



národní  
úložiště  
šedé  
literatury

## **Overdetermined Absolute Value Equations**

Rohn, Jiří  
2019

Dostupný z <http://www.nusl.cz/ntk/nusl-394153>

Dílo je chráněno podle autorského zákona č. 121/2000 Sb.

Tento dokument byl stažen z Národního úložiště šedé literatury (NUŠL).

Datum stažení: 05.05.2024

Další dokumenty můžete najít prostřednictvím vyhledávacího rozhraní [nusl.cz](http://www.nusl.cz) .



**Institute of Computer Science**  
**Academy of Sciences of the Czech Republic**

## **Overdetermined Absolute Value Equations**

Jiří Rohn  
<http://uivtx.cs.cas.cz/~rohn>

Technical report No. V-1265

04.04.2019



**Institute of Computer Science**  
**Academy of Sciences of the Czech Republic**

## **Overdetermined Absolute Value Equations**

Jiří Rohn<sup>1</sup>

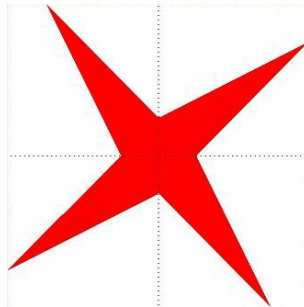
<http://uivtx.cs.cas.cz/~rohn>

Technical report No. V-1265

04.04.2019

Abstract:

We consider existence, uniqueness and computation of a solution of an absolute value equation in the overdetermined case.<sup>2</sup>



Keywords:

Absolute value equations, overdetermined system.

---

<sup>1</sup>This work was supported with institutional support RVO:67985807.

<sup>2</sup>Above: logo of interval computations and related areas (depiction of the solution set of the system  $[2, 4]x_1 + [-2, 1]x_2 = [-2, 2]$ ,  $[-1, 2]x_1 + [2, 4]x_2 = [-2, 2]$  (Barth and Nuding [1])).

# 1 Introduction

The absolute value equation

$$Ax + B|x| = b \tag{1.1}$$

has been studied so far for the square case only ( $A, B \in \mathbb{R}^{n \times n}$ ). In this report we consider the rectangular case ( $A, B \in \mathbb{R}^{m \times n}$ ); the assumption (2.1) made below ensures that  $m \geq n$ , so that in fact we investigate the overdetermined case only.

Notation used:  $|x|$  is the entrywise absolute value of  $x$ ,  $\rho$  denotes the spectral radius,  $I$  is the identity matrix and  $A^\dagger$  stands for the Moore-Penrose inverse of  $A$ .

# 2 The result

We shall handle the questions of existence, uniqueness and computation of a solution in frame of a single theorem.

**Theorem 1.** *Let  $A, B \in \mathbb{R}^{m \times n}$  satisfy*

$$\text{rank}(A) = n \tag{2.1}$$

and

$$\rho(|A^\dagger B|) < 1. \tag{2.2}$$

Then for each  $b \in \mathbb{R}^m$  the sequence  $\{x^i\}_{i=0}^\infty$  generated by

$$x^0 = A^\dagger b, \tag{2.3}$$

$$x^{i+1} = -A^\dagger B|x^i| + A^\dagger b \quad (i = 0, 1, 2, \dots) \tag{2.4}$$

tends to a limit  $x^*$ , and we have:

- (i) if  $Ax^* + B|x^*| = b$ , then  $x^*$  is the unique solution of (1.1),
- (ii) if  $Ax^* + B|x^*| \neq b$ , then (1.1) possesses no solution.

*Proof.* For clarity, we divide the proof into several steps.

(a) From (2.4) we have

$$|x^{i+1} - x^i| \leq |A^\dagger B| |x^i - x^{i-1}|$$

for each  $i \geq 1$  and since  $|A^\dagger B|^j \rightarrow 0$  as  $j \rightarrow \infty$  due to (2.2), proceeding as in the proof of Theorem 1 in [2] we prove that  $\{x^i\}$  is a Cauchian sequence, thus it is convergent,  $x^i \rightarrow x^*$ . Taking the limit in (2.4) we obtain that  $x^* = -A^\dagger B|x^*| + A^\dagger b$ , i.e.,  $x^*$  solves the equation

$$x + A^\dagger B|x| = A^\dagger b. \tag{2.5}$$

(b) Assume that  $\tilde{x}$  also solves (2.5). Then

$$|x^* - \tilde{x}| \leq |A^\dagger B| |x^* - \tilde{x}|,$$

hence

$$(I - |A^\dagger B|)|x^* - \tilde{x}| \leq 0$$

and premultiplying this inequality by the inverse of  $I - |A^\dagger B|$  which is nonnegative due to (2.2) results in

$$|x^* - \tilde{x}| \leq 0,$$

hence  $x^* = \tilde{x}$  which means that  $x^*$  is the unique solution to (2.5).

(c) We prove that if  $x$  solves (1.1), then  $x = x^*$ . Indeed, in that case it also solves the preconditioned equation

$$A^\dagger Ax + A^\dagger B|x| = A^\dagger b \tag{2.6}$$

and since  $A^\dagger = (A^T A)^{-1} A^T$  due to (2.1),  $A^\dagger A = I$  and  $x$  solves (2.5) so that  $x = x^*$ .

(d) If  $Ax^* + B|x^*| = b$ , then  $x^*$  is a solution of (1.1) and it is unique by (c).

(e) If  $Ax^* + B|x^*| \neq b$ , then existence of a solution  $x$  to (1.1) would mean that  $x = x^*$  by (c), hence  $Ax^* + B|x^*| = b$ , a contradiction.  $\square$

We have this immediate consequence.

**Theorem 2.** *Under conditions (2.1) and (2.2) the equation (1.1) possesses for each  $b \in \mathbb{R}^m$  at most one solution.*

## Bibliography

- [1] W. Barth and E. Nuding, *Optimale Lösung von Intervallgleichungssystemen*, Computing, 12 (1974), pp. 117–125.
- [2] J. Rohn, V. Hooshyarbakhsh, and R. Farhadsefat, *An iterative method for solving absolute value equations and sufficient conditions for unique solvability*, Optimization Letters, 8 (2014), pp. 35–44.