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Abstract:

We describe a transformation of the Euler equations from the conservative form to the variables of pressure, temperature and mass flow, which are preferred in the applications of the system control. This model is used to describe steam and flue gas flow in two pipes coupled by a wall with finite thickness. Then, we deal with the numerical solution of the system and suggest using a finite-volume AUSM-type scheme. A note on a model of the wall and an injection cooler is also included. Finally, we present a comparison with a theoretically computed temperature profile for a stationary state.

Keywords:

Heat exchangers, Steam flow, Boiler model

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Introduction

In this article, results of development of a model of a heat exchanger

1 Euler Equations in “Real” Variables

Recall that the system of Euler equations for one-dimensional flow through a pipe with constant cross-section reads as

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}q = 0, \quad (1.1)$$

$$\frac{\partial}{\partial t}q + \frac{\partial}{\partial x}\left(p + \frac{q^2}{\rho}\right) = -\frac{1}{2}\zeta\frac{q^2}{\rho}, \quad (1.2)$$

$$\frac{\partial}{\partial t}E + \frac{\partial}{\partial x}\left((p + E)\frac{q}{\rho}\right) = Q, \quad (1.3)$$

which have yet to be completed with the equation of state $p = p(\rho, E, q)$. The additional right side term $-\frac{1}{2}\zeta\frac{q^2}{\rho}$ should describe a pressure drop due to the turbulent friction (see [6]).

1.1 Transformations to “Real” State Variables

Next, we deal with the conversion of Euler equation in conservative variables into real variables consisting of pressure, temperature and mass flux:

Equation of continuity

Supposing that density is function of pressure and temperature only ($\rho = \rho(p, T)$), after computing the derivatives we obtain

$$\frac{\partial\rho}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial\rho}{\partial T}\frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x}. \quad (1.4)$$

Equation of energy

Deriving the equation of energy, we consider following basic thermodynamical relations for enthalpy

$$H = U + pV \quad (1.5)$$

and internal energy

$$E = U + \frac{1}{2}\rho V v^2. \quad (1.6)$$

Because the Euler equations are related to the unit volume ($V = 1$), we immediately get that

$$E = H + \frac{1}{2}\rho v^2 - p. \quad (1.7)$$

We substitute for E in the equation of energy:

$$\frac{\partial}{\partial t}\left(H + \frac{1}{2}\rho v^2 - p\right) + \frac{\partial}{\partial x}\left(pv + v\left(H + \frac{1}{2}\rho v^2 - p\right)\right) = Q. \quad (1.8)$$

Then, using the relations

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2\right) = \frac{1}{2}v^2\frac{\partial}{\partial t}\rho + \rho v\frac{\partial}{\partial t}v, \quad (1.9)$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho v^3\right) = \frac{1}{2}v^2\frac{\partial}{\partial x}(\rho v) + \rho v^2\frac{\partial}{\partial x}v. \quad (1.10)$$

and the equation of continuity, after performing the differentiations we obtain

$$\frac{\partial H}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial H}{\partial T} \frac{\partial T}{\partial t} - \frac{\partial p}{\partial t} + \rho v \frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(vH) + \rho v^2 \frac{\partial v}{\partial x} = Q. \quad (1.11)$$

Performing the differentiation in the equation of momentum

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(p + \rho v^2) = -\frac{1}{2}\zeta \rho v^2, \quad (1.12)$$

using the equation of continuity and multiplying by v we obtain

$$\rho v \frac{\partial v}{\partial t} + v \frac{\partial p}{\partial x} + \rho v^2 \frac{\partial v}{\partial x} = -\frac{1}{2}\zeta \rho v^3. \quad (1.13)$$

Subtracting (1.13) from (1.11) yields

$$\frac{\partial H}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial H}{\partial T} \frac{\partial T}{\partial t} - \frac{\partial p}{\partial t} = \frac{q}{\rho} \frac{\partial p}{\partial x} + \frac{1}{2}\zeta \frac{1}{\rho^2} q^3 - \frac{\partial}{\partial x}(vH) + Q. \quad (1.14)$$

Then, it's possible to compute the derivatives of $\frac{\partial p}{\partial t}$ and $\frac{\partial T}{\partial t}$ from the equations (1.4) and (1.14):

$$\%_1 \frac{\partial p}{\partial t} = -\frac{\partial H}{\partial T} \frac{\partial q}{\partial x} - \frac{\partial \rho}{\partial T} \%_2, \quad (1.15)$$

$$\%_1 \frac{\partial T}{\partial t} = -\left(1 - \frac{\partial H}{\partial p}\right) \frac{\partial q}{\partial x} + \frac{\partial \rho}{\partial p} \%_2, \quad (1.16)$$

where

$$\%_1 = \frac{\partial \rho}{\partial p} \frac{\partial H}{\partial T} + \left(1 - \frac{\partial H}{\partial p}\right) \frac{\partial \rho}{\partial T}, \quad (1.17)$$

$$\%_2 = \frac{q}{\rho} \frac{\partial p}{\partial x} + \frac{1}{2}\zeta \frac{q^3}{\rho^2} - \frac{\partial}{\partial x} \left(\frac{q}{\rho} H\right) + Q. \quad (1.18)$$

These two equations together with equation of momentum (1.2) and state equations

$$\left[\rho, \hat{H}, \frac{\partial \rho}{\partial p}, \frac{\partial \rho}{\partial T}, \frac{\partial \hat{H}}{\partial p}, \frac{\partial \hat{H}}{\partial T} \right] = f(p, T) \quad (1.19)$$

form the evolution equations in state variables (p, T, q) . Values of $\frac{\partial H}{\partial p}$ and $\frac{\partial H}{\partial T}$ can be easily computed using the Leibniz rule:

$$\frac{\partial H}{\partial p} = \frac{\partial \rho}{\partial p} \hat{H} + \rho \frac{\partial \hat{H}}{\partial p}, \quad (1.20)$$

$$\frac{\partial H}{\partial T} = \frac{\partial \rho}{\partial T} \hat{H} + \rho \frac{\partial \hat{H}}{\partial T}. \quad (1.21)$$

1.2 Approximation of the steam properties

To complete the system of equations we have to provide the equations of state. In the case of steam, the equations we are using are based on the Steam Properties Package [9]. However, to increase computation speed, we currently use a polynomial approximation which seems to be precise enough. Moreover, having a polynomial approximation, we are able to compute partial derivatives very easily.

For flue gas we use a state equation of an ideal nitrogen, because it constitutes the largest portion of it, determining the behavior of the mixture. Thus, for enthalpy we consider the relation

$$\hat{H} = c_p T \quad (1.22)$$

and for density it holds that

$$\rho = \frac{pM}{RT}, \quad (1.23)$$

where

$$M = 0.02801 \text{ kg mol}^{-1}$$

is the molar mass of nitrogen,

$$c_p = 1037.0 \text{ J kg}^{-1} \text{ K}$$

specific thermal capacity for constant pressure and

$$R = 8.3144 \text{ J mol}^{-1} \text{ K}^{-1}$$

molar gas constant.

2 Numerical Solution of the Steam/Flue Gas Flow

Taking the real variable system and replacing the spatial derivatives with differences, we obtain the following set of equations

$$\frac{dp_i}{dt} = \frac{1}{\%_1} \left(- \left(\frac{\partial \rho}{\partial T} \Big|_i \hat{H}_i + \rho_i \frac{\partial \hat{H}}{\partial T} \Big|_i \right) \Delta_x q_i - \frac{\partial \rho}{\partial T} \Big|_i \%_2 \right), \quad (2.1)$$

$$\frac{dT_i}{dt} = \frac{1}{\%_1} \left(- \left(1 - \left(\frac{\partial \rho}{\partial p} \Big|_i \hat{H}_i + \rho_i \frac{\partial \hat{H}}{\partial p} \Big|_i \right) \right) \Delta_x q_i + \frac{\partial \rho}{\partial p} \Big|_i \%_2 \right), \quad (2.2)$$

$$\frac{dq_i}{dt} = \Delta_x p_i + \Delta_x \left(\frac{q_i^2}{\rho_i} \right) - \frac{1}{2} \zeta \frac{q_i^2}{\rho_i}, \quad (2.3)$$

where

$$\%_1 = \frac{\partial \rho}{\partial p} \Big|_i \left(\frac{\partial \rho}{\partial T} \Big|_i \hat{H}_i + \rho_i \frac{\partial \hat{H}}{\partial T} \Big|_i \right) + \left(1 - \left(\frac{\partial \rho}{\partial p} \Big|_i \hat{H}_i + \rho_i \frac{\partial \hat{H}}{\partial p} \Big|_i \right) \right) \frac{\partial \rho}{\partial T} \Big|_i, \quad (2.4)$$

$$\%_2 = \frac{q_i}{\rho_i} \Delta_x p_i + \frac{1}{2} \zeta \frac{q_i^3}{\rho_i^2} + \Delta_x (q_i \hat{H}_i) - Q, \quad (2.5)$$

$$\Delta_x \{ \cdot \} = \frac{\{ \cdot \}_R - \{ \cdot \}_L}{\Delta x} \quad (2.6)$$

and $\{ \cdot \}_L, \{ \cdot \}_R$ denote values at the left and right volume interface.

These values are determined using the AUSM formulas with Van Leer Mach number splitting. First, recall that local sound speed and local Mach number compute as

$$a = \sqrt{\kappa \frac{p}{\rho}}, \quad (2.7)$$

$$M = \frac{q}{\rho a}. \quad (2.8)$$

Then, the convective terms at the volume interface are approximated as follows:

$$(vx)_{1/2} = \frac{1}{2} M_{1/2} (x_L a_L + x_R a_R) - \frac{1}{2} |M_{1/2}| (x_R a_R - x_L a_L), \quad (2.9)$$

where

$$M_{1/2} = M_L^+ + M_R^-, \quad (2.10)$$

$$M_{\pm} = \begin{cases} \pm \frac{1}{4} (M \pm 1)^2 & |M| \leq 1; \\ \frac{1}{2} (M \pm |M|) & \text{otherwise.} \end{cases} \quad (2.11)$$

For pressure terms, we use second order polynomial expansion of the characteristic speeds:

$$p_{1/2} = p_L^+ + p_R^-, \quad (2.12)$$

$$p^\pm = \begin{cases} \frac{\rho}{4}(M \pm 1)^2(2 \mp M) & |M| \leq 1; \\ \frac{\rho}{2}(M \pm |M|)/M & \text{otherwise.} \end{cases} \quad (2.13)$$

3 Pipe Wall Model and the Thermal Transfer

3.1 Heat transfer equations

The exchanger pipe wall is described by a partial differential equation of the heat transfer

$$S_3 \left(\rho_3 c_3 \frac{\partial T_3}{\partial t} - \frac{\partial}{\partial x} \left(\lambda \frac{\partial T_3}{\partial x} \right) \right) = -Q, \quad (3.1)$$

with S_3 being the wall cross-section, c_3 specific heat capacity, T_3 wall temperature, λ thermal conductivity and Q linear heat transfer density out of the wall.

For linear heat transfer density, we consider following relations:

$$Q_1 = o_1 \alpha_1 (T_3 - T_1), \quad (3.2)$$

$$Q_2 = o_2 \alpha_2 (T_3 - T_2), \quad (3.3)$$

$$Q = Q_1 + Q_2. \quad (3.4)$$

Formulas for the transmission coefficient α are presented in [5]. An empirical relation for an aligned tube bundle is

$$\alpha = \frac{Nu \lambda_f}{D_f}, \quad Nu = 0.202 K_1 Re^{0.64} Pr^{0.4}, \quad Re = \frac{u_f D_f}{\nu}, \quad Pr = \frac{\lambda}{\rho c \nu} \quad (3.5)$$

where ν is the cinematic viscosity, D_f outer tube diameter, u_f fluid velocity and K_1 fitting constant. Re , Pr and Nu are Reynolds number, Prandtl number and Nusselt number, respectively.

3.2 Numerical solution

The heat equation describing the wall can be easily solved using method of lines. This method has also an advantage that it can be incorporated into MATLAB/Simulink framework very easily.

The heat transfer equation is discretized in space, yielding the following set of ordinary differential equations (indices omitted):

$$\frac{dT_i}{dt} = \frac{\lambda}{\rho c} \left(\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} - \frac{Q_i}{S \Delta x} \right). \quad (3.6)$$

The wall model can be eventually simplified yet, because the thermal transfer coefficient λ and the wall cross-section are fairly small and thermal transfer along the wall can be neglected. In that case, the (discretized) wall model simply reduces to

$$\frac{dT_i}{dt} = -\frac{\lambda}{\rho c} \frac{Q_i}{S \Delta x}. \quad (3.7)$$

4 Model of an Injection Cooler

Injection cooler model we currently use follows the idea of [5]. We neglect the pressure drop in the cooler element so the model reflects only dynamics of output temperature change.

Mass balance in the cooler implies immediately (in the following, all the mass flows are specified in kg s^{-1} , rather than $\text{kg s}^{-1} \text{m}^{-2}$)

$$V \frac{d\rho}{dt} = q_{\text{in}} + q_w - q_{\text{out}} \quad (4.1)$$

and from the energy balance

$$V \frac{d}{dt}(\rho \hat{H}) = q_{\text{in}} \hat{H}_{\text{in}} + q_{\text{w}} \hat{H}_{\text{w}} - q_{\text{out}} \hat{H}. \quad (4.2)$$

Multiplying (4.1) by \hat{H} and subtracting it from (4.2) we obtain

$$V \rho \frac{d\hat{H}}{dt} = q_{\text{in}} \hat{H}_{\text{in}} + q_{\text{w}} \hat{H}_{\text{w}} - (q_{\text{in}} + q_{\text{w}}) \hat{H}. \quad (4.3)$$

For steady state, enthalpy of the mixture is

$$\hat{H} = \frac{q_{\text{in}} \hat{H}_{\text{in}} + q_{\text{w}} \hat{H}_{\text{w}}}{q_{\text{in}} + q_{\text{w}}}. \quad (4.4)$$

Using the state equations $\rho = \rho(p, T)$ and $\hat{H} = \hat{H}(p, T)$, we can rewrite (4.1) and (4.3) to the following form:

$$V \begin{bmatrix} \frac{\partial \rho}{\partial p} & \frac{\partial \rho}{\partial T} \\ \rho \frac{\partial \hat{H}}{\partial p} & \rho \frac{\partial \hat{H}}{\partial T} \end{bmatrix} \begin{bmatrix} \frac{\partial p}{\partial t} \\ \frac{\partial T}{\partial t} \end{bmatrix} = \begin{bmatrix} q_{\text{in}} + q_{\text{w}} - q_{\text{out}} \\ q_{\text{in}} \hat{H}_{\text{in}} + q_{\text{w}} \hat{H}_{\text{w}} - (q_{\text{in}} + q_{\text{w}}) \hat{H} \end{bmatrix}. \quad (4.5)$$

As the hydraulic resistance of the cooler can be omitted we may assume that $p = p_{\text{in}}$. When we also neglect the terms containing $\frac{dp}{dt}$, the system (4.5) can be rewritten in the form

$$\begin{bmatrix} V \frac{\partial \rho}{\partial T} & -1 \\ V \rho \frac{\partial \hat{H}}{\partial T} & \hat{H} - \hat{H}_{\text{in}} \end{bmatrix} \begin{bmatrix} \frac{dT}{dt} \\ q_{\text{in}} \end{bmatrix} = \begin{bmatrix} q_{\text{w}} - q_{\text{out}} \\ q_{\text{w}}(\hat{H}_{\text{w}} - \hat{H}) \end{bmatrix}. \quad (4.6)$$

From the first line we get

$$q_{\text{in}} = q_{\text{out}} - q_{\text{w}} + V \frac{\partial \rho}{\partial T} \frac{dT}{dt}. \quad (4.7)$$

Substitution of (4.7) into the second equation of (4.6) gives

$$V \left((\hat{H} - \hat{H}_{\text{in}}) \frac{\partial \rho}{\partial T} + \rho \frac{\partial \hat{H}}{\partial T} \right) \frac{dT}{dt} = (\hat{H}_{\text{w}} - \hat{H}_{\text{in}}) q_{\text{w}} - (\hat{H} - \hat{H}_{\text{in}}) q_{\text{out}}. \quad (4.8)$$

In many cases we can neglect the cooler dynamics at all and use a static model. Linearized expression for the specific enthalpy is

$$\hat{H} - \hat{H}_{\text{in}} = \hat{H}(p_{\text{in}}, T) - \hat{H}(p_{\text{in}}, T_{\text{in}}) = \frac{\partial \hat{H}}{\partial T} (T - T_{\text{in}}). \quad (4.9)$$

With $\frac{dT}{dt} = 0$, (4.8) then simplifies to

$$T = T_{\text{in}} - \frac{\hat{H}_{\text{in}} - \hat{H}_{\text{w}}}{\frac{\partial \hat{H}}{\partial T}} \frac{q_{\text{w}}}{q_{\text{out}}} \quad (4.10)$$

and the mass outflow is simply

$$q_{\text{out}} = q_{\text{in}} + q_{\text{w}}. \quad (4.11)$$

5 Stationary Flow in Heat Exchangers

When we impose additional simplifying assumptions, it is possible to treat the problem of finding the stationary state by solving a system of ordinary differential equations. Using [4], we are able to find temperature profiles in parallel heat exchangers, and compare the theoretical profiles with the results obtained by our numerical simulation.

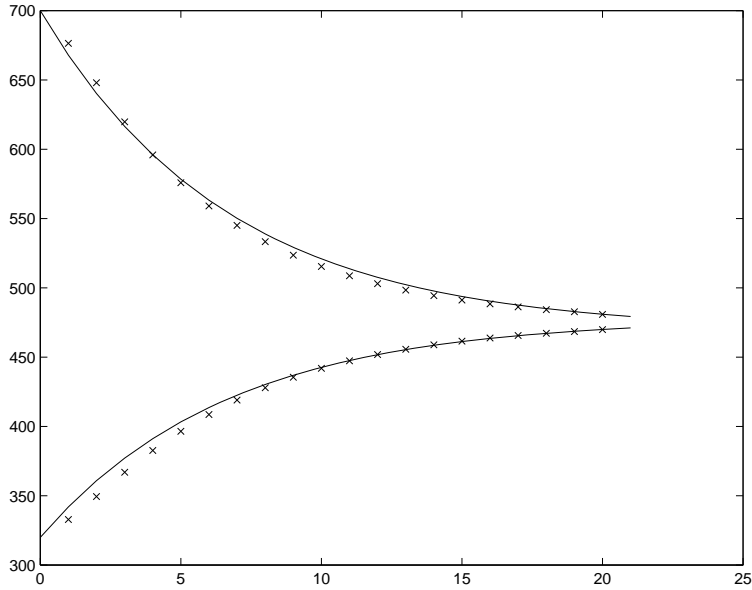


Figure 5.1: Sample stationary profiles compared to the theoretical ones. Thermal transfer coefficient was increased here so the exponential character of the profiles can be demonstrated.

Let's denote mass fluxes in the pipes a and b by q_a and q_b . We assume that the velocity and all thermodynamical quantities except temperature T are constant. We look for a stationary solution to the transport equations

$$S_a \rho_a c_a \left(\frac{\partial T_a}{\partial t} + v_a \frac{\partial T_a}{\partial x} \right) = Q_a, \quad (5.1)$$

$$S_b \rho_b c_b \left(\frac{\partial T_b}{\partial t} + v_b \frac{\partial T_b}{\partial x} \right) = Q_b, \quad (5.2)$$

where we don't consider any heat capacity of the wall, so just a direct Newton-type heat exchange occurs between the two media:

$$Q_a = k(T_b - T_a), \quad Q_b = k(T_a - T_b). \quad (5.3)$$

The constant k denotes the linear heat transfer coefficient, e.g $k = \alpha o$, where α is the heat transfer coefficient and o denotes an effective perimeter of the pipe. Performing the laplacian transformation, it's possible to find the following solution:

$$T_a(x) = \frac{T_{a1} - T_{a2}}{1 - e^{-CL}} (e^{-Cx} - 1) + T_{a1}, \quad (5.4)$$

$$T_b(x) = \frac{T_{b1} - T_{b2}}{1 - e^{-CL}} (e^{-Cx} - 1) + T_{b1} \quad (5.5)$$

with

$$C = k \left(\frac{1}{c_a q_a} + \frac{1}{c_b q_b} \right), \quad (5.6)$$

for $x \in [0, L]$.

An example of the stationary temperature profiles is shown in figure 5.1. The setup under which this result was obtained resembles the real exchanger, except that the thermal transfer coefficient was increased here to emphasize the exponential character of the profiles.

6 Conclusion

The current version of the heat exchanger model has been tested and the full system of equations without the direct link between inputs and outputs performs well even in the setup with more heat exchangers with injection coolers between them.

During development of the model, a routine for extraction of the experimental data was also written, to allow comparison between real and computed values. The results were summarized in the appendix of [8].

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