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Abstract

As experimental scientists strive to understand the inner workings of more and more complex systems, classification of the interactions between the components of such systems is gaining much importance. Many biological, geophysical and atmospheric processes can be analyzed in the framework of nonlinear dynamical systems. An important subclass are oscillatory or quasi-oscillatory systems which can be coupled in various ways leading to a rich spectrum of cooperative behavior. One of the most important types of such behaviors is *synchronization*. Many forms of synchronization have been discovered to date, among them phase synchronization which occurrs in weakly coupled oscillators. Up to now, instead of direct detection of phase synchronization, much work has been devoted to quantifying phase dependence from bivariate time series of a pair of oscillatory processes. In this paper we introduce a selection of available methods for quantification of phase dependence and describe the first detector of phase synchronization from bivariate time series. The efficiency of the method is demonstrated on a model system and the method is compared with existing approaches to analysis of synchronization.

1. Introduction

Increasingly, complex biological, biochemical, meteorological and geophysical systems have become the focus of intensive experimental and theoretical research. Many of these systems can be characterized as coupled networks of nonlinear oscillators or quasi-oscillators. The complexity of their behavior typically arises from nontrivial interactions between more or less discrete components. A higher level of understanding of the function of the systems is facilitated by a more detailed analysis of the various types of behaviors induced by the coupling between their components.

There is a variety of ways that two oscillatory systems can be directly or indirectly coupled. Certain types of coupling between two systems lead (under favorable conditions) to a specific type of cooperative behavior termed "synchronization". Synchronization has been first described in the 17th century by Christian Huygens and who observed the phenomenon of mutual adjustment of motions of two pendulums hanging from a common beam. Performing further experiments he found that this is not a random effect but one that is brought about by a connection between the two pendulum clocks — in this case the very slight motion of the beam transferring forces between the two clocks. He called this interaction "sympathy" and published these (and other) findings in the monograph Horologium Oscillatorium [7].

The term synchronization has since come to represent a multitude of phenomena and much effort has been spent differentiating between its various forms. The simplest form is called *complete synchronization* and is found in coupled identical systems when both systems move along coincident trajectories after reaching a steady state [5]. *Generalized synchronization* requires that a smooth map exists between the trajectories of both oscillators [2]. More recently, *phase synchronization* resulting from weak coupling has been discovered as a form of synchronization that occurrs even in oscillating systems exhibiting deterministic chaos [1, 20, 19]. *Phase* is an observable which efficiently describes the motion of an oscillatory system: it indicates the current position of the dynamical system on its limit cycle. It is an increasing variable which grows by a fixed amount (usually

2π) for every completed cycle of the periodic motion. Phase is generally not directly available and must be first obtained from an observable of the system by a process called "phase extraction".

Experimentally, synchronization has been found for example in the human cardiorespiratory system [4, 21], in the Solar system [17], in meteorological systems [16] or in neural signals [15, 22].

Synchronization can occur in pairs of oscillators with similar natural frequencies which then lock at a 1:1 ratio. It is also possible that a pair of systems sychronizes at different ratios, for example a parent walking with a child alongside may make one large step while the child makes two smaller steps, every second step of the child aligning with the large step of the parent. Such a system synchronizes at a ratio 2:1. In general, synchronization at ratios different from 1:1 is called *higher order synchronization* and the ratio of frequencies at which the systems synchronize is termed the *locking ratio*.

In experimental practice, the only information available about the investigated systems are the recorded time series. In this case it is necessary to apply methods developed in the context of non-linear time series analysis and to infer if the systems synchronize or not. Unfortunately without additional knowledge or the option to interact with the given systems, it is not possible to decide with certainty if two systems synchronize. At best, either an index characterizing the strength of phase dependence can be estimated or an inference regarding the synchronization state can be made with a desired level of significance.

The rest of this paper is organized as follows: the next section deals with two frequently used methods of computing a "synchronization index" and describes the proposed synchronization detector; the following section details experiments testing the methods and comments on the results and the paper closes with a brief discussion and a conclusion.

2. Methods

In this section the sequence of steps required for processing time series of the original observable to obtain a result indicating the synchronization state is described. First, phase must be extracted from the time series. The phase is then used as input into the actual synchronization analysis methods which supply the final result: either a computed index or a decision.

2.1. Phase Extraction

There are multiple ways of extracting phase, each of which is suited to a particular situation. Instantaneous phase can be obtained from using the Hilbert transform [12] or the Wavelet transform [11]. If the time series of the observable is too noisy to obtain a reliable instantaneous phase signal, the marked-events method may yield better results [25]. In the following, the phase time series (obtained by one of the methods above) of the coupled systems will be denoted ϕ_1 and ϕ_2. It should be noted that the extraction methods usually provide a "wrapped" phase time series which is confined to the interval $\langle 0, 2\pi)$ but synchronization methods may work with an "unwrapped" definition, where 2π is added to the phase after a cycle is completed to produce an increasing phase. In the following experiments it will be specified which methods work with which definition of phase.

2.2. Synchronization indices

Because of the variety of synchronization phenomena, different synchronization analysis methods have been proposed, a comparison and overview of methods for analyzing phase synchronization is in [10]. These methods however usually estimate a "degree of synchronization", which should more aptly be called the "degree of phase dependence" and their result is typically a normalized synchronization index. It has however proven difficult to make a decision as to whether two systems are synchronized based on the values of such indices. We propose a new approach to the problem of detecting phase synchronization by constructing method which provides a decision whether two systems are synchronized with a pre-selected level of statistical significance.

In this section we first describe two frequently used methods in quantifying phase synchronization in systems of coupled non-linear oscillators: mean phase coherence and mutual information. In the rest of the section the new phase synchronization detection method is introduced.

2.2.1 Mean Phase Coherence: The mean phase coherence (MPC) [6] is defined as

$$R = \Big| \frac{1}{N} \sum_{j=1}^{N} e^{i \Delta \phi(j)} \Big| = 1 - \mathrm{CV}, \qquad (1)$$

where $\Delta\phi(j) = n\phi_1(j) - m\phi_2(j)$ is the difference of the "unwrapped" phases scaled by the locking ratio $m{:}n$ and CV denotes the circular variance [13], a well-known measure of point spread in circular statistics.

The ratio m:n should be set to the expected phase synchronization ratio. The function

$$\Delta\phi(j) = n\phi_1(j) - m\phi_2(j)$$

is important and describes the evolution of the difference of the scaled phases. If the systems are synchronized, this function should be constant (assuming there is no noise induced into the system). The mean of the derivative of the continuous version of the function $\Delta\phi(j)$ denotes the scaled relative phase velocity of the two systems. If the systems are synchronized, this should be exactly 0 indicating that m cycles of the second systems correspond to n cycles of the first system.

The result R is a synchronization index with values in the interval $\langle 0, 1\rangle$. The value of 0 indicates independent phases while 1 indicates completely synchronous motion.

The MPC quantifies the "spread" of the phase differences. If all of phase differences are tightly coupled together for a given time series, the value of MPC will be high. If, on the other hand, the phase differences exhibit high fluctuations, the value of MPC will be low. This can be seen from the relationship between MPC and the circular variance CV [13].,

2.2.2 Mutual Information: Mutual information [24] characterizes the statistical dependence of random variables. The phase time series are interpreted as realizations of an ergodic stochastic process. Under this assumption, the probability density function (PDF) of the variables can be estimated from a single realization. Using Φ_1, Φ_2 to denote the stochastic processes we can write

$$\begin{aligned} I(\Phi_1;\Phi_2) &= \iint p(\Phi_1,\Phi_2)\log\frac{p(\Phi_1,\Phi_2)}{p(\Phi_1)p(\Phi_2)} \\ &= H(\Phi_1) + H(\Phi_2) - H(\Phi_1,\Phi_2). \end{aligned} \quad (2)$$

If the two systems are uncoupled and behave independently, the mutual information (MI) of the two variables should be close to 0. In practice, however, contamination by noise and insufficient data to estimate the PDF reliably cause the value of MI to fluctuate. A systematic error is also introduced by similarities in the dynamics of the two systems as MI quantifies not only dependencies in the variables resulting from coupling between the systems but also dependencies resulting from common dynamics.

For the purpose of evaluation, mutual information can be normalized by $\min\{H(\Phi_1), H(\Phi_2)\}$ yielding an index of phase dependence in the interval $\langle 0, 1\rangle$. The value of 0 indicates that the random variables Φ_1, Φ_2 are independent and the value of 1 indicates that a functional relationship exists between the variables. In general a stronger connection between the PDFs of the processes will produce a higher value of MI.

Use of mutual information requires an effective tool to estimate the marginal PDF of each stochastic variable and also the joint PDF. This is currently the most challenging problem in applying information-theoretic functionals to time series analysis. An effective PDF estimator must capture the salient features of the PDF while being as resistant to noise as possible. There are many ways of estimating the PDF, a comprehensive review is in [23].

As an alternative to using (2), mutual information can be directly estimated from some statistics of the data. This is the approach used in this paper. One of the most promising estimators of mutual information, the Kraskov-Grassberger-Stögbauer method [9] of estimating mutual information from nearest neighbor distances is applied. The work is based on earlier efforts of Kozachenko and Leonenko [8] on asymptotically unbiased estimators of entropy.

2.3. Bootstrap Synchronization Detection

The above methods do not use any mathematical definition of synchronization as a basis for detecting the presence of synchronization. Rather they provide an index related to the mutual dependence of the phases of the systems. On the other hand, the method proposed in this section is based on a mathematical definition of phase synchronization. There are currently two widely accepted definitions of phase synchronization which respect the possible influence of noise on the systems and are therefore practically applicable:

$$|m\phi_1(t) - n\phi_2(t)| = |\Delta\phi(t)| < \delta, \quad (3)$$

which allows the phase time series to fluctuate slightly. This allows a pair of synchronized systems to be labeled as such even in the presence of some noise. This condition states that the phase difference between the two time series is bounded. This is a theoretically sound definition but unfortunately it cannot be tested on time series of finite length as every such time series satisfies (3) for $\delta = \sup\{|\Delta\phi|\}$. The other, slightly weaker condition, states that two systems are phase synchronized if their mean phase velocities are equal

$$m\langle\dot{\phi}_1\rangle = n\langle\dot{\phi}_2\rangle, \quad (4)$$

where $\langle ... \rangle$ denotes the time average. In the following, we show that this condition can be tested on a finite time series. First, it is necessary to prove that if the mean frequencies are not equal (systems are not synchronized) then the phase difference time series has a non-zero gradient and vice-versa. Henceforth we will work with sampled time series. This will be indicated by the use of the variable i to index the time series ϕ_1, ϕ_2 and $\Delta\phi$. We note that the condition (4) can be rewritten as

$$\langle \dot{\Delta\phi} \rangle = 0, \qquad (5)$$

Using least squares linear regression we may write

$$\Delta\phi(i) = at(i) + b + \epsilon(i) \qquad (6)$$

where a and b are chosen to minimize $\chi^2 = \sum_i \epsilon(i)^2$ [3]. As a corollary to this we have that mean $\epsilon(i)$ is zero. Subtracting the equation for $\Delta\phi(i)$ from the equation for $\Delta\phi(i+1)$ and rearranging gives

$$\frac{\Delta\phi(i+1) - \Delta\phi(i)}{t(i+1) - t(i)} = a + \frac{\epsilon(i+1) - \epsilon(i)}{t(i+1) - t(i)} \qquad (7)$$

Averaging over all samples (assuming equidistant sampling) we obtain

$$\frac{\langle \Delta\phi(i+1) - \Delta\phi(i) \rangle}{\Delta t} = a, \qquad (8)$$

where $\Delta t = t(i+1) - t(i)$. If we sampled with infinite density we would be able to take the limit $\Delta t \to 0$ to arrive at

$$\langle \dot{\Delta\phi} \rangle = a \qquad (9)$$

We note that the above shows that no matter what the actual evolution of the phase difference is, a linear trend will be present if the systems are unsynchronized. The phase locking condition (4) can thus be restated as $a = 0$. In real time series, noise and fluctuations will invariably cause the value of a to be slightly different from zero. The question is whether the gradient a is significantly non-zero. In this way, the problem of detecting synchronization has been transformed into the problem of estimating the significance of a gradient in the phase difference time series.

In general it is not possible to assume that a will have any particular distribution. This fact makes the construction of a direct statistical test of the value of a very difficult. In this work we propose not to test the value of a directly but to estimate its significance in an indirect fashion. This requires that a least-squares fit of a horizontal line $\Delta\phi(i) = c + \eta(i)$ is performed, where again c is chosen to minimize $\chi^2 = \sum_i \eta(i)^2$. In this case c is simply the mean of $\Delta\phi(i)$. We now compare the sample of errors of the original fit (6) and that of the errors of the horizontal line fit. If there is no real gradient in the time series $\Delta\phi(i)$ then the value of a is the result of random fluctuations and the distributions of the errors of both of the fits should be the *same*. If on the other hand there is a trend in the time series and the value of a explains part of the variance of the errors than the distributions will be different. There is a standard test that determines if two samples are drawn from the same distribution — the Kolmogorov-Smirnov test [14]. The test is a standard hypothesis test with the null hypothesis being that the samples are drawn from the same distribution. This means that the proposed synchronization detector assumes that the processes are synchronized and tries to reject this assumption using evidence from the data. This is a completely new approach to detecting synchronized states.

As described above, the method requires a high volume of data, on the order of hundreds or thousands of cycles to reliably differentiate between synchronized and unsynchronized states. This is because of long correlations in the time series which cause the appearance of spurious gradients in short time series. We use a simple solution which breaks these long correlations if there is no gradient but preserves the autocorrelation of the signal if there is a significant trend: time indices are sorted by the magnitude of the associated phase difference values. This step leads to a significant reduction in the frequency of false negatives and improves the efficiency of the method greatly. As it will be shown in the next section, time series only tens of periods long are now necessary for reliable detection even for higher locking ratios, such as those occurring in the cardiorespiratory system.

3. Experiments

Numerical tests on model systems are a prerequisite to the application of any method to experimental data. Experimental data suffer from a number of problems which make the task of synchronization detection (indeed of any type of interaction analysis) difficult. The main problem is generally stationarity: the methods require as much data as possible to provide reasonable estimates, on the other hand using time series that are too long may violate the assumption of stationarity of the system. Experimental time series are burdened with noise signals of multiple origins (measurement, thermal, quantization).

Testing on model systems under many different conditions does not ensure that the method will work well in practice but succesfull tests under a wide range of conditions indicate that the method should work well. Such tests also show how the effectivity of the method changes with respect to different parameters.

In this work we investigate the problem of detecting higher-order synchronization at the ratio of frequencies 1:4. The ratio has been carefully selected to match the frequency ratio of the heart-beat period to the breathing cycle period. The problem of detecting synchronization in the cardiorespiratory system has been examined by some authors, e.g. [4].

3.1. Phase synchronization in noisy systems

The simplest possible nonlinear oscillator is the *phase oscillator*. A linearly coupled pair of symmetrically coupled phase oscillators is described by the differential equations

$$\begin{aligned} \dot{\phi}_1 &= \omega_1 + b\cos(\phi_1) + \epsilon\sin(m\phi_2 - n\phi_1) + \eta_1 \\ \dot{\phi}_2 &= \omega_2 + b\cos(\phi_2) + \epsilon\sin(n\phi_1 - m\phi_2) + \eta_2, \end{aligned} \tag{10}$$

where $\omega_{1,2}$ represent the natural frequencies of the systems, b is the coefficient of the nonlinear term, ϵ represents the strength of coupling and $\eta_{1,2}$ are uncorrelated Gaussian noise terms. In this paper we show the synchronization results for a pair of oscillators with the frequency ratio approximately 1:4. When the systems synchronize, the definition (4) should hold.

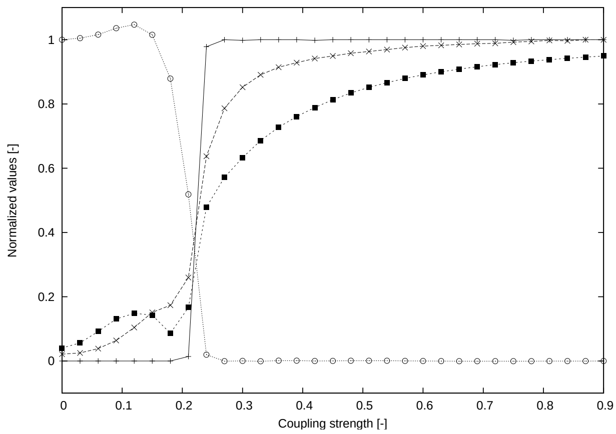

Figure 1: Comparison of synchronization analysis algorithms. Circles denote the DPV, pluses denote the detection rate of the bootstrap synchronization detector, crosses indicates the normalized mutual information estimated using kNN and full squares denote the mean phase coherence. Time series length is 2048 data points with about 60 points per period of the faster system (≈ 18 periods).

In Fig. 1 we plot the function $\langle n\dot{\phi}_1 - m\dot{\phi}_2\rangle$, here called the difference of scaled phase velocities (DPV) together with the results of the introduced synchronization analysis algorithms. The DPV is "normalized" by dividing all of its values by the value attained with non-existent coupling, this was done so that the shape of the DPV was clearly seen in the figure. The standard deviation of the inserted uncorrelated Gaussian noise was set to 0.02.

The synchronization transition region is approximately at the coupling strength 0.23 and is indicated by the phase difference velocity rapidly approaching 0.

3.2. Arnold tongues in phase oscillators

The second test is a reconstruction of one of the "Arnold tongues" for the system of coupled phase oscillators. The Arnold tongue refers to the region of synchronization of the coupled model system in the parameter space. We investigate coupled phase oscillator model (10), where the frequencies $\omega_{1,2}$ are set to

$$\begin{aligned} \omega_1 &= 1 + \Delta f \\ \omega_2 &= 4 - \Delta f \end{aligned} \tag{11}$$

where Δf is the frequency mismatch. The standard deviation of the inserted Gaussian uncorrelated noise was set to 0.02. The coupling strength ϵ spanned the interval $\langle 0, 0.5\rangle$, and the frequency mismatch was varied in the interval $\langle -0.2, 0.2\rangle$.

Fig. 2 shows the difference in scaled phase velocities (DPV) adjusted for the locking ratio 1:4. In the synchronized region,this difference should be 0 indicating that there are exactly four cycles of the faster system for one cycle of the slower system, the shape of the region resembles a tongue, hence the name of the region. In the figure, there are other flat regions with nonzero DPV. These regions correspond to synchronization in *different* ratios than 1:4. A synchronization analysis algorithm with adequate specificity should not be sensitive to the parameter combinations inside these regions.

In Fig. 3 it can be clearly seen that the Bootstrap synchronization detector is able to identify the region of 1:4 synchronization clearly. The interface between the synchronized and unsynchronized regions is sharply defined indicating that the detector is sensitive even near the transition between regions.

Figs. 4 and 5 show how the value of the synchronization indices varies with the frequency mismatch and coupling strength. It can be discerned that the highest values of the indices are in the synchronization region. However, we note that the values are not constant inside the region, thus rendering eventual thresholding more difficult and that there are non-zero values outside the synchronization region. These correspond to the other (secondary) plateaus in Fig. 2. Because the methods are sensitive to synchronization outside the pre-selected ratio, there is a danger of incorrectly accepting states synchronized at different ratios as states synchronized at the given ratio.

To the best of our knowledge there is currently no procedure which would reliably compute a threshold discriminating between the synchronized and unsynchronized states for either of the indices (MPC and MI) based on a single bivariate time-series.

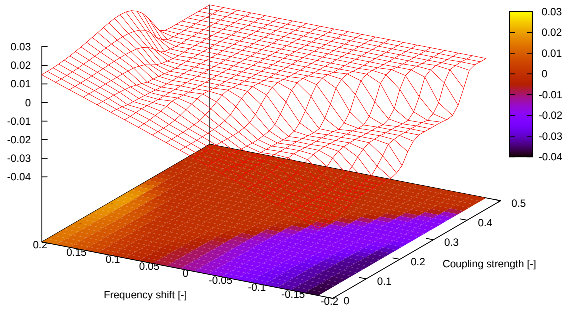

Figure 2: Difference of scaled phase velocities (DPV) plotted as a function of the frequency mismatch and of the coupling strength. The synchronization region is clearly seen as a plateau where the value of DPV is 0.

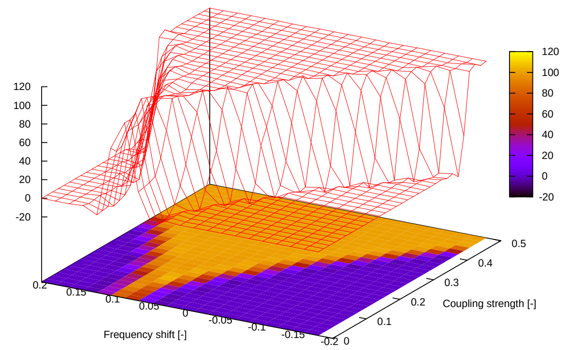

Figure 3: Detection rates of the bootstrap synchronization detector in percent. Comparing this image with Fig. 2, it is clearly seen that the region of synchronization is detected with excellent precision.

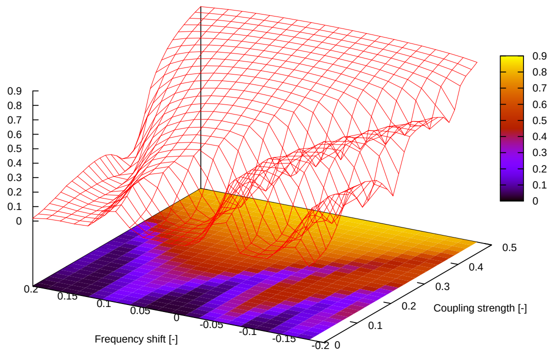

Figure 4: The values of the mean phase coherence. MPC shows the highest values in the synchronization region, however non-zero values are also outside the region and the value of the MPC index R varies widely even inside the synchronization region.

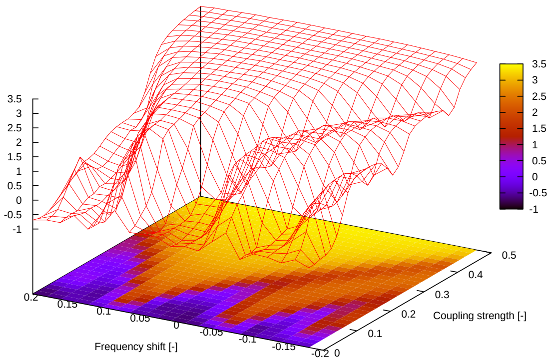

Figure 5: The values of mutual information estimated using the kNN method. The value of MI clearly attains its highest values in the synchronization region but also shows non-zero values outside the 1:4 synchronization region. The estimate is more stable inside the synchronization region than the MPC estimate, cf. Fig. 4.

4. Discussion

Phase synchronization is in general difficult to detect solely using information contained in the time series of observables. The main problem is that synchronization is a *process* [18] that manifests itself in the time series and causes phase locking. However detecting phase locking in a time series does not automatically imply that the two systems are synchronized. A simple example is of two identical oscillators with the same initial conditions and no coupling. Without the influence of noise, the two systems will evolve along coincident trajectories and from the time series they would appear to be perfectly locked. The problem of detecting phase synchronization from time-series therefore remains a problem of a statistical nature.

The influence of noise on the quality of detection is twofold. A small amount of noise may break static correlations (resulting from common dynamics) such as those described in the last paragraph. A large amount of noise may prevent synchronization alltogether or cause difficulties in detection of synchronization. A pervasive type of problem is called *phase slipping*, phase slipping occurs when sufficiently strong noise perturbs the the states of the two systems so that one of the systems loses a cycle and “slips” behind. A unified approach to treating phase slips is not agreed upon at present. In this work we have not investigated the problem of phase slips, suffice it to note that phase slips adversely affect all synchronization analysis methods. The problem is discussed in [26].

5. Conclusion

In this work the notion of phase synchronization between nonlinear oscillatory systems has been introduced. Frequently used methods to “detect” synchronization have been introduced and their drawbacks have been described. Subsequently a new synchronization detection method — the Bootstrap synchronization detector, has been introduced. Numerical experiments similar in nature to the problem of detecting synchronization in the cardiorespiratory system have been performed. The effectiveness of the proposed method has been demonstrated and compared to existing approaches. The method will be applied in analysis of cardiorespiratory and neural data within the project BRACCIA.

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