX IS NOT USED

? KBF (0) ?

TYPE OF SIMPLE BOUNDS:

- 0 - NO SIMPLE BOUNDS
- 1 - ONE SIDED SIMPLE BOUNDS
- 2 - TWO SIDED SIMPLE BOUNDS

? KBC (0) ?

TYPE OF GENERAL CONSTRAINTS:

- 0 - NO GENERAL CONSTRAINTS
- 1 - ONE SIDED GENERAL CONSTRAINTS
- 2 - TWO SIDED GENERAL CONSTRAINTS

? EXTREM (L) ?

TYPE OF OPTIMIZATION

- L - LOCAL OPTIMIZATION
- G - GLOBAL OPTIMIZATION

? NORMF (0) ?

SCALING SPECIFICATION FOR VARIABLES:

- 0 - NO SCALING IS PERFORMED
- 1 - SCALING FACTORS ARE DETERMINED AUTOMATICALLY
- 2 - SCALING FACTORS ARE SUPPLIED BY USER

? INPUTDATA (N) ?

READ INPUT VALUES OF X (Y OR N)



\_\_\_\_\_ ? TEST (N) ? \_\_\_\_\_  
STANDARD TEST OF EXTERNAL SUBROUTINES:  
N - NO TEST  
Y - PERFORM TEST BEFORE SOLUTION  
A - PERFORM TEST AFTER SOLUTION  
O - PERFORM TEST WITHOUT SOLUTION

\_\_\_\_\_ ? KOUT (0) ? \_\_\_\_\_  
LEVEL OF TEXT FILE OUTPUT:  
ABS(KOUT)=0 - NO PRINT OR PAPER SAVING PRINT  
ABS(KOUT)=1 - STANDARD PRINT OF ITERATIONS  
ABS(KOUT)=2 - ADDITIONAL PRINT OF STEPSIZE SELECTION  
ABS(KOUT)=3 - ADDITIONAL PRINT OF DIRECTION DETERMINATION  
AND VARIABLE METRIC UPDATE  
ABS(KOUT)=4 - ADDITIONAL PRINT OF CONSTRAINT HANDLING  
ABS(KOUT)=5 - ADDITIONAL PRINT OF NUMERICAL DIFFERENTIATION  
KOUT<0 - ADDITIONAL PRINT OF DATA AND OPTIONS IN THE HEADING

\_\_\_\_\_ ? LOUT (1) ? \_\_\_\_\_  
LEVEL OF TEXT FILE OUTPUT:  
0 - NO PRINT  
1 - COPY OF THE BASIC SCREEN OUTPUT  
-1 - PAPER SAVING PRINT

\_\_\_\_\_ ? MOUT (-2) ? \_\_\_\_\_  
LEVEL OF BASIC SCREEN OUTPUT:  
ABS(MOUT)=0 - NO OUTPUT  
ABS(MOUT)=1 - FINAL OUTPUT  
ABS(MOUT)=2 - ADDITIONAL OUTPUT IN EACH ITERATION  
ABS(MOUT)=3 - ADDITIONAL FINAL OUTPUT OF LINEAR OR  
QUADRATIC PROGRAMMING  
ABS(MOUT)=4 - ADDITIONAL OUTPUT IN EACH ITERATION  
OF LINEAR OR QUADRATIC PROGRAMMING  
MOUT<0 - FINAL OUTPUT WITH TERMINATION CRITERION

1

\_\_\_\_\_ ? NOUT (0) ? \_\_\_\_\_  
LEVEL OF BASIC SCREEN OUTPUT:  
0 - BASIC FINAL OUTPUT  
1 - EXTENDED FINAL OUTPUT

1

\_\_\_\_\_ ? MSELECT (1) ? \_\_\_\_\_

SELECTION OF OPTIMIZATION METHOD

1 - AUTOMATICAL SELECTION OF METHOD

2 - MANUAL SELECTION OF METHOD

3 - MANUAL SELECTION OF METHOD AND IMPORTANT PARAMETERS

4 - MANUAL SELECTION OF METHOD AND ALL PARAMETERS

\_\_\_\_\_ ? LAPACK (N) ? \_\_\_\_\_

USE LAPACK SUBROUTINES

N - ONLY UFO SUBROUTINES

Y - CONNECTION TO LAPACK POSSIBLE

\_\_\_\_\_ ? OUTPUT ( ) ? \_\_\_\_\_

USER SUPPLIED OUTPUT:

HERE THE RESULTS OBTAINED IN THE OPTIMIZATION PROCESS  
CAN BE USED FOR ADDITIONAL COMPUTATIONS AND FOR A  
SPECIFIC OUTPUT.

\_\_\_\_\_ ? OUTPUTDATA (N) ? \_\_\_\_\_

WRITE OUTPUT VALUES OF X (Y OR N)

UFO PREPROCESSOR STOP

Each question is represented by one frame which contains the contents of the question (name of the macrovariable that has to be defined), the default value (in brackets) and an explanation of the requirement. If no default value is wanted, the corresponding value or text has to be typed. The dialogue can be ended by pressing the key <!> .

The result of the UFO preprocessor action is the following source program (reported in a slightly shortened form) consisting of global declarations, input specifications, problem definition, method realization and control variables adjustment:

```
*
*  -----
*  GLOBAL DECLARATIONS
*  -----
*
INTEGER ITIME
INTEGER IMD
INTEGER IX(1)
REAL*8 UXVDOT
REAL*8 GF(2)
REAL*8 X(2)
REAL*8 HD(2)
REAL*8 HF(2*(2+1)/2)
REAL*8 S(2)
```

```

REAL*8 ALF
REAL*8 BET
REAL*8 XO(2)
REAL*8 GO(2)
INTEGER IMB
*
*   commons placed here were omitted
*   since they require a large space
*
*   -----
*   END OF DECLARATIONS
*   -----
*
OPEN (2,FILE='P.OUT',STATUS='UNKNOWN')
OPEN (3,FILE='P.DIM',STATUS='UNKNOWN')
CALL UYCLEA
CALL UYINTP
*
*   -----
*   METHOD (1)
*   -----
*
CALL UYINT1
CALL UOTES1('VM','L','I','1','B','FF
&    ','D',NF)
X(1)=-1.2DO
X(2)=1.0DO
CALL UYCLST
WRITE(3,('( 'PROBLEM: NEXT =',I8)') NEXT
IF (NF.GT.2) THEN
CALL UOERR2('UZLMIN',80,NF,2)
CALL UOERR4
ITERM=-80
TXFU='LACK SPC'
ENDIF
WRITE(3,('( 'NUMBER OF VARIABLES:          NF =',I8)') NF
M=NF*(NF+1)/2
IF (ITERM.LT.0) STOP
CALL UYTIM1(ITIME)
NDECF=0
IF (ITERM.NE.0) GO TO 11200
CALL UOOFU1(NF,NA,NAL,MAL,NC,NCL,MCL,EPS0,EPS1,EPS2,EPS3,EPS4,EPS5
&    ,EPS6,EPS7,EPS8,EPS9,ETA0,ETA1,ETA2,ETA3,ETA4,ETA5,ETA6,ETA7,E
&    TA8,ETA9,ALF1,ALF2,ALF3,BET1,BET2,BET3,GAM1,GAM2,GAM3,DEL1,DEL
&    2,DEL3,RPF1,RPF2,RPF3,RGF1,RGF2,RGF3,FMIN,XMAX,XDEL,REXP,MET,M
&    ET1,MET2,MET3,MES,MES1,MES2,MES3,MOT,MOT1,MOT2,MOT3,MOS,MOS1,M
&    OS2,MOS3,MEP,MEP1,MEP2,MEP3,MEG,MEG1,MEG2,MEG3,MEX,MEX1,MEX2,M
&    EX3,MED,MED1,MED2,MED3,MCG,MCG1,MFP,MFP1,MPF,MPF1,MGF,MGF1,MLP
&    ,MLP1,MQP,MQP1,MEQ,MEQ1,MSG,MSG1,KSF,KCF,KSA,KCA,KSC,KCC,KTERS
&    ,INITD,INITS,INITH,IREM,IADD,IRES1,IRES2,MRED,IRAN1,IRAN2,ISAM
&    1,ISAM2,KINP,IPRN)
*

```

```

* -----
* VARIABLE METRIC METHOD
* TEMPLATE : U1FDU1
* -----
*
    ASSIGN 11130 TO IMD
    CALL UYPRO1('UXFU',1)
    CALL UYPRO2(FMIN,FO)
11110 CONTINUE
*
* -----
* MODEL DESCRIPTION
* -----
*
11600 CALL UF1FO1(NF,GF,GF,FF,F)
      GOTO (11640,11610,11620) ISB+1
11610 CONTINUE
      ASSIGN 11710 TO IMB
11700 CONTINUE
      NFV=NFV+1
      FF=1.0D2*(X(1)**2-X(2))**2+(X(1)-1.0D0)**2
      GOTO IMB
11710 CONTINUE
      GOTO 11600
11620 CONTINUE
      CALL UFOGS2(NF,X,IX,X,GF,FF,HD,R,SNORM,1.0D-15,1.0D-15,2,1)
      GOTO (11600,11630) ISB+1
11630 CONTINUE
      ASSIGN 11910 TO IMB
      GOTO 11700
11910 CONTINUE
      GO TO 11620
11640 CONTINUE
*
* -----
* END OF MODEL DESCRIPTION
* -----
*
      GO TO IMD
11130 CONTINUE
      CALL UYTRUG(NF,N,X,GF,GF,UMAX,GMAX)
      CALL UO2FU3(NF,M,NA,NC,X,GF,HF,X,X,F,DMAX,GMAX)
      CALL UYFUT1(N,F,FO,UMAX,GMAX,DMAX,ITES,IRES1,IRES2,INEW)
      IF(ITERM.NE.0) GOTO 11190
11140 CONTINUE
      ASSIGN 11140 TO IMD
      CALL UUDSD1(N,HF,1)
      GOTO (11150,11110) ISB+1
11150 CONTINUE
      IF(ITERM.NE.0) GOTO 11190
      CALL UYCPD(NF,IX,HF,HD,MCG1)
      CALL UYTRUH(NF,N,X,HF)

```

```

*
* -----
* DIRECTION DETERMINATION
* TEMPLATE : UDGLG1
* -----
*
CALL UOD1D1
IF (IDECLF.LT.0) THEN
IDECLF=9
INF=0
ENDIF
TDXX(1:4)=' INV '
IF (IDECLF.EQ.0) THEN
*
* INVERSION
*
ALF=ETA2
CALL UXDPGF(N,HF,INF,ALF,BET)
CALL UXDPGI(N,HF)
NDECLF=NDECLF+1
IDECLF=9
ELSE IF (IDECLF.EQ.9) THEN
ELSE
ITERD=-1
TDXX='BAD DEC9'
CALL UOERR1('UDDL1',1)
GO TO 12630
ENDIF
GNORM=SQRT(UXVDOT(N,GF,GF))
*
* NEWTON LIKE STEP
*
CALL UXDSMM(N,HF,GF,S)
CALL UXVNEG(N,S,S)
INITD=MAX(ABS(INITD),1)
ITERD=1
IF(INF.EQ.0) THEN
TDXX(5:8)=' POS'
ELSEIF(INF.LT.0) THEN
TDXX(5:8)=' ZER'
ELSE
TDXX(5:8)=' NEG'
ENDIF
SNORM=SQRT(UXVDOT(N,S,S))
NRED=INF
CALL UOD1D5(ALF,BET,INF)
12630 CALL UOD1D2(N,GF,S)
*
* -----
* END OF DIRECTION DETERMINATION
* -----
*

```

```

      IF (KD.GT.0) P=UXVDOT(N,GF,S)
      CALL UD1TL1(NF,N,GF,S,EPS0,ALF1,ALF2,R,P,GNORM,SNORM,RMIN,RMAX,XMA
&      X,XDEL,MES,INITD,INITH)
      IF(ITERM.NE.0) GOTO 11190
      IF(IREST.NE.0) GOTO 11140
      CALL UYTRUS(NF,X,X,XO,GF,GO,S,S,RO,FP,FO,F,PO,P,CMAX,CMAXO)
11170 CONTINUE
      ASSIGN 11170 TO IMD
      CALL USOLO1(EPS1,RO,RP,R,FO,FP,F,PO,PP,FMIN,FMAX,PAR1,PAR2,RMAX,RM
&      IN,SNORM,MODE,KTERS,MES,MES1,MES2,INITS,MRED)
      GOTO (11174,11172) ISB+1
11172 CONTINUE
      CALL UXVDIR(NF,R,S,XO,X)
      GOTO 11110
11174 CONTINUE
      IF (ITERS.LE.0) THEN
      CALL UYZERO(NF,X,XO,R,F,FO,FF,P,PO,MOT3)
      IF(IDIR.EQ.0) THEN
      CALL UYRES1(TSXX)
      CALL UYSET1
      GO TO 11140
      ELSE IF (MOT3.EQ.0) THEN
      CALL UYSET1
      GO TO 11140
      ELSE
      ITERD=0
      ENDIF
      ENDIF
      IF(KD.GT.LD) THEN
      ASSIGN 11180 TO IMD
      GO TO 11110
      ENDIF
11180 CONTINUE
      TXFU=TUXX
      CALL UYUPSD(NF,X,IX,XO,GF,GO,HD,P,MCG1)
      CALL UYTRUD(NF,X,X,XO,GF,GO,R,F,FO,P,PO,DMAX)
      CALL UUDBI1(N,HF,S,XO,GO,R,PO,F,FO,P,1.0D 60,8)
      IF(IDIR.EQ.0) THEN
      IF(ITERH.NE.0) CALL UYRES1('UPDATE ')
      GOTO 11130
      ELSE
      GOTO 11140
      ENDIF
11190 CONTINUE
      IF(ITERM.LT.0) TXFU=TDXX
      CALL UYEPI1(1)
11200 CONTINUE
      CALL UOERR3(KOUT,LOUT,MOUT,ITERM,IER)
      CALL UO1FU2(NF,NA,NC,X,X,X,X,FF,F,FO,DMAX,GMAX,XMAX,EPS0,EPS1,EPS2
&      ,EPS3,EPS4,EPS5,BET1,BET2,GAM1,GAM2,ETA1,ETA2,MET,MET1,MET2,ME
&      T3,MOT,MOT1,MOT2,MOT3,MES,MES1,MES2,MES3,MOS,MOS1,MOS2,MOS3,IN
&      ITD,INITS,INITH,IRES1,KTERS,IPRN)

```

13599 CONTINUE

\*

\*

-----  
END OF METHOD (1)

\*

\*

CALL UYTIM2(ITIME)

CLOSE (2)

CLOSE (3)

END

\*

\*

-----  
INITIATION OF METHOD (1)

\*

\*

SUBROUTINE UYINT1

\*

commons placed here were omitted

since they require a large space

\*

REAL\*8 XDELS,RPF1S,RPF2S,RPF3S,RGF1S,RGF2S,RGF3S

COMMON/UMCLST/ XDELS,RPF1S,RPF2S,RPF3S,RGF1S,RGF2S,RGF3S

ETA0=1.0D-15

ETA9=1.0D 60

ITR=6

IRD=5

IWR=2

\*

many other assignments follow which were

omitted since they require a large space

\*

END

\*

\*

-----  
INITIATION OF PROBLEM

\*

\*

SUBROUTINE UYINTP

\*

commons placed here were omitted

since they require a large space

\*

NF=2

IEXT=0

KCF=2

KSF=1

KBF=0

KBC=0

NORMF=0

KDF=0

KDA=-1

KDC=-1

```

KDE=-1
KDY=-1
END
*
* -----
* BROYDEN CLASS OF VARIABLE METRIC UPDATES
* TEMPLATE : UUDBI1
* -----
SUBROUTINE UUDBI1(N,H,S,XO,GO,R,PO,F,FO,P,ETA9,MET)
*
* commons placed here were omitted
* since they require a large space
*
REAL*8 H(N*(N+1)/2),S(N),XO(N),GO(N),R,PO,ETA9
REAL*8 F,FO,P
REAL*8 AA,CC
COMMON /UMFUN1/ AA,CC
REAL*8 UXVDOT,UNFUN1
REAL*8 DIS,POM,POM3,POM4,A,B,C,GAM,RHO,PAR
REAL*8 DEN
INTEGER IUPDT
LOGICAL L1,L3
EXTERNAL UNFUN1
IF (MET.LE.0) GO TO 22
CALL UOU1D1(N,XO,GO)
IF (IDECF.NE.9) THEN
ITERH=-1
TUXX='BAD DEC9'
CALL UOERR1('UUDBI2',1)
GO TO 22
ENDIF
L1=ABS(4).GE.3.OR.ABS(4).EQ.2.AND.NIT.EQ.KIT
L3=.NOT.L1
*
* DETERMINATION OF THE PARAMETERS A, B, C
*
B=UXVDOT(N,XO,GO)
IF (B.LE.ZERO) THEN
ITERH=2
TUXX='B - NEG.'
GO TO 22
ENDIF
CALL UXDSMM(N,H,GO,S)
A=UXVDOT(N,GO,S)
IF (A.LE.ZERO) THEN
ITERH=1
TUXX='A - NEG.'
GO TO 22
ENDIF
IF (MET.GE.4.OR.L1) THEN
IF (ITERD.NE.1) THEN
MET=1
C=ZERO

```



```

ELSE
C=-R*PO
IF (C.LE.ZERO) THEN
ITERH=3
TUXX='C - NEG.'
GO TO 22
ENDIF
ENDIF
ELSE
C=ZERO
ENDIF
*
* DETERMINATION OF THE PARAMETER RHO (NONQUADRATIC PROPERTIES)
*
IF (FO-F+P.EQ.0) THEN
RHO=ONE
ELSE
RHO=HALF*B/(FO-F+P)
ENDIF
IF(RHO.LE.1.0D-2) RHO=ONE
IF(RHO*1.0D-2.GE.ONE) RHO=ONE
AA=A/B
CC=C/B
IUPDT=0
IF (L1) THEN
*
* DETERMINATION OF THE PARAMETER GAM (SELF SCALING)
*
IF (C.LE.ZERO) THEN
PAR=A/B
POM3=0.8D 0
POM4=8.0D 0
ELSE
PAR=SQRT(A/C)
POM3=0.7D 0
POM4=6.0D 0
ENDIF
GAM=RHO/PAR
IF (NIT.NE.KIT) THEN
L3=GAM.LT.POM3.OR.GAM.GT.POM4
ENDIF
ENDIF
IF (L3) THEN
GAM=ONE
PAR=RHO/GAM
ENDIF
*
* NEW UPDATE
*
POM=ONE/(AA*CC)
IF (POM.LT.ONE) THEN
DEN=MAX(POM+1.0D-15,SQRT(C/A))

```

```

        POM=(DEN-POM)/(ONE-POM)
        TUXX='NEW      '
        GO TO 20
        ENDIF
17 CONTINUE
*
*   BFGS UPDATE
*
        POM=ONE
        DIS=PAR+AA
        CALL UXVDIR(N,-DIS,XO,S,XO)
        DIS=ONE/(B*DIS)
        CALL UXDSMU(N,H,DIS,XO)
        CALL UXDSMU(N,H,-DIS,S)
        TUXX='BFGS      '
        GO TO 21
20 CONTINUE
*
*   GENERAL UPDATE
*
        DEN=PAR+POM*AA
        DIS=POM/DEN
        CALL UXDSMU(N,H,(PAR*DIS-ONE)/A,S)
        CALL UXVDIR(N,-DIS,S,XO,S)
        CALL UXDSMU(N,H,DEN/B,S)
21 CONTINUE
        ITERH=0
        IF (GAM.EQ.ONE) GO TO 22
*
*   SCALING
*
        CALL UXDSMS(N,H,GAM)
22 CONTINUE
        CALL UOU1D2(N,H,S,RHO,GAM,PAR,A,B,C,POM,ETA9)
        RETURN
        END

```

The results (screen output) obtained by using this source program have the following form:

```

0 NIT=  40 NFV= 138 NFG=   0 GRAD TOL F= .5038712822E-13 G= .828D-05
FF =   .5038712822D-13
X  =   .1000000098D+01   .1000000177D+01

```

## B The BEL interpreter

The BEL (Batch Editor Language) interpreter, developed as a part of the UFO project, is especially determined for the generation of computer programs, batch editing of texts, preparation of print files, filtering of text files etc. The BEL interpreter allows us to generate a prescribed output file from the input file (template) which is a mixture of text lines and special instructions.

### B.1 General description

The BEL interpreter, realized by code `BEL.EXE`, requires input text file `BEL.TEM`, output text file `BEL.OUT` and an internal table of symbols. The input text file (template) consists of standard text lines together with the BEL instructions. The output text file contains a text generated by the BEL interpreter. The table of symbols contains names and values of the macrovariables used.

Although the BEL interpreter can be used in various general applications, it was developed especially for the generation of FORTRAN programs. It is:

1. Interpreter, since instructions contained in the input text are interpreted and immediately realized.
2. Batch editor, since it serves for editing batch files.
3. Macroprocessor, since it makes it possible to define or modify special macrovariables which can be substituted into the processed text.

The macrovariable can be an integer constant, a logical constant, a string of characters, a set of text lines, a set of BEL instructions, even a text file.

The BEL instructions, contained in the input text file, can be of two types:

1. Directives, i.e. control instructions and instructions for manipulation with the table of symbols. These instructions begin with the special character `CHDIR`. In the subsequent text, we will suppose that `CHDIR='$'` (`'$'` is the default value).
2. Substitutions, i.e. instructions for substituting macrovariables into the text. These instructions begin with the special character `CHSUB`. In the subsequent text, we will suppose that `CHSUB='$'` (`'$'` is the default value).

The BEL interpreter works in the following way:

1. The line of the input file is read.
2. The line is recognized and if the character `CHSUB` is found, a pertinent substitution is realized.
3. If the first character (different from blank) is `CHDIR`, the line is a directive line. The recognized directive is realized.

This process is repeated until directives `$STOP`, `$EXIT` or the end of the input file is found. Note that we suppose that `CHSUB` and `CHDIR` have the same values. This is allowed, since the correct meaning is recognized from the context.

At the end of this section, we stress some specific features and advantages of the BEL interpreter.

1. The substitution is recursive. The depth of recursion (the number of nested files) only depends on the declared work space size.
2. Substitution is allowed in both the text lines and the directives.
3. The names and values of macrovariables can have an arbitrary length which again only depends on the declared work space size.

4. The set of directives is relatively small with a consistent syntax. It contains all important instructions (\$IF-\$ELSEIF-\$ELSE-\$ENDIF, \$DO-\$ENDDO, \$REPEAT-\$UNTIL etc.)
5. The control parameters (CHDIR, CHSUB etc.) can be changed during the work of the BEL interpreter. This makes it possible to generate a program written in the BEL language which can be immediately processed.
6. The optional parameter of the BEL interpreter (e.g. the number of the I/O unit) can be changed during its work. This possibility enables us to change the input file without recursive nesting.
7. The BEL interpreter is a fully portable device. It can be implemented in an arbitrary system containing FORTRAN 77 compiler.

## B.2 List of instructions

### Substitutions:

\$INTEGER	- Substitute by the absolute label computed from the relative label.
\$NAME, \$(NAME)	- Substitute by the value of the macrovariable NAME.
\$DATA(NAME)	- Substitute by a new item from the list of items which is a value of the macrovariable NAME.
\$DEF(NAME)	- Substitute by '.TRUE.' if the macrovariable NAME is defined in the table of symbols. Otherwise substitute by '.FALSE.'
\$INT(NAME)	- Substitute by '.TRUE.' if the value of the macrovariable NAME is an integer constant. Otherwise substitute by '.FALSE.'
\$LOG(NAME)	- Substitute by '.TRUE.' if the value of the macrovariable NAME is a logical constant. Otherwise substitute by '.FALSE.'
\$REAL(NAME)	- Substitute by '.TRUE.' if the value of the macrovariable NAME is a real constant. Otherwise substitute by '.FALSE.'
\$\$	- Substitute '\$' (replace '\$\$' by '\$'). This makes possible to insert the character CHSUB into the text.

### Directives:

\$ADD	- Add a value to a macrovariable.
\$ADD, \$ENDADD	- Add text lines to a macrovariable.
\$CLEAR	- Clear value of a macrovariable which is a list of items type.
\$DO, \$ENDDO	- Cycle.
\$ERASE	- Erase a macrovariable from the table of symbols.
\$EXIT	- Return from the deepest nested file or termination of the BEL interpreter work if there are no nested files.
\$HELP, \$CHECK	- Set a default value to a macrovariable which has not been previously defined.
\$IF, \$ELSEIF,	
\$ELSE, \$ENDIF	- Conditioned instruction.
\$INCLUDE	- Insert a macrovariable or a text file into the output file.
\$OPTION	- Change some optional parameter of the BEL interpreter.
\$REM	- Remark.
\$REPEAT, \$UNTIL	- Cycle.
\$RESTORE	- Adjust the list of items pointer to the first item.
\$REWIND	- Rewind the file on a given unit.
\$SET	- Set a value to a macrovariable.
\$SET, \$ENDSET	- Set text lines to a macrovariable.
\$STOP	- Termination of the BEL interpreter work.
\$SUBST	- Substitute a text file into the input file.

### B.3 Special characters

The following special characters are important for the BEL interpreter work:

- \$ - CHSUB (Substitution Character) - this is the first character in every substitution. If '\$' should be inserted into the text, we have to use '\$\$'.
- \$ - CHDIR (Directive Character) - if the first character on the line is CHDIR, then the line is a directive line (CHSUB and CHDIR are distinguished by the context).
- & - CHCON (Continuation Character) - if the last character on the line is CHCON, then it is assumed that the logical line continues on the next physical line.
- ;  
; - CHEOL (End Of Line Character) - this character specifies the end of the logical line if it does not coincide with the end of the physical line. This makes it possible to write several logical lines by using the same physical line.
- \ - CHDS (Data Separator Character) - this character separates individual items in the list of items type macrovariable.

The use of special characters can be demonstrated by the following simple example. Assume that the input text has the form

```
$A='Paul\Peter\Jane\Mary'  
This is a list of my brothers and sis&  
ters:  
$DO(I=1,4); $DATA(A); $ENDDO
```

Then the output from the BEL interpreter has the form

```
This is a list of my brothers and sisters:  
Paul  
Peter  
Jane  
Mary
```

The special characters can be changed by the directive \$OPTION. But no special character has to be the alphabet or the digit. Moreover, different special characters have to differ (with the exception of CHSUB and CHDIR).

### B.4 Description of instructions

This section contains a detailed description of the syntax and action of individual BEL instructions. The following definitions will be used:

- <digit> ::= 0 | 1 | 2 | 3 | ..... | 9
- <alphabet> ::= A | B | C | D | ..... | Z
- <character> ::= an arbitrary character with the exception of apostrophe
- <integer constant> ::= (+ | -) <digit> {<digit>}
- <logical constant> ::= .TRUE. | .FALSE.
- <macroname> ::= <alphabet> {<alphabet> | <digit>}
- <string of characters> ::= '{<character> | }'
- <text> ::= <string of characters> '{; <string of characters>}'
- <list of items> ::= <string of characters> '{\ <string of characters>}'

## **Substitutions:**

### **\$INTEGER**

#### Syntax:

The type of INTEGER is an integer constant. Although it can have an arbitrary value, an application to the UFO source program generation requires it to be positive and lower than LABEL2 (see the directive \$OPTION).

#### Action:

The integer constant INTEGER is a relative label in a given template. The absolute label, substituted into the UFO source program, is computed by the formula  $LABEL=LABEL1+K*LABEL2$ , where LABEL1 and LABEL2 are options of the BEL interpreter (see the directive \$OPTION) and K is a serial number of the application of the directive \$SUBST.

#### Example:

```
$10
```

generates

```
10010
```

if the main template is used or

```
10110
```

after the first application of the directive \$SUBST.

### **\$NAME, \$(NAME)**

#### Syntax:

The type of NAME is a macroname. This substitution has two forms, either \$NAME or \$(NAME). The latter form is required if the substitution appears inside a continuous string of characters to separate the NAME from the adjacent text.

#### Action:

The string '\$NAME' is replaced by the value of the macrovariable NAME.

#### Example:

```
$A='UFO'
```

```
$A SYSTEM
```

generates

```
UFO SYSTEM
```

### **\$DATA(NAME)**

#### Syntax:

The type of NAME is a macroname.

#### Action:

The string '\$DATA(NAME)' is replaced by the next item of the list of items which is a value of the macrovariable NAME. If the next item does not exist, the list of items pointer is returned to the first item. Additional information is contained in the description of the directive \$RESTORE.

#### Example:

```
$LIST='ITEM1\ITEM2\ITEM3'
```

```
$DATA(LIST)
```

```
$DATA(LIST)
```

\$DATA(LIST)  
\$DATA(LIST)

generates

ITEM1  
ITEM2  
ITEM3  
ITEM1

### **\$DEF(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

If the macrovariable NAME is defined in the table of symbols, the string '\$DEF(NAME)' is replaced by the logical constant .TRUE., otherwise it is replaced by the logical constant .FALSE..

Example:

\$A=10  
\$DEF(A)

generates

.TRUE.

### **\$INT(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

If the value of the macrovariable NAME is an integer constant, the string '\$INT(NAME)' is replaced by the logical constant .TRUE., otherwise it is replaced by the logical constant .FALSE..

Example:

\$A=-25  
\$INT(A)

generates

.TRUE.

### **\$LOG(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

If the value of the macrovariable NAME is a logical constant, the string '\$LOG(NAME)' is replaced by the logical constant .TRUE., otherwise it is replaced by the logical constant .FALSE..

Example:

\$A=.FALSE.  
\$LOG(A)

generates

.TRUE.

## **\$REAL(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

If the value of the macrovariable NAME is a real constant (i.e. a string of characters which satisfies the syntactic rules for FORTRAN real constants), the string '\$REAL(NAME)' is replaced by the logical constant .TRUE., otherwise it is replaced by the logical constant .FALSE..

Example:

```
$A='-0.09D-12'
```

```
$REAL(A)
```

generates

```
.TRUE.
```

## **\$\$**

Action:

The string '\$\$' is replaced by the character '\$'. This substitution allows us to insert the character '\$' into the generated text or into the macrovariable.

Example:

```
$I='NAME'
```

```
$$DEF($I)
```

generates

```
DEF(NAME)
```

## **Directives:**

### **\$ADD(NAME1,NAME2 or VALUE)**

Syntax:

The type of NAME1 and NAME2 is a macroname.

The type of VALUE is an integer constant or a logical constant or a string of characters.

Action:

The value of the macrovariable NAME2 or the VALUE is added to the value of the macrovariable NAME1 (the resulting value of the macrovariable NAME1 is \$NAME1\$NAME2 in the first case).

Example:

```
$NAME='TOM'
```

```
$ADD(NAME,' JONES')
```

```
Name: $NAME
```

generates

```
Name: TOM JONES
```

### **\$ADD(NAME)**

**TEXT**

### **\$ENDADD**

Syntax:

The type of NAME is a macroname.

The type of TEXT is a text.



Action:

The TEXT is added to the value of the macrovariable NAME.

Example:

```
$SET(A)
  Day: 31
$ENDSET
$ADD(A)
  Month: December
  Year: 1998
$ENDADD
$A
```

generates

```
  Day: 31
  Month: December
  Year: 1998
```

Remark: Only substitutions are realized in the text TEXT (not directives).

### **\$CLEAR(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

This directive clears a list-of-items-type value of the macrovariable NAME, i.e. it deletes all duplications of items. Small and capital letters of items are not distinguished.

Example:

```
$DECL='N\IX(N)\N\M\ I\J\N\M'
$CLEAR(DECL)
$END='$DATA(DECL)'
$REPEAT
  $I='$DATA(DECL)'
  INTEGER $I
$UNTIL(I=END)
```

generates

```
  INTEGER IX(N)
  INTEGER M
  INTEGER I
  INTEGER J
  INTEGER N
```

### **\$DO(NAME=INDEX1,INDEX2,INDEX3)**

**TEXT**

### **\$ENDDO**

Syntax:

The type of NAME is a macroname.

The type of INDEX1, INDEX2, INDEX3 is a macroname or an integer constant.

The type of TEXT is a text.

Action:

This directive has a similar meaning as the statement DO in the FORTRAN language:

NAME is the cycle counter.

INDEX1 is the initial value of the cycle counter.

INDEX2 is the final value of the cycle counter.

INDEX3 is the change of the cycle counter after a cycle step.

If INDEX3 is not present, the default value INDEX1=1 is assumed.

The cycle counter NAME does not have to be changed in the cycle step.

The value INDEX3 does not have to be equal to 0.

The body of the cycle is terminated by \$ENDDO.

If INDEX1>INDEX2 and INDEX3>0 or INDEX1<INDEX2 and INDEX3<0, then the cycle is not realized.

Cycles can be nested. The maximum depth of nested cycles is 20.

Example:

```
$A='X\Y\Z'  
$DO(I=1,5,2)  
  A($I,1)=C($I)+$DATA(A)  
$ENDDO
```

generates

```
A(1,1)=C(1)+X  
A(3,1)=C(3)+Y  
A(5,1)=C(5)+Z
```

### **\$ERASE(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

The macrovariable NAME is erased from the table of symbols.

Example:

```
$A=1  
$DEF(A)  
$ERASE(A)  
$DEF(A)
```

generates

```
.TRUE.  
.FALSE.
```

### **\$EXIT**

Action:

The directive \$EXIT has the same meaning as the end of the file achievement. If the nested files are processed (see the description of the directive \$SUBST), the directive \$EXIT realizes return to the higher level file (if the higher level file does not exist, then \$EXIT has the same meaning as \$STOP).

### **\$HELP**

**TEXT**

### **\$CHECK(NAME,DEFAULT,TYPE,LEVEL,TRANSFER)**

Syntax:

The type of TEXT is a text.

The type of NAME is a macroname.

The type of `DEFAULT` is either a macroname or an integer constant or a logical constant or a string of characters.

The type of `TYPE` is either a list of items or one of the strings `INT` (integer), `LOG` (logical), `REAL` (real).

The type of `LEVEL` is an integer constant.

The type of `TRANSFER` is a logical constant.

Action:

The text `TEXT` appears on the screen if the dialogue mode is used. The value of the macrovariable `$NAME` is checked to have the type `TYPE`. If the macrovariable `$NAME` is not defined or if it has a wrong value, the value `DEFAULT` is used. The value of `LEVEL` gives the lowest level of the dialogue (1,2,3 or 4) from which the text `TEXT` appears on the screen. The value of `TRANSFER` specifies transfer of the variable `$NAME` into the UFO source program (`YES` if transfer is accepted or `NO` if transfer is suppressed).

Example:

```
$HELP
```

```
  TYPE OF THE HESSIAN MATRIX:
```

```
    D - DENSE
```

```
    B - SPARSE WITH KNOWN (PARTITIONED) STRUCTURE
```

```
    S - SPARSE WITH KNOWN (GENERAL) STRUCTURE
```

```
    N - HESSIAN MATRIX IN NOT USED
```

```
$CHECK(HESF,'N','D\B\S\N',1,NO)
```

### **\$IF(CONDITION) LINE**

Syntax:

The `CONDITION` can be of the following types:

The type of `CONDITION` is a macroname and a value of `CONDITION` is a logical constant.

The type of `CONDITION` is a logical constant (`.TRUE.` or `.FALSE.`).

The type of `CONDITION` is a string of the form `PART1<operator>PART2`.

The type of `PART1` and `PART2` can be a macroname or an integer constant or a logical constant or a string (values of `PART1` and `PART2` have to be of the same type) and `<operator>` can have the following forms:

= equal to

<> not equal to

< less than (for integer values only)

<= less than or equal to (for integer values only)

> greater than (for integer values only)

>= greater than or equal to (for integer values only)

`LINE` is either a text line or a directive.

Action:

If the condition `CONDITION` is satisfied, `LINE` is inserted into the output file (if it is a text line) or carried out (if it is a directive). If the values of `PART1` and `PART2` are strings, then small and capital letters are not distinguished and blanks are ignored.

Example:

```
$A='J O H N'
```

```
$IF(A='John') Yes
```

```
$IF(A<>'Mary') No
```

generates

```
  Yes
```

```
  No
```

```

$IF(CONDITION1)
  TEXT1
$ELSEIF(CONDITION2)
  TEXT3
  .
  .
$ELSE
  TEXT
$ENDIF

```

Syntax:

CONDITION1 and CONDITION2 have the same syntax and meaning as CONDITION in the previous case. The number of repeated \$ELSEIF is not limited, \$ELSEIF or \$ELSE can be omitted.

Action:

This directive has a similar meaning as the conditioned statement IF-ELSEIF-ELSE-ENDIF in the FORTRAN language. The conditioned statements can be nested. The maximum depth of nested conditioned statements is 20.

Example:

```

$A=10
$L=.FALSE.
$IF(A=10)
  A = A + 1
  B = B + 1
  $IF(L)
    C = C + 1
  $ENDIF
$ELSE
  WRITE(*,*) I
$ENDIF

```

generates

```

  A = A + 1
  B = B + 1

```

**\$INCLUDE(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

The directive \$INCLUDE(NAME) is a special case of substitution. This directive makes it possible to insert (into the generated text) one or more lines, which were previously assigned to the macrovariable NAME. In contrast to the standard substitution \$NAME, the inserted lines are not processed by the BEL interpreter, so the directives are not carried out.

Example:

```

$SET(LINES)
  $ADD(A)
  X = Y + Z
  CALL SUB(X)
  $ENDADD
$ENDSET
$INCLUDE(LINES)

```

generates

```
$ADD(A)
X = Y + Z
CALL SUB(X)
$ENDADD
```

### **\$INCLUDE('FILE')**

Syntax:

The type of FILE is a string.

Action:

The directive \$INCLUDE('FILE') is a special case of substitution. This directive makes it possible to insert (into the generated text) the text which is stored in the file with the name FILE. The inserted text is not processed by the BEL interpreter, so the directives are not carried out.

Example:

```
$INCLUDE('C:\UFO\UMCOMN.I')
```

includes FORTRAN common blocks into the generated text (these common blocks are stored in the file C:\UFO\UMCOMN.I).

### **\$OPTION(OPTIONNAME=NAME or VALUE)**

Syntax:

OPTIONNAME is a selected name from the table of optional parameters (see below).

The type of NAME is a macroname. The value of NAME has to be an integer constant or a logical constant or a string of character and has to correspond to the type of OPTIONNAME.

The type of VALUE has to be an integer constant or a logical constant or a string of character and has to correspond to the type of OPTIONNAME.

Action:

This directive makes us possible to change selected optional parameter of the BEL interpreter. Optional parameters are contained in the following table.

Name	Type	Default	Description
CHDIR	char.	'\$'	see B.3
CHEOL	char.	','	see B.3
CHCON	char.	'&'	see B.3
CHDS	char.	'\'	see B.3
FN1	char.	' '	first part of the file name
FN2	char.	'I '	last part of the file name
ILNLEN	int.	80	physical length of the input line
OLNLEN	int.	80	physical length of the output line
IUNIT	int.	-	No. of the input file unit
OUNIT	int.	-	No. of the output file unit
INUNIT	int.	-	No. of the \$INCLUDE files unit
IIUNIT	int.	-	No. of the interactive mode input unit
OIUNIT	int.	-	No. of the interactive mode output unit
DIALOG	int.	1	level of dialogue (0 or 1 or 2)
MODERW	int.	1	READ/WRITE mode (1 or 2 or 3)
LABEL1	int.	10000	initial label
LABEL2	int.	100	difference between two consecutive labels
LSUBS	log.	.TRUE.	substitutions carried out
LOUT	log.	.TRUE.	output file created
LSMLET	log.	.TRUE.	small letters used in instructions
LFORTO	log.	.TRUE.	output in standard FORTRAN format
LFRFMT	log.	.TRUE.	input in free FORTRAN format (used only if LFORTO=.TRUE.)
SIFDEC	log.	.FALSE.	using the SIF decoder
DIALGR	log.	.FALSE.	using the graphic dialogue

## **\$REM**

Action:

The rest of the line (following after \$REM) is ignored by the BEL interpreter. The directive \$REM is used for remarks.

## **\$REPEAT**

**TEXT**

## **\$UNTIL(CONDITION)**

Syntax:

The type of TEXT is text.

CONDITION has the same syntax and meaning as that in the directive \$IF(...).

Action:

This directive has a similar meaning as the statement REPEAT-UNTIL in the PASCAL language:

The cycle is terminated whenever the condition CONDITION is satisfied (at least one realization is carried out).

Cycles can be nested. The maximum depth of nested cycles is 20.

Example:

```
$N=10
$REAL='X($N)\G($N)\H($N,$N)\.END.'
$REPEAT
  $I='$DATA(REAL)'
  $IF(I<>'.END.') REAL $I
$UNTIL(I='.END.')
```

generates

```
REAL X(10)
REAL G(10)
REAL H(10,10)
```

### **\$RESTORE(NAME)**

Syntax:

The type of NAME is a macroname.

Action:

The directive \$RESTORE(NAME) can only be used if the value of the macrovariable NAME is a list of items. Such a macrovariable uses a pointer which points out the next called item. The directive \$RESTORE adjust this pointer to point out the first item of the list (if the end of this list is found, the pointer is adjusted to point out the first item without applying the directive \$RESTORE).

Example:

```
$A='X\Y\Z'
  $DATA(A)
  $DATA(A)
$RESTORE(A)
  $DATA(A)
```

generates

```
X
Y
X
```

### **\$REWIND(UNIT)**

Syntax:

The type of UNIT is an integer constant.

Action:

The file opened on the unit with the number UNIT is rewound, so it can again be read from the first record (numbering of I/O units is used in the FORTRAN language).

### **\$NAME1 = NAME2 or VALUE**

#### **\$SET(NAME1 = NAME2 or VALUE)**

Syntax:

The type of NAME1 and NAME2 is a macroname.

The type of VALUE is an integer constant or a logical constant or a string of characters.

This directive has two forms. The latter form is used if the macroname is identical with a directive (e.g. \$SET(REM='REMARK')).

Action:

The new macrovariable with the name NAME1 and the value equal to the value of the macrovariable NAME2 or constant VALUE is inserted into the table of symbols. If the macrovariable NAME1 has already been defined in the table of symbols, then it is changed.

### **\$SET(NAME)**

**TEXT**

#### **\$ENDSET**

Syntax:

The type of NAME is a macroname.

The type of TEXT is text.

Action:

The macrovariable NAME is inserted into the table of symbols with the value TEXT. If the macrovariable NAME has already been defined in the table of symbols, then it is changed.

Example:

```
$ENDTEST=100
$SET(INIT)
  CALL EIUD01(NF,X,FMIN,XMAX,NEXT,IEXT,IERR)
  IF (IERR.NE.0) GO TO $ENDTEST
$ENDSET
$INIT
```

generates

```
  CALL EIUD01(NF,X,FMIN,XMAX,NEXT,IEXT,IERR)
  IF (IERR.NE.0) GO TO 100
```

Remark: Only substitutions are realized in the text TEXT (not directives).

### **\$STOP**

Action:

The directive \$STOP terminates the BEL interpreter work.

### **\$SUBST('FILE')**

Syntax:

The type of FILE is a string.

Action:

This directive performs the following actions:

The new reference label is computed (using the parameters LABEL1 and LABEL2 of the BEL interpreter).

The file with the name FILE is opened.

This file is processed by the BEL interpreter.

The file with the name FILE is closed.

The old reference label is restored.

This directive is similar to the directive \$INCLUDE('FILE'). But the inserted text is now processed by the BEL interpreter. All substitutions and directives are carried out. The directive \$SUBST('FILE') serves for dividing large texts into segments and makes it possible to generate texts by using conditioned branching. This is advantageously used for generation of the UFO source program where templates corresponding to individual subroutines are such segments.

Example:

```
$SUBST('C:\UFO\PROBLEM.UFO')
```

inserts a template, written in the UFO control language, into the generated text (this template is stored in the file C:\UFO\PROBLEM.UFO).

## **B.5 Error messages**

The BEL interpreter checks input templates and detects errors in the text written by the batch editing language. A typical error message has the form



```
+++ FATAL ERROR:      9
   FILE:  UZMETH.I
   LINE:   295
   CAUSE:  MACRO NOT DEFINED
   NAME:   COLLECTION
+++ EXECUTION ABORTED
```

Here UZMETH.I is the template where the error was detected, 9 is the error number, 295 is the number of line containing incorrect text, MACRO NOT DEFINED is the explanation and COLLECTION is the name of undefined macrovariable.

The following table presents all UFO error messages (error numbers and explanations):

```
 1 - INPUT FILE DOES NOT EXIST OR I/O ERROR
 2 - MAX. LENGTH OF BUFFER EXCEEDED
 4 - MAX. LENGTH OF SYMBOL TABLE EXCEEDED
 5 - ATTEMPT TO APPEND TO READ ONLY VARIABLE
 6 - ATTEMPT TO APPEND TO UNDEFINED VARIABLE
 7 - ATTEMPT TO OVERWRITE OR DELETE UNDEFINED VARIABLE
 8 - ATTEMPT TO OVERWRITE OR DELETE RO OR RA VARIABLE
 9 - MACRO NOT DEFINED
10 - OUTPUT HARDWARE ERROR
11 - WRONG FORMAT OF FORTRAN LINE
14 - MAX. NUMBER OF POINTERS TO SYMBOL TABLE EXCEEDED
15 - UNEXPECTED CHDS
16 - MAX. VALUE OF LABEL EXCEEDED
17 - UNEXPECTED CHDIR
20 - SYNTAX ERROR
21 - '(' EXPECTED
22 - '(' EXPECTED
23 - MACRO NAME EXPECTED
24 - APOSTROPHE EXPECTED
25 - EOL EXPECTED
26 - INTEGER CONSTANT EXPECTED
27 - MEMORY EXHAUSTED
28 - INCLUDED FILE DOES NOT EXIST OR I/O ERROR
29 - '=' EXPECTED
30 - IDENTIFIER, STRING, INTEGER OR LOGICAL CONSTANT EXPECTED
31 - ', ' EXPECTED
32 - ERROR IN DIRECTIVE OPTION
33 - UNEXPECTED 'ENDSET'
34 - UNEXPECTED 'ENDADD'
35 - UNEXPECTED 'ENDIF'
36 - UNEXPECTED 'ELSE'
37 - UNEXPECTED 'ELSEIF'
38 - TOO MANY NESTED 'IF-ENDIF'
39 - REWIND - I/O ERROR
40 - TOO MANY NESTED 'SUBST(FILE)'
41 - SUBSTITUTED FILE DOES NOT EXIST OR I/O ERROR
42 - TOO MANY NESTED 'REPEAT-UNTIL'
43 - UNEXPECTED 'UNTIL'
44 - IN DO(I=IND1,IND2,IND3) IS IND3=0
45 - UNEXPECTED 'ENDDO'
46 - TOO MANY NESTED 'DO-ENDDO'
```

- 47 - UNEXPECTED END OF FILE
- 48 - AUTOMATIC DIFFERENTIATION ERROR

## B.6 Organization and compilation of the UFO source modules

The BEL interpreter driven by the special input template UZDPRE.I can be used for preprocessing UFO source modules. The UFO source modules \*.I usually consist of two parts. The first part is a template written in the UFO control language. This template, which begins with substitution \$INTERFACE and terminates by directive \$EXIT, serves for generation of the UFO source program by the UFO control language preprocessor (described in Section B.7). The second part is an actual source module which can contain statements of the UFO control language, e.g., substitutions \$FLOAT, \$P, \$DBLE for defining the precision of the final Fortran 77 source code (single or double). The BEL interpreter driven by template UZDPRE.I skips the interface template and starts the processing on the line containing remark \*IMPLEMENTATION. If the first part is missing, the remark \*IMPLEMENTATION can be omitted. If the second part is missing, the macrovariable \$INTERFACE and directive \$EXIT need not be used. The more details can be obtained by reading template UZDPRE.I:

```

$REM +-----+
$REM +  TEMPLATE:  UZDPRE                                01/12/89  +
$REM +                                                                 +
$REM +  PURPOSE:  DRIVER FOR UFO SOURCE-MODULE PREPROCESSOR      +
$REM +-----+

$REM +----+ INSTALATION +-----+

$P='D'
$FLOAT='REAL*8'
$DBLE='DBLE'
$GRAPHICS=6
$OPENFILE=.TRUE.
$OPTION(FN2='.I')

$REM +----+ INITIATION +-----+

$OPTION(LFRFMT=.FALSE.)
$NEXT=0
$SET(INTERFACE)
  $OPTION(LOUT=.FALSE.)
  $OPTION(LSUBS=.FALSE.)
  $OPTION(LFORTO=.FALSE.)
  $OPTION(CHDIR='*')

$ENDSET
$IF($DEF(PREDAT))
  $PREDAT
$ENDIF

$REM +----+ GRAPHIC INTERFACE +-----+

$SET(FIFAC)
  $IF(GRAPHICS=0)
  $ELSEIF(GRAPHICS=1)

```

```

        INCLUDE 'FGRAPH.FI'
        INCLUDE 'FLIB.FI'
$ELSEIF(GRAPHICS=4)
        INCLUDE 'FGRAPH.FI'
        INCLUDE 'FLIB.FI'
$ELSE
$ENDIF
$ENDSET
$SET(FDECL)
$IF(GRAPHICS=0)
$ELSE
$IF(GRAPHICS=1)
        INCLUDE 'FGRAPH.FD'
        INCLUDE 'FLIB.FD'
$ELSEIF(GRAPHICS=4)
        INCLUDE 'FGRAPH.FD'
        INCLUDE 'FLIB.FD'
$ELSE
        USE DFLIB
$ENDIF
        STRUCTURE /MYITEM/
        CHARACTER*10 NAZEV
        INTEGER POZICE
        INTEGER*2 DELKA
        INTEGER BARVA
        INTEGER HI
        ENDSTRUCTURE
$ENDIF
$ENDSET

$REM +-----+ INITIATION OF SPECIAL MACROVARIABLES +-----+

$SUBST('UZSETM')

$OPTION(IUNIT=9)

```

The first several statements of this template are system dependent and define an individual UFO installation. Here \$FLOAT='REAL\*8', \$P='D', \$DBLE='DBLE' correspond to double precision Fortran 77 source code and \$GRAPHICS=6 defines the graphics library used. Option \$OPTION(FN2='.I') defines extensions of source modules which have names \*.I.

Another part of template UZDPRE.I contains the initiation of important macrovariables used. Option \$OPTION(LFRMT=.FALSE.) means that the input file is written in the free Fortran format (with possible non-Fortran characters). Options contained in macrovariable \$INTERFACE have the following meaning: \$OPTION(LOUT=.FALSE.) means that the interface will be ignored, \$OPTION(LSUBS=.FALSE.) means that substitutions are not carried out, \$OPTION(LFORTO=.FALSE.) means that the temporary output is written in the free format (with possible non-Fortran characters) and \$OPTION(CHDIR='\*') means that the directive character CHDIR will be '\*' (thus remark \*IMPLEMENTATION is interpreted as the directive, which switches the BEL interpreter into the normal mode (with substitutions and records into the output file).

Finally, \$FIFAC and \$FDECL define the graphic interfaces and \$OPTION(IUNIT=9) means that the input unit IUNIT is changed in such a way that the subsequent input data are read from file BEL.DAT (where source module \*.I has to be copied).

If a source module \*.I consists of two parts, the above process has one disadvantage. If the interface template contains remark

```
* remark
```

which should be copied into the UFO source program, then \$OPTION(CHDIR='\*') causes that it is understood as a directive. Therefore the above remark has to be replaced by the sequence of statements

```
$CHPOM='$$'
$IF(.FALSE.)
*IF(.TRUE.)
*CHPOM='*'
*OPTION(CHDIR='$')
$ENDIF
* remark
$OPTION(CHDIR=CHPOM)
```

in the interface template. This is unnecessary if the processed template has only one part and substitution \$INTERFACE is not used.

The output file BEL.OUT obtained by the BEL interpreter contains a resulting source module written in the standard Fortran format, which can be copied into corresponding file \*.FOR and compiled. All required steps, i.e., copying \*.I into BEL.DAT, processing of BEL interpreter, copying BEL.OUT into \*.FOR and compilation \*.FOR to \*.OBJ is realized by using the batch file PRE.BAT. The object module \*.OBJ is added to the UFO library of object modules by using the batch file UF.BAT.

## B.7 The UFO control language preprocessor

The UFO system is organized in such a way that the UFO source program P.FOR (or P.F) need not be written in Fortran 77 immediately. Instead, the procedure written in the UFO control language is supplied and the UFO control language preprocessor (UFOCLP), which is the BEL interpreter driven by the special input template UZDCLP.I, generates the resulting Fortran 77 source program. Besides template UZDCLP.I, UFOCLP uses many additional templates: user supplied template P.UFO, system templates UZ\*.I, drivers for individual classes of methods U0\*.I, U1\*.I, U2\*.I and other templates, the interfaces of files \*.I, which realize various parts of the source program. Basic properties of UFOCLP can be understood by reading template UZDCLP.I:

```
$REM +-----+
$REM +   TEMPLATE: UZDCLP                               01/12/98   +
$REM +                                                                 +
$REM +   PURPOSE:  DRIVER FOR UFO CONTROL LANGUAGE PREPROCESSOR   +
$REM +-----+

$REM +-----+ INSTALATION +-----+

$OPTION(MODERW=3)
$OPTION(ILNLEN=200)
$OPTION(OLNLEN=200)
$OPTION(FN2='.I')
$P='D'
$USERUNIT=12
$OUTPUTUNIT=11
$FLOAT='REAL*8'
$DBLE='DBLE'
```

```

$GRAPHICS=6
$OPTION(DIALGR=.TRUE.)

$REM +-----+ OUTPUT UNIT FOR ERROR MESSAGES +-----+
$SET(ERROR)
$OPTION(OUNIT=6)
ERROR:
$ENDSET

$REM +-----+ INITIATION +-----+

$OPTION(LSUBS=.FALSE.)

$REM +-----+ INITIATION OF THE SYSTEM MACROVARIABLES +-----+

$SET(BATCH)
  $OPTION(DIALOG=0)
  $OPTION(DIALGR=.FALSE.)
  $DIA=0
$ENDSET

$SET(GDIALOGUE)
  $OPTION(DIALOG=1)
  $OPTION(DIALGR=.TRUE.)
  $DIA=1
$ENDSET

$SET(TDIALOGUE)
  $OPTION(DIALOG=1)
  $OPTION(DIALGR=.FALSE.)
  $DIA=1
$ENDSET

$DIALOGUE=GDIALOGUE

$SET(STANDARD)
  $SUBST('UZSTAN')
$ENDSET

$SET(METHOD)
  $SUBST('UZMETH')
$ENDSET

$SET(TSTART)
  CALL UYTIM1(ETIME)
  $ADD(INTEGER,'\ETIME')
$ENDSET

$SET(TSTOP)
  CALL UYTIM2(ETIME)
$ENDSET

```

```

$NUMBERS='1\2\3\4\5\6\7\8\9\0'

$SET(VARERASE)
  CALL UYCLEA
$ENDSET

$SET(INITIATION)
  $SUBST('UZINIT')
$ENDSET

$SET(LABEL)
  $L CONTINUE
  $IF(TRACE='YES') $SUBST('UZTPNT')
$ENDSET

$INTERFACE='$REM'

$SUBST('UZSETM')

$OPTION(MODERW=2)

$INTEGER='.END'
$INTEGER2='.END'
$LOGICAL='.END'
$REAL='.END'
$REAL4='.END'

$OPTION(MODERW=1)
$DIA=1

$SET(MODERASE)
  $SUBST('UZMDER')
$ENDSET
$SET(METERASE)
  $SUBST('UZMTER')
$ENDSET
$SET(RUNERASE)
  $SUBST('UZMTER')
  $LMET=-1
$ENDSET

$SET(GLOBAL)
  $OPTION(LFORTO=.FALSE.)
  $$SUBST('UZDECL')
  $OPTION(LFORTO=.TRUE.)
  $ERASE(GLOBAL)
  $SET(UFOINPUT)
  $SUBST('UZINPT')
$ENDSET
$IF($DEF(DELETION))
  OPEN(7,FILE='P.PER',STATUS='REPLACE')

```

```

        CLOSE (7)
        $ERASE(DELETION)
    $ENDIF
$ENDSET

$REM $INPUT=' '

$SET(SUBROUTINES)

$ENDSET

$NEXT=0
$METHODINIT=.FALSE.
$OPTION(LSUBS=.TRUE.)

$REM +-----+ 1. PASS +-----+

$TMPUNIT=$OUNIT
$INITUNIT=$IUNIT
$OPTION(IUNIT=USERUNIT)
$OPTION(LFORTO=.FALSE.)
$$SUBST('UZIPAR')
$$SUBROUTINES

$REM +-----+ 2. PASS +-----+

$OPTION(LFORTO=.TRUE.)
$OPTION(OUNIT=OUTPUTUNIT)
$REWIND(TMPUNIT)
$OPTION(IUNIT=TMPUNIT)

```

The first several statements of this template are system dependent and define an individual UFO installation. Here \$FLOAT='REAL\*8', \$P='D', \$DBLE='DBLE' correspond to double precision Fortran 77 source code and \$GRAPHICS=6 defines the graphics library used (option \$OPTION(DIALGR=.TRUE.) determines the graphic dialogue). Option \$OPTION(MODERW=3) means that the subsequent macrovariables cannot be rewritten in the symbol table, ILNLEN and OLNLEN are lengths of input line and output line, respectively and \$OPTION(FN2='I') defines extensions of intermediate templates which have names \*.I. Option \$OPTION(LSUBS=.FALSE.) means that substitutions are not carried out.

Another part of template UZDCLP.I contains initiation of system macrovariables:

\$BATCH	- Switch to the batch mode.
\$DIALOGUE	- Switch to the default dialogue mode (text or graphic).
\$TDIALOGUE	- Switch to the text dialogue mode.
\$GDIALOGUE	- Switch to the graphic dialogue mode.
\$STANDARD	- Standard frame of the UFO source program.
\$METHOD	- Generation of the optimization method.
\$TSTART	- Start of the time measurement.
\$TSTOP	- Termination of the time measurement and print of the measured time.
\$VARERASE	- Clearing the common variables.
\$INITIATION	- Initiation of the global variables.
\$LABEL	- Labelling in the UFO source program with possible tracing.
\$MODERASE	- Cancellation of the current model.
\$METERASE	- Cancellation of the current method.

\$GLOBAL - Global declarations.  
 \$\$SUBROUTINES - User supplied subroutines.

The substituted template UZSETM.I contains a definition of additional important macrovariables, e.g., \$UK-MAI1, \$UKMCI1, \$SETAG, \$SETCG. Options \$OPTION(MODERW=2) and \$OPTION(MODERW=1) mean that the subsequent macrovariables will be appended or rewritten in the symbol table, respectively.

After the initiation of system macrovariables, the first pass of UFOCLP is started. Thus the input unit IUNIT is changed in such a way that the subsequent input data are read from file P.UFO and a temporary output is written into BEL.OUT. Option \$OPTION(LFORTO=.FALSE.) means that the temporary output is written in the free format (with possible non-Fortran characters) and statements \$\$SUBST('UZIPAR'), \$\$SUBROUTINES mean that subroutines containing initiation of parameters and user supplied subroutines will be added to the UFO source program in the second pass.

In the second pass, the output temporary file P.UFO is rewound and the input unit IUNIT is changed in such a way that the subsequent input data are read from file P.UFO. The output unit OUNIT is changed in such a way, that the final source program is written into P.FOR. Option \$OPTION(LFORTO=.TRUE.) means that the UFO source program is written in the standard Fortran format.

## B.8 Basic templates and the UFO source program generation

This section contains information, which can be useful for UFO administrators and builders. As it is shown in Section B.7, the generation of the UFO source program P.FOR (or P.F) starts by reading template P.UFO, where basic macrovariables defining the problem and the method used are defined and a frame of the UFO source program is described. The standard frame of the UFO source program is determined by substitution of macrovariable \$STANDARD (synonym for template UZSTAN.I) realizing the sequence \$GLOBAL, \$INITIATION, \$INPUT, \$TSTART, \$METHOD, \$TSTOP, \$OUTPUT. Here \$INPUT contains user's input data (defined by using \$SET(INPUT) and \$ENDSET), \$OUTPUT contains user's output data (defined by using \$SET(OUTPUT) and \$ENDSET), \$TSTART and \$TSTOP are points denoting start and stop of the time measurement, \$METHOD, \$INITIATION, \$MODERASE, \$METERASE are synonyms for templates UZMETH.I, UZINIT.I, UZMDER.I, UZMTER.I and \$GLOBAL realizes global declarations by using template UZDECL.I. Global declarations are defined by list of items \$INTEGER, \$INTEGER2, \$LOGICAL, \$REAL, \$REAL4 filled up in various templates by requirements of individual methods.

The UFO source program is generated by system templates UZ\*.I, which call additional templates (interfaces of the UFO source modules \*.I, see Section B.6). The basic system templates are:

UZPROB.I - Problem definition. Here macrovariables described in Section 2 are determined.  
 UZMETH.I - Method selection. Here input specifications are defined, possible tests of external subroutines are performed and global or local optimization method (by using templates UZGMIN.I or ULGMIN.I) is specified.  
 UZGMIN.I - Global optimization method selection. Here a particular global optimization method and its parameters described in Section 3 are determined. Template UZGMIN.I usually calls template UZLMIN.I  
 UZLMIN.I - Local optimization method selection. Here a particular local optimization method and its parameters described in Section 3 are determined.  
 UZLINS.I - Step-size selection. Here a suitable line-search or trust-region method and its parameters described in Section 3 are determined.  
 UZMODN.I - Objective function definition with the choice of a way for computation of derivatives. If \$NUMDER=1, first and second derivatives are computed by using the objective function. If \$NUMDER=2, first and second derivatives are computed by using the approximating functions. This template usually calls template UZMODL.I.  
 UZMODL.I - Objective function definition. Here a suitable module is called, which is determined by the model used (described in Section 2). Also a structure of this model, which can be dense, sparse, partitioned, is taken into account.

There are many other system templates, which has to be modified only in case the UFO system is substantially changed (e.g., templates UZMOD\*.I, UZ\*MOD.I, UZ\*SET.I with '\*' replaced by A,C,E,F,Y used for



objective function definition or templates UZMDER.I and UZMTER.I for cancellation of the current model and method, respectively.) Template UZLMIN.I calls drivers for individual classes of methods, which have the form:

Udmscn.I

where character "d" corresponds to the degree of derivatives used (0, 1, 2), character "m" denotes the model used:

- F - general objective function,
- L - linear objective function,
- Q - quadratic objective function,
- S - sum of squares or powers of approximating functions,
- A - sum of absolute values of approximating functions,
- M - maximum of values of approximating functions (minimax),

character "s" denotes the problem structure:

- D - methods for dense problems,
- S - methods for sparse problems,
- B - methods for partially separable problems,
- L - limited memory methods,

character "c" corresponds to the type of constraints:

- U - unconstrained problems,
- L - linearly constrained problems,
- N - nonlinearly constrained problems,
- E - problems with equality constraints,
- I - interior point methods for general nonlinear programming problems,
- F - nonsmooth equation methods for general nonlinear programming problems

and character "n" corresponds to the driver number (1-6).

A typical driver template has the following form:

```
$INTERFACE UOFDU1
```

97/12/01

```
$N='N'
$ADD(INTEGER, '\IMD')
$HELP
  TYPE OF METHOD:
    P - PATTERN SEARCH
    S - SIMPLEX METHOD
$CHECK(TYPE, 'P', 'P\S', 2, NO)
$IF(TYPE='P') $TYP='PS'
$IF(TYPE='S') $TYP='SM'
$IF(TYPE='P')
  $HELP
    NUMBER OF METHOD:
      1 - BASIC PATTERN SEARCH WITH SORTING
$CHECK(NUMBER, '1', '1', 2, NO)
$ELSE
  $HELP
    NUMBER OF METHOD:
      1 - BASIC SIMPLEX METHOD
$CHECK(NUMBER, '1', '1\2', 2, NO)
```

```

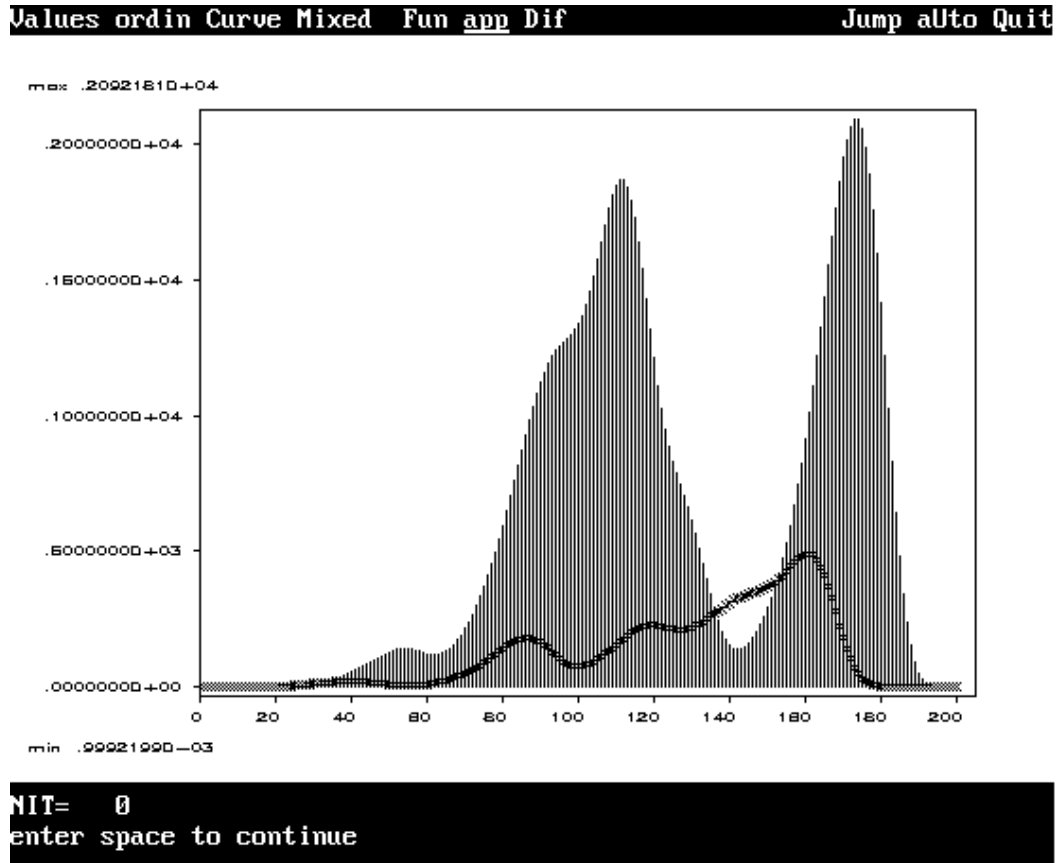
$ENDIF
$ISNF=0
$SUBST('UOOFU1')
*
* -----
* HEURISTIC METHOD
* TEMPLATE: UOFDU1
* -----
*
$LMD=$10
ASSIGN $20 TO IMD
CALL UYPRO1('UXFU',0)
CALL UYPRO2(FMIN,FO)
N=NF
$L=$10 ; $LABEL
$KD=0
$SUBST('UZMODL')
GO TO IMD
$L=$20 ; $LABEL
$SUBST('UO2FU3')
$SUBST('UYFUT1')
IF(ITERM.NE.0) GOTO $50
$L=$30 ; $LABEL
ASSIGN $30 TO IMD
*
* -----
* DIRECTION DETERMINATION
* TEMPLATE : UDD$(TYP)$NUMBER
* -----
*
$SUBST('UDD$(TYP)$NUMBER')
*
* -----
* END OF DIRECTION DETERMINATION
* -----
*
GOTO ($40,$10) ISB+1
$L=$40 ; $LABEL
ASSIGN $40 TO IMD
$SUBST('USO$(TYP)$NUMBER')
GOTO ($45,$10) ISB+1
$L=$45 ; $LABEL
TXFU=TUXX
IF(ITERM.EQ.0) GOTO $20
TXFU=TDXX
$L=$50 ; $LABEL
CALL UYEPI1(0)
$SUBST('UO1FU2')
$ADD(INIT)
NXFU='UOFDU1'
$ENDADD
$EXIT

```

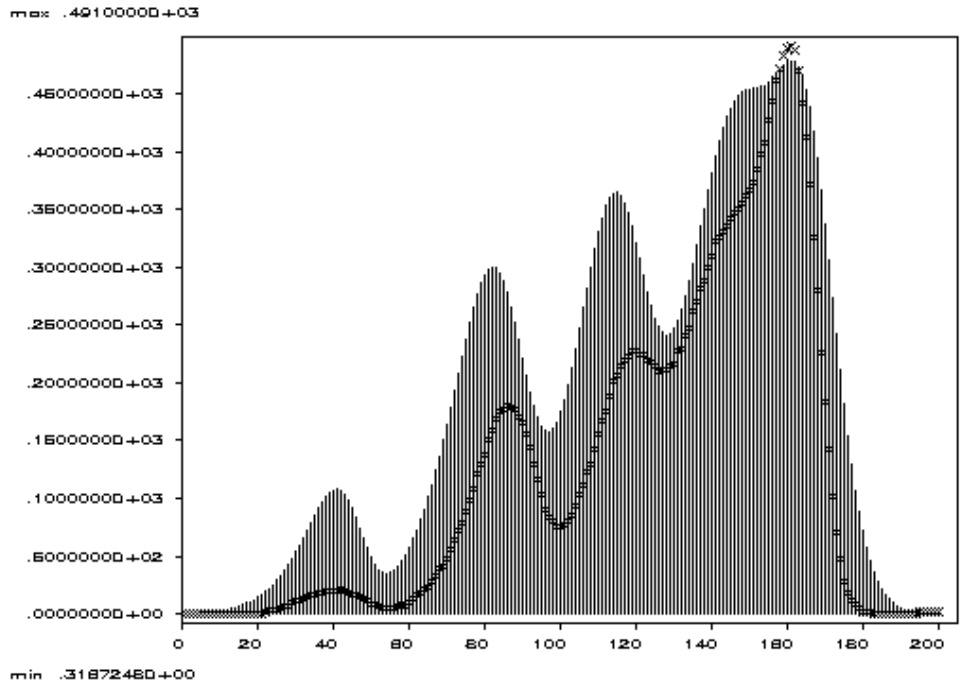
The first part of this template contains definition of important macrovariables (parameters of the method selected), e.g., \$TYPE and \$NUMBER. The second part contains instructions for corresponding parts of the UFO source program. Here the objective function is defined by UZMODL.I, direction is determined by UDDPS1.I or UDDSM1.I and step-size is selected by USOPS1.I or USOSM1.I. Statements \$L=\$10, \$L=\$20 etc. define labels 11110, 11120 etc. in the generated source program and substitution \$LABEL defines possible tracing. Modules U00FU1.I, U01FU2.I and U02FU3 serve for printing intermediate and final results. Modules UYPR01.I, UYPR02.I, UYEPI1.I serve for defining and clearing system variables and UYFUT1.I contains termination criteria. The form of the UFO source program is also influenced by macrovariables \$INIT, \$SINIT, \$SINID and \$CONST, which are substituted into the UFO source program by system templates UZINIT.I, UZINPT.I and UZMETH.I. The text from \$INIT is substituted into SUBROUTINE UYINT1, which is a part of the UFO source program. The texts from \$SINIT and \$SINID are substituted before and after user's input \$INPUT, respectively. The text from \$CONST is substituted in such a way that statements of a generated method follow immediately.

## C Graphic screen output

### C.1 Nonlinear regression

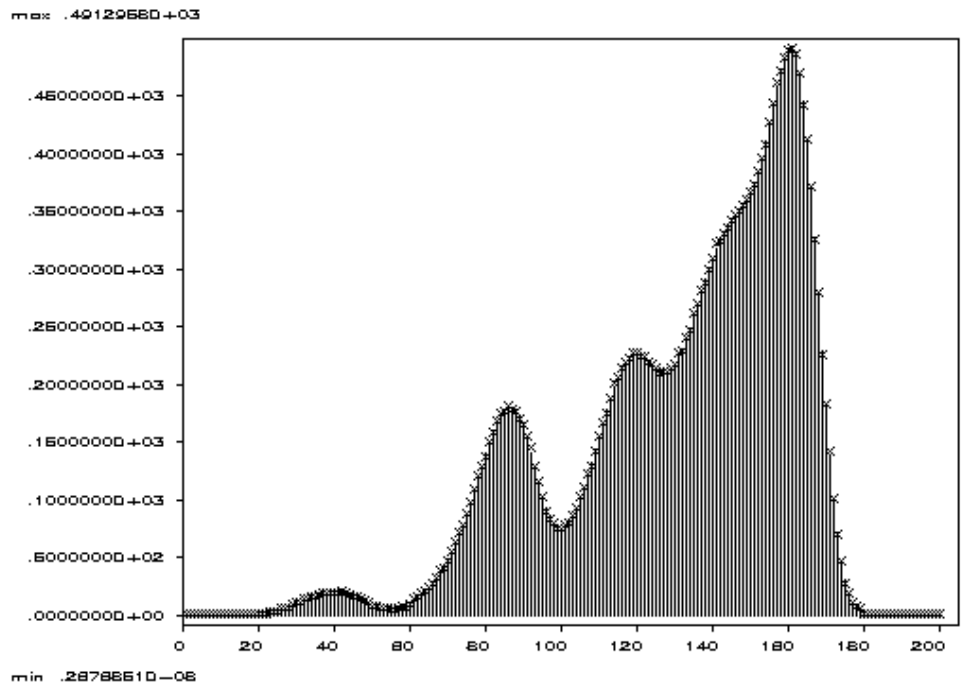


Nonlinear regression: Iteration 0



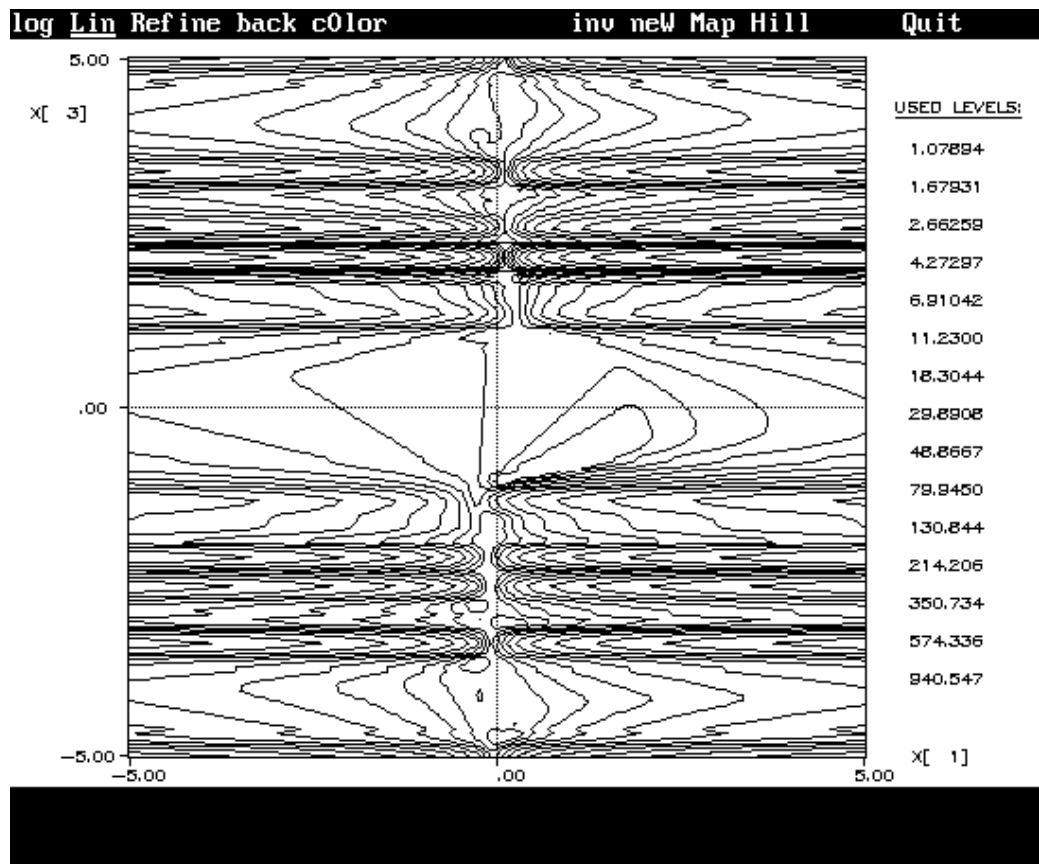
NIT= 6  
enter space to continue

Nonlinear regression: Iteration 6

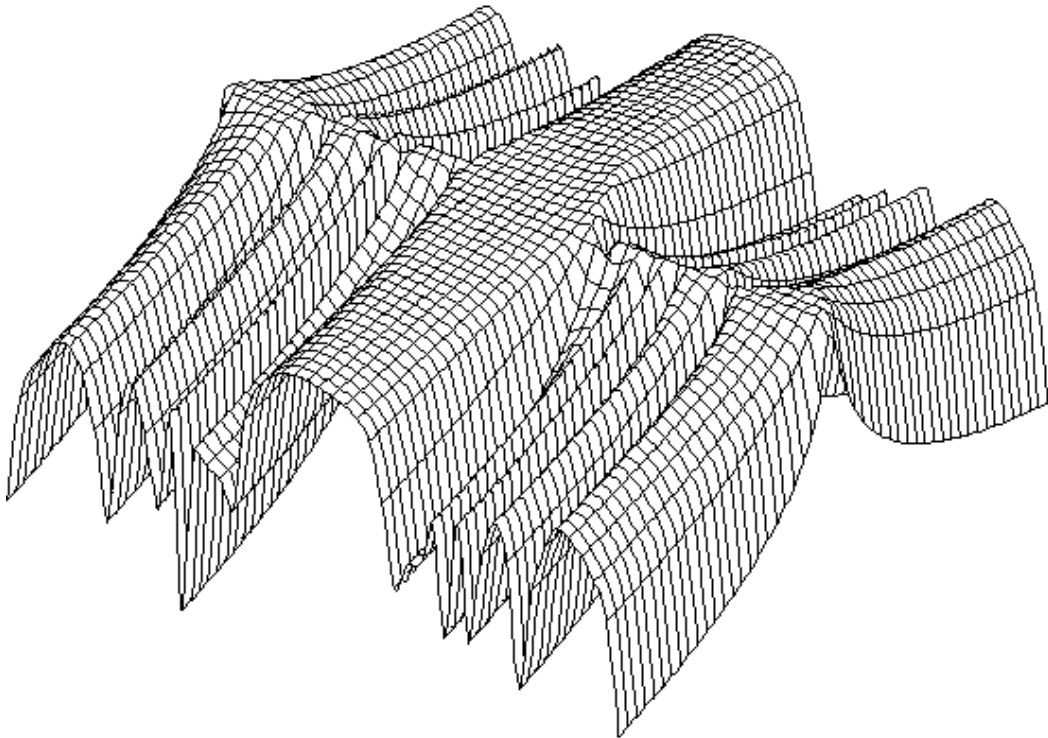


**Nonlinear regression: Final solution**

## C.2 Nonlinear minimax optimization

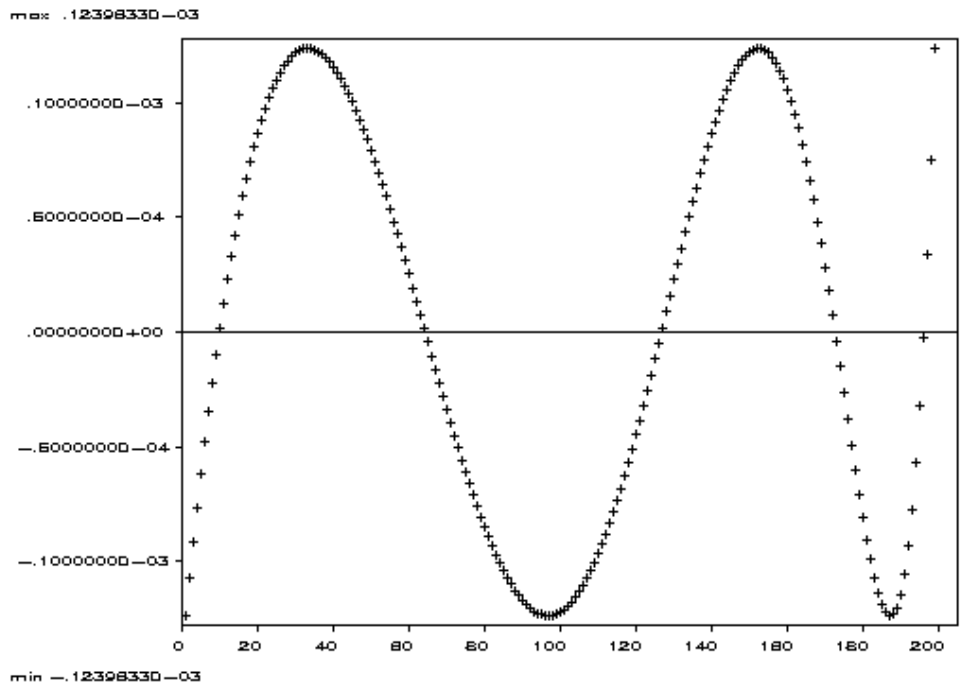


Nonlinear minimax optimization: Isolines



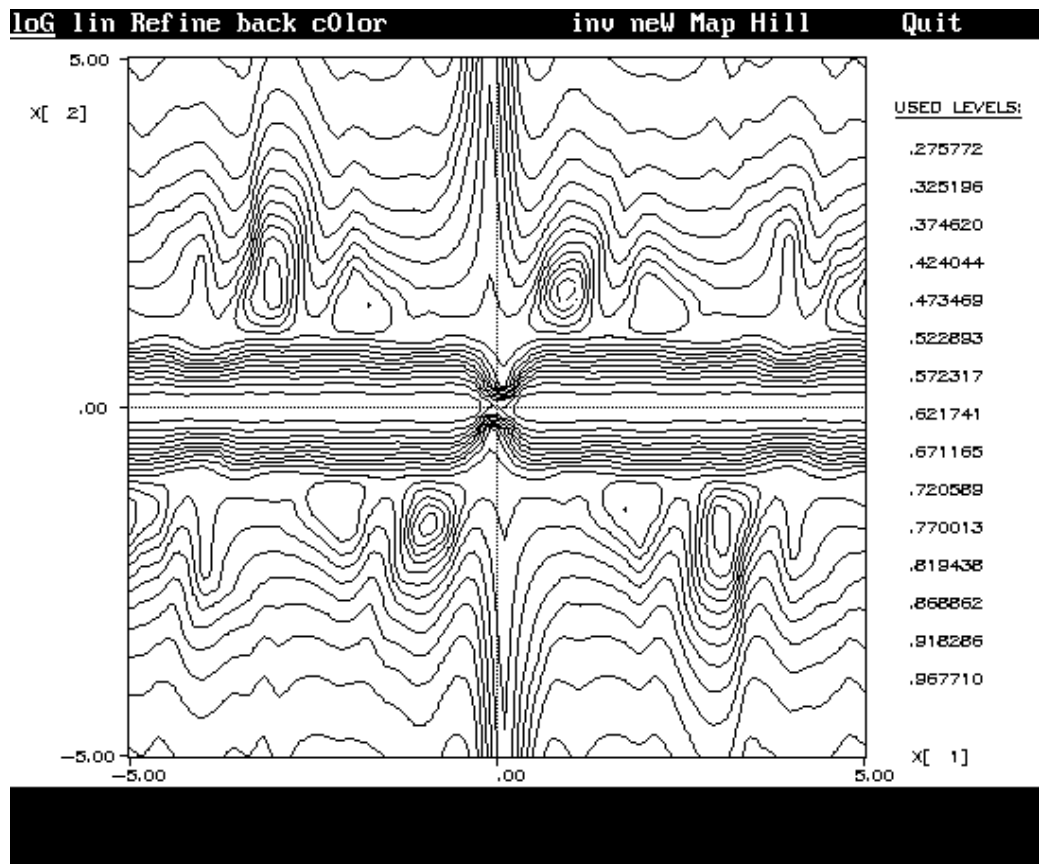
**Nonlinear minimax optimization: Surface**



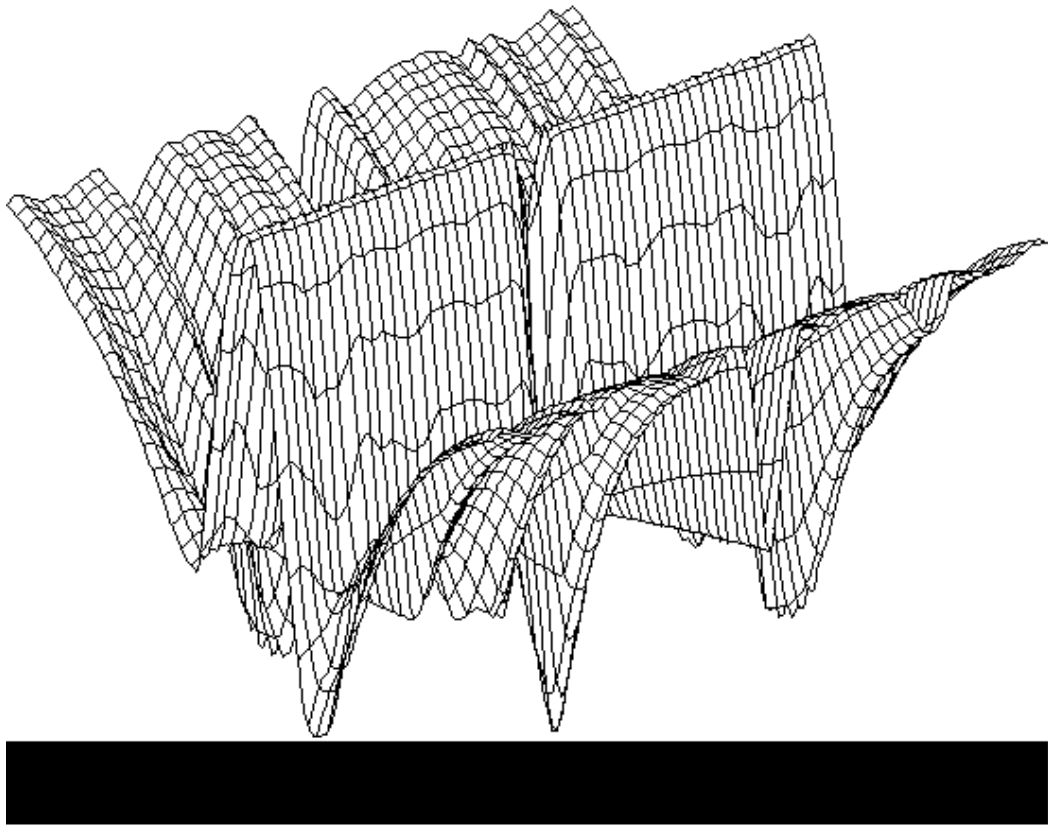


Nonlinear minimax optimization: Graph

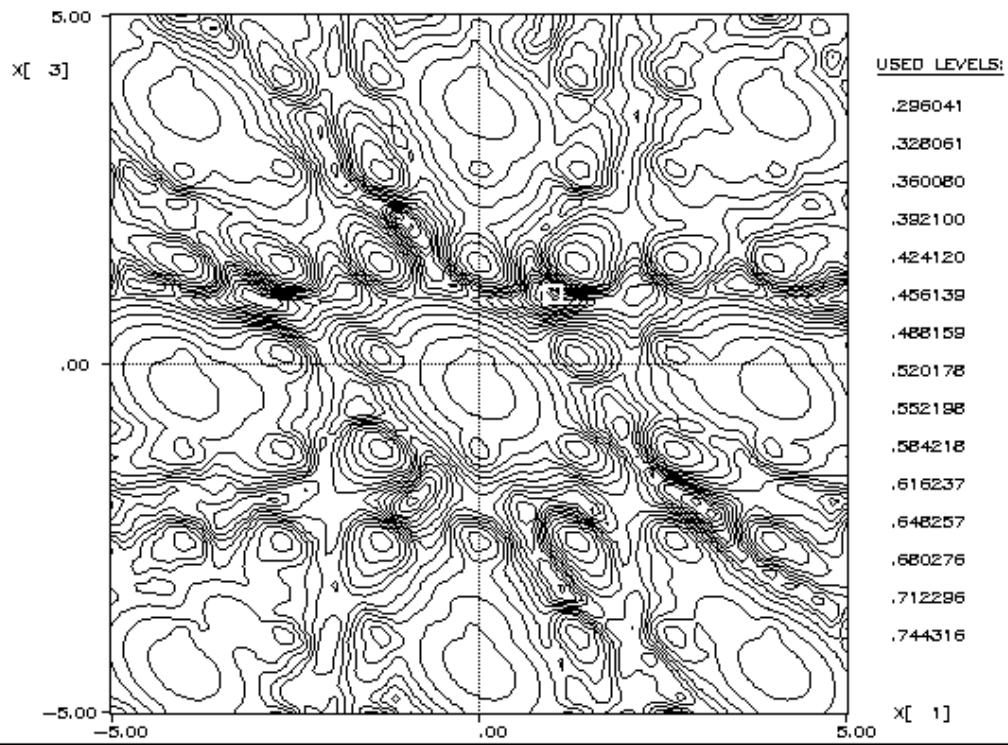
### C.3 Transformer network design



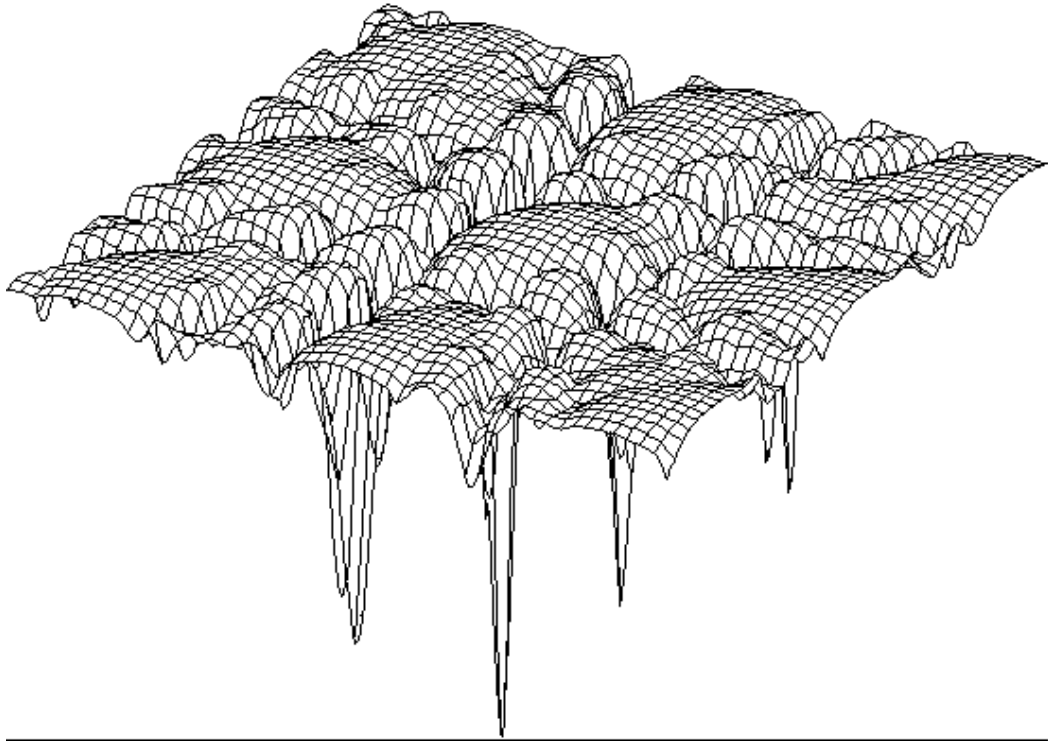
Transformer network design: Isolines



Transformer network desogn : Surface

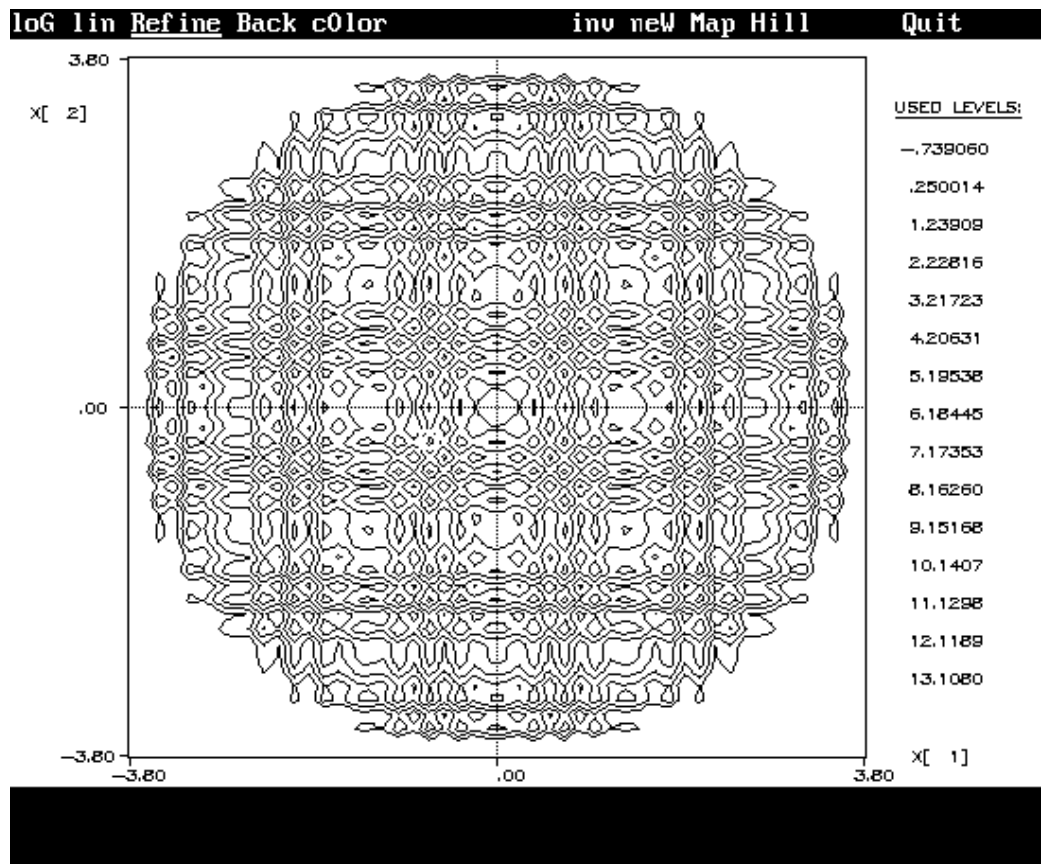


Transformer network design: Isolines



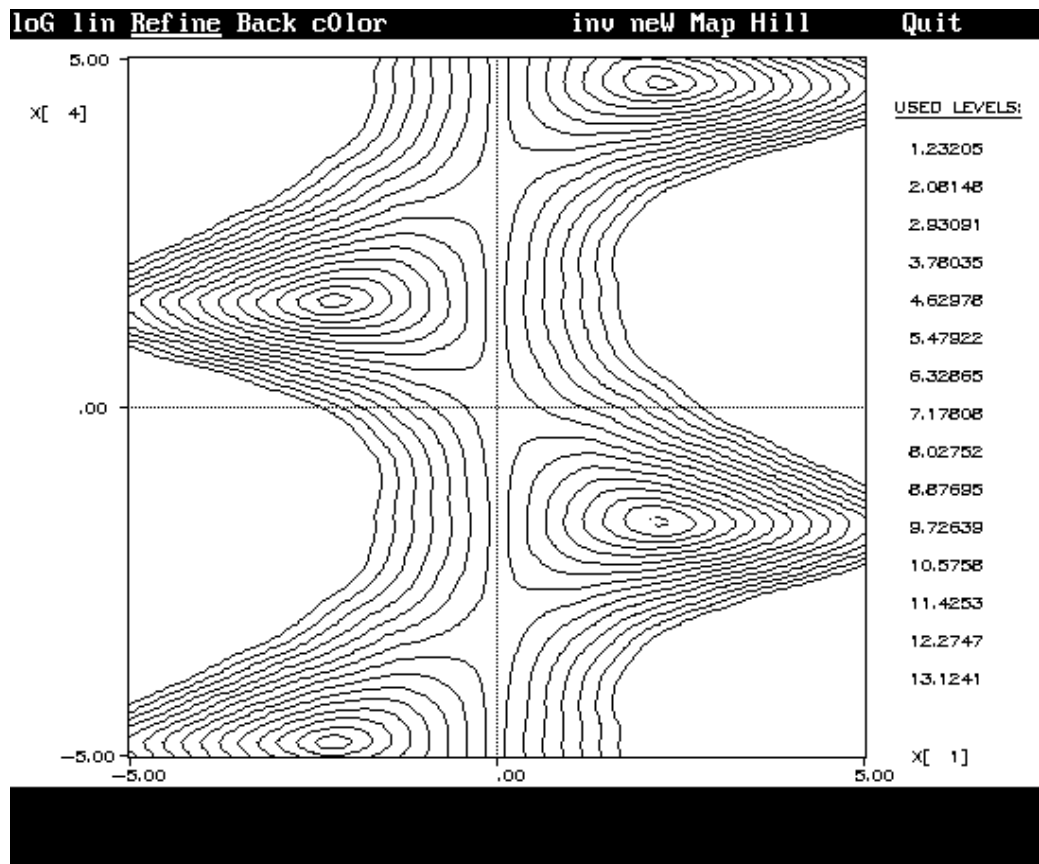
**Transformer network design: Surface**

## C.4 Global optimization



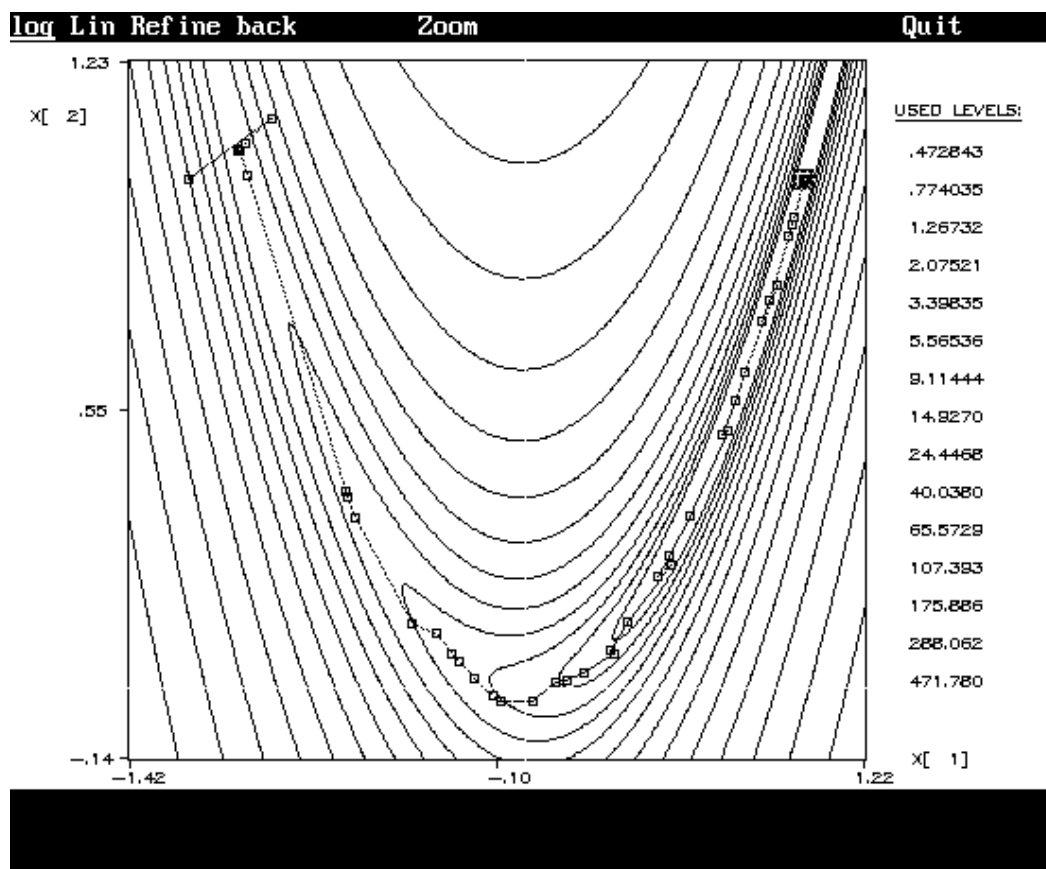
Global optimization: Isolines

## C.5 Nonsmooth optimization



**Nonsmooth optimization: Isolines**

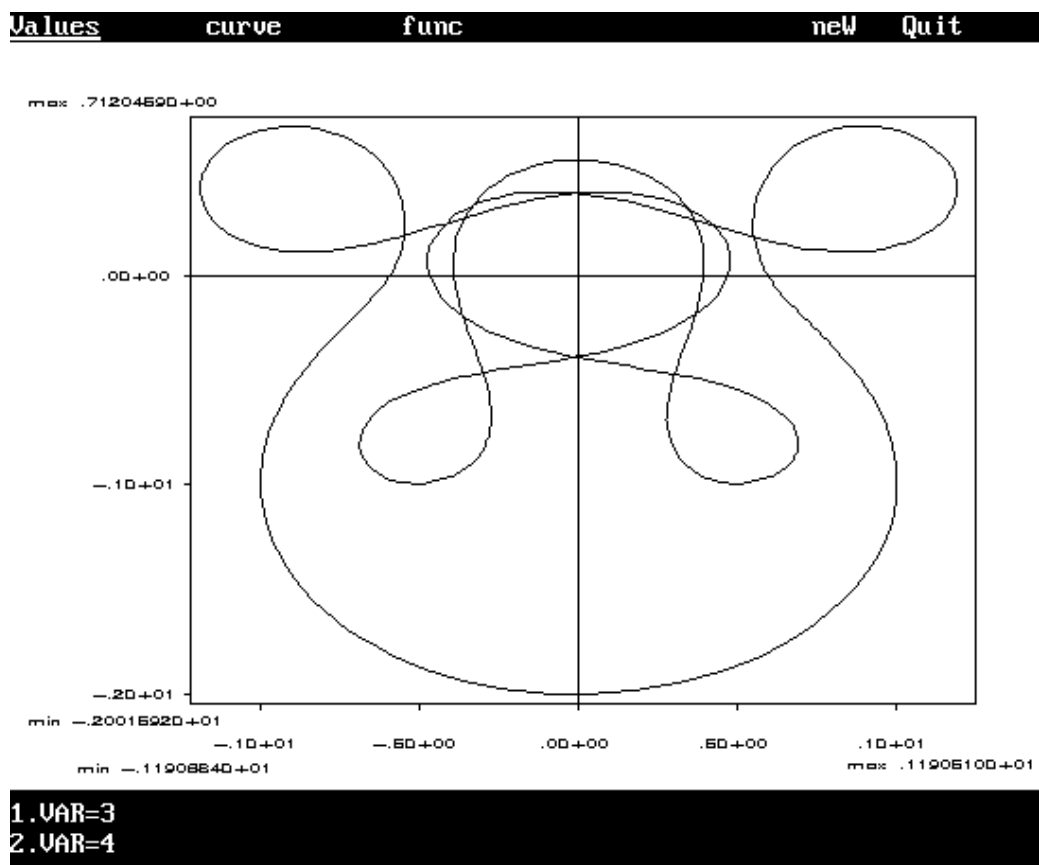
## C.6 Rosenbrock function



Rosenbrock function: Path of iterations

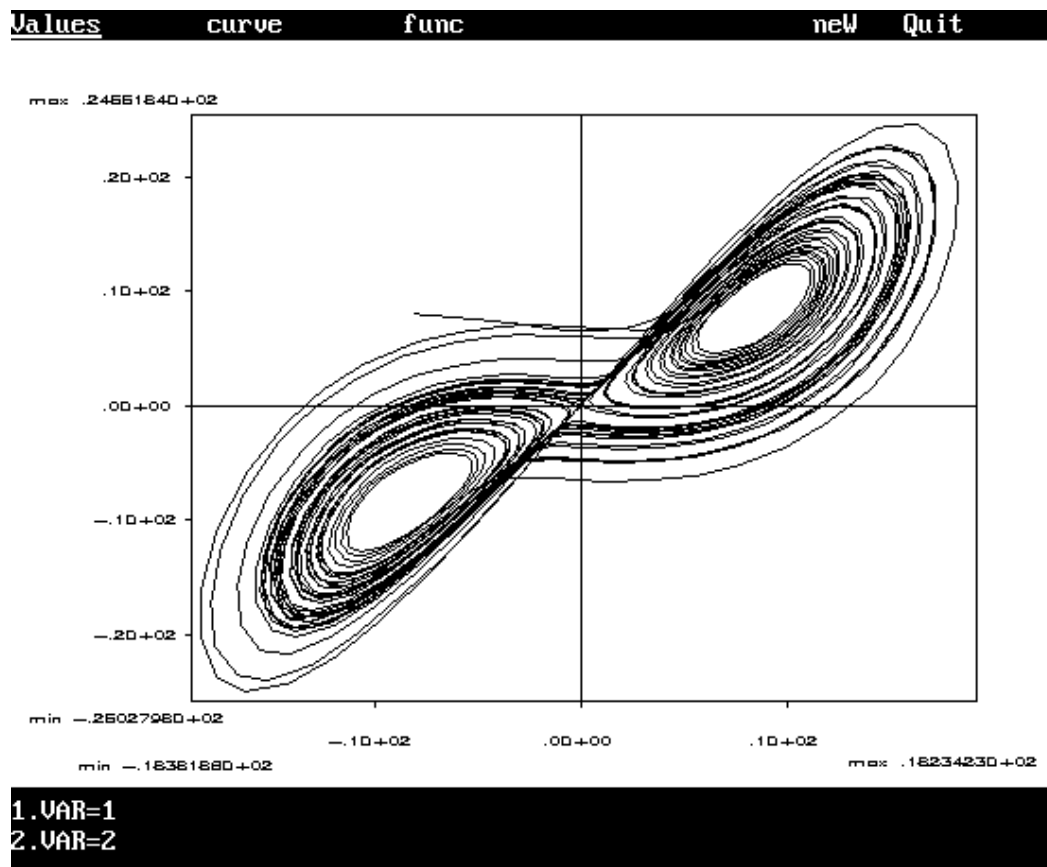


## C.7 Ordinary differential equations

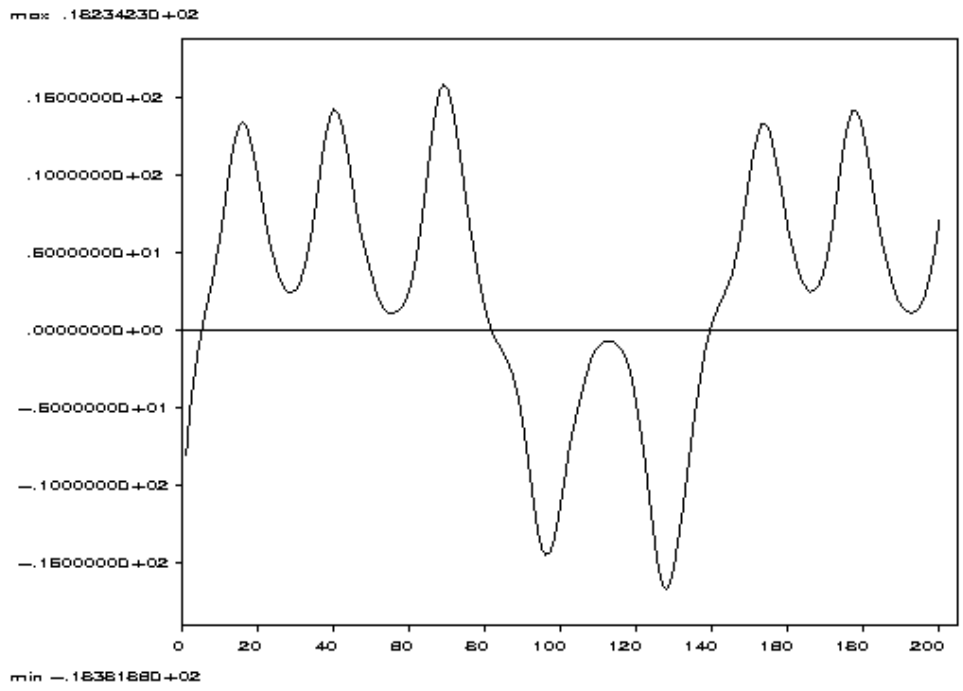


Ordinary differential equations: Orbit

## C.8 The Lorenz attractor



The Lorenz attractor: Orbit



**The Lorenz attractor: Trajectory**