Problems for Nonlinear Least Squares and Nonlinear Equations

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Abstract:

This report contains a description of subroutines which can be used for testing large-scale optimization codes. These subroutines can easily be obtained from the web page `http://www.cs.cas.cz/~luksan/test.html`. Furthermore, all test problems contained in these subroutines are presented in the analytic form.

Keywords:
large-scale optimization, least squares, nonlinear equations, test problems

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1 Introduction

This report describes two groups of subroutines which contain problems for testing optimization codes:

1. Subroutines `TIUD24`, `TAFU24`, `TAGU24` which contain 84 general problems for testing codes for sum of squares minimization. We want to find a minimum of the special objective function

$$F(x) = \frac{1}{2} \sum_{k=1}^{n_A} f_k^2(x), \quad x \in R^n,$$

   where $f_k(x)$, $1 \le k \le n_A$, are twice continuously differentiable partial functions.

2. Subroutines `TIUD37`, `TAFU37`, `TAGU37`, which contain 64 general problems for testing codes for systems of nonlinear equations. We want to find a solution to the nonlinear system

$$f_k(x) = 0, \quad 1 \le k \le n,$$

   where $f_k(x)$, $1 \le k \le n$, are twice continuously differentiable partial functions.

All subroutines are written in the standard Fortran 77 language. Their names are derived from the following rule:

- The first letter is `T` - test subroutines.
- The second letter is either `I` - initiation, or `F` - objective function, or `A` - partial function, or `C` - constraint function.
- The third letter is either `U` - initiation of unconstrained problem, or `N` - initiation of nonlinearly constrained problem, or `F` - computation of the function value, or `G` - computation of the function gradient.
- The fourth letter is either `U` - universal subroutine, or `S` - subroutine for general sparse problems, or `B` - subroutine for partially separable problems.

The last two digits determine a given collection (numbering corresponds to the UFO system [23], which contains similar collections).

Initiation subroutines use the following parameters (array dimensions are given in parentheses):

`N`	input	number of variables
`NA`	output	number of partial functions (or equations)
`X(N)`	output	vector of variables
`FMIN`	output	lower bound of the objective function value
`XMAX`	output	maximum stepsize
`NEXT`	input	number of the problem selected
`IERR`	output	error indicator (0 - correct data, 1 - `N` is too small)

Although N is an input parameter, it can be changed by the initiation subroutine when its value does not satisfy the required conditions. For example, most of the problems require N to be even or a multiple of a positive integer.

Evaluation subroutines use the following parameters (array dimensions are given in parentheses):

N	input	number of variables
X(N)	input	vector of variables
KA	input	index of the partial function (or equation) selected
FA	output	value of the partial function (or equation) selected
GA(N)	output	gradient of the partial function (or equation) selected
NEXT	input	number of the problem selected

2 Test problems for dense nonlinear least squares

Calling statements have the form

```
CALL TIUD24(N,NA,X,FMIN,XMAX,NEXT,IERR)
CALL TAFU24(N,KA,X,FA,NEXT)
CALL TAGU24(N,KA,X,GA,NEXT)
```

with the following significance

- TIUD24 – initiation of vector of variables N, X and the number of functions in the sum of squares NA
- TAFU24 – evaluation of the KA-th partial function value FA at point X
- TAGU24 – evaluation of the KA-th partial function gradient GA at point X

The following table gives typical sizes of problems in case we choose N=200.

NEXT	N	NA	NEXT	N	NA	NEXT	N	NA	NEXT	N	NA
1	200	398	22	200	398	43	196	196	64	200	201
2	200	594	23	200	200	44	196	196	65	200	202
3	200	396	24	200	200	45	196	196	66	200	200
4	200	495	25	200	200	46	196	196	67	200	200
5	200	200	26	199	199	47	200	200	68	200	200
6	200	200	27	200	200	48	200	200	69	200	200
7	200	398	28	200	200	49	200	200	70	200	200
8	200	1000	29	200	200	50	200	200	71	200	200
9	200	594	30	200	200	51	200	200	72	200	200
10	200	398	31	200	200	52	200	200	73	200	200
11	200	398	32	200	200	53	200	200	74	200	200
12	200	396	33	200	200	54	200	200	75	200	200
13	200	462	34	200	200	55	200	200	76	200	200
14	200	396	35	200	200	56	200	200	77	200	200
15	200	396	36	200	200	57	200	200	78	200	200
16	200	396	37	200	200	58	200	200	79	200	200
17	200	200	38	200	200	59	196	196	80	200	200
18	200	200	39	200	200	60	196	196	81	200	200
19	200	200	40	196	196	61	200	201	82	200	597
20	200	200	41	196	196	62	200	200	83	200	597
21	200	462	42	196	196	63	200	200	84	200	398

We seek a local minimum of the function

$$F(x) = \frac{1}{2} \sum_{k=1}^{n_A} f_k^2(x), \quad x \in R^n,$$

from the starting point $\bar{x}$. For positive integers k and l, we use the notation $\mathrm{div}(k,l)$ for integer division, i.e., maximum integer not greater than k/l, and $\mathrm{mod}(k,l)$ for the remainder after integer division, i.e., $\mathrm{mod}(k,l) = l(k/l - \mathrm{div}(k,l))$. The description of individual problems follows.

Problem 2.1. Chained Rosenbrock function [21].

$$\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) \quad , \quad \mathrm{mod}(k,2) = 1, \\
f_k(x) &= x_i - 1 \quad , \quad \mathrm{mod}(k,2) = 0, \\
n_A &= 2(n-1), \quad i = \mathrm{div}(k+1,2), \\
\bar{x}_l &= -1.2, \quad \mathrm{mod}(l,2) = 1, \qquad \bar{x}_l = 1.0, \quad \mathrm{mod}(l,2) = 0.
\end{aligned}$$

Problem 2.2. Chained Wood function [36].

$$\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) \quad , \quad \mathrm{mod}(k,6) = 1, \\
f_k(x) &= x_i - 1 \quad , \quad \mathrm{mod}(k,6) = 2, \\
f_k(x) &= \sqrt{90}\,(x_{i+2}^2 - x_{i+3}) \quad , \quad \mathrm{mod}(k,6) = 3,
\end{aligned}$$

$$
\begin{aligned}
f_k(x) &= x_{i+2} - 1 \quad , \quad \mathrm{mod}(k,6) = 4, \\
f_k(x) &= \sqrt{10}\,(x_{i+1} + x_{i+3} - 2) \quad , \quad \mathrm{mod}(k,6) = 5, \\
f_k(x) &= (x_{i+1} - x_{i+3})/\sqrt{10} \quad , \quad \mathrm{mod}(k,6) = 0, \\
n_A &= 3(n-2), \quad i = 2\,\mathrm{div}(k+5,6) - 1, \\
\bar{x}_l &= -3, \quad \mathrm{mod}(l,2) = 1, \quad l \le 4, \qquad \bar{x}_l = -2, \quad \mathrm{mod}(l,2) = 1, \quad l > 4, \\
\bar{x}_l &= -1, \quad \mathrm{mod}(l,2) = 0, \quad l \le 4, \qquad \bar{x}_l = 0, \quad \mathrm{mod}(l,2) = 0, \quad l > 4.
\end{aligned}
$$

Problem 2.3. Chained Powell singular function [21].

$$
\begin{aligned}
f_k(x) &= x_i + 10x_{i+1} \quad , \quad \mathrm{mod}(k,4) = 1, \\
f_k(x) &= \sqrt{5}\,(x_{i+2} - x_{i+3}) \quad , \quad \mathrm{mod}(k,4) = 2, \\
f_k(x) &= (x_{i+1} - 2x_{i+2})^2 \quad , \quad \mathrm{mod}(k,4) = 3, \\
f_k(x) &= \sqrt{10}\,(x_i - x_{i+3})^2 \quad , \quad \mathrm{mod}(k,4) = 0, \\
n_A &= 2(n-2), \quad i = 2\,\mathrm{div}(k+3,4) - 1, \\
\bar{x}_l &= 3, \quad \mathrm{mod}(l,4) = 1, \qquad \bar{x}_l = -1, \quad \mathrm{mod}(l,4) = 2, \\
\bar{x}_l &= 0, \quad \mathrm{mod}(l,4) = 3, \qquad \bar{x}_l = 1, \quad \mathrm{mod}(l,4) = 0.
\end{aligned}
$$

Problem 2.4. Chained Cragg and Levy function [36].

$$
\begin{aligned}
f_k(x) &= (\exp(x_i) - x_{i+1})^2 \quad , \quad \mathrm{mod}(k,5) = 1, \\
f_k(x) &= 10(x_{i+1} - x_{i+2})^3 \quad , \quad \mathrm{mod}(k,5) = 2, \\
f_k(x) &= \frac{\sin^2(x_{i+2} - x_{i+3})}{\cos^2(x_{i+2} - x_{i+3})} \quad , \quad \mathrm{mod}(k,5) = 3, \\
f_k(x) &= x_i^4 \quad , \quad \mathrm{mod}(k,5) = 4, \\
f_k(x) &= x_{i+3} - 1 \quad , \quad \mathrm{mod}(k,5) = 0, \\
n_A &= 5(n-2)/2, \quad i = 2\,\mathrm{div}(k+4,5) - 1, \\
\bar{x}_l &= 1 \;\; , l = 1 \;\; , \;\; \bar{x}_l = 2 \;\; , \;\; 2 \le l \le n.
\end{aligned}
$$

Problem 2.5. Generalized Broyden tridiagonal function [21].

$$
\begin{aligned}
f_k(x) &= (3 - 2x_k)\,x_k + 1 - x_{k-1} - x_{k+1}, \\
n_A &= n, \quad x_0 = x_{n+1} = 0, \\
\bar{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 2.6. Generalized Broyden banded function [21].

$$
\begin{aligned}
f_k(x) &= (2 + 5x_k^2)x_k + 1 + \sum_{j=k_1}^{k_2} x_j(1 + x_j), \\
n_A &= n, \quad k_1 = \max(1, k-5), \quad k_2 = \min(n, k+1), \\
\bar{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 2.7. Chained Freudenstein and Roth function [36].

$$\begin{aligned}
f_k(x) &= x_i + x_{i+1}((5 - x_{i+1})x_{i+1} - 2) - 13 \ , \quad \mathrm{mod}(k,2) = 1, \\
f_k(x) &= x_i + x_{i+1}((1 + x_{i+1})x_{i+1} - 14) - 29, \quad \mathrm{mod}(k,2) = 0, \\
n_A &= 2(n-1), \quad i = \mathrm{div}(k+1,2), \\
\bar{x}_l &= 0.5, \quad 1 \le l < n, \qquad \bar{x}_l \ = \ -2, \quad l = n.
\end{aligned}$$

Problem 2.8. Wright and Holt zero residual problem [37].

$$\begin{aligned}
f_k(x) &= (x_i^a - x_j^b)^c, \\
a &= 1, \qquad k \le m/2, \qquad a \ = \ 2, \quad k > m/2, \\
b &= 5 - \mathrm{div}(k, m/4), \qquad c \ = \ \mathrm{mod}(k,5) + 1, \\
n_A &= 5n, \quad i = \mathrm{mod}(k, n/2) + 1, \quad j = i + n/2, \\
\bar{x}_l &= \sin^2(l), \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.9. Toint quadratic merging problem [36].

$$\begin{aligned}
f_k(x) &= x_i + 3x_{i+1}(x_{i+2} - 1) + x_{i+3}^2 - 1 \quad , \quad \mathrm{mod}(k,6) = 1, \\
f_k(x) &= (x_i + x_{i+1})^2 + (x_{i+2} - 1)^2 - x_{i+3} - 3 \quad , \quad \mathrm{mod}(k,6) = 2, \\
f_k(x) &= x_i x_{i+1} - x_{i+2} x_{i+3} \quad , \quad \mathrm{mod}(k,6) = 3, \\
f_k(x) &= 2x_i x_{i+2} + x_{i+1} x_{i+3} - 3 \quad , \quad \mathrm{mod}(k,6) = 4, \\
f_k(x) &= (x_i + x_{i+1} + x_{i+2} + x_{i+3})^2 + (x_i - 1)^2 \quad , \quad \mathrm{mod}(k,6) = 5, \\
f_k(x) &= x_i x_{i+1} x_{i+2} x_{i+3} + (x_{i+3} - 1)^2 - 1 \quad , \quad \mathrm{mod}(k,6) = 0, \\
n_A &= 3(n-2), \quad i = 2\,\mathrm{div}(k+5,6) - 1, \\
\bar{x}_l &= 5, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.10. Chained exponential problem [21].

$$\begin{aligned}
f_k(x) &= 4 - \exp(x_i) - \exp(x_{i+1}) \quad , \quad \mathrm{mod}(k,2) = 1, \ \ i = 1, \\
f_k(x) &= 8 - \exp(3x_{i-1}) - \exp(3x_i) \\
&+ \ 4 - \exp(x_i) - \exp(x_{i+1}) \quad , \quad \mathrm{mod}(k,2) = 1, \ \ 1 < i < n, \\
f_k(x) &= 8 - \exp(3x_{i-1}) - \exp(3x_i) \quad , \quad \mathrm{mod}(k,2) = 1, \ \ i = n, \\
f_k(x) &= 6 - \exp(2x_i) - \exp(2x_{i+1}) \quad , \quad \mathrm{mod}(k,2) = 0, \\
n_A &= 2(n-1), \quad i = \mathrm{div}(k+1,2), \\
\bar{x}_l &= 0.2, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.11. Chained serpentine function [22].

$$\begin{aligned}
f_k(x) &= 10(2x_i/(1 + x_i^2) - x_{i+1}) \quad , \quad \mathrm{mod}(k,2) = 1, \\
f_k(x) &= x_i - 1 \quad , \quad \mathrm{mod}(k,2) = 0, \\
n_A &= 2(n-1), \quad i \ = \ \mathrm{div}(k+1,2), \\
\bar{x}_l &= -0.8, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.12. Chained and modified problem HS47 [22].

$$\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) &&, \quad \mathrm{mod}(k,6) = 1,\\
f_k(x) &= x_{i+2} - 1 &&, \quad \mathrm{mod}(k,6) = 2,\\
f_k(x) &= (x_{i+3} - 1)^2 &&, \quad \mathrm{mod}(k,6) = 3,\\
f_k(x) &= (x_{i+4} - 1)^3 &&, \quad \mathrm{mod}(k,6) = 4,\\
f_k(x) &= x_i^2 x_{i+3} + \sin(x_{i+3} - x_{i+4}) - 10 &&, \quad \mathrm{mod}(k,6) = 5,\\
f_k(x) &= x_{i+1} + x_{i+2}^4 x_{i+3}^2 - 20 &&, \quad \mathrm{mod}(k,6) = 0,\\
n_A &= 6(\mathrm{div}(n-5,3)+1), \quad i \;=\; 3\,\mathrm{div}(k+5,6) - 2,\\
\bar{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.13. Chained and modified problem HS48 [22].

$$\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) &&, \quad \mathrm{mod}(k,7) = 1,\\
f_k(x) &= 10(x_{i+1}^2 - x_{i+2}) &&, \quad \mathrm{mod}(k,7) = 2,\\
f_k(x) &= (x_{i+2} - x_{i+3})^2 &&, \quad \mathrm{mod}(k,7) = 3,\\
f_k(x) &= (x_{i+3} - x_{i+4})^2 &&, \quad \mathrm{mod}(k,7) = 4,\\
f_k(x) &= x_i + x_{i+1}^2 + x_{i+2} - 30 &&, \quad \mathrm{mod}(k,7) = 5,\\
f_k(x) &= x_{i+1} - x_{i+2}^2 + x_{i+3} - 10 &&, \quad \mathrm{mod}(k,7) = 6,\\
f_k(x) &= x_i x_{i+4} - 10 &&, \quad \mathrm{mod}(k,7) = 0,\\
n_A &= 7(\mathrm{div}(n-5,3)+1), \quad i \;=\; 3\,\mathrm{div}(k+6,7) - 2,\\
\bar{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.14. Sparse signomial function [22].

$$\begin{aligned}
f_k(x) &= y_j - \sum_{p=1}^{3} (p^2/j) \prod_{q=1}^{4} \mathrm{sign}(x_{i+q}) |x_{i+q}|^{q/(pj)},\\
n_A &= 4(\mathrm{div}(n-4,2)+1), \quad i \;=\; 2\,\mathrm{div}(k+3,4) - 2, \quad j \;=\; \mathrm{mod}(k-1,4)+1,\\
\bar{x}_l &= -0.8 \quad, \quad \mathrm{mod}(l,4) = 1, \quad y_1 = 14.4,\\
\bar{x}_l &= 1.2 \quad, \quad \mathrm{mod}(l,4) = 2, \quad y_2 = 6.8,\\
\bar{x}_l &= -1.2 \quad, \quad \mathrm{mod}(l,4) = 3, \quad y_3 = 4.2,\\
\bar{x}_l &= 0.8 \quad, \quad \mathrm{mod}(l,4) = 0, \quad y_4 = 3.2.
\end{aligned}$$

Problem 2.15. Sparse exponential function [22].

$$\begin{aligned}
f_k(x) &= y_j - \sum_{p=1}^{3} (p^2/j) \exp\left(\sum_{q=1}^{4} x_{i+q} q/(pj) \right),\\
n_A &= 4(\mathrm{div}(n-4,2)+1), \quad i \;=\; 2\,\mathrm{div}(k+3,4) - 2, \quad j \;=\; \mathrm{mod}(k-1,4)+1,\\
\bar{x}_l &= -0.8 \quad, \quad \mathrm{mod}(l,4) = 1, \quad y_1 = 35.8,\\
\bar{x}_l &= 1.2 \quad, \quad \mathrm{mod}(l,4) = 2, \quad y_2 = 11.2,\\
\bar{x}_l &= -1.2 \quad, \quad \mathrm{mod}(l,4) = 3, \quad y_3 = 6.2,\\
\bar{x}_l &= 0.8 \quad, \quad \mathrm{mod}(l,4) = 0, \quad y_4 = 4.4.
\end{aligned}$$

Problem 2.16. Sparse trigonometric function [22].

$$
\begin{aligned}
f_k(x) &= y_j - \sum_{q=1}^{4}[(-1)^q jq^2 \sin(x_{i+q}) + j^2 q \cos(x_{i+q})], \\
n_A &= 4(\text{div}(n-4,2)+1), \quad i \;=\; 2\,\text{div}(k+3,4)-2, \quad j \;=\; \text{mod}(k-1,4)+1, \\
\bar{x}_i &= -0.8 \quad , \quad \text{mod}(l,4)=1, \quad y_1 = 30.6, \\
\bar{x}_i &= 1.2 \quad , \quad \text{mod}(l,4)=2, \quad y_2 = 72.2, \\
\bar{x}_i &= -1.2 \quad , \quad \text{mod}(l,4)=3, \quad y_3 = 124.4, \\
\bar{x}_i &= 0.8 \quad , \quad \text{mod}(l,4)=0, \quad y_4 = 187.4.
\end{aligned}
$$

Problem 2.17. Countercurrent reactors problem 1 [7] (modified).

$$
\begin{aligned}
f_k(x) &= \alpha - (1-\alpha)x_{k+2} - x_k(1+4x_{k+1}) && , \; k=1, \\
f_k(x) &= -(2-\alpha)x_{k+2} - x_k(1+4x_{k-1}) && , \; k=2, \\
f_k(x) &= \alpha x_{k-2} - (1-\alpha)x_{k+2} - x_k(1+4x_{k+1}) && , \; \text{mod }(k,2)=1 \;, \; 2<k<n-1, \\
f_k(x) &= \alpha x_{k-2} - (2-\alpha)x_{k+2} - x_k(1+4x_{k-1}) && , \; \text{mod }(k,2)=0 \;, \; 2<k<n-1, \\
f_k(x) &= \alpha x_{k-2} - x_k(1+4x_{k+1}) && , \; k=n-1, \\
f_k(x) &= \alpha x_{k-2} - (2-\alpha) - x_k(1+4x_{k-1}) && , \; k=n, \\
n_A &= n, \quad \alpha \;=\; 1/2, \\
\overline{x}_l &= 0.1 \;\; , \text{ mod }(l,8)=1, \quad \overline{x}_l \;=\; 0.2 \;\; , \text{ mod }(l,8)=2, \\
\overline{x}_l &= 0.3 \;\; , \text{ mod }(l,8)=3, \quad \overline{x}_l \;=\; 0.4 \;\; , \text{ mod }(l,8)=4, \\
\overline{x}_l &= 0.5 \;\; , \text{ mod }(l,8)=5, \quad \overline{x}_l \;=\; 0.4 \;\; , \text{ mod }(l,8)=6, \\
\overline{x}_l &= 0.3 \;\; , \text{ mod }(l,8)=7, \quad \overline{x}_l \;=\; 0.2 \;\; , \text{ mod }(l,8)=0.
\end{aligned}
$$

Problem 2.18. Tridiagonal system [20].

$$
\begin{aligned}
f_k(x) &= 4(x_k - x_{k+1}^2) && , \; k=1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + 4(x_k - x_{k+1}^2) && , \; 1<k<n, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) && , \; k=n, \\
n_A &= n, \quad \overline{x}_l = 12, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 2.19. Structured Jacobian problem [14].

$$
\begin{aligned}
f_k(x) &= -2x_k^2 + 3x_k - 2x_{k+1} + 3x_{n-4} - x_{n-3} \\
&- \; x_{n-2} + 0.5x_{n-1} - x_n + 1 && , \; k=1, \\
f_k(x) &= -2x_k^2 + 3x_k - x_{k-1} - 2x_{k+1} + 3x_{n-4} - x_{n-3} \\
&- \; x_{n-2} + 0.5x_{n-1} - x_n + 1 && , \; 1<k<n, \\
f_k(x) &= -2x_k^2 + 3x_k - x_{k-1} + 3x_{n-4} - x_{n-3} \\
&- \; x_{n-2} + 0.5x_{n-1} - x_n + 1 && , \; k=n, \\
n_A &= n, \quad \overline{x}_l = -1, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 2.20. Modified discrete boundary value problem [22].

$$
\begin{aligned}
f_k(x) &= 2x_k + (1/2)h^2(x_k + hk + 1)^3 - x_{k-1} - x_{k+1} + 1, \\
n_A &= n, \quad h = 1/(n+1), \quad x_0 = x_{n+1} = 0, \\
\bar{x}_l &= l(l-1)h, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 2.21. Chained and modified problem HS53 [22].

$$
\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) &&, \quad \mathrm{mod}(k,7) = 1, \\
f_k(x) &= x_{i+1} + x_{i+2} - 2 &&, \quad \mathrm{mod}(k,7) = 2, \\
f_k(x) &= x_{i+3} - 1 &&, \quad \mathrm{mod}(k,7) = 3, \\
f_k(x) &= x_{i+4} - 1 &&, \quad \mathrm{mod}(k,7) = 4, \\
f_k(x) &= x_i + 3x_{i+1} &&, \quad \mathrm{mod}(k,7) = 5, \\
f_k(x) &= x_{i+2} + x_{i+3} - 2x_{i+4} &&, \quad \mathrm{mod}(k,7) = 6, \\
f_k(x) &= 10(x_{i+1}^2 - x_{i+4}) &&, \quad \mathrm{mod}(k,7) = 0, \\
n_A &= 7(\mathrm{div}(n-5,3) + 1) &&, \quad i = 3\,\mathrm{div}(k+6,7) - 2, \\
\bar{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 2.22. Attracting-Repelling problem [22].

$$
\begin{aligned}
f_k(x) &= x_1 - 1 &&, \quad k = 1, \\
f_k(x) &= 10(x_i^2 - x_{i+1}), &&, \quad k > 1, \quad \mathrm{mod}(k,2) = 0, \\
f_k(x) &= 2\exp(-(x_i - x_{i+1})^2) + \exp(-2(x_{i+1} - x_{i+2})^2) &&, \quad k > 1, \quad \mathrm{mod}(k,2) = 1, \\
n_A &= 2(n-1), \quad i = \mathrm{div}(k,2), \\
\bar{x}_l &= -1.2 \;, \quad \mathrm{mod}(l,2) = 1, \quad \bar{x}_l = 1.0 \;, \quad \mathrm{mod}(l,2) = 0.
\end{aligned}
$$

Problem 2.23. Countercurrent reactors problem 2 [7] (modified).

$$
\begin{aligned}
f_k(x) &= x_1 - (1 - x_1)x_{k+2} - \alpha(1 + 4x_{k+1}) &&, \; k = 1, \\
f_k(x) &= -(1 - x_1)x_{k+2} - \alpha(1 + 4x_k) &&, \; k = 2, \\
f_k(x) &= \alpha x_1 - (1 - x_1)x_{k+2} - x_k(1 + 4x_{k-1}) &&, \; k = 3, \\
f_k(x) &= x_1 x_{k-2} + (1 - x_1)x_{k+2} - x_k(1 + 4x_{k-1}) &&, \; 3 < k < n-1, \\
f_k(x) &= x_1 x_{k-2} - x_k(1 + 4x_{k-1}) &&, \; k = n-1, \\
f_k(x) &= x_1 x_{k-2} - (1 - x_1) - x_k(1 + 4x_{k-1}) &&, \; k = n, \\
n_A &= n, \quad \alpha = 0.414214, \\
\overline{x}_i &= 0.1 \;, \; \mathrm{mod}\,(i,8) = 1, \quad & \overline{x}_i &= 0.2 \;, \; \mathrm{mod}\,(i,8) = 2, \\
\overline{x}_i &= 0.3 \;, \; \mathrm{mod}\,(i,8) = 3, \quad & \overline{x}_i &= 0.4 \;, \; \mathrm{mod}\,(i,8) = 4, \\
\overline{x}_i &= 0.5 \;, \; \mathrm{mod}\,(i,8) = 5, \quad & \overline{x}_i &= 0.4 \;, \; \mathrm{mod}\,(i,8) = 6, \\
\overline{x}_i &= 0.3 \;, \; \mathrm{mod}\,(i,8) = 7, \quad & \overline{x}_i &= 0.2 \;, \; \mathrm{mod}\,(i,8) = 0.
\end{aligned}
$$

Problem 2.24. Trigonometric system [35].

$$\begin{aligned} f_k(x) &= 5-(l+1)(1-\cos x_k)-\sin x_k-\sum_{j=5l+1}^{5l+5}\cos x_j, \\ n_A &= n, \quad l=\mathrm{div}(k-1,5), \\ \overline{x_i} &= 1/n, \quad 1\le i\le n. \end{aligned}$$

Problem 2.25. Trigonometric - exponential system (trigexp 1) [35].

$$\begin{aligned} f_k(x) &= 3x_k^3+2x_{k+1}-5+\sin(x_k-x_{k+1})\sin(x_k+x_{k+1}) \;, \quad k=1, \\ f_k(x) &= 3x_k^3+2x_{k+1}-5+\sin(x_k-x_{k+1})\sin(x_k+x_{k+1}) \\ &+ 4x_k-x_{k-1}\exp(x_{k-1}-x_k)-3 \;, \quad 1<k<n, \\ f_k(x) &= 4x_k-x_{k-1}\exp(x_{k-1}-x_k)-3 \;, \quad k=n, \\ n_A &= n, \quad \overline{x}_i=0, \quad 1\le i\le n. \end{aligned}$$

Problem 2.26. Trigonometric - exponential system (trigexp 2) [35].

$$\begin{aligned} f_k(x) &= 3(x_k-x_{k+2})^3-5+2x_{k+1}, \\ &+ \sin(x_k-x_{k+1}-x_{k+2})\sin(x_k+x_{k+1}-x_{k+2}) \;, \ \mathrm{mod}\ (k,2)=1,\ k=1, \\ f_k(x) &= -6(x_{k-2}-x_k)^3+10-4x_{k-1} \\ &- 2\sin(x_{k-2}-x_{k-1}-x_k)\sin(x_{k-2}+x_{k-1}-x_k) \\ &+ 3(x_k-x_{k+2})^3-5+2x_{k+1} \\ &+ \sin(x_k-x_{k+1}-x_{k+2})\sin(x_k+x_{k+1}-x_{k+2}) \;, \ \mathrm{mod}\ (k,2)=1,\ 1<k<n, \\ f_k(x) &= -6(x_{k-2}-x_k)^3+10-4x_{k-1} \\ &- 2\sin(x_{k-2}-x_{k-1}-x_k)\sin(x_{k-2}+x_{k-1}-x_k) \;, \ \mathrm{mod}\ (k,2)=1,\ k=n, \\ f_k(x) &= 4x_k-(x_{k-1}-x_{k+1})\exp(x_{k-1}-x_k-x_{k+1})-3 \;, \ \mathrm{mod}\ (k,2)=0, \\ n_A &= n, \quad \overline{x}_i=1, \quad 1\le i\le n. \end{aligned}$$

Problem 2.27. Singular Broyden problem [14].

$$\begin{aligned} f_k(x) &= ((3-2x_k)x_k-2x_{k+1}+1)^2 \;, \ k=1, \\ f_k(x) &= ((3-2x_k)x_k-x_{k-1}-2x_{k+1}+1)^2 \;, \ 1<k<n, \\ f_k(x) &= ((3-2x_k)x_k-x_{k-1}+1)^2 \;, \ k=n, \\ n_A &= n, \quad \overline{x}_i=-1, \quad 1\le i\le n. \end{aligned}$$

Problem 2.28. Five-diagonal system [20].

$$\begin{aligned} f_k(x) &= 4(x_k-x_{k+1}^2)+x_{k+1}-x_{k+2}^2 \;, \ k=1, \\ f_k(x) &= 8x_k(x_k^2-x_{k-1})-2(1-x_k) \\ &+ 4(x_k-x_{k+1}^2)+x_{k+1}-x_{k+2}^2 \;, \ k=2, \\ f_k(x) &= 8x_k(x_k^2-x_{k-1})-2(1-x_k) \\ &+ 4(x_k-x_{k+1}^2)+x_{k-1}^2-x_{k-2}+x_{k+1}-x_{k+2}^2 \;, \ 2<k<n-1, \end{aligned}$$

$$
\begin{aligned}
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} && , \; k = n-1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + x_{k-1}^2 - x_{k-2} && , \; k = n, \\
n_A &= n, \quad \overline{x}_i = -2, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 2.29. Seven-diagonal system [20].

$$
\begin{aligned}
f_k(x) &= 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 + x_{k+2} - x_{k+3}^2 && , \; k = 1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 + x_{k+1} - x_{k+2}^2 + x_{k+2} - x_{k+3}^2 && , \; k = 2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 \\
&+ x_{k-2}^2 + x_{k+2} - x_{k+3}^2 && , \; k = 3, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 \\
&+ x_{k-2}^2 + x_{k+2} - x_{k-3} - x_{k+3}^2 && , \; 3 < k < n-2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 \\
&+ x_{k-2}^2 + x_{k+2} - x_{k-3} && , \; k = n-2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} \\
&+ x_{k-2}^2 - x_{k-3} && , \; k = n-1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + x_{k-1}^2 - x_{k-2} \\
&+ x_{k-2}^2 - x_{k-3} && , \; k = n, \\
n_A &= n, \quad \overline{x}_i = -3, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 2.30. Extended Freudenstein and Roth function [6].

$$
\begin{aligned}
f_k &= x_k + ((5 - x_{k+1})x_{k+1} - 2)x_{k+1} - 13 && , \; \mathrm{mod}(k,2) = 1, \\
f_k &= x_{k-1} + ((x_k + 1)x_k - 14)x_k - 29 && , \; \mathrm{mod}(k,2) = 0, \\
n_A &= n, \\
\overline{x}_i &= 90 \;\; , \;\; \mathrm{mod}(i,2) = 1, \qquad \overline{x}_i \; = \; 60 \;\; , \;\; \mathrm{mod}(i,2) = 0.
\end{aligned}
$$

Problem 2.31. Broyden tridiagonal problem [27].

$$
\begin{aligned}
f_k(x) &= x_k(0.5x_k - 3) + 2x_{k+1} - 1 &&, \ k = 1,\\
f_k(x) &= x_k(0.5x_k - 3) + x_{k-1} + 2x_{k+1} - 1 &&, \ 1 < k < n,\\
f_k(x) &= x_k(0.5x_k - 3) - 1 + x_{k-1} &&, \ k = n,\\
n_A &= n, \quad \overline{x}_i = -1, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 2.32. Extended Powell badly scaled function [27].

$$
\begin{aligned}
f_k(x) &= 10000\, x_k\, x_{k+1} - 1 &&, \quad \mathrm{mod}(k,2) = 1,\\
f_k(x) &= \exp(-x_{k-1}) + \exp(-x_k) - 1.0001 &&, \quad \mathrm{mod}(k,2) = 0,\\
n_A &= n,\\
\overline{x}_i &= 0 \ , \ \mathrm{mod}(i,2) = 1, \qquad \overline{x}_i = 1 \ , \quad \mathrm{mod}(i,2) = 0.
\end{aligned}
$$

Problem 2.33. Extended Wood problem [15].

$$
\begin{aligned}
f_k(x) &= -200x_k(x_{k+1} - x_k^2) - (1 - x_k) &&, \quad \mathrm{mod}(k,4) = 1,\\
f_k(x) &= 200(x_k - x_{k-1}^2) + 20.2(x_k - 1) + 19.8(x_{k+2} - 1) &&, \quad \mathrm{mod}(k,4) = 2,\\
f_k(x) &= -180x_k(x_{k+1} - x_k^2) - (1 - x_k) &&, \quad \mathrm{mod}(k,4) = 3,\\
f_k(x) &= 180(x_k - x_{k-1}^2) + 20.2(x_k - 1) + 19.8(x_{k-2} - 1) &&, \quad \mathrm{mod}(k,4) = 0,\\
n_A &= n,\\
\overline{x}_i &= -3, \ \ \mathrm{mod}(i,2) = 1, \quad \overline{x}_i = -1, \ \ \mathrm{mod}(i,2) = 0.
\end{aligned}
$$

Problem 2.34. Tridiagonal exponential problem [6].

$$
\begin{aligned}
f_k(x) &= x_k - \exp(\cos(k(x_k + x_{k+1}))) &&, \ k = 1,\\
f_k(x) &= x_k - \exp(\cos(k(x_{k-1} + x_k + x_{k+1}))) &&, \ 1 < k < n,\\
f_k(x) &= x_k - \exp(\cos(k(x_{k-1} + x_k))) &&, \ k = n,\\
n_A &= n, \quad \overline{x}_i = 1.5, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 2.35. Discrete boundary value problem [27].

$$
\begin{aligned}
f_k(x) &= 2x_k + 0.5h^2(x_k + hk + 1)^3 - x_{k+1} &&, \ k = 1,\\
f_k(x) &= 2x_k + 0.5h^2(x_k + hk + 1)^3 - x_{k-1} - x_{k+1} &&, \ 1 < k < n,\\
f_k(x) &= 2x_k + 0.5h^2(x_k + hk + 1)^3 - x_{k-1} &&, \ k = n,\\
n_A &= n, \quad h = 1/(n+1),\\
\overline{x}_i &= ih(ih - 1), \quad 1 \le i \le n.
\end{aligned}
$$

Problem 2.36. Brent problem [5].

$$
\begin{aligned}
f_k(x) &= 3x_k(x_{k+1} - 2x_k) + x_{k+1}^2/4 &&, \ k = 1,\\
f_k(x) &= 3x_k(x_{k+1} - 2x_k + x_{k-1}) + (x_{k+1} - x_{k-1})^2/4 &&, \ 1 < k < n,\\
f_k(x) &= 3x_k(20 - 2x_k + x_{k-1}) + (20 - x_{k-1})^2/4 &&, \ k = n,\\
n_A &= n, \quad \overline{x}_i = 10, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 2.37. Troesch problem [31].

$$\begin{aligned}
f_k(x) &= 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k+1} & , \ k = 1, \\
f_k(x) &= 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k-1} - x_{k+1} & , \ 1 < k < n, \\
f_k(x) &= 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k-1} - 1 & , \ k = n, \\
n_A &= n, \quad \rho = 10, \quad h \ = \ 1/(n+1), \\
\overline{x}_i &= 1, \quad 1 \le i \le n.
\end{aligned}$$

Problem 2.38. Flow in a channel [4].

This is a finite difference analogue of the following nonlinear ordinary differential equation

$$u'''' = R(u'u'' - uu'''), \quad R = 500$$

over a unit interval Ω with boundary conditions $u(0) = 0$, $u'(0) = 0$, $u(1) = 1$, $u'(1) = 0$. We use standard 5-point finite differences on a uniform grid having 5000 internal nodes. The initial approximate solution is a discretization of $u_0(x) = (x - 1/2)^2$.

Problem 2.39. Swirling flow [4].

This is a finite difference analogue of the following system of two nonlinear ordinary differential equations

$$\begin{aligned}
u'''' + R(uu''' + vv') &= 0 \\
v'' + R(uv' + u'v) &= 0, \quad R = 500
\end{aligned}$$

over a unit interval Ω with boundary conditions $u(0) = u'(0) = u(1) = u'(1) = 0$, $v(0) = -1$, $v(1) = 1$. We use standard 5-point finite differences on a uniform grid having 2500 internal nodes. The initial approximate solution is a discretization of $u_0(x) = (x - 1/2)^2$ and $v_0(x) = x - 1/2$.

Problem 2.40. Bratu problem [16].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u + R \exp(u) = 0, \quad R = 6.8$$

over a unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 2.41. Poisson problem 1 [14].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u = \frac{u^3}{1 + x^2 + y^2}$$

over a unit square Ω with Dirichlet boundary conditions $u(0, y) = 1$, $u(1, y) = 2 - \exp(y)$, $u(x, 0) = 1$, $u(x, 1) = 2 - \exp(x)$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = -1$.

Problem 2.42. Poisson problem 2 [25].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u + \sin(2\pi u) + \sin\left(2\pi \frac{\partial u}{\partial x}\right) + \sin\left(2\pi \frac{\partial u}{\partial y}\right) + f(x,y) = 0,$$

where $f(x,y) = 1000((x-1/4)^2 + (y-3/4)^2)$, over a unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x,y) = 0$.

Problem 2.43. Porous medium problem [11].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u^2 + R\left(\frac{\partial u^3}{\partial x} + f(x,y)\right) = 0, \quad R = 50,$$

where $f(1/71, 1/71) = 1$ and $f(x,y) = 0$ for $(x,y) \neq (1/71, 1/71)$, over a unit square Ω with Dirichlet boundary conditions $u(0,y) = 1$, $u(1,y) = 0$, $u(x,0) = 1$, $u(x,1) = 0$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x,y) = 1 - xy$.

Problem 2.44. Convection-difussion problem [18].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u - Ru\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) + f(x,y) = 0, \quad R = 20,$$

where $f(x,y) = 2000x(1-x)y(1-y)$, over a unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x,y) = 0$.

Problem 2.45. Nonlinear biharmonic problem [24].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta\Delta u + R\left(\max(0,u) + \mathrm{sign}(x - \frac{1}{2})\right) = 0, \quad R = 500$$

over a unit square Ω with the boundary conditions $u = 0$ on $\partial\Omega$ and $\partial u(0,y)/\partial x = 0$, $\partial u(1,y)/\partial x = 0$, $\partial u(x,0)/\partial y = 0$, $\partial u(x,1)/\partial y = 0$. We use standard 13-point finite differences on a shifted uniform grid having 50×50 internal nodes [16]. The initial approximate solution is a discretization of $u_0(x,y) = 0$.

Problem 2.46. Driven cavity problem [16].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta\Delta u + R\left(\frac{\partial u}{\partial y}\frac{\partial \Delta u}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial \Delta u}{\partial y}\right) = 0, \quad R = 500$$

over a unit square Ω with the boundary conditions $u = 0$ on $\partial\Omega$ and $\partial u(0, y)/\partial x = 0$, $\partial u(1, y)/\partial x = 0$, $\partial u(x, 0)/\partial y = 0$, $\partial u(x, 1)/\partial y = 1$. We use standard 13-point finite differences on a shifted uniform grid having 50×50 internal nodes [16]. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 2.47.

$$\begin{aligned} f_k(x) &= 2x_k - x_{k+1} - x_{k-1} \\ &\quad + h^2\left(x_k^3 + 2.10^{-4}(2.10^{-4}a_2 - 1)x_k - 10^9\exp(-3.10^4 a_2)\right), \\ n_A &= n, \quad h = 1/(n+1), \quad a_1 = h\,k, \quad a_2 = (a_1 - 1/2)^2, \\ x_0 &= x_{n+1} = 0, \quad \overline{x}_l = 5\min(lh, 1 - lh), \quad 1 \le l \le n. \end{aligned}$$

Problem 2.48.

$$\begin{aligned} f_k(x) &= 2x_k - x_{k+1} - x_{k-1} \\ &\quad + h^2\left(x_k^3\exp(x_k) + 5.10^8\exp(-10^4 a_2)\sqrt{|a_1 - 1/2|}(x_{k+1} - x_{k-1}) + a_3\right), \\ n_A &= n, \quad h = 1/(n+1), \quad a_1 = h\,k, \quad a_2 = (a_1 - 1/2)^2, \quad a_3 = 10^6\text{sign}(a_1 - 1/2), \\ x_0 &= x_{n+1} = 0, \quad \overline{x}_l = 5\min(lh, 1 - lh), \quad 1 \le l \le n. \end{aligned}$$

Problem 2.49. [30], Problem 202 in [32].

$$\begin{aligned} f_k(x) &= x_k - \frac{x_{k+1}^2}{10}, \quad 1 \le k < n, \\ f_k(x) &= x_k - \frac{x_1^2}{10}, \quad k = n, \\ n_A &= n, \quad \overline{x}_l = 2, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.50. [10], Problem 206 in [32].

$$\begin{aligned} f_k(x) &= x_{k-1} - 2x_k + x_{k+1} - h^2\exp(x_k), \quad 1 \le k \le n, \\ n_A &= n, \quad h = 1/(n+1), \quad x_0 = x_{n+1} = 0, \\ \overline{x}_l &= 1, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.51. [9], Problem 207 in [32].

$$\begin{aligned} f_k(x) &= (3 - x_k/10)x_k + 1 - x_{k-1} - 2x_{k+1}, \quad 1 \le k \le n, \\ x_0 &= x_{n+1} = 0, \\ n_A &= n, \quad \overline{x}_l = -1, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.52. [9], Problem 208 in [32].

$$\begin{aligned} f_k(x) &= (1 + x_k^2)x_k + 1 - \sum_{i \in I_k}(x_i + x_i^2), \quad 1 \le k \le n, \\ I_k &= \{i : i \ne k, \max(1, k-3) \le i \le \min(n, k+3)\}, \\ n_A &= n, \quad \overline{x}_l = -1, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.53. [26], Problem 212 in [32].

$$\begin{aligned} f_k(x) &= x_k \qquad\qquad\qquad\qquad, \quad k = 1\,, \\ f_k(x) &= \cos(x_{k-1}) + x_k - 1), \quad 1 < k \le n\,, \\ n_A &= n, \quad \overline{x}_l = 1/2, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 2.54. [1], Problem 213 in [32].

$$\begin{aligned} f_k(x) &= 2x_k + h^2(x_k + \sin(x_k)) - x_{k-1} - x_{k+1}, \quad 1 \le k \le n\,, \\ n_A &= n, \quad h = 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1\,, \\ \overline{x}_l &= 1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 2.55. Broyden banded function [27],

$$\begin{aligned} f_k(x) &= x_k(2 + 5x_k^2) + 1 - \sum_{i \in I_k} x_i(1 + x_i), \quad 1 \le k \le n\,, \\ I_k &= \{i : i \ne k, \max(1, k-5) \le i \le \min(n, k+1)\}\,, \\ n_A &= n, \quad \overline{x}_l = -1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 2.56. Ascher and Russel boundary value problem [3].

$$\begin{aligned} f_k(x) &= 2x_k - 2h^2\left(x_k^2 + \frac{x_{k+1} - x_{k-1}}{2h}\right) - x_{k-1} - x_{k+1}, \quad 1 \le k \le n\,, \\ n_A &= n, \quad h = 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1/2\,, \\ \overline{x}_l &= 1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 2.57. Allgower and Georg boundary value problem [2] (modified).

$$\begin{aligned} f_k(x) &= 2x_k + 0.3h^2\left[\exp(20(x_k + 25(kh-1))) - \exp(-20(x_k + 25kh)) - t_k\right] \\ &\quad -x_{k-1} - x_{k+1}\,, \\ t_k &= \operatorname{sign}(kh - 0.5), \quad k \ge 1\,, \\ n_A &= n, \quad h = 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 25\,, \\ \overline{x}_l &= 1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 2.58. Potra and Rheinboldt boundary value problem [29].

$$\begin{aligned} f_k(x) &= 2x_k - x_{k-1} - x_{k+1} + h^2(x_k^2 + x_k + 0.1x_{k+n/2}^2 - 1.2)\ , \quad 1 \le k < n/2\,, \\ f_k(x) &= 2x_k - x_{k-1} + h^2(x_k^2 + x_k + 0.1x_{k+n/2}^2 - 1.2)\ , \quad k = n/2\,, \\ f_k(x) &= 2x_k - x_{k+1} + h^2(0.2x_{k-n/2}^2 + x_k^2 + 2x_k - 0.6), \quad k = n/2 + 1\,, \\ f_k(x) &= 2x_k - x_{k-1} - x_{k+1} + h^2(0.2x_{k-n/2}^2 + x_k^2 + 2x_k - 0.6), \quad n/2 + 1 < k \le n\,, \\ n_A &= n, \quad h = 1/(n/2 + 1), \quad x_0 = x_{n+1} = 0\,, \\ \overline{x}_l &= lh(1 - lh), \quad \overline{x}_{l+n/2} = \overline{x}_l, \quad 1 \le l \le n/2\,. \end{aligned}$$

Problem 2.59. Modified Bratu problem [13].

$$\begin{aligned} f_k(x) &= 4x_k - x_{k-1} - x_{k+1} - x_{k-\sqrt{n}} - x_{k+\sqrt{n}} + h^2\exp(x_k), \quad 1 \le k \le n\,, \\ n_A &= n, \quad h = 1/(\sqrt{n} + 1), \quad x_l = 0 \quad \text{for} \quad l < 1 \quad \text{or} \quad l > n, \\ \overline{x}_l &= 1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 2.60. Nonlinear Dirichlet problem [13].

$$\begin{aligned}
f_k(x) &= 4x_k - x_{k-1} - x_{k+1} - x_{k-\sqrt{n}} - x_{k+\sqrt{n}} + h^2 x_k^2 - y_k, \quad 1 \le k \le n\,, \\
n_A &= n, \quad h = 1/(\sqrt{n}+1), \quad x_l = 0 \quad \text{for} \quad l < 1 \quad \text{or} \quad l > n, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.61. Watson function [27].

$$\begin{aligned}
f_k(x) &= \sum_{i=2}^{n} (i-1) x_i t_k^{i-2} - \left(\sum_{i=1}^{n} x_i t_k^{i-1} \right)^2 - 1, \quad t_k = k/(n-1), \quad 1 \le k \le n-1, \\
f_n(x) &= x_1, \\
f_{n+1}(x) &= x_2 - x_1^2 - 1, \\
n_A &= n+1, \quad \overline{x}_l = 0, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.62. Extended Rosenbrock function [27].

$$\begin{aligned}
f_k(x) &= 10(x_k^2 - x_{k+1}) \quad , \quad \mathrm{mod}(k,2) = 1 \\
f_k(x) &= x_{k-1} - 1 \quad , \quad \mathrm{mod}(k,2) = 0 \\
n_A &= n, \\
\bar{x}_l &= -1.2, \quad \mathrm{mod}(l,2) = 1, \qquad \bar{x}_l = 1.0, \quad \mathrm{mod}(l,2) = 0
\end{aligned}$$

Problem 2.63. Extended Powell singular function [27].

$$\begin{aligned}
f_k(x) &= x_k + 10x_{k+1} \quad , \quad \mathrm{mod}(k,4) = 1, \\
f_k(x) &= \sqrt{5}\,(x_{k+1} - x_{k+2}) \quad , \quad \mathrm{mod}(k,4) = 2, \\
f_k(x) &= (x_{k-1} - 2x_k)^2 \quad , \quad \mathrm{mod}(k,4) = 3, \\
f_k(x) &= \sqrt{10}\,(x_{k-3} - x_k)^2 \quad , \quad \mathrm{mod}(k,4) = 0, \\
n_A &= n, \\
\bar{x}_l &= 3, \quad \mathrm{mod}(l,4) = 1, \qquad \bar{x}_l = -1, \quad \mathrm{mod}(l,4) = 2, \\
\bar{x}_l &= 0, \quad \mathrm{mod}(l,4) = 3, \qquad \bar{x}_l = 1, \quad \mathrm{mod}(l,4) = 0.
\end{aligned}$$

Problem 2.64. Penalty function 1 [27].

$$\begin{aligned}
f_k(x) &= \frac{1}{\sqrt{100000}}\,(x_k - 1), \quad 1 \le k \le n, \\
f_k(x) &= \sum_{i=1}^{n} x_i^2 - \frac{1}{4}, \quad k = n+1\,, \\
n_A &= n+1 \quad \overline{x}_l = l, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.65. Variably dimensioned function [27].

$$\begin{aligned}
f_k(x) &= x_k - 1\,, \quad 1 \le k \le n, \\
f_k(x) &= \sum_{i=1}^{n} i(x_i - 1), \quad k = n+1, \\
f_k(x) &= \left(\sum_{i=1}^{n} i(x_i - 1) \right)^2, \quad k = n+2, \\
n_A &= n+2, \quad \overline{x}_l = 1 - l/n, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.66. Brown almost linear function [27].

$$\begin{aligned}
f_k(x) &= x_k + \sum_{i=1}^{n} x_i - (n+1)\,, \quad 1 \le k \le n, \\
f_k(x) &= \left(\prod_{i=1}^{n} x_i \right) - 1, \quad k = n, \\
n_A &= n, \\
\overline{x}_l &= 1/2, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.67. Discrete integral equation function [27].

$$\begin{aligned}
f_k(x) &= x_k + \frac{h}{2} \left[(1-kh) \sum_{i=1}^{k} ih(x_i + ih + 1)^3 + kh \sum_{i=k+1}^{n} (1-ih)(x_i + ih + 1)^3 \right], \\
n_A &= n, \quad h = 1/(n+1)\,, \\
\overline{x}_l &= lh(lh-1), \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.68. Broyden tridiagonal function [27].

$$\begin{aligned}
f_k(x) &= (3 - 2x_k)x_k - x_{k-1} - 2x_{k+1} + 1, \\
n_A &= n, \quad x_0 = x_{n+1} = 0\,, \\
\overline{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}$$

Problem 2.69. [33], Problem 201 in [32].

$$\begin{aligned}
f_k(x) &= 1 - x_k \qquad\qquad\qquad , \quad k = 1\,, \\
f_k(x) &= 10(k-1)(x_k - x_{k-1})^2, \quad 1 < k \le n\,, \\
n_A &= n, \quad \overline{x}_l = -1.2, \quad 1 \le l < n, \quad \overline{x}_l = -1, \quad l = n\,.
\end{aligned}$$

Problem 2.70. [30], Problem 202 in [32].

$$\begin{aligned}
f_k(x) &= x_k - \frac{x_{k+1}^2}{10}, \quad 1 \le k < n\,, \\
f_k(x) &= x_k - \frac{x_1^2}{10}, \quad k = n\,, \\
n_A &= n, \quad \overline{x}_l = 2, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.71. [17], Problem 205 in [32].

$$\begin{aligned}
f_k(x) &= x_k - \frac{\sum_{i=1}^{n} x_i^3 + k}{2n}, \quad 1 \le k \le n\,, \\
n_A &= n, \quad \overline{x}_l = 3/2, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.72. [26], Problem 209 in [32].

$$\begin{aligned}
f_k(x) &= x_k^2 - 1 \qquad\qquad\quad , \quad k = 1\,, \\
f_k(x) &= x_{k-1}^2 + \log x_k - 1, \quad 1 < k \le n\,, \\
n_A &= n, \quad \overline{x}_l = 1/2, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.73. Trigonometric function [27].

$$\begin{aligned} f_k(x) &= n - \sum_{i=1}^{n} \cos(x_i) + k(1 - \cos(x_k)) - \sin(x_k) \\ n_A &= n, \quad \overline{x}_l = 1/n, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.74. Problem 221 in [32].

$$\begin{aligned} f_k(x) &= x_k - 1 + k\left(\sum_{i=1}^{n} i(x_i - 1)\right)\left(1 + 2\left(\sum_{i=1}^{n} i(x_i - 1)\right)^2\right) \\ n_A &= n, \quad \overline{x}_l = 1 - l/n, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.75. Gheri and Mancino problem [12].

$$\begin{aligned} f_k(x) &= 14nx_k + \left(k - \frac{n}{2}\right)^3 + \\ &\quad + \sum_{i=1, i\ne k}^{n} z_{ki}\left[\sin^5 \log(z_{ki}) + \cos^5 \log(z_{ki})\right], \quad 1 \le k \le n, \\ z_{ki} &= \sqrt{x_i^2 + k/i}, \quad 1 \le k \le n, \quad 1 \le i \le n, \\ \overline{x}_l &= -f_l(0)\frac{a+b}{2ab}, \quad 1 \le l \le n. \\ n_A &= n, \quad a = 14n - 6(n-1), \quad b = 14n + 6(n-1). \end{aligned}$$

Problem 2.76. Ortega and Rheinboldt problem [28].

$$\begin{aligned} f_k(x) &= \left(1 - \frac{1}{8n}\right)x_k - 1 - \sum_{i=1}^{n} a_{ki}x_kx_i, \quad 1 \le k \le n, \\ a_{ki} &= \frac{k}{2n}\,\frac{1}{2(k+i)}, \quad 1 \le i < n, \\ a_{ki} &= \frac{k}{2n}\,\frac{1}{4(k+i)}, \quad i = n, \\ n_A &= n, \quad \overline{x}_l = 1, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.77. Ascher and Russel boundary value problem [3].

$$\begin{aligned} f_k(x) &= x_k - \frac{kh}{2} - 2h^2\sum_{i=1}^{k} i(1-kh)\left(x_i^2 + \frac{x_{i+1} - x_{i-1}}{2h}\right) - \\ &\quad -2h^2\sum_{i=k+1}^{n} k(1-ih)\left(x_i^2 + \frac{x_{i+1} - x_{i-1}}{2h}\right), \quad 1 \le k \le n, \\ h &= 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1/2, \\ n_A &= n, \quad \overline{x}_l = 1, \quad 1 \le l \le n. \end{aligned}$$

Problem 2.78.

$$\begin{aligned} f_k(x) &= x_k - 1 - \frac{1}{5n}x_k\left(1 + \frac{k}{k+n}x_n + 2\sum_{i=1}^{n-1}\frac{k}{k+i}x_i\right), \\ n_A &= n, \\ \overline{x}_l &= 1, \quad \mathrm{mod}(l,2) = 1, \quad \overline{x}_l = 3, \quad \mathrm{mod}(l,2) = 0. \end{aligned}$$

Problem 2.79. Problem NONDQUAR in [8].

$$\begin{aligned}
f_k(x) &= x_k - 1, && k = 1 \\
f_k(x) &= (x_k + x_{k+1} + x_n)^2, && 1 < k < n, \\
f_k(x) &= x_{k-1} - x_k, && k = n \\
n_A &= n, \\
\bar{x}_l &= 1, \quad \mathrm{mod}(l,2) = 1, \quad \bar{x}_l = -1, \quad \mathrm{mod}(l,2) = 0.
\end{aligned}$$

Problem 2.80.

$$\begin{aligned}
f_k(x) &= \sum_{i=2}^{n} (i-1) \left(\frac{k}{n-2} \right)^{i-2} x_i - \left(x_1 + \sum_{i=2}^{n} \left(\frac{k}{n-2} \right)^{i-1} x_i \right)^2 - 1, \quad 1 \le k \le n-2, \\
f_{n-1}(x) &= x_1, \\
f_n(x) &= x_2 - x_1^2 - 1, \\
n_A &= n, \quad \bar{x}_l = 0, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.81. Problem SINQUAD in [8].

$$\begin{aligned}
f_k(x) &= (x_k - 1)^2, && k = 1 \\
f_k(x) &= \sin(x_k - x_n) + x_k^2 - x_1^2, && 1 < k < n, \\
f_k(x) &= x_k^2 - x_1^2, && k = n \\
n_A &= n, \quad \bar{x}_l = 1/10, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.82. [19], Problem EDENSCH in [8].

$$\begin{aligned}
f_k(x) &= (x_i - 2)^2, && \mathrm{mod}(k,3) = 1, \\
f_k(x) &= x_i x_{i+1} - 2x_{i+1}, && \mathrm{mod}(k,3) = 2, \\
f_k(x) &= x_{i+1} + 1, && \mathrm{mod}(k,3) = 0, \\
n_A &= 3(n-1), \quad i = \mathrm{div}(k-1,3) + 1 \\
\bar{x}_l &= 0, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.83. Problem GENHUMPS in [8].

$$\begin{aligned}
f_k(x) &= x_i/\sqrt{20}, && \mathrm{mod}(k,3) = 1, \\
f_k(x) &= \sin(20x_i)\sin(20x_{i+1}), && \mathrm{mod}(k,3) = 2, \\
`f_k(x) &= x_{i+1}/\sqrt{20}, && \mathrm{mod}(k,3) = 0, \\
n_A &= 3(n-1), \quad i = \mathrm{div}(k-1,3) + 1 \\
\bar{x}_1 &= -506.2, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 2.84. Problem ERRINROS (modified) in [8].

$$\begin{aligned}
f_k(x) &= x_i - 16x_{i+1}^2(1.5 + \sin(i+1))^2, && \mathrm{mod}(k,2) = 1, \\
f_k(x) &= x_i - 1, && \mathrm{mod}(k,2) = 0, \\
n_A &= 2(n-1), \quad i = \mathrm{div}(k-1,2) + 1 \\
\bar{x}_l &= -1, \quad 1 \le l \le n\,.
\end{aligned}$$

3 Test problems for dense systems of nonlinear equations

Calling statements have the for

```
CALL TIUD37(N,NA,X,FMIN,XMAX,NEXT,IERR)
CALL TAFU37(N,KA,X,FA,NEXT)
CALL TAGU37(N,KA,X,GA,NEXT)
```

with the following significance

`TIUD37` - initiation of vector of variables `N`, `X` and the number of equations `NA`
`TAFU37` - evaluation of the `KA`-th partial function value `FA` at point `X`
`TAGU37` - evaluation of the `KA`-th partial function gradient `GA` at point `X`

We seek a solution to the nonlinear system

$$f_k(x) = 0, \quad 1 \le k \le n$$

from the starting point $\bar{x}$. For positive integers k and l, we use the notation $\mathrm{div}(k,l)$ for integer division, i.e., maximum integer not greater than k/l, and $\mathrm{mod}(k,l)$ for the remainder after integer division, i.e., $\mathrm{mod}(k,l) = l(k/l - \mathrm{div}(k,l))$. The description of individual problems follows.

Problem 3.1. Countercurrent reactors problem 1 [7] (modified).

$$\begin{aligned}
f_k(x) &= \alpha - (1-\alpha)x_{k+2} - x_k(1+4x_{k+1}) &&, \ k=1,\\
f_k(x) &= -(2-\alpha)x_{k+2} - x_k(1+4x_{k-1}) &&, \ k=2,\\
f_k(x) &= \alpha x_{k-2} - (1-\alpha)x_{k+2} - x_k(1+4x_{k+1}) &&, \ \mathrm{mod}\ (k,2)=1\ ,\ \ 2<k<n-1,\\
f_k(x) &= \alpha x_{k-2} - (2-\alpha)x_{k+2} - x_k(1+4x_{k-1}) &&, \ \mathrm{mod}\ (k,2)=0\ ,\ \ 2<k<n-1,\\
f_k(x) &= \alpha x_{k-2} - x_k(1+4x_{k+1}) &&, \ k=n-1,\\
f_k(x) &= \alpha x_{k-2} - (2-\alpha) - x_k(1+4x_{k-1}) &&, \ k=n,\\
\alpha &= 1/2,\\
\overline{x}_i &= 0.1 \quad , \ \mathrm{mod}\ (i,8)=1,\\
\overline{x}_i &= 0.2 \quad , \ \mathrm{mod}\ (i,8)=2 \ \text{or} \ \mathrm{mod}\ (i,8)=0,\\
\overline{x}_i &= 0.3 \quad , \ \mathrm{mod}\ (i,8)=3 \ \text{or} \ \mathrm{mod}\ (i,8)=7,\\
\overline{x}_i &= 0.4 \quad , \ \mathrm{mod}\ (i,8)=4 \ \text{or} \ \mathrm{mod}\ (i,8)=6,\\
\overline{x}_i &= 0.5 \quad , \ \mathrm{mod}\ (i,8)=5.
\end{aligned}$$

Problem 3.2. Countercurrent reactors problem 2 [7] (modified).

$$\begin{aligned}
f_k(x) &= x_1 - (1-x_1)x_{k+2} - \alpha(1+4x_{k+1}) &&, \ k=1,\\
f_k(x) &= -(1-x_1)x_{k+2} - \alpha(1+4x_k) &&, \ k=2,\\
f_k(x) &= \alpha x_1 - (1-x_1)x_{k+2} - x_k(1+4x_{k-1}) &&, \ k=3,\\
f_k(x) &= x_1 x_{k-2} + (1-x_1)x_{k+2} - x_k(1+4x_{k-1}) &&, \ 3<k<n-1,
\end{aligned}$$

$$
\begin{aligned}
f_k(x) &= x_1x_{k-2} + x_k(1+4x_{k-1}) && , \ k=n-1, \\
f_k(x) &= x_1x_{k-2} - (1-x_1) - x_k(1+4x_{k-1}) && , \ k=n, \\
\alpha &= 0.414214,
\end{aligned}
$$

$$
\begin{aligned}
\overline{x}_i &= 0.1 \ , \ \text{mod}\,(i,8)=1, & \overline{x}_i &= 0.2 \ , \ \text{mod}\,(i,8)=2, \\
\overline{x}_i &= 0.3 \ , \ \text{mod}\,(i,8)=3, & \overline{x}_i &= 0.4 \ , \ \text{mod}\,(i,8)=4, \\
\overline{x}_i &= 0.5 \ , \ \text{mod}\,(i,8)=5, & \overline{x}_i &= 0.4 \ , \ \text{mod}\,(i,8)=6, \\
\overline{x}_i &= 0.3 \ , \ \text{mod}\,(i,8)=7, & \overline{x}_i &= 0.2 \ , \ \text{mod}\,(i,8)=0.
\end{aligned}
$$

Problem 3.3. Trigonometric system [35].

$$
\begin{aligned}
f_k(x) &= 5 - (l+1)(1-\cos x_k) - \sin x_k - \sum_{j=5l+1}^{5l+5} \cos x_j, \\
l &= \text{div}(k-1,5), \\
\overline{x_i} &= 1/n, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 3.4. Trigonometric - exponential system (trigexp 1) [35].

$$
\begin{aligned}
f_k(x) &= 3x_k^3 + 2x_{k+1} - 5 + \sin(x_k - x_{k+1})\sin(x_k + x_{k+1}) \ , && k=1, \\
f_k(x) &= 3x_k^3 + 2x_{k+1} - 5 + \sin(x_k - x_{k+1})\sin(x_k + x_{k+1}) \\
&+ \ 4x_k - x_{k-1}\exp(x_{k-1} - x_k) - 3 && , \quad 1<k<n, \\
f_k(x) &= 4x_k - x_{k-1}\exp(x_{k-1} - x_k) - 3 && , \quad k=n, \\
\overline{x}_i &= 0, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 3.5. Trigonometric - exponential system (trigexp 2) [35].

$$
\begin{aligned}
f_k(x) &= 3(x_k - x_{k+2})^3 - 5 + 2x_{k+1}, \\
&+ \ \sin(x_k - x_{k+1} - x_{k+2})\sin(x_k + x_{k+1} - x_{k+2}) && , \ \text{mod}\,(k,2)=1, \ k=1, \\
f_k(x) &= -6(x_{k-2} - x_k)^3 + 10 - 4x_{k-1} \\
&- \ 2\sin(x_{k-2} - x_{k-1} - x_k)\sin(x_{k-2} + x_{k-1} - x_k) \\
&+ \ 3(x_k - x_{k+2})^3 - 5 + 2x_{k+1} \\
&+ \ \sin(x_k - x_{k+1} - x_{k+2})\sin(x_k + x_{k+1} - x_{k+2}) && , \ \text{mod}\,(k,2)=1, \ 1<k<n, \\
f_k(x) &= -6(x_{k-2} - x_k)^3 + 10 - 4x_{k-1} \\
&- \ 2\sin(x_{k-2} - x_{k-1} - x_k)\sin(x_{k-2} + x_{k-1} - x_k) && , \ \text{mod}\,(k,2)=1, \ k=n, \\
f_k(x) &= 4x_k - (x_{k-1} - x_{k+1})\exp(x_{k-1} - x_k - x_{k+1}) - 3 && , \ \text{mod}\,(k,2)=0, \\
\overline{x}_i &= 1, \quad 1 \le i \le n.
\end{aligned}
$$

Problem 3.6. Singular Broyden problem [14].

$$
\begin{aligned}
f_k(x) &= ((3-2x_k)x_k - 2x_{k+1} + 1)^2 && , \ k=1, \\
f_k(x) &= ((3-2x_k)x_k - x_{k-1} - 2x_{k+1} + 1)^2 && , \ 1<k<n, \\
f_k(x) &= ((3-2x_k)x_k - x_{k-1} + 1)^2 && , \ k=n, \\
\overline{x}_i &= -1, \quad 1 \le i \le n
\end{aligned}
$$

Problem 3.7. Tridiagonal system [20].

$$\begin{array}{rcll} f_k(x) &=& 4(x_k - x_{k+1}^2) & ,\ k=1, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + 4(x_k - x_{k+1}^2) & ,\ 1<k<n, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & ,\ k=n, \\ \overline{x}_i &=& 12, \quad 1 \le i \le n. & \end{array}$$

Problem 3.8. Five-diagonal system [20].

$$\begin{array}{rcll} f_k(x) &=& 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 & ,\ k=1, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 & ,\ k=2, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 & ,\ 2<k<n-1, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} & ,\ k=n-1, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + x_{k-1}^2 - x_{k-2} & ,\ k=n, \\ \overline{x}_i &=& -2, \quad 1 \le i \le n. & \end{array}$$

Problem 3.9. Seven-diagonal system [20].

$$\begin{array}{rcll} f_k(x) &=& 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 + x_{k+2} - x_{k+3}^2 & ,\ k=1, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 + x_{k+1} - x_{k+2}^2 + x_{k+2} - x_{k+3}^2 & ,\ k=2, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 & \\ &+& x_{k-2}^2 + x_{k+2} - x_{k+3}^2 & ,\ k=3, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 & \\ &+& x_{k-2}^2 + x_{k+2} - x_{k-3} - x_{k+3}^2 & ,\ 3<k<n-2, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 & \\ &+& x_{k-2}^2 + x_{k+2} - x_{k-3} & ,\ k=n-2, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & \\ &+& 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} & \\ &+& x_{k-2}^2 - x_{k-3} & ,\ k=n-1, \\ f_k(x) &=& 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + x_{k-1}^2 - x_{k-2} & \\ &+& x_{k-2}^2 - x_{k-3} & ,\ k=n, \\ \overline{x}_i &=& -3, \quad 1 \le i \le n. & \end{array}$$

Problem 3.10. Structured Jacobian problem [14].

$$f_k(x) = -2x_k^2 + 3x_k - 2x_{k+1} + 3x_{n-4} - x_{n-3}$$

$$
\begin{aligned}
& - \quad x_{n-2} + 0.5x_{n-1} - x_n + 1 \qquad , \; k = 1, \\
f_k(x) &= -2x_k^2 + 3x_k - x_{k-1} - 2x_{k+1} + 3x_{n-4} - x_{n-3} \\
& - \quad x_{n-2} + 0.5x_{n-1} - x_n + 1 \qquad , \; 1 < k < n, \\
f_k(x) &= -2x_k^2 + 3x_k - x_{k-1} + 3x_{n-4} - x_{n-3} \\
& - \quad x_{n-2} + 0.5x_{n-1} - x_n + 1 \qquad , \; k = n, \\
\overline{x}_i &= -1, \quad i \geq 1.
\end{aligned}
$$

Problem 3.11. Extended Freudenstein and Roth function [6].

$$
\begin{aligned}
f_k &= x_k + ((5 - x_{k+1})x_{k+1} - 2)x_{k+1} - 13 \; , \quad \text{mod}(k, 2) = 1, \\
f_k &= x_{k-1} + ((x_k + 1)x_k - 14)x_k - 29 \quad , \quad \text{mod}(k, 2) = 0, \\
\overline{x}_i &= 90 \; , \quad \text{mod}(i, 2) = 1, \qquad \overline{x}_i = 60 \; , \quad \text{mod}(i, 2) = 0.
\end{aligned}
$$

Problem 3.12. Extended Powell singular problem [27].

$$
\begin{aligned}
f_k(x) &= x_k + 10x_{k+1} \qquad , \quad \text{mod}(k, 4) = 1, \\
f_k(x) &= \sqrt{5}(x_{k+1} - x_{k+2}) \quad , \quad \text{mod}(k, 4) = 2, \\
f_k(x) &= (x_{k-1} - 2x_k)^2 \qquad , \quad \text{mod}(k, 4) = 3, \\
f_k(x) &= \sqrt{10}(x_{k-3} - x_k)^2 \quad , \quad \text{mod}(k, 4) = 0, \\
\overline{x}_i &= 3 \; , \quad \text{mod}(i, 4) = 1, \qquad \overline{x}_i = -1 \; , \quad \text{mod}(i, 4) = 2, \\
\overline{x}_i &= 0 \; , \quad \text{mod}(i, 4) = 3, \qquad \overline{x}_i = 1 \; , \quad \text{mod}(i, 4) = 0.
\end{aligned}
$$

Problem 3.13. Extended Cragg and Levy problem [27].

$$
\begin{aligned}
f_k(x) &= (\exp(x_k) - x_{k+1})^2 \quad , \quad \text{mod}(k, 4) = 1, \\
f_k(x) &= 10(x_k - x_{k+1})^3 \qquad , \quad \text{mod}(k, 4) = 2, \\
f_k(x) &= \tan^2(x_k - x_{k+1}) \qquad , \quad \text{mod}(k, 4) = 3, \\
f_k(x) &= x_k - 1 \qquad\qquad , \quad \text{mod}(k, 4) = 0, \\
\overline{x}_i &= 1 \; , \quad \text{mod}(i, 4) = 1, \qquad \overline{x}_i = 2 \; , \quad \text{mod}(i, 4) \neq 1.
\end{aligned}
$$

Problem 3.14. Broyden tridiagonal problem [27].

$$
\begin{aligned}
f_k(x) &= x_k(0.5x_k - 3) + 2x_{k+1} - 1 \qquad , \; k = 1, \\
f_k(x) &= x_k(0.5x_k - 3) + x_{k-1} + 2x_{k+1} - 1 \; , \; 1 < k < n, \\
f_k(x) &= x_k(0.5x_k - 3) - 1 + x_{k-1} \qquad , \; k = n, \\
\overline{x}_i &= -1, \quad 1 \leq i \leq n.
\end{aligned}
$$

Problem 3.15. Generalized Broyden banded problem [27].

$$
\begin{aligned}
f_k(x) &= (2 + 5x_k^2)x_k + 1 + \sum_{i=k_1}^{k_2} x_i(1 + x_i), \\
k_1 &= \max\,(1, k - 5), \quad k_2 = \min\,(n, k + 1), \\
\overline{x}_i &= -1, \quad 1 \leq i \leq n.
\end{aligned}
$$

Problem 3.16. Extended Powell badly scaled function [27].

$$\begin{aligned}
f_k(x) &= 10000\, x_k\, x_{k+1} - 1 && , \quad \text{mod}(k,2) = 1,\\
f_k(x) &= \exp(-x_{k-1}) + \exp(-x_k) - 1.0001 && , \quad \text{mod}(k,2) = 2,\\
\overline{x}_i &= 0 \;, \quad \text{mod}(i,2) = 1, \qquad \overline{x}_i = 1 && , \quad \text{mod}(i,2) = 0.
\end{aligned}$$

Problem 3.17. Extended Wood problem [15].

$$\begin{aligned}
f_k(x) &= -200x_k(x_{k+1} - x_k^2) - (1 - x_k) && , \quad \text{mod}(k,4) = 1,\\
f_k(x) &= 200(x_k - x_{k-1}^2) + 20(x_k - 1) + 19.8(x_{k+2} - 1) && , \quad \text{mod}(k,4) = 2,\\
f_k(x) &= -180x_k(x_{k+1} - x_k^2) - (1 - x_k) && , \quad \text{mod}(k,4) = 3,\\
f_k(x) &= 180(x_k - x_{k-1}^2) + 20.2(x_k - 1) + 19.8(x_{k-2} - 1) && , \quad \text{mod}(k,4) = 4,\\
\overline{x}_i &= -3, \quad \text{mod}(i,2) = 1, \quad \overline{x}_i = -1, \quad \text{mod}(i,2) = 0.
\end{aligned}$$

Problem 3.18. Tridiagonal exponential problem [6].

$$\begin{aligned}
f_k(x) &= x_k - \exp(\cos(k(x_k + x_{k+1}))) && , \; k = 1,\\
f_k(x) &= x_k - \exp(\cos(k(x_{k-1} + x_k + x_{k+1}))) && , \; 1 < k < n,\\
f_k(x) &= x_k - \exp(\cos(k(x_{k-1} + x_k))) && , \; k = n,\\
h &= 1/(n+1),\\
\overline{x}_i &= 1.5, \quad 1 \le i \le n.
\end{aligned}$$

Problem 3.19. Discrete boundary value problem [27].

$$\begin{aligned}
f_k(x) &= 2x_k + 0.5h^2(x_k + hk)^3 - x_{k+1} && , \; k = 1,\\
f_k(x) &= 2x_k + 0.5h^2(x_k + hk)^3 - x_{k-1} - x_{k+1} && , \; 1 < k < n,\\
f_k(x) &= 2x_k + 0.5h^2(x_k + hk)^3 - x_{k-1} && , \; k = n,\\
h &= 1/(n+1),\\
\overline{x}_i &= ih(ih - 1), \quad 1 \le i \le n.
\end{aligned}$$

Problem 3.20. Brent problem [5].

$$\begin{aligned}
f_k(x) &= 3x_k(x_{k+1} - 2x_k) + x_{k+1}^2/4 && , \; k = 1,\\
f_k(x) &= 3x_k(x_{k+1} - 2x_k + x_{k-1}) + (x_{k+1} - x_{k-1})^2/4 && , \; 1 < k < n,\\
f_k(x) &= 3x_k(20 - 2x_k + x_{k-1}) + (20 - x_{k-1})^2/4 && , \; k = n,\\
\overline{x}_i &= 10, \quad 1 \le i \le n.
\end{aligned}$$

Problem 3.21. Troesch problem [31].

$$\begin{aligned}
f_k(x) &= 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k+1} && , \; k = 1,\\
f_k(x) &= 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k-1} - x_{k+1} && , \; 1 < k < n,\\
f_k(x) &= 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k-1} - 1 && , \; k = n,\\
\rho &= 10, \quad h = 1/(n+1),\\
\overline{x}_i &= 1, \quad 1 \le i \le n.
\end{aligned}$$

Problem 3.22. Flow in a channel [4].

This is a finite difference analogue of the following nonlinear ordinary differential equation

$$u'''' = R(u'u'' - uu'''), \quad R = 500$$

over unit interval Ω with boundary conditions $u(0) = 0$, $u'(0) = 0$, $u(1) = 1$, $u'(1) = 0$. We use standard 5-point finite differences on a uniform grid having 5000 internal nodes. The initial approximate solution is a discretization of $u_0(x) = (x - 1/2)^2$.

Problem 3.23. Swirling flow [4].

This is a finite difference analogue of the following system of two nonlinear ordinary differential equations

$$\begin{aligned} u'''' + R(uu''' + vv') &= 0 \\ v'' + R(uv' + u'v) &= 0, \quad R = 500 \end{aligned}$$

over unit interval Ω with boundary conditions $u(0) = u'(0) = u(1) = u'(1) = 0$, $v(0) = -1$, $v(1) = 1$. We use standard 5-point finite differences on an uniform grid having 2500 internal nodes. The initial approximate solution is a discretization of $u_0(x) = (x - 1/2)^2$ and $v_0(x) = x - 1/2$.

Problem 3.24. Bratu problem [16].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u + R\exp(u) = 0, \quad R = 6.8$$

over unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 3.25. Poisson problem 1 [14].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u = \frac{u^3}{1 + x^2 + y^2}$$

over unit square Ω with Dirichlet boundary conditions $u(0, y) = 1$, $u(1, y) = 2 - \exp(y)$, $u(x, 0) = 1$, $u(x, 1) = 2 - \exp(x)$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = -1$.

Problem 3.26. Poisson problem 2 [25].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u + \sin(2\pi u) + \sin\left(2\pi\frac{\partial u}{\partial x}\right) + \sin\left(2\pi\frac{\partial u}{\partial y}\right) + f(x, y) = 0,$$

where $f(x, y) = 1000((x - 1/4)^2 + (y - 3/4)^2)$, over unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 3.27. Porous medium problem [11].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u^2 + R\left(\frac{\partial u^3}{\partial x} + f(x, y)\right) = 0, \quad R = 50,$$

where $f(1/71, 1/71) = 1$ and $f(x, y) = 0$ for $(x, y) \neq (1/71, 1/71)$, over unit square Ω with Dirichlet boundary conditions $u(0, y) = 1$, $u(1, y) = 0$, $u(x, 0) = 1$, $u(x, 1) = 0$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 1 - xy$.

Problem 3.28. Convection-difussion problem [18].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u - Ru\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) + f(x, y) = 0, \quad R = 20,$$

where $f(x, y) = 2000x(1 - x)y(1 - y)$, over unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 3.29. Nonlinear biharmonic problem [24].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta\Delta u + R\left(\max(0, u) + \text{sign}(x - \frac{1}{2})\right) = 0, \quad R = 500$$

over unit square Ω with the boundary conditions $u = 0$ on $\partial\Omega$ and $\partial u(0, y)/\partial x = 0$, $\partial u(1, y)/\partial x = 0$, $\partial u(x, 0)/\partial y = 0$, $\partial u(x, 1)/\partial y = 0$. We use standard 13-point finite differences on a shifted uniform grid having 50×50 internal nodes [16]. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 3.30. Driven cavity problem [16].

This is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta\Delta u + R\left(\frac{\partial u}{\partial y}\frac{\partial \Delta u}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial \Delta u}{\partial y}\right) = 0, \quad R = 500$$

over unit square Ω with the boundary conditions $u = 0$ on $\partial\Omega$ and $\partial u(0, y)/\partial x = 0$, $\partial u(1, y)/\partial x = 0$, $\partial u(x, 0)/\partial y = 0$, $\partial u(x, 1)/\partial y = 1$. We use standard 13-point finite differences on a shifted uniform grid having 50×50 internal nodes [16]. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 3.31.

$$\begin{aligned} f_k(x) &= 2x_k - x_{k+1} - x_{k-1} \\ &\quad + h^2\left(x_k^3 + 2.10^{-4}(2.10^{-4}a_2 - 1)x_k - 10^9\exp(-3.10^4 a_2)\right), \\ h &= 1/(n+1), \quad a_1 = h\,k, \quad a_2 = (a_1 - 1/2)^2, \\ x_0 &= x_{n+1} = 0, \quad \overline{x}_l = 5\min(lh, 1-lh), \quad 1 \le l \le n. \end{aligned}$$

Problem 3.32.

$$\begin{aligned} f_k(x) &= 2x_k - x_{k+1} - x_{k-1} \\ &\quad + h^2\left(x_k^3\exp(x_k) + 5.10^8\exp(-10^4 a_2)\sqrt{|a_1 - 1/2|}(x_{k+1} - x_{k-1}) + a_3\right), \\ h &= 1/(n+1), \quad a_1 = h\,k, \quad a_2 = (a_1 - 1/2)^2, \quad a_3 = 10^6\text{sign}(a_1 - 1/2), \\ x_0 &= x_{n+1} = 0, \quad \overline{x}_l = 5\min(lh, 1-lh), \quad 1 \le l \le n. \end{aligned}$$

Problem 3.33. [33], Problem 201 in [32].

$$\begin{aligned} f_k(x) &= 1 - x_k \qquad\qquad , \quad k = 1, \\ f_k(x) &= 10(k-1)(x_k - x_{k-1})^2, \quad 1 < k \le n, \\ \overline{x}_l &= -1.2, \quad 1 \le l < n, \quad \overline{x}_l = -1, \quad l = n. \end{aligned}$$

Problem 3.34. [30], Problem 202 in [32].

$$\begin{aligned} f_k(x) &= x_k - \frac{x_{k+1}^2}{10}, \quad 1 \le k < n, \\ f_k(x) &= x_k - \frac{x_1^2}{10}, \quad k = n, \\ \overline{x}_l &= 2, \quad 1 \le l \le n. \end{aligned}$$

Problem 3.35. Brown almost linear function [27].

$$\begin{aligned} f_k(x) &= x_k + \sum_{i=1}^n x_i - (n+1), \quad 1 \le k < n, \\ f_k(x) &= \left(\prod_{i=1}^n x_i\right) - 1, \quad k = n, \\ \overline{x}_l &= 1/2, \quad 1 \le l \le n. \end{aligned}$$

Problem 3.36. Extended Rosenbrock function [27].

$$\begin{aligned} f_k(x) &= 1 - x_{k-1} \qquad , \quad \text{mod}(k,2) = 1, \\ f_k(x) &= 10(x_k - x_{k-1}^2) \quad , \quad \text{mod}(k,2) = 0, \\ \bar{x}_l &= -1.2, \quad \text{mod}(l,2) = 1, \qquad \bar{x}_l = 1.0, \quad \text{mod}(l,2) = 0. \end{aligned}$$

Problem 3.37. [17], Problem 205 in [32].

$$\begin{aligned} f_k(x) &= x_k - \frac{\sum_{i=1}^n x_i^3 + k}{2n}, \quad 1 \le k \le n, \\ \overline{x}_l &= 3/2, \quad 1 \le l \le n. \end{aligned}$$

Problem 3.38. [10], Problem 206 in [32].

$$\begin{aligned} f_k(x) &= x_{k-1} - 2x_k + x_{k+1} - h^2 \exp(x_k), \quad 1 \le k \le n\,, \\ h &= 1/(n+1), \quad x_0 = x_{n+1} = 0\,, \\ \overline{x}_l &= 0, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.39. [9], Problem 207 in [32].

$$\begin{aligned} f_k(x) &= (3 - x_k/10)x_k + 1 - x_{k-1} - 2x_{k+1}, \quad 1 \le k \le n\,, \\ x_0 &= x_{n+1} = 0\,, \\ \overline{x}_l &= -1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.40. [9], Problem 208 in [32].

$$\begin{aligned} f_k(x) &= (1 + x_k^2)x_k + 1 - \sum_{i \in I_k} (x_i + x_i^2), \quad 1 \le k \le n\,, \\ I_k &= \{i : i \ne k, \max(1, k-3) \le i \le \min(n, k+3)\}\,, \\ \overline{x}_l &= -1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.41. [26], Problem 209 in [32].

$$\begin{aligned} f_k(x) &= x_k^2 - 1 \qquad\qquad\qquad, \quad k = 1\,, \\ f_k(x) &= x_{k-1}^2 + \log x_k - 1, \quad 1 < k \le n\,, \\ \overline{x}_l &= 1/2, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.42. [26], Problem 212 in [32].

$$\begin{aligned} f_k(x) &= x_k \qquad\qquad\qquad\qquad, \quad k = 1\,, \\ f_k(x) &= \cos(x_{k-1}) + x_k - 1\,, \quad 1 < k \le n\,, \\ \overline{x}_l &= 1/2, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.43. [1], Problem 213 in [32].

$$\begin{aligned} f_k(x) &= 2x_k + h^2(x_k + \sin(x_k)) - x_{k-1} - x_{k+1}, \quad 1 \le k \le n\,, \\ h &= 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1\,, \\ \overline{x}_l &= 1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.44. Broyden banded function [27].

$$\begin{aligned} f_k(x) &= x_k(2 + 5x_k^2) + 1 - \sum_{i \in I_k} x_i(1 + x_i), \quad 1 \le k \le n\,, \\ I_k &= \{i : i \ne k, \max(1, k-5) \le i \le \min(n, k+1)\}\,, \\ \overline{x}_l &= -1, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.45. Discrete integral equation function [27].

$$\begin{aligned} f_k(x) &= x_k + \frac{h}{2}\left[(1 - kh)\sum_{i=1}^{k} ih(x_i + ih + 1)^3 + kh \sum_{i=k+1}^{n} (1 - ih)(x_i + ih + 1)^3\right], \\ h &= 1/(n+1)\,, \\ \overline{x}_l &= lh(lh - 1), \quad 1 \le l \le n. \end{aligned}$$

Problem 3.46. Trigonometric function [27].

$$
\begin{aligned}
f_k(x) &= n - \sum_{i=1}^{n} \cos(x_i) + k(1 - \cos(x_k)) - \sin(x_k), \\
\overline{x}_l &= 1/n, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 3.47. Problem 221 in [32].

$$
\begin{aligned}
f_k(x) &= x_k - 1 + k\left(\sum_{i=1}^{n} i(x_i - 1)\right)\left(1 + 2\left(\sum_{i=1}^{n} i(x_i - 1)\right)^2\right), \\
\overline{x}_l &= 1 - l/n, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 3.48. Broyden tridiagonal function [27].

$$
\begin{aligned}
f_k(x) &= (3 - 2x_k)x_k - x_{k-1} - 2x_{k+1} + 1, \\
x_0 &= x_{n+1} = 0\,, \\
\overline{x}_l &= -1, \quad 1 \le l \le n.
\end{aligned}
$$

Problem 3.49. Gheri and Mancino problem [12].

$$
\begin{aligned}
f_k(x) &= 14nx_k + \left(k - \frac{n}{2}\right)^3 + \\
&\quad + \sum_{i=1, i \ne k}^{n} z_{ki}\left[\sin^5(\log(z_{ki})) + \cos^5(\log(z_{ki}))\right], \quad 1 \le k \le n\,, \\
z_{ki} &= \sqrt{x_i^2 + k/i}, \quad 1 \le k \le n, \quad 1 \le i \le n\,, \\
\overline{x}_l &= -f_l(0)\frac{a + b}{2ab}, \quad 1 \le l \le n\,. \\
a &= 14n - 6(n - 1), \quad b = 14n + 6(n - 1)\,.
\end{aligned}
$$

Problem 3.50. Ortega and Rheinboldt problem [28].

$$
\begin{aligned}
f_k(x) &= \left(1 - \frac{1}{8n}\right)x_k - 1 - \sum_{i=1}^{n} a_{ki}x_k x_i, \quad 1 \le k \le n\,, \\
a_{ki} &= \frac{k}{2n}\,\frac{1}{2(k + i)}, \quad 1 \le i < n\,, \\
a_{ki} &= \frac{k}{2n}\,\frac{1}{4(k + i)}, \quad i = n\,, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}
$$

Problem 3.51. Troesch problem [31] (modified).

$$
\begin{aligned}
f_k(x) &= 2x_k + 3h\,\frac{\exp(3x_k) - \exp(-3x_k)}{2} - x_{k-1} - x_{k+1}, \quad 1 \le k \le n\,, \\
h &= 1/(n + 1), \quad x_0 = 0, \quad x_{n+1} = 1\,, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}
$$

Problem 3.52. Ascher and Russel boundary value problem [3].

$$\begin{aligned}
f_k(x) &= 2x_k - 2h^2\left(x_k^2 + \frac{x_{k+1}-x_{k-1}}{2h}\right) - x_{k-1} - x_{k+1}, \quad 1 \le k \le n\,, \\
h &= 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1/2\,, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 3.53. Ascher and Russel boundary value problem [3].

$$\begin{aligned}
f_k(x) &= x_k - \frac{kh}{2} - 2h^2 \sum_{i=1}^{k} i(1-kh)\left(x_i^2 + \frac{x_{i+1}-x_{i-1}}{2h}\right) - \\
&\quad -2h^2 \sum_{i=k+1}^{n} k(1-ih)\left(x_i^2 + \frac{x_{i+1}-x_{i-1}}{2h}\right), \quad 1 \le k \le n\,, \\
h &= 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1/2\,, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 3.54. Allgower and Georg boundary value problem [2] (modified).

$$\begin{aligned}
f_k(x) &= 2x_k + 0.3h^2\left[\exp(20(x_k + 25(kh-1))) - \exp(-20(x_k + 25kh)) - t_k\right] \\
&\quad -x_{k-1} - x_{k+1}\,, \\
t_k &= \operatorname{sign}(kh - 0.5), \quad k \ge 1\,, \\
h &= 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 25\,, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 3.55. Potra and Rheinboldt boundary value problem [29].

$$\begin{aligned}
f_k(x) &= 2x_k - x_{k-1} - x_{k+1} + h^2(x_k^2 + x_k + 0.1x_{k+n/2}^2 - 1.2)\ , \quad 1 \le k < n/2\,, \\
f_k(x) &= 2x_k - x_{k-1} + h^2(x_k^2 + x_k + 0.1x_{k+n/2}^2 - 1.2)\ , \quad k = n/2\,, \\
f_k(x) &= 2x_k - x_{k+1} + h^2(0.2x_{k-n/2}^2 + x_k^2 + 2x_k - 0.6), \quad k = n/2+1\,, \\
f_k(x) &= 2x_k - x_{k-1} - x_{k+1} + h^2(0.2x_{k-n/2}^2 + x_k^2 + 2x_k - 0.6), \quad n/2+1 < k \le n\,, \\
h &= 1/(n/2+1), \quad x_0 = x_{n+1} = 0\,, \\
\overline{x}_l &= lh(1-lh), \quad \overline{x}_{l+n/2} = \overline{x}_l, \quad 1 \le l \le n/2\,.
\end{aligned}$$

Problem 3.56. Modified Bratu problem [13].

$$\begin{aligned}
f_k(x) &= 4x_k - x_{k-1} - x_{k+1} - x_{k-\sqrt{n}} - x_{k+\sqrt{n}} + h^2\exp(x_k), \quad 1 \le k \le n\,, \\
h &= 1/(\sqrt{n}+1), \quad x_l = 0 \quad \text{for} \quad l < 1 \quad \text{or} \quad l > n, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 3.57. Nonlinear Dirichlet problem [13].

$$\begin{aligned}
f_k(x) &= 4x_k - x_{k-1} - x_{k+1} - x_{k-\sqrt{n}} - x_{k+\sqrt{n}} + h^2x_k^2 - y_k, \quad 1 \le k \le n\,, \\
h &= 1/(\sqrt{n}+1), \quad x_l = 0 \quad \text{for} \quad l < 1 \quad \text{or} \quad l > n, \\
\overline{x}_l &= 1, \quad 1 \le l \le n\,.
\end{aligned}$$

Problem 3.58.

$$\begin{aligned} f_k(x) &= x_k - 1 - \frac{1}{5n} x_k \left(1 + \frac{k}{k+n} x_n + 2\sum_{i=1}^{n-1} \frac{k}{k+i} x_i\right), \\ \overline{x}_l &= 1, \quad \mathrm{mod}(l,2)=1, \quad \overline{x}_l = 3, \quad \mathrm{mod}(l,2)=0. \end{aligned}$$

Problem 3.59. Problem NONDQUAR in [8].

$$\begin{aligned} f_k(x) &= x_k - 1, & k=1 \\ f_k(x) &= (x_k + x_{k+1} + x_n)^2, & 1<k<n, \\ f_k(x) &= x_{k-1} - x_k, & k=n \\ \overline{x}_l &= 1, \quad \mathrm{mod}(l,2)=1, \quad \overline{x}_l = -1, \quad \mathrm{mod}(l,2)=0. \end{aligned}$$

Problem 3.60.

$$\begin{aligned} f_k(x) &= \sum_{i=2}^{n} (i-1)\left(\frac{k}{n-2}\right)^{i-2} x_i - \left(x_1 + \sum_{i=2}^{n} \left(\frac{k}{n-2}\right)^{i-1} x_i\right)^2 - 1, \quad 1 \le k \le n-2, \\ f_{n-1}(x) &= x_1, \\ f_n(x) &= x_2 - x_1^2 - 1, \\ \overline{x}_l &= 0, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.61. [19], Problem LIARWHD in [8].

$$\begin{aligned} f_k(x) &= 4(x_k^2 - x_1)^2 + (x_k - 1)^2 \\ \overline{x}_l &= 4, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.62. Problem SINQUAD in [8].

$$\begin{aligned} f_k(x) &= (x_k - 1)^2, & k=1 \\ f_k(x) &= \sin(x_k - x_n) + x_k^2 - x_1^2, & 1<k<n, \\ f_k(x) &= x_k^2 - x_1^2, & k=n \\ \overline{x}_l &= 1/10, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.63. Problem SPARSQUR in [8].

$$\begin{aligned} f_k(x) &= \sqrt{k}\left(x_k^2 + x_{i_1}^2 + x_{i_2}^2 + x_{i_3}^2 + x_{i_4}^2 + x_{i_5}^2\right), \\ i_1 &= \mathrm{mod}(2k-1,n)+1, \quad i_2 = \mathrm{mod}(3k-1,n)+1, \quad i_3 = \mathrm{mod}(5k-1,n)+1, \\ i_4 &= \mathrm{mod}(7k-1,n)+1, \quad i_5 = \mathrm{mod}(11k-1,n)+1, \\ \overline{x}_l &= 1/2, \quad 1 \le l \le n\,. \end{aligned}$$

Problem 3.64. Problem SPARSINE in [8].

$$\begin{aligned} f_k(x) &= \sqrt{k}\,(\sin(x_k) + \sin(x_{i_1}) + \sin(x_{i_2}) + \sin(x_{i_3}) + \sin(x_{i_4}) + \sin(x_{i_5})\,, \\ i_1 &= \mathrm{mod}(2k-1,n)+1, \quad i_2 = \mathrm{mod}(3k-1,n)+1, \quad i_3 = \mathrm{mod}(5k-1,n)+1, \\ i_4 &= \mathrm{mod}(7k-1,n)+1, \quad i_5 = \mathrm{mod}(11k-1,n)+1, \\ \overline{x}_l &= 1/2, \quad 1 \le l \le n\,. \end{aligned}$$

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