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Jiřina, Marcel
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Pod Vodárenskou věží 2, 182 07 Prague 8, phone: 6605 3350, fax: (+4202) 86 58 57 89,
e-mail:marcel@cs.cas.cz

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Abstract:

For statement of particle's energy detected in particle detectors an approximation method is developed. The method is based on linear regression of variables which are, in fact, quadrats of geometric variables as radius or center distance from the origin.

Keywords:
regression approximation, circular regression, particle track, Bremsstrahlung

1 Introduction

In this report we deal with problem of finding tracks in full scan TRT. The tracks looked for are circular tracks either going from the origin (primary vertex) or circular tracks having radius smaller than is the distance of tracks's centre from the origin.

The tracks of the second kind may originate from lot of different processes, and among them from Bremstrahlung phenomenon. To isolate a case of Bremstrahlung it is necessary to find both parts of the track – the fast one, which can be found in silicon detector and the second one, slow, which is usually visible in TRT. Our searching of Bremstrahlung tracks is based on finding slow part of track in TRT, then forming hypotheses for fast part of the track and finally in approving some of these hypotheses using hits in silicon detector.

Using data from TRT detector only the algorithm forms hypotheses for the first part of the track giving possible approximate values of p_{T1} (up to 6 values). Also corresponding angles of hits in individual silicon layers and starting angles are computed.

In the second part the algorithm approves or rejects the track hypotheses leaving at most one (the best) for each track.

The point of view to the problem of finding tracks is more geometrical than physical. Of course, there is simple relation of circle radius R and transversal momentum p_T of electron or positron moving in magnetic field 2 tesla in ATLAS detector [1]

$$p_{T[G_eV]} = 0.006R_{[cm]}. \tag{1.1}$$

In discussion of method and its testing we limit ourselves to barrel only, more exactly, to one half of barrel.

For searching tracks in TRT several methods are known:

- methods based on Kalman filter [2] [3] [4]
- histogramming method [2]
- method used in Alice detector [5], [6]

To identify Bremstrahlung, the wave algorithm [7] has been used.

2 Problem formulation

Given TRT barrel geometry (i.e. radii of straw layers, angular position of the first straw in each layer) and list of hits for each event. The straws are ordinarily numbered from 1 to approx. 50.000 starting from inner layer successively one by one to the outer layer. In the list of hits there is ordinal number of hitted straw and intensity of hit–either 1 (low) or 2 (high). The list may contain a drift time, but we do not use it. For testing using simulated data there is also KINE number of the track to which particular hit belongs.

For each event there should be also data from silicon detector – similarly as in TRT: geometry parameters and list of hits with necessary parameters. **The task** is to reconstruct tracks. It means to find for each track a set of hits which form it and state global parameters of the track

- p_T (or radius R, see (1.1)),
- the angle of the track in the origin, if it is a single track coming from origin (primary vertex),
- or the distance ρ of the centre from origin if $\rho > R$.

In the last case it is assumed that track identified is the slow part of track after Bremstrahlung. Based on ρ and R we have to form hypotheses for possible fast (the first) part of track – i.e before Bremstrahlung arise.

If Si detector data are given, we have to test these hypotheses with respect to data, approve valid hypotheses and give more exact parameters of Bremstrahlung track, especially p_{T1}, initial angle p_{T2}, Bremstrahlung point, and, finally, list of hits in Si detector and in TRT which form such a track.

3 Method

The method used is based on wave algorithm [7] in its part of searching track candidate. When the track candidate (the set of hits possibly forming a track) is found, it is tested by new approach. This approach is essentially different from approach in the report cited, where assumption that the track goes through the origin (primary vertex) has been used.

3.1 Outline of the procedure

1. Find a set of hits in TRT, which form (possibly) a circular track. This set of hits we call simply a **track candidate**. The track candidate must obey some simple rules as to have enough hits or do not have "breaks"-missing hits in more than - say - in 10 successive layers. For details see [7].
2. Test the track candidate for circularity without prior assumption that track goes through the origin. The computation will give several values
   - track radius R_2
   - track radius error $\triangle R_2$
   - distance ρ_2 of centre of the circle from the origin
3. When evaluating these values one can get
   - $\rho_2 < R2$ (with respect to track radius error, i.e. more exactly $\rho_2 < R_2 + \triangle R_2$). Then the origin lies inside the circle and the track candidate cannot form a track we are searching for.
   - $\rho_2 \geq R_2$. Then if
     - $\triangle R_2 <$ Straw Radius, the error is small enough to accept track candidate as a **track**. Two cases are possible:
       * Either $|R_2 - \rho_2| <$Straw Radius, then we consider the track as a track going through origin,
       * or $\rho_2 > R_2 + \triangle R_2$ and then it can be a track after Bremstrahlung. In this case the first part of track (from origin to Bremstrahlung point) is looked for
     - $\triangle R_2 \geq$Straw Radius. Then the hit from track candidate which has the largest error is omitted from the track candidate. Then the parameters $R_2, \triangle R_2, \rho_2$ are computed again and newly evaluated according to this procedure.

3.2 Procedure details

3.2.1 Track tolerance, worst case and statistical hit error

From TRT geometry[1] follows that average distance of axes of neighbour straws in given layer is 0.67 cm (0.66–0.68 cm). Straw diameter is 0.4 cm, but evaluating drift values one can find, that radius is a little bit smaller – straw radius is 0.1949 cm (maximal value of drift time). If there is a hit then a particle went around the centre of hitted straw in tolerance less than 0.1949 cm from one or other side.

From it follows simply, that if given a circle, the hits can be accepted if a distance of centre of corresponding straw from the circle is less than 0.1949 cm. If not, the hit is rejected, i.e. excluded from track candidate. If the track candidate is formed of only good hits (i.e. hits belonging really to the track), then worst case error is just 0.1949 cm – we consider the track as practically perpendicular to the circle of straws in a layer:

$$\epsilon_{\text{worst case}} = R_{\text{straw}},$$

where R_{straw} is effective straw radius, i.e.R_{straw} =0.1949 cm.

The position of straw with respect to real track is random. If only good hits are considered, the individual deviations of straw centers from the track may vary from $-R_{\text{straw}}$ to $+R_{\text{straw}}$ with homogenous distribution. Let us consider L_1 norm for deviation. Then mean error in this norm is

$$\epsilon_{L_1} = \frac{1}{2} R_{\text{straw}}.$$

One can consider L_2 norm, and in this case

$$\epsilon_{L_2} = \frac{1}{3} R_{\text{straw}}.$$

In fact, we found that the tracks in simulated events are a little broader than it corresponds to this case. In fact, electron of $p_T > 0.5 GeV$ never hits two neighbour straws in one layer. On the other hand it hits straws which have distance of its centre from ideal track larger than R_{straw}. It corresponds to R_{straw} larger than 0.1949 cm. Then we must use in these consideration R_{straw} a little hit larger than 0.1949 cm: 0.3 cm.

3.2.2 Circular regression

Let n points be given in a plane in polar coordinates $(\rho_1, \phi_1), (\rho_2, \phi_2), \ldots (\rho_n, \phi_n)$. These points we wish to approximate by a circle so that the error measured as suitable measure of distance of points from the approximating circle was minimal. The circle k looked for is given by its radius R and centre $S = (\rho, \phi)$, then $k = (R, \rho, \phi)$.

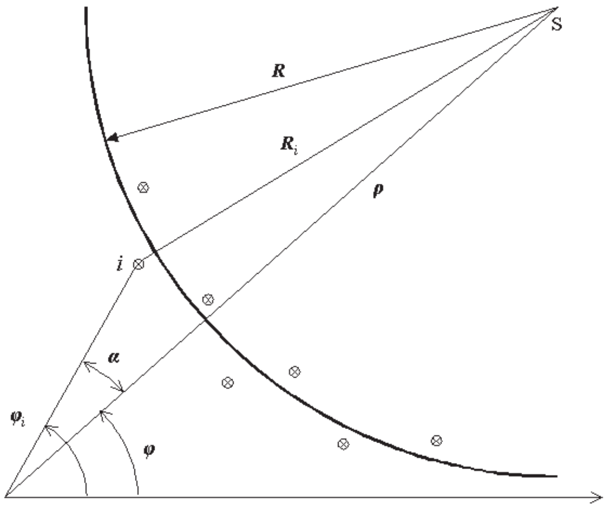

Fig. 1

It holds (see Fig. 1) $\alpha = \phi_i - \phi$ and according to cosinus theorem in triangle OS_i there is

$$R_i^2 = \rho_i^2 + \rho^2 - 2\rho_i \rho \cos \alpha. \tag{3.1}$$

We define an error by formula

$$\epsilon = \sum_{i=1}^{n} (R_i^2 - R^2)^2 \tag{3.2}$$

and let $A_i = R_i^2 - R^2$. (The error is then measured by sum of distances squared and the distance we measure not by difference but by difference of squares; nonetheles $|A_i|$ is a metrics).

As there is

$$|R_i - R| \ll R,$$

one can write $R_i = R + \delta_i$, where δ_i is small number with respect to $R, |\delta_i| \ll R$. Then

$$R_i^2 - R^2 = 2R\delta_i + \delta_i^2$$

and with sufficient exactness

$$R_i^2 - R^2 = 2R\delta_i. \tag{3.3}$$

Because R is a constant then, under the assumption that δ_i is a small number, it is a linear relation. In (3.1) we have then standard sum of squares of differences of the second power of track radius because with respect to (3.2) there is

$$\epsilon = 4R^2 \sum_{i=1}^{n} \delta_i. \tag{3.4}$$

Function $\epsilon = \epsilon(R, \rho, \phi)$ is continuous and differentiable. To find its local minimum, we find its partial derivatives with respect to R, ρ, ϕ and formulas found we set equal to zero. We will get successively (we write $\sum$ instead of $\sum_{i=1}^{n}$):

$$\epsilon = \sum [\rho_i^2 + \rho^2 - R^2 - 2\rho_i \rho \cos(\phi_i - \phi)]^2.$$

$$\begin{aligned} \frac{\partial \epsilon}{\partial R} &= \sum 2A_i(-2R) \\ \frac{\partial \epsilon}{\partial \rho} &= \sum 2A_i(2\rho - 2\rho_i \cos(\phi_i - \phi)) \\ \frac{\partial \epsilon}{\partial \phi} &= \sum 2A_i(-2\rho_i \rho \sin(\phi_i - \phi)) \end{aligned}$$

these derivatives are equal to zero in local minimal. Then

$$\sum A_i = 0 \tag{3.5}$$

$$\sum A_i(\rho - \rho_i \cos(\phi_i - \phi)) = 0 \tag{3.6}$$

$$\sum A_i(\rho - \rho_i \sin(\phi_i - \phi)) = 0 \tag{3.7}$$

We rewrite (3.5) using (3.4) in form:

$$\sum A_i \rho_i \cos(\phi_i - \phi) = 0 \tag{3.8}$$

Using formulas for sinus and cosinus of difference of angles we get equations (3.5) a (3.6) in form

$$\cos\phi \sum A_i \rho_i (\sin\phi_i + \cos\phi_i) + \sin\phi \sum A_i \rho_i (\sin\phi_i - \cos\phi_i) = 0 \tag{3.9}$$

$$\cos\phi \sum A_i \rho_i (\sin\phi_i - \cos\phi_i) + \sin\phi \sum A_i \rho_i (\sin\phi_i + \cos\phi_i) = 0 \tag{3.10}$$

From sum and difference of equations (3.8) a (3.9) one gets after a little algebra

$$\sum A_i \rho_i (\sin\phi_i - \cos\phi_i) = 0 \tag{3.11}$$

$$\sum A_i \rho_i (\sin\phi_i + \cos\phi_i) = 0 \tag{3.12}$$

Let us denote

$$\begin{aligned} a_i &= \rho_i(\sin\phi_i - \cos\phi_i) \\ b_i &= \rho_i(\sin\phi_i + \cos\phi_i). \end{aligned}$$

Then the equations (3.4), (3.10) and (3.11) can be rewritten in matrix form:

$$\begin{pmatrix} n & \sum \rho_i \cos\phi_i & \sum \rho_i \sin\phi_i \\ \sum a_i & \sum a_i\rho_i \cos\phi_i & \sum a_i\rho_i \sin\phi_i \\ \sum b_i & \sum b_i\rho_i \cos\phi_i & \sum b_i\rho_i \sin\phi_i \end{pmatrix} \cdot \begin{pmatrix} \rho^2 - R^2 \\ 2\rho\cos\phi \\ 2\rho\sin\phi \end{pmatrix} = \begin{pmatrix} -\sum \rho_i^2 \\ -\sum \rho_i^2 a_i^2 \\ -\sum \rho_i^2 b_i^2 \end{pmatrix} \tag{3.13}$$

Solving these equations we should get

$$\begin{aligned} \rho^2 - R^2 &= A = \beta_1 \\ 2\rho\cos\phi &= B = \beta_2 \\ 2\rho\sin\phi &= C = \beta_3 \end{aligned} \tag{3.14}$$

and from it

$$\rho\frac{1}{2}\sqrt{B^2 + C^2}, \qquad R = \sqrt{\rho^2 - A}, \qquad \cos\phi = B/(2\rho).$$

The system (3.12) can be considered as sum of n equations in form

$$\begin{pmatrix} 1 & \rho_i \cos\phi_i & \rho_i \sin\phi_i \\ a_i & a_i\rho_i \cos\phi_i & a_i\rho_i \sin\phi_i \\ b_i & b_i\rho_i \cos\phi_i & b_i\rho_i \sin\phi_i \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} -\rho_i^2 \\ -a_i\rho_i^2 \\ -b_i\rho_i^2 \end{pmatrix} \tag{3.15}$$

All these equations are singular because the second and the third row are multiples of the first row. For equation (3.12) it does not hold generally but it will be bad conditioned equation. For this reason we consider only the first equation of (3.14). Such equations one can write n and it is possible to write them in standard form of system of linear regression equations with respect to vector $\beta = (A, B, C)^t$:

$$X \cdot \beta = y.$$

Solving it we will get (under assumption of regularity of matrix X^tX):

$$\beta = (X^tX)^{-1}X^ty.$$

Error estimations: For estimation of error of regression coefficients the theorem holds [8], [9]. Lemma: Under assumption of linear independent columns of matrix X it holds:

1. regression coefficients $\beta = (\beta_1, \beta_2, \ldots \beta_n)$ are normally distributed
2. regression coefficients have variance matrix

$$\sigma^2 \cdot (X^tX)^{-1} \tag{3.16}$$

3. unbiased estimation of parameter σ^2 is

$$s^2 = \frac{S_e}{r - m}, \tag{3.17}$$

where

$$S_e = (y - y')^t(y - y') = y^ty - \beta^tX^ty, \tag{3.18}$$

and where $y' = X\beta$.

Lemma[8], [9]
Let v_{ij} be element of matrix

$$(X^tX)^{-1}, \tag{3.19}$$

$i, j = 1, 2, \ldots m$. Then random variable

$$T_i = (b_i - \beta_i)\sqrt{s^2 v_{ii}}$$

has distribution t_{r-m}.

Lemma [8]: The half width of confidence interval on level $1-\alpha$ for β_i is

$$\frac{k_i}{2} \leq t_{r-m}(\alpha) \cdot \sqrt{s^2 v_{ii}}. \tag{3.20}$$

We set (3.16) and (3.18) to (3.19) and thus we have just all m values $k_i/2$.

In our case $m = 3$ (R, ρ, ϕ).

For practical computation we use formula (3.19) and assumption that β_i is normally distributed and its σ correponds to half width of confidence interval on level $0.683 = 1 - 0.317; \alpha = 0.317$, then

$$\sigma_i \leq t_{r-m}(0.317)\sqrt{s^2 v_{ii}}$$

here for $r = m \geq 7$ value of $T_{r-m}(0.317)$ is between 1 and 1.0767. In the following we use value 1, then

$$\sigma_i = \sqrt{s^2 v_{ii}},$$

where we use for v_{ii} eq. (3.18) and for s^2 successively (3.17) a (3.16).

We will get σ_i for A,B,C and we recompute them to $\sigma_\rho, \sigma_R, \sigma_\phi$ using assumption that σ is small enough with respect to mean value and curvature of the function near mean value is small so that distribution will keep its normality after transformation, and only the mean value and σ will be changed.

Then after a little algebra

$$\begin{aligned}
\frac{\partial\rho}{\partial B} &= \frac{B}{4\rho} \\
\frac{\partial\rho}{\partial C^2} &= \frac{C}{4\rho} \\
\sigma_\rho^2 &= (\frac{B\sigma_B}{4\rho})^2 + (\frac{C\sigma_C}{4\rho})^2
\end{aligned}$$

$\frac{B\sigma_b}{4\rho}$ and $\frac{C\sigma_c}{4\rho}$ are marginal distributions to corresponding directions.

$$\begin{aligned}
\frac{\partial R}{\partial\rho} &= \frac{\rho}{R} \\
\frac{\partial R}{\partial A} &= -\frac{1}{2R} \\
\sigma_R^2 &= (\frac{\rho}{R}\sigma_\rho^2)^2 + (\frac{\sigma_A}{2R})^2
\end{aligned}$$

$$\phi = arccos\frac{B}{2\rho}$$

$$\begin{aligned}
\frac{\partial\phi}{\partial B} &= \frac{-1}{\sqrt{1-(\frac{B}{2\rho})^2}} \cdot \frac{1}{2\rho} \\
\frac{\partial\phi}{\partial\rho} &= \frac{B}{2\rho^2\sqrt{1-(\frac{B}{2\rho})^2}} \\
\sigma_\phi^2 &= (\frac{-1}{2\rho\sqrt{1-(\frac{B}{2\rho})^2}} \cdot \sigma_{\rho B})^2 + (\frac{B}{2\rho^2\sqrt{1-(\frac{B}{2\rho})^2}} \cdot \sigma_\rho)^2
\end{aligned}$$

$$\sigma_\phi^2 = \frac{1}{4\rho^2(1-(\frac{B}{2\rho})^2)} \cdot (\sigma_B^2 + \frac{B^2}{\rho^2}\sigma_\rho^2)$$
$$\sigma_\phi^2 = \frac{1}{4\rho^2 - B^2}(\sigma_B^2 + \frac{B^2}{\rho^2}\sigma_\rho^2),$$

and, at the same time, the following condition must hold:

$$\rho^2 > B^2.$$

We are interested in circle radius which corresponds to tangential momentum p_T of a particle according to formula

$$p_T = 0.006R,$$

and tangent angle ψ of the track in the point nearest to the origin. In Fig. 2 is seen that it holds

$$\psi = \phi + \pi/2$$

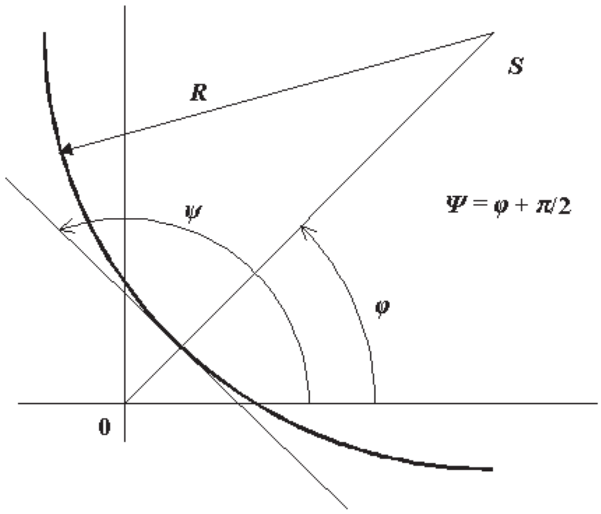

Fig. 2

Circular regression - simplified case when the approximation circle goes through the origin.

If the approximation circle goes through the origin, then the distance ρ of the centre from the origin is equal to radius of the circle R. Equation (1.1) changes its form to

$$R_i^2 = \rho_i^2 + R^2 - 2\rho_i R\cos\alpha. \tag{3.21}$$

Then there is

$$\epsilon = \sum_{i=1}^{n}(\rho_i^2 - 2\rho_i R\cos\alpha)^2$$

and

$$A_i = \rho_i(\rho_i - 2R\cos\alpha).$$

Instead of three unknown variables we have two only, R and ϕ. It holds

$$\frac{\partial\epsilon}{\partial R} = \sum 2A_i(2\rho - 2\rho_i\cos(\phi_i - \phi))$$
$$\frac{\partial\epsilon}{\partial\phi} = \sum 2A_i(-2\rho_i R\sin(\phi_i - \phi))$$

and we get a system

$$\begin{pmatrix} \sum a_i & \sum a_i\rho_i\cos\phi_i & \sum a_i\rho_i\sin\phi_i \\ \sum b_i & \sum b_i\rho_i\cos\phi_i & \sum b_i\rho_i\sin\phi_i \end{pmatrix} \cdot \begin{pmatrix} 2\rho\cos\phi \\ 2\rho\sin\phi \end{pmatrix} = \begin{pmatrix} -\sum \rho_i^2 a_i^2 \\ -\sum \rho_i^2 b_i^2 \end{pmatrix}$$

From it we should get values of

$$\begin{aligned} 2\rho\cos\phi &= B \\ 2\rho\sin\phi &= C \end{aligned} \tag{3.22}$$

and finally

$$\rho = \frac{1}{2}\sqrt{B^2 + C^2}, \qquad \cos\phi = B/(2\rho)$$

and due to assumption mentioned there is $R = \rho$. For different angle ψ of the track and momentum p_T it holds

$$\begin{aligned} \psi &= \phi + \frac{\pi}{2} \\ p_T &= 0.006R. \end{aligned}$$

With respect to badly conditioned system matrix we use again linear regression.

The sign of track radius: Let us introduce following convention. Let ϕ be angular position of the centre of the circle which approximates the track, and α be the angular position of last (most outer) hit. Then if

$$\triangle\phi = \phi - \alpha \in (0, \frac{\pi}{2}\rangle$$

then $R > 0$, else $R < 0$. (It is understood that $\triangle\phi$) is minimal oriented angle between circular track centre and the last hit of the track, see Fig. 3.

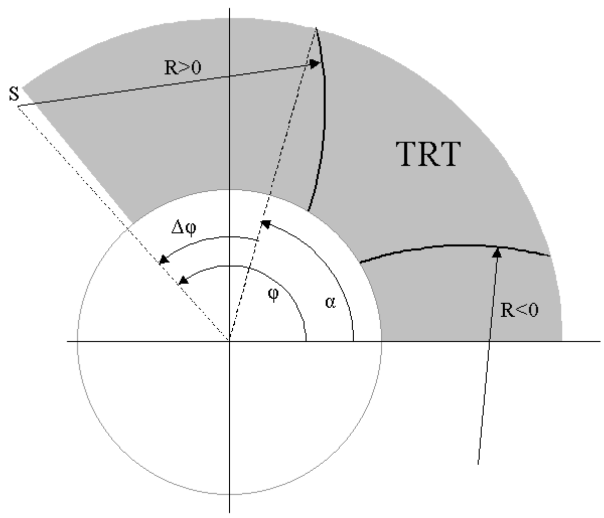

Fig. 3

3.2.3 A little pessimism about error

Let us consider three hits only: the first (1), medium hit (2) and the last hit (3), and let straw diameter be taken into account. Then we can construct three circles as shown in Fig. 4.

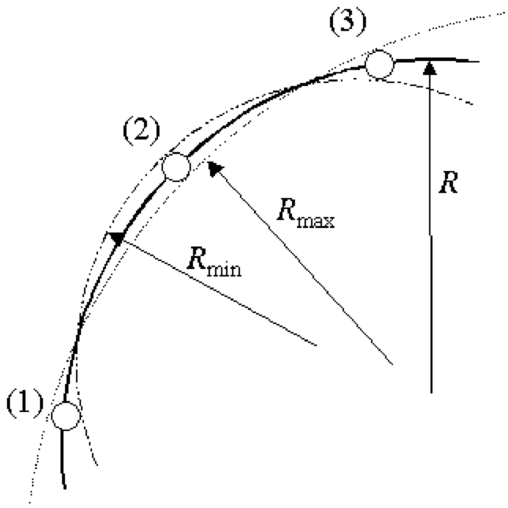

Fig. 4

In Fig.5 is easily seen that

$$\sin\gamma_s = \frac{d}{R} \qquad \sin\alpha = \frac{R_{\text{straw}}}{d/2}$$
$$\sin\gamma_m = \frac{d}{R_{\text{min}}} \qquad \gamma_m = \gamma_s + \alpha$$

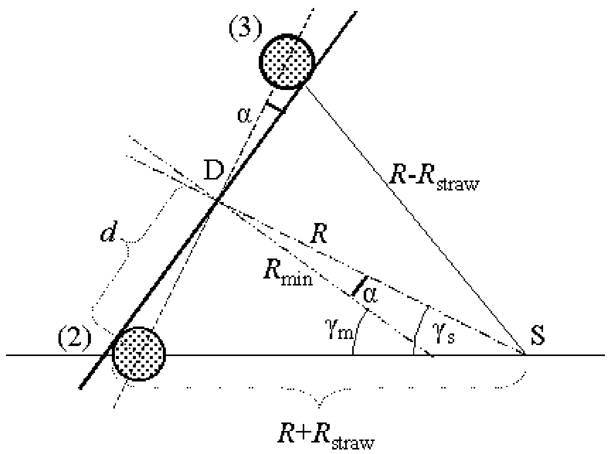

Fig. 5

Form it, after a little algebra and using approximately $\cos\alpha = 1, \cos\gamma_s = 1$ we get

$$\frac{\sin\gamma_s}{\sin\gamma_m} = \frac{1}{1 + 2\frac{R - R_{\text{straw}}}{d^2}}$$

On the other hand simply
$$\frac{\sin\gamma_s}{\sin\gamma_m} = \frac{R\text{min}}{R}$$
and combining last two equations there is
$$R_{\text{min}} = \frac{R}{1 + 2\frac{R - R_{\text{straw}}}{d^2}}$$
Similarly we get
$$R_{\text{max}} = \frac{R}{1 - \frac{R - R_{\text{straw}}}{d^2}}$$
In the end
$$R_{\text{max}} - R_{\text{min}} = 2\epsilon_R = 2 \cdot R \frac{4\frac{R - R_{\text{straw}}}{d^2}}{1 - 4(\frac{R - R_{\text{straw}}}{d^2})^2}.$$
Example:
$$d \sim \frac{1}{2}(106 - 56)cm = 25cm, R \sim 100cm, R_{\text{straw}} \sim 0,2cm.$$
Then
$$\epsilon_R = 12.8cm.$$

The worst case error of track radius is then rather large, nearly 13% and thus it has no sense to use it for any other estimation. Similarly one cannot use regression model error which is approximately of the same size.

Therefore, we will use the regression approximation of circle centre coordinations as "good" (fixed centre assumption), and we will compute average distance of all track candidate hits as track radius. Then individual deviations of hits from this average radius can be computed and finally we can get mean quadratic error of radius. This error is two orders of magnitude less than error of regression approximation or than worst case error. Moreover, knowing individual deviations of hits, one can exclude hit having largest deviation in process of "clearing" the track candidate, see Chap. 3.2.6.

3.2.4 Considerations about regression error

Using fixed centre assumption, the individual error of i-th hit is
$$\epsilon_i = R_i^2 - R^2,$$
where
$$R_i = \rho_i^2 + \rho^2 - 2\rho_i\rho\cos(\phi_i - \phi) \tag{3.23}$$
(see(3.1)). Note that ϵ_i has dimensionality cm^2. The mean quadratic error of squared track radius is given by
$$\epsilon_0^2 = \frac{1}{n}\sum_{i=1}^{n}(R_i^2 - R^2).$$
Let
$$R_i = R + \delta_i \tag{3.24}$$
where δ_i is individual deviation of i-th hit from mean track circle. Then
$$\epsilon_i = R_i^2 - R^2 = 2R\delta_i + \delta_i^2 \approx 2R\delta_i \tag{3.25}$$
and then
$$\delta_i = \frac{\epsilon_i}{2R}. \tag{3.26}$$

The mean value of $\epsilon_i, i = 1, 2, \ldots, n$ is the mean quadratic error ϵ_0 of R^2. Due to (3.26), the mean value of $\delta_i, i = 1, 2, \ldots, n$ is

$$\delta_0 = \frac{\epsilon_0}{2R}$$

and δ_0 is mean quadratic error of track radius R (with uncertainty given by approximation used in (3.25)).

For practical computation we use relation

$$\delta_o = \frac{1}{2R}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(R_i^2 - R^2)^2}.$$

Transversal momentum error The mean quadratic error δ_0 of track radius R gives, at the same time, the mean quadratic error of p_t, due to formula $p_T[GeV] = 0.006R_{[cm]}$, then $\delta_{p_{T[GeV]}} = 0.006\delta_{0[cm]}$.

3.2.5 Cleaning the track

In cleaning process we exclude from track candidate successively hits with largest individual error until for all remaining hits absolute individual errors are less than straw radius:

$$\delta_i = R_{\text{straw}}$$

$i = 1, 2, \ldots, N_0$ of remaining hits of track candidate. The individual errors δ_i follow from (3.24) using (3.23).

One bad hit case: Let in the track candidate be n "good" hits and one bad hit. The good hits in average correspond to radius R. The bad hit to the radius, say, $R + \delta$. The radius R' computed from all $n + 1$ hits will be

$$R' = \frac{1}{n+1}(n \cdot R + R + \delta) = R + \frac{\delta}{n+1}.$$

From it follows that for large n we need not count essential influence of one hit to track radius R. Let 2δ be approximately span of straws in a layer and $\delta \doteq 2R_{\text{straw}}$, see Fig 6.

Then

$$\frac{\delta}{n+1} = \frac{2R_{\text{straw}}}{n+1}.$$

Minimal number of hits in track must be N_{min}. The largest δ will be

$$\delta = \frac{R_{\text{straw}}}{N_{\text{min}} + 1}$$

and for $N_{\text{min}} = 10 \quad \delta = 0.1 \cdot R_{\text{straw}} \doteq 0.02cm$. It is seen, that bad hit near the track can cause only very slight error in track radius or in p_T.

Cleaning procedure:

1. Given track candidate.
2. Compute track parameters R, ρ, ϕ.
3. Compute individual deviations δ_i for all hits in the track candidate.
4. If there is a hit with largest δ_i and its $\delta_i > R_{\text{straw}}$, then exclude this hit from the track candidate. Go to 1.
5. If track candidate follows other conditions for track, accept it as resulting track; stop.

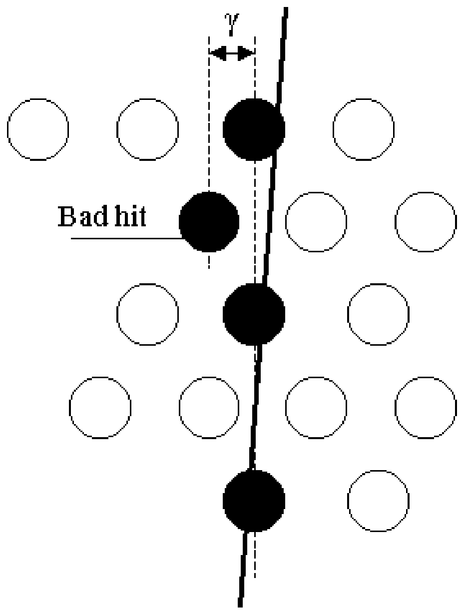

Fig. 6

Filling up procedure

1. For layer No.1 to the last layer do: if there is no hit in this layer and there is a hit in distance (centre of straw with respect to track circle) less than R_{straw}, then add this straw to the track.

2. Apply cleaning procedure. Stop.

Note. Filling up procedure is not used in the programme for computation speed.

3.2.6 Evaluation procedure

The next step is testing the results of computation. The most essential parameter is mean square deviation δ_0 of track radius computed directly according to formula

$$\delta_o = \sqrt{\frac{1}{n}\delta_i^2},$$

where δ_i are computed according to (3.23) and (3.24).

Testing procedure

- if $\delta_0 > R_{\text{straw}}$ then
  - if $|R_2| > \rho + \delta_0$ then it is a bad circle, $T = -1$, end
  - else it is perhaps good circle but with rather large error. Exclude the worst hit and go back to the computation of δ_0 without that hit; $T = 0$.
- else it is sufficiently exact circle
  - if $|R_2| > \rho + \delta_0$ then it is a bad circle, $T = -2$, end
  - else it is a good and exact circle
    * if $|R_2| < 70$ then circle is too small, $T = -4$, end
    * if $||R_2| - \rho| \leqq R_{\text{straw}}$ then the circle goes through the origin, $T = 1$, end

* else it is possibly the second Bremstrahlung circle. Perform Bremstrahlung evaluation and assign proper value to T, end

The output informative value is T
$T = -4$: the circle is exact, but too small
$T = -3$: for bad situation in testing Bremstrahlung, see later
$T = -2$: the circle is exact but bad so that origin lies inside the circle
$T = -1$: the case as previous, but error is large
$T = 0$: error δ_0 is too large. Decrease it by omitting the worst hit and repeating the whole error evaluation procedure
$T = 1$: exact circle going through origin (the track originates in primary vertex)
$T > 1$: results of testing Bremstrahlung, see later.

3.3 Bremstrahlung

If the track identified and tested is sufficiently exactly circular and origin lies outside this circle, then the track may originate from Bremstrahlung. Bremstrahlung may arise only where there is some mass. This is on layers of silicon detector or everywhere in inner part of TRT. Using this fact we can compute hypotheses about primary, i.e. the first part of electron's or positron's track. In the case of Bremstrahlung the track (the B-track) consists of two parts. The first part of B-track has larger p_T (larger radius R_1) than the second part of B-track, begins in the origin and ends in some layer of silicon detector or in TRT. Let us denote this point B. In the point B some energy is lost, but electron of positron continues its move in the same direction but with smaller p_T and then the B-track has now smaller radius. From it and from elementary geometrical rules follows that Bremstrahlung point B, the centre S_2 of the second part of the B-track and the centre S_1 of the first part of the B-track must lie on the same straight time as shown in Fig. 7. Moreover, the first part of the B-track must start nearly exactly in the origin O due to short lifetime of intermediate particles.

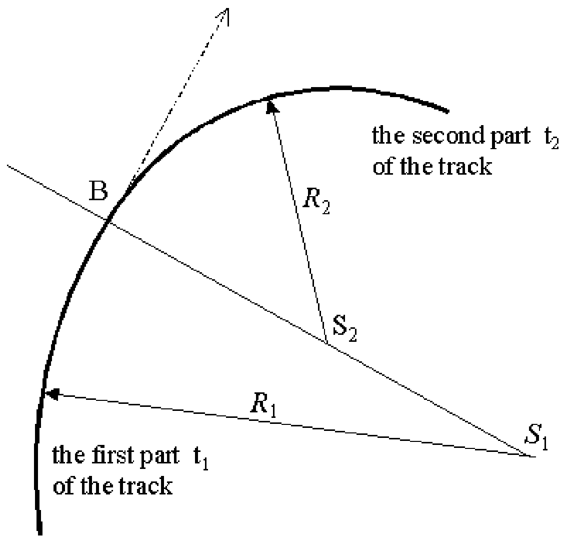

Fig. 7

3.3.1 The slowest and fastest fast part of B-track

Once the second (slow) part of B-track is identified, a question arises what are all possible fast tracks.

Limit case – the infinitely fast first part of the B-track

Let us assume, that electron's or positron's speed is infinite and then the first part of the track is a straight line as shown in Fig. 8. In this case the first part t_1 is a straight line and it is tangent to the slow part t_2 of the B-track. In this case the Bremstrahlung point B is (for given t_2) in shortest distance from the origin O. Then the distance $\overline{OB}$ is minimal among all possible t_1 parts of the B-track. The triangle OBS_2 has right angle in point B and then from Pythagoras theorem follows:

$$\overline{OB_{min}} = d_{B_\infty} = \sqrt{\rho^2 - R_2}. \tag{3.27}$$

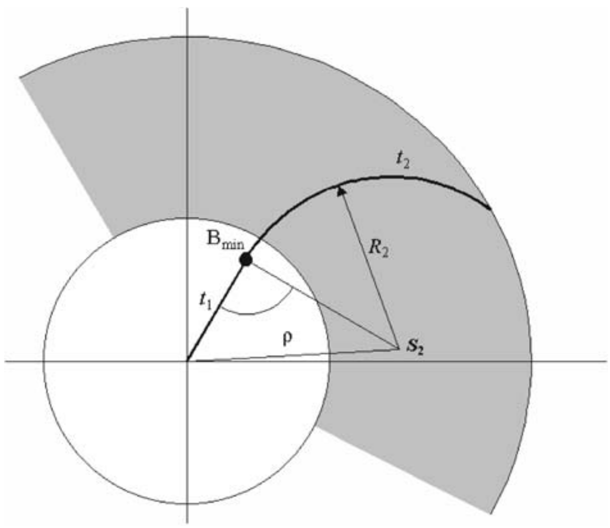

Fig. 8

The slowest first part of the B-track

When the second part t_2 of (possible) B-track is identified, we know also its first hit. It is a hit, which is nearest of all hits of t_2. Its distance from origin is given by radius of corresponding layer and let us denote it as $R_{\text{layer}H1}$. At the same time the Bremstrahlung point B cannot be farther from the origin than the first hit. So, maximal distance of B-point from origin is

$$\overline{OB_{max}} = d_{B_0} = R_{\text{layer}H1} \tag{3.28}$$

The real Bremstrahlung point must lie somewhere between d_{B_∞} and d_{B_0}.

3.3.2 Bremstrahlung point B in general position

Let the second part of the track t_2 (the triple (R_2, ρ_2, ϕ_2)) and the distance of B-point from the origin d_B be given. The task is, what radius R_1, or p_{T1}, has the first part of the track t_1.

In the triangle OS_2Bi, see Fig. 9, is easily seen that

$$\rho_2^2 = R_2^2 + d_B^2 - 2R_2 d_B \cdot \cos\delta$$

In the triangle OS_1C simply $\cos\delta = d_B/(2R_1)$. Eliminating $\cos\delta$ one gets finally

$$R_1 = \frac{R_2 d_B^2}{R_2^2 + d_B^2 - \rho_2^2}. \tag{3.29}$$

$$p_{T1_{[GeV]}} = 0.006 R_{1_{[cm]}} \tag{3.30}$$

The other unknown value is the angular position ψ_B of the Bremstrahlung point.

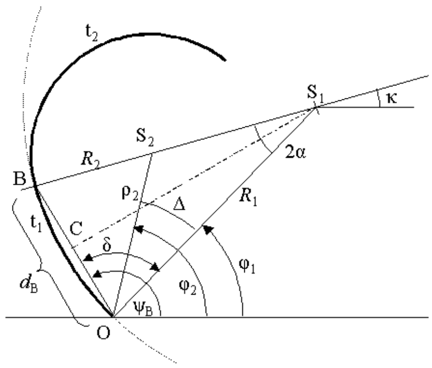

Fig. 9

In Fig. 9 in the triangle OS_2B cosinus theorem holds

$$R_2^2 = \rho_2^2 + d_B^2 - 2\rho_2 d_B \cdot \cos\gamma$$

and then

$$\gamma = \arccos\frac{\rho_2^2 + d_B^2 - R_2^2}{2\rho_2 d_B}; \qquad \gamma \in \langle 0, \frac{\pi}{2}) \tag{3.31}$$

$$\psi_B = \phi_2 + \gamma \cdot \text{sign}R_2. \tag{3.32}$$

3.3.3 Algorithm for forming Bremstrahlung hypotheses

1. Compute the minimal distance $d_{B\infty}$ of Bremstrahlung point, see eq. (3.27). Given d_{B0}, see (3.28).

2. Set successively d_B equal to radii $R_{Si2}, \ldots$ of individual silicon layers starting from the second layer (R_{Si2}) or from the most inner layer k for which

   $$R_{Sik} \geqq d_{B\infty}$$

   and until

   $$R_{Sik} \leqq d_{B0}.$$

   Because the Bremstrahlung may arise also in TRT we can consider its individual inner layers which have radii less or equal to d_{B0} as some other layers above the last layer of Si detector. These layers with radii $R_{Si8}, R_{Si9}, \ldots$ can be used in the following computations the same way as real silicon detector layers. Of course, as Bremstrahlung may arise anywhere in TRT, we will get only some discrete hypothetical samples, but "layers" in TRT one can choose arbitrarily dense.

3. For each d_B according to point 2

   (a) compute the radius R_1 of the first part of the B-track according to formula (3.29) and p_{T1} according to (3.30),

(b) set successively d_L equal to radii $R_{Si1}, R_{Si2}, \ldots$ (including possibly also radii of layers in TRT as mentioned in point 2) starting from $R_{Sik1}(k_2 = 1)$ and until $R_{Sik2} \leqq R_{Sik}$.

4. For each d_L according to point 3(b) and then each d_B according to point 2. compute angular position ψ_B of the point where electron or positron went through the corresponding layer according to formula

$$\psi_B = \phi_2 + \gamma \cdot \mathrm{sign} R_2.$$

where

$$\gamma = \arccos \frac{\rho_2^2 + d_L^2 - R_2^2}{2\rho_2 d_L}; \qquad \gamma \in \langle 0, \frac{\pi}{2})$$

(see (3.32) and (3.31))

Thus we have set of possible radii R_1 and p_{T1} of the first part t_1 of the B-track and for each this radius or p_T we have angular position ψ_B where t_1 goes through individual layers of silicon detector (and also layers in TRT if we use them and if it is possible).

3.3.4 Classification of results

This part is, in fact, continuation of evaluation procedure described in 3.2.6. Here we deal with part when possible existence of Bremstrahlung was identified.

There:

... it is possibly the second Bremstrahlung circle:

- if $d_B < d_{b\infty}$ then there exist a hit in TRT which is nearer to origin then is the shortest distance where Bremstrahlung can arise and then this is not the Bremstrahlung case, $T = -3$, end
- set T=number of accepted hits of track
- if $R_{Si1} > D_{B0}$, then there is no way how to certify Bremstrahlung: no necessary data exist, set T=1000 + number accepted hits, end
- if $R_{Si7} > D_{B\infty}$ (or $R_{Si_{last}} < D_{B\infty}$ if some layers in TRT were also established) then Bremstrahlung may arise outside the silicon detector (or above last layer considered), T=2000+number of accepted hits, end

Else compute parameters of all possible first parts of B-track according to Chap. 3.3.3. Finish.

4 Conclusion

The algorithm developed is based on idea to use instead of distances their squares. Thus most of dependencies related to circular track in elementary particle detectors changes to linear. It was shown that especially linear regression of squared variables is applicable in this case. The whole algorithm of elementary particle track identification has essentially three steps, identification of track using circular regression and rather rough data from TRT detector, forming track hypotheses using geometrical conditions for central as well as "broken", i.e. bremsstrahlung tracks and verification and selection of the best of these hypotheses for each track using the circular regression once more using data from both TRT and silicon detector.

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