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Technical Report No. 868

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Abstract:

The algorithm for searching tracks of electrons and positrons in transition radiation detector (TRT) is described and its behavior shown. The algorithm is based on searching a new hit near to the hit which is already supposed as a member of the track candidate in the supposed direction. The searching procedure reminds Huygen's principle. After the track candidate is found, several criteria are tested, especially whether the track is really circular, goes through the origin and has sufficiently large radius (the pT) - the conditions for the tracks which may arise from b-physics events.

Keywords:

particle track, track searching, b-physics, linear regression

Contents:

I. Introduction

The Task

From physics point of view the B-physics studies include the search for and measurement of so called CP violation through decays $B_d^0 \to J/\psi K_S^0$, $B_d^0 \to \pi^+\pi^-$ and $B_S^0 \to J/\psi \phi$. The other decay $J/\psi \to e^+e^-$ produces electrons which can be identified using transition radiation tracker (TRT) [1]. There are several other b-physics decay processes which produce electrons and positrons. Thus the b-physics events are identified through identification of special electrons in TRT.

Standard algorithms as XKALMAN [2] or the one used in DELPHI detector [3], as well as in ALICE detector [4] - [6], use a transform of circular TRT to polar coordinates, which changes the circular tracks going through the origin to straight lines, and searches these straight lines systematically in all possible directions and places or build the track starting from the initial hit, search for next hit which can be a candidate and at the same time with filtering procedure the outliers are rejected.

In this report a new algorithm is proposed. It reminds a little Huygen's principle, but the wave spreads only in limited sector of possible directions. After a set of hits which may, but need not form a track, is found (so we get the track set), the track approximation is computed and outlier hits are deleted from the set to get track candidate (the tracks found). These track candidates are again tested with respect to other criteria and so we get tracks accepted.

In Fig. 1a an so called event in TRT is seen. The inner radius of this part of detector is 63 cm, the outer radius 107 cm. TRT consists of „straws“, a thin 4 mm in diameter detecting elements, which can be easily seen in Fig. 1b. There are total 50 048 straws in 73 concentric layers in the TRT. In Fig. 1a there is lot of tracks shown. Each track represents a trajectory of a particle, electron or positron. Because there is axial magnetic field of 2 T (tesla), the trajectories of these charged particles form a part of a circle. For particles arising from b-physics events these circles go through the centre of detector, the origin, more exactly nearly through the origin. The distance can be several milimeters only, otherwise the track cannot arise from b-physics event.

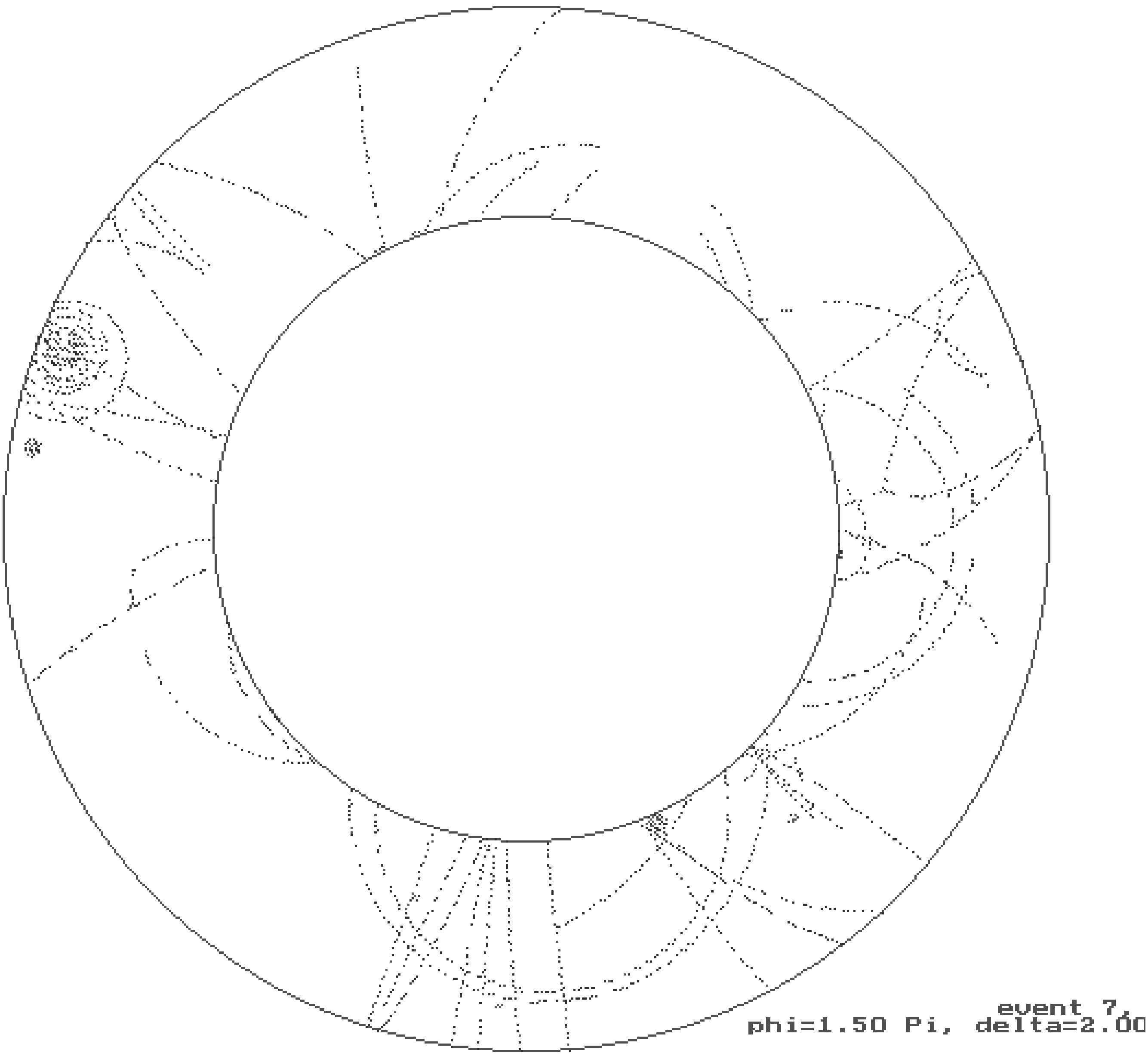

Figure. 1a. The cross section of TRT with trajectories shown.

The radius of a trajectory depends on the velocity, and then on the transversal momentum pT of the particle. Hence holds the formula

$$pT = 300H\rho\,,$$

where H is intensity of magnetic field and ρ track radius. There is a simple relation between the track radius and pT:

$$pT[GeV] = 0.006\rho[cm].$$

To the minimal value of pT 0.5 corresponds a track of radius ρ 83 cm. So, we are looking for tracks which:

1. have radius at least 83 cm
2. go through the origin.

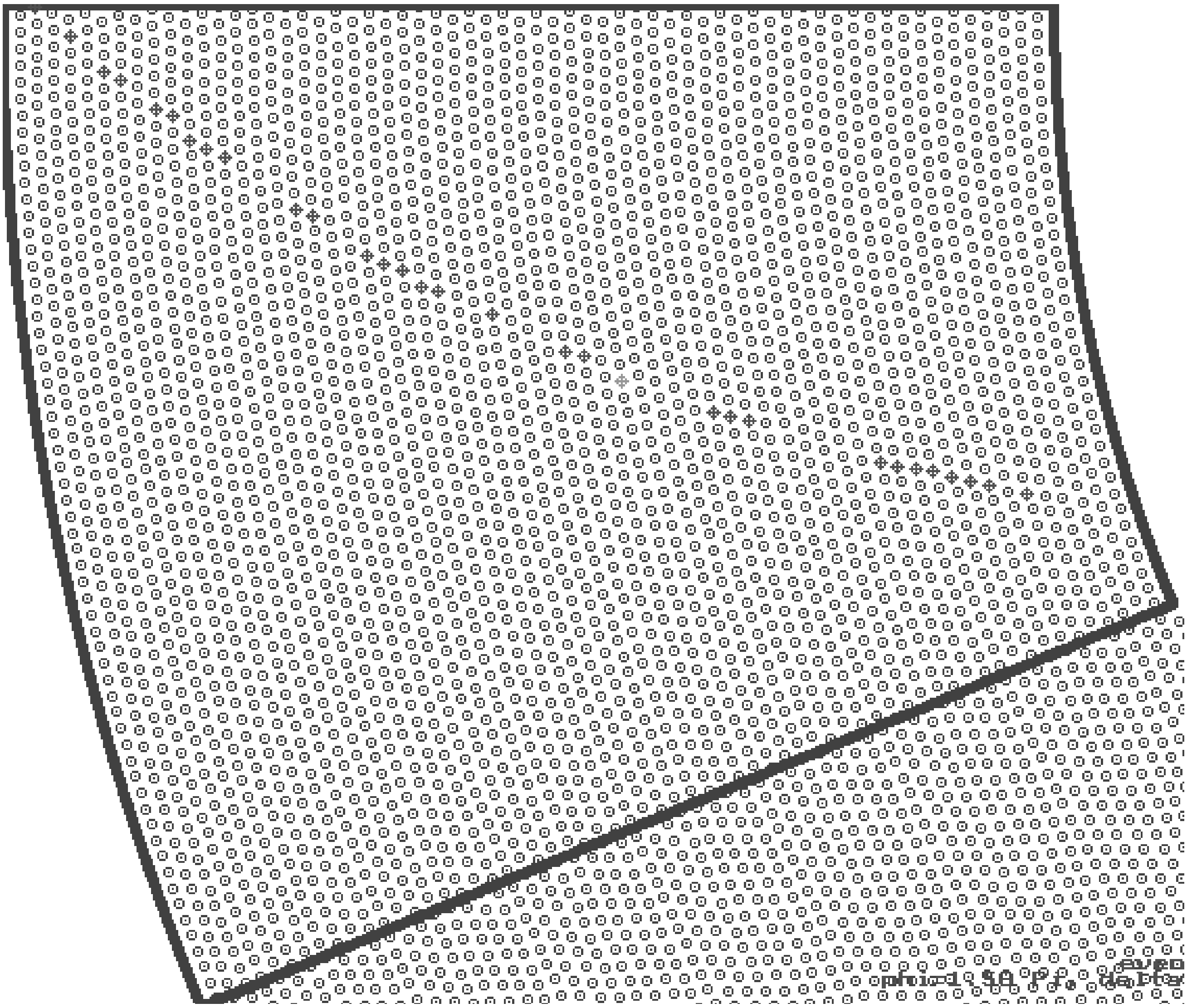

Figure 1b. A part of TRT cross section with individual „straws“ seen and one track (set of „hitted“ straws shown - the straws with crosses).

II.The algorithm

The track set building

The algorithm uses the information on straws position which is given in polar coordinates. When a hit is given which is supposed to be already a member of the track candidate (the „core“ straw) a new hit is searched in the supposed direction and not too far from the core straw. The area which must besearched forms a „cone“ or a triangle; on the top is the core straw and the height of the cone or triangle corresponds to the maximal allowed distance (counted in number of layers) of hits on an acceptable track. Then it is necessary to search only a small part of straws in the TRT as seen in Fig. 2a.

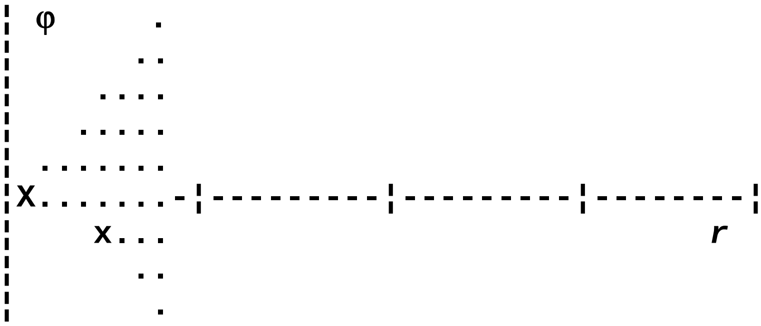

Fig. 2a. The „cone“. On the left side is the inner layer of the TRT, on the right side the last outer layer of the TRT. By „X“ the core straw is denoted, dots show the straws searched, and „x“ is a hit which has been found.

The maximal angle of the „cone“ corresponds to the smallest track radius, and hence to the smallest pT. Thus the track candidates of too low pT or not enough dense are automatically excluded already during the track searching.

When some hits in a cone are found, these hits are used as a new core straw and new cone is searched as shown in Fig. 2b.

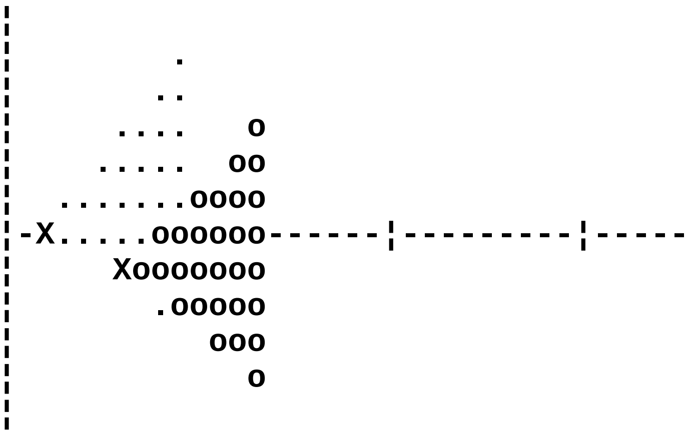

Figure 2b. The second core straw and new cone.

The process of finding hits in cones continues until the lats layer of the TRT is reached. The all hits found forms a track set. The procedure is shown in Fig. 3 where is also shown the direction of the track and it is easily seen the hits which are members of the track candidate.

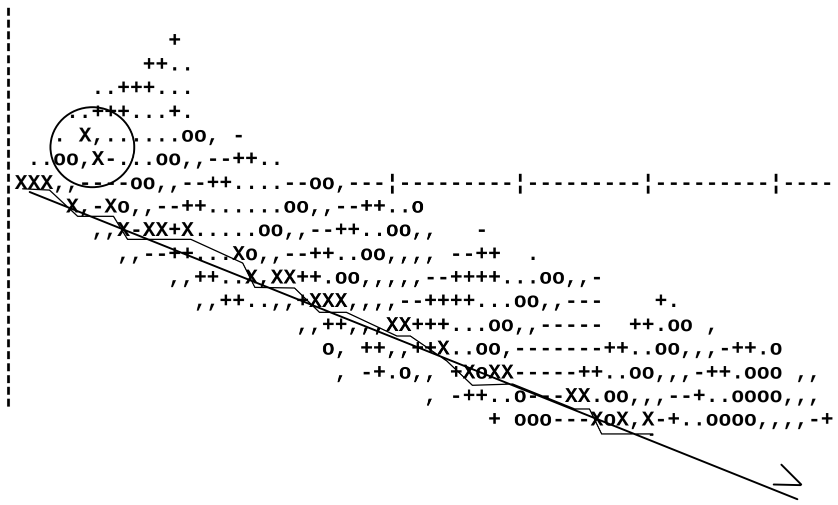

Figure 3. The track building.

At the same time in a circle we see two hits of the track set which has been found during the track building but apparently does not belong to the track. We will discuss it later. It is obvious that during the track building we search straws only near the preceding hit.

During the searching procedure some conditions are followed: The track should

- have sufficient number of hits (10)
- start not far than in (7)th layer
- end not earlier than in (65)th layer of 73
- not to be broken for more than (8) layers

which is illustrated in Fig. 4.

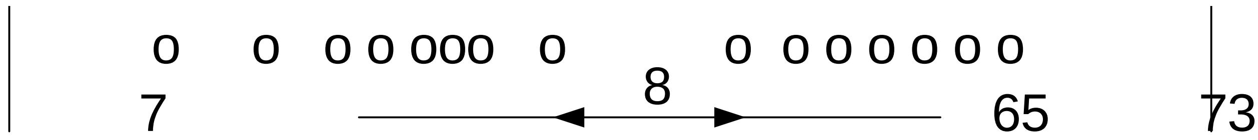

Figure 4. The conditions for track building.

Backtracking and testing

There may found some hits in the track set which does not belong to the track formed or there may be several real tracks in the track set found during one track set building procedure. To exclude the randomly scattered hits and to split the track set of hits to individual track candidates the backtracking is used. Simply, each core straw is a predecessor for hits in its corresponding cone. The backtracking starts from the last hit found and goes backwards according to predecessors. The sequence of predecestors thus formed is shown in Fig. 3 by thinner broken line. Thus a simple track is formed and its hits are excluded from the track set. If the track set is not empty, the procedure is repeated and another track is formed, if the conditions above are fulfilled, etc.

After the track is formed, i.e. the set of straws which forms it is found, the track must be tested whether it fulfills essential conditions for the tracks which may arise from b-physics events and wich has been mentioned in the end of Chap. 2. The first condition, i.e. radius of the track should be larger than 83 cm which corresponds to minimal transversal momentum pT = 0.5 GeV, is in essence fulfilled due to correspondingly tuned angle of the cone, but it must be checked in relation with the second condition that the track should go through origin.

Let us derive the procedure for verification of these conditions. We start from the fact that the track is a set of *n* straws and for each straw (*i*) several parameters are known:

- the layer to which it belongs and then the distance from the origin r_i,
- its angular position ϕ_i, and others,

but now only these two parameters will be used. Because in polar coordinates the part of circle of relatively large radius which corresponds to the part seen in TRT is nearly the straight line, we can use a linear regression. Let for straw *i* holds

$$\phi_i = \phi_0 + \alpha r_i \ .$$

Then *n* such equations for straw 1 till *n* can be written in the form

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \cdots \\ \phi_n \end{pmatrix} = \begin{pmatrix} r_1 & 1 \\ r_1 & 1 \\ \cdots & \cdots \\ r_n & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \phi_0 \end{pmatrix} \ ,$$

simply

$$\mathbf{y} = \mathbf{X}.\beta\ .$$

This is shown in Fig. 5.

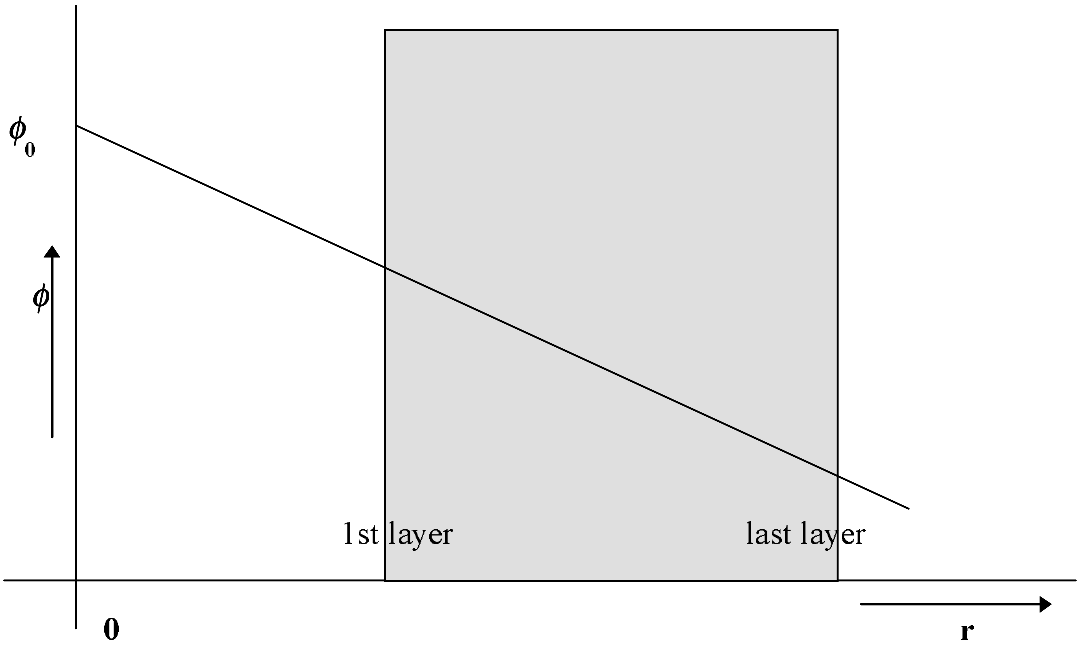

Figure 5. One track goint through the origin in TRT in polar coordinates.

Least mean squares solution of this equation is

$$\beta = \begin{pmatrix} \alpha \\ \phi_0 \end{pmatrix} = \left(\mathbf{X}^t \mathbf{X}\right)^{-1} \mathbf{X}^t y \quad .$$

For evaluation of error the following procedure is used. Let ϕ_0 be fixed and for each straw the radius of the track ρ_i is computed by the formula

$$\rho_i = \frac{r_i}{2\sin(\phi_0 - \phi_i)} \ .$$

This formula follows from simple geometry as shown in Fig. 6.

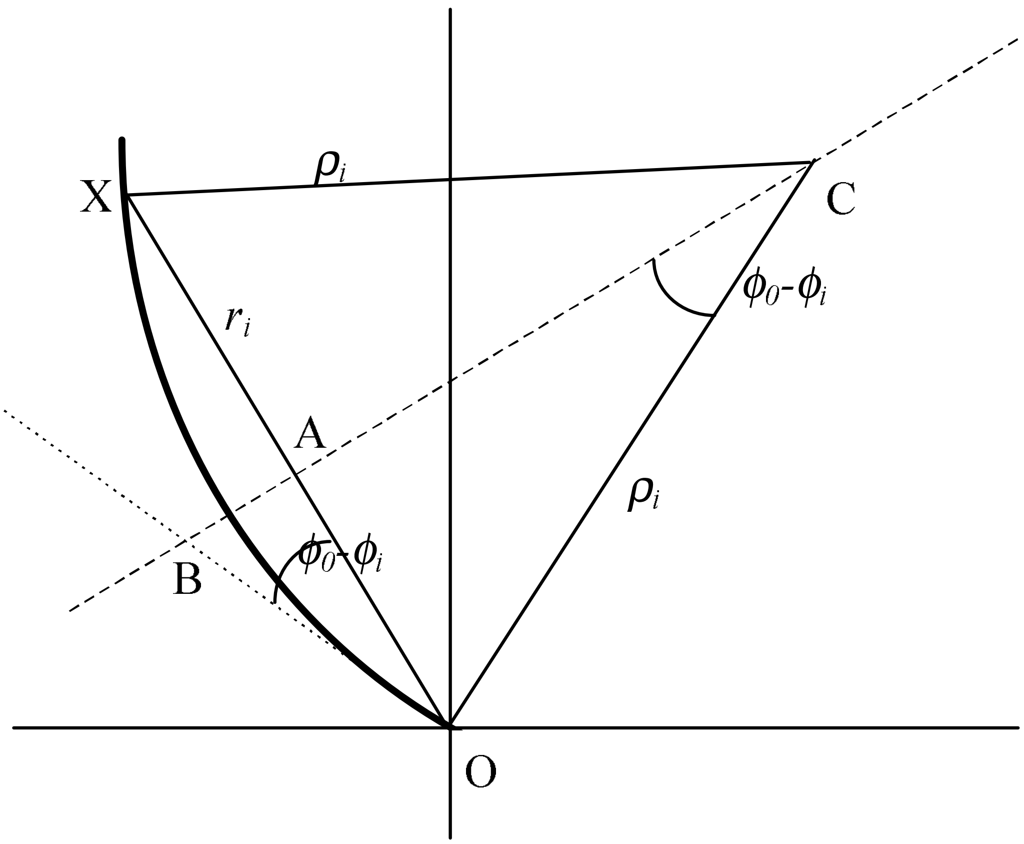

Figure 6. Illustration of geometry for the formula for track radius. The bold circular line is the track, the dotted line the tangent to the track in the origin, the dashed line divides the triangle 0CX to halves. The triangle 0CA is similar to the triangle 0BA and then the angles 0CA and A0B are of the same size $\phi_0 - \phi_i$.

This value is supposed to have standard normal distribution and then the mean value and mean squared deviation σ^2 is computed. The ratio $\sigma / \overline{\rho}$, more exactly its absolute value, gives the relative error for level 0.68 (N(0,1) for $\pm\sigma$).

Analysis of transform based algorithm for track searching

The tracks we are searching for are characterized in x, y plane by circular shape, where the circle goes through the primary vertex - origin. In fact, we see only part of these circles in TRT and they are formed by individual hitted straws. Each such a circle is characterized by two parameters, the pT and by the tangential angle in the vertex, ψ. The pT corresponds to the radius ρ of the circle by formula pT[GeV] = 0.006ρ [cm], i.e. ρ [cm] = 167pT[GeV]. Thus, 83 cm corresponds to 0.5 GeV. Each hit is characterized by its polar coordinates, φ and r. If the polar coordinates are used as orthogonal coordinates (φ horizontally and r vertically) the circle going through the vertex forms a line (approximately a straight line, but with very smal error). The line is given by equation (see Fig. 6b.)

$$r = 2\rho \sin(\varphi - \psi) \ .$$

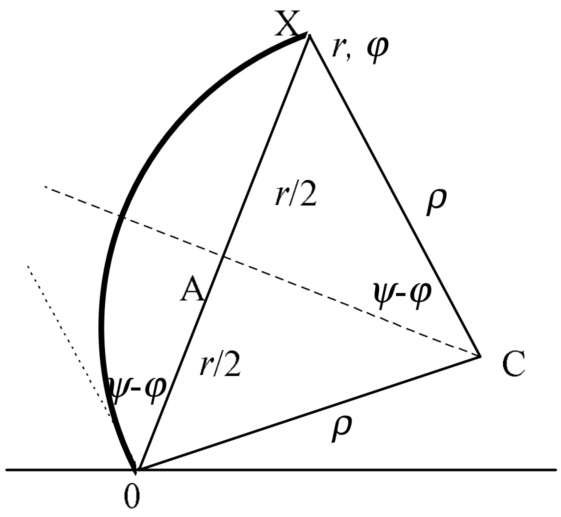

Fig. 6b.

A bit of geometry

Let us consider a single hit with coordinates r_0, φ_0. Through this hit and the vertex can be drawn many circles (a bunch of tracks) with different radii and different tangential angles in the vertex. In orthogonal coordinates φ, r these circles form a bunch of lines all going through the hit mentioned, i.e. the point (r_0, φ_0). General equation of these lines follows from equations $r = 2\rho\sin(\varphi - \psi)$ and $r_0 = 2\rho\sin(\varphi_0 - \psi)$ from which

we get $\psi = \varphi - \arcsin\frac{r}{2\rho}$ and $\psi = \varphi_0 - \arcsin\frac{r_0}{2\rho}$. Eliminating ψ, there is

$$\arcsin\frac{r}{2\rho} = (\varphi - \varphi_0) + \arcsin\frac{r_0}{2\rho}\ .$$

Approximately $\arcsin x = x$ for small x and then we get the approximate equation

$r = r_0 + 2(167pT)(\varphi - \varphi_0)$ after a little algebra.)

Changing dependent and independent variables r and φ we get

$$\varphi = \arcsin(r(\tfrac{1}{333pT})) + \varphi_0 - \arcsin(r_0(\tfrac{1}{333pT}))\ .$$

($\varphi = r(\frac{1}{333pT}) + \varphi_0 - r_0(\frac{1}{333pT})$ approximately.)

This equation of bunch of lines going through the point (φ_0, r_0) is governed by a coefficient $\frac{1}{333pT}$ and the shift which is just the tangential angle of a circle, and which is different for each pT and then for each circle:

$$\psi = \varphi_0 - \arcsin(\tfrac{r_0}{333pT})\ . \qquad (1)$$

In fact, this is a nearly linear dependence (if approximated by $\psi = \varphi_0 - \frac{r_0}{333pT}$) with respect to $x = \frac{1}{pT}$ and with fixed coefficients φ_0, r_0 given by the hit considered. The equation above describes a single line as shown in Fig. 6c instead of a bunch of lines.

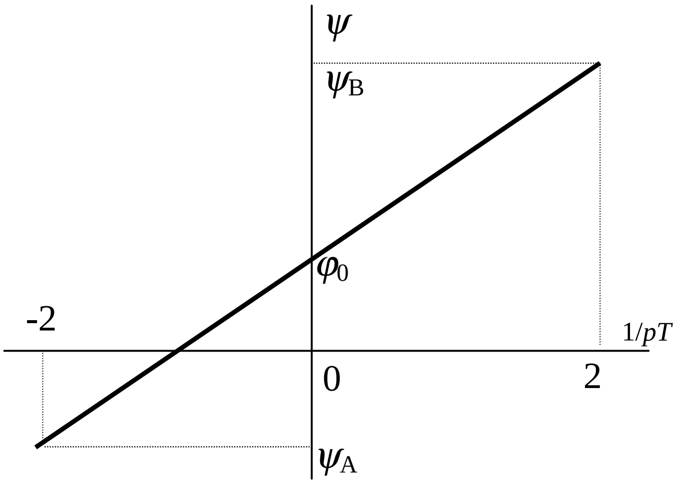

Fig. 6c

Finally, the values ψ_A and ψ_B (see Fig. 6c) for $|pT| > 0.5$ GeV are computed from simple equations

$$\psi_A = \varphi_0 - \arcsin(0.006 r_0) \cong \varphi_0 - 0.006 r_0 \ ,$$

$$\psi_B = \varphi_0 + \arcsin(0.006 r_0) \cong \varphi_0 + 0.006 r_0 \ . \qquad (2)$$

The error of approximation is calculated in the table below:

r_0 cm	$0.006{*}r_0 = X$	arcsin(X)	arcsin(X) - X	(arcsin(X) - X)/arcsin(X)
56.34	0.33804	0.344833518	0.006793518	1,97%
105.98	0.63588	0.689148195	0.053268195	7,73%

This systematic error can be lowered using a value 0.006306 instead of 0.006 in approximating formulae; the error is then -3.03% and +3.02% for minimal and maximal radii.

If there is another hit (φ_1, r_1), then there is another equation

$$\psi = \varphi_1 - \arcsin(r_1(\tfrac{1}{333pT})) \qquad (3)$$

and a similar line as above can be drawn.

Let there is a single circle going through vertex and through both hits (φ_0, r_0) and (φ_1, r_1). Then we can solve the system of (1) and (3) and get the parameters of the circle (or track), ψ and $\frac{1}{333pT}$, i.e. pT.

According to overlapping intervals (ψ_A, ψ_B) for different straws we seek for group of straws which may but need not form a track. The place of maximally overlapping intevals let be characterized by angle j .

In Fig. 6c there are shown three lines corresponding to three hits. Supposing some tollerance of the angle and its normalisation and transform to integer in limits 0 to NSLLL we have the angle as an integer j , and its tollerance say from j-1 to j+1. We can estimate the limits pTA and pTB of pT for each straw, see the rising line in Fig. 6c. Using (3) we put

$$\psi = \varphi_1 - \arcsin(r_1(\tfrac{1}{333pT})) = j\,\tfrac{2\pi}{NSLLL} \ .$$

From it there is

$$pT = \frac{-r_0}{333\sin(\frac{j2\pi}{NSLLL} - \varphi_0)}$$

and simply

$$pTA = -r_0 / 333 \sin(\tfrac{2\pi(J+1)}{NSLLL} - \varphi_0) \quad \text{and} \quad pTB = -r_0 / 333 \sin(\tfrac{2\pi(J-1)}{NSLLL} - \varphi_0)\,.$$

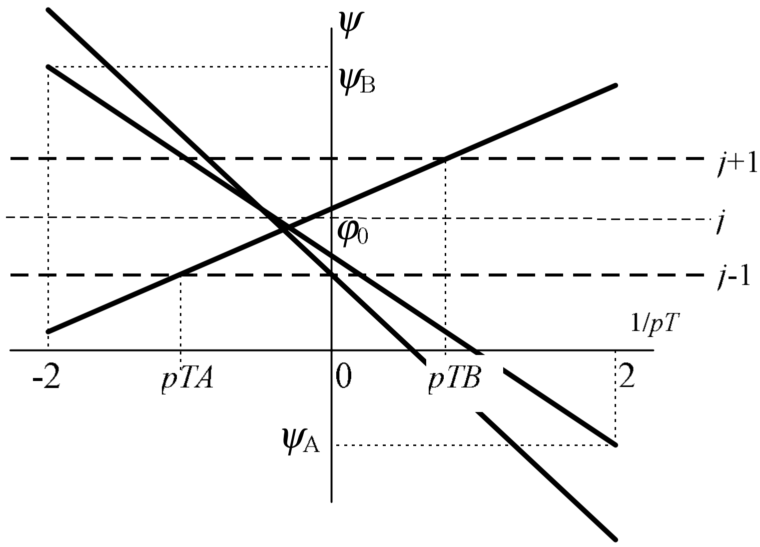

Fig. 6c

Conditions for lines to mutually intersect

The overlapping intervals does not mean that the lines really intersect. Let there are two lines *a* and *b* with starting angles ψ_{A1}, ψ_{A2}, and end angles ψ_{B1}, ψ_{B2}, fulfilling the conditions $\psi_{A1} < \psi_{A2}$, $\psi_{B1} > \psi_{B2}$.

Algorithm

Note. Because all straws in one layer have the same distance from the primary vertex, the angular difference $\arcsin(0.006 r_0) \cong 0.006 r_0$ is the same for all straws in the layer.

The following algorithm does not rely on any sorting; the hitted straws can have an arbitrary position in the file of hitted straws.

To understand the algorithm, imagine a large matrix of as many rows as there are straws in the last (largest) layer of the TRT and columns corresponding to individual hits. Thus rows would correspond to some discretization of the angular interval (0, 2π). We fill each column by 1s in all entries which would correspond to interval from ψ_A and ψ_B for particular straw. This column corresponds to horizontal projection of the line in Fig. A to a vertical line. We sum horizontally these projections to find local maxima, i.e. the places where larger number of lines probably intersect and thus the corresponding hits may lie on the same track.

0) There is an array of hitted straws. For each straw there is its position StrawPos (0..HitsTotal) in this array, its StrawNumber, i.e. its „absolute“ position 0..~50 000, and the corresponding layer or radius.
1) Form array for „row sums“ RowSums[0..NSLLL-1], initially all zeros.
2) For each straw

   compute ψ_A and ψ_B (PsiA, and PsiB, respectively - save these values for use in the step 7) according to eq. (2). In fact, these variables are converted to integers after normalisation by number of straws in the last (largest) layer NSLLL (divide by 2π and multiply by NSLLL).

for i from PsiA to PsiB do RowSums[i]++

(It is necessary to check the cases when PsiA or PsiB are outside the original interval (0, 2π) or (0, NSLLL-1).)

3) Locate one from all local maxima in RowSums which is larger than or equal to HitsCountMin (the minimal number of hits which must form a track); its index in RowSums is j.

Make the array Tracks empty.

For each straw (from all straws) do

if (PsiA<=j AND j<=PsiB) then write the corresponding straw into the Tracks.

(It is necessary to check the cases when PsiA or PsiB are outside the original interval (0, 2π) or (0, NSLLL-1).)

It was grouping with respect to ψ.

4) Now we group the straws with respect to pT:

Form an Array[0..99] of integers, initially all zeros.

For each straw in the Tracks with parameters φ_0, r_0

compute $pTA = -r_0 / 333\sin(\frac{2\pi(J+1)}{NSLLL} - \varphi_0)$ and $1/pTa = 25/pTA + 50$

and $pTB = -r_0 / 333\sin(\frac{2\pi(J-1)}{NSLLL} - \varphi_0)$ and $1/pTb = 25/pTB + 50$.

The $1/pTa$ and $1/pTb$ are both integers, $1/pTa < 1/pTb$.

For i = $1/pTa$ to $1/pTb$ do Array[i]++ ;

(It is necessary to check the cases when 1/pTa or 1/pTb are outside the original interval.)

Find a maximum in the Array; its position (index) let be k

For each straw in the Tracks do

if $k \notin (1/pTa, 1/pTb)$ then remove the straw from the Tracks.

5) Test elementary conditions for a track (number of straws, not a broken line). All straws in Tracks now should have rather close values of pT and ψ but lie in different layers.

6) Test the track formed by the straws recorded in the Tracks using linear regression and compute its pT and ψ. If the error is sufficiently low, denote the track as accepted. (This step corresponds to the similar step in the wave algorithm.)

Test the individual straws errors. If larger than e.g. 3σ, exclude the straw and repeat the procedure of this step.

7) (Optional) If the track is accepted then remove the corresponding straws from RowSums as follows:

For all straws of the track do

for i=PsiA to PsiB do RowSums[i]--

(It is necessary to check the cases when PsiA or PsiB are outside the original interval (0, 2π) or (0, NSLLL-1).)

8) repeat 3) to 7) until all local maxima are exhausted.

III. Results

For tests was used 37 events generated using PYTHIA program in CERN.

The track finding ability of the algorithm is shown in Figs. 7 and 8 where a part of event and the tracks found are depicted, respectively.

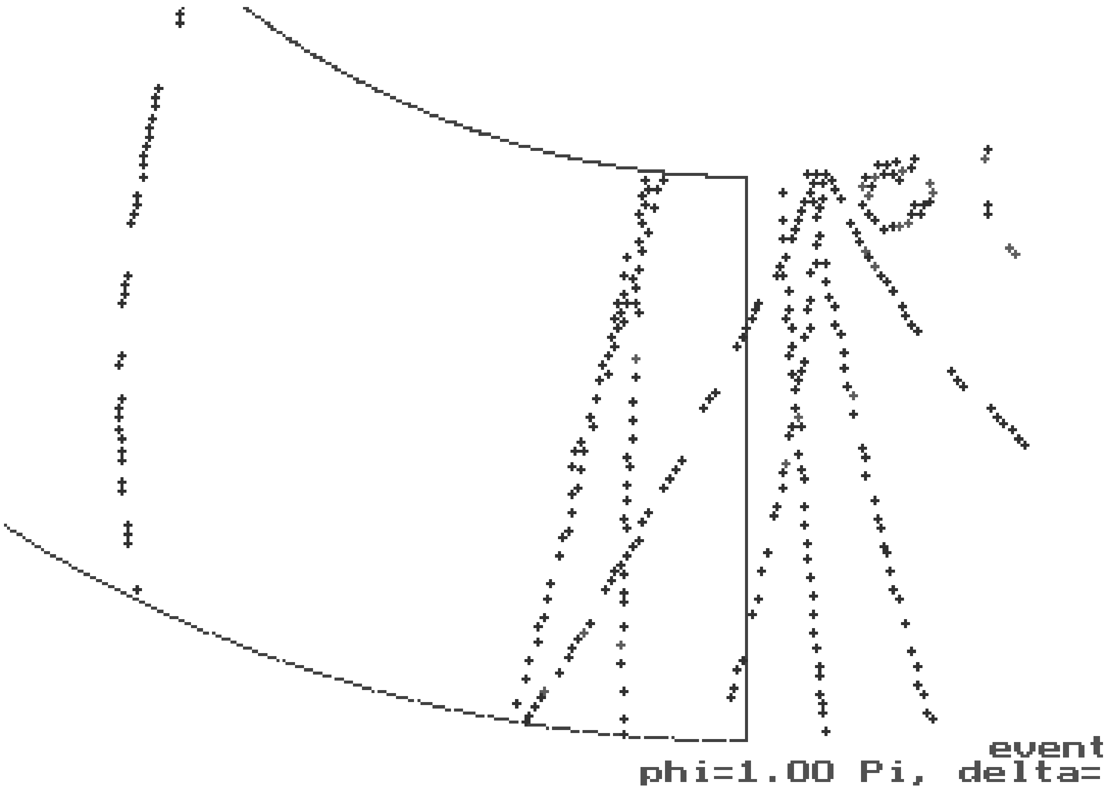

Figure 7. A part of event as seen in TRT.

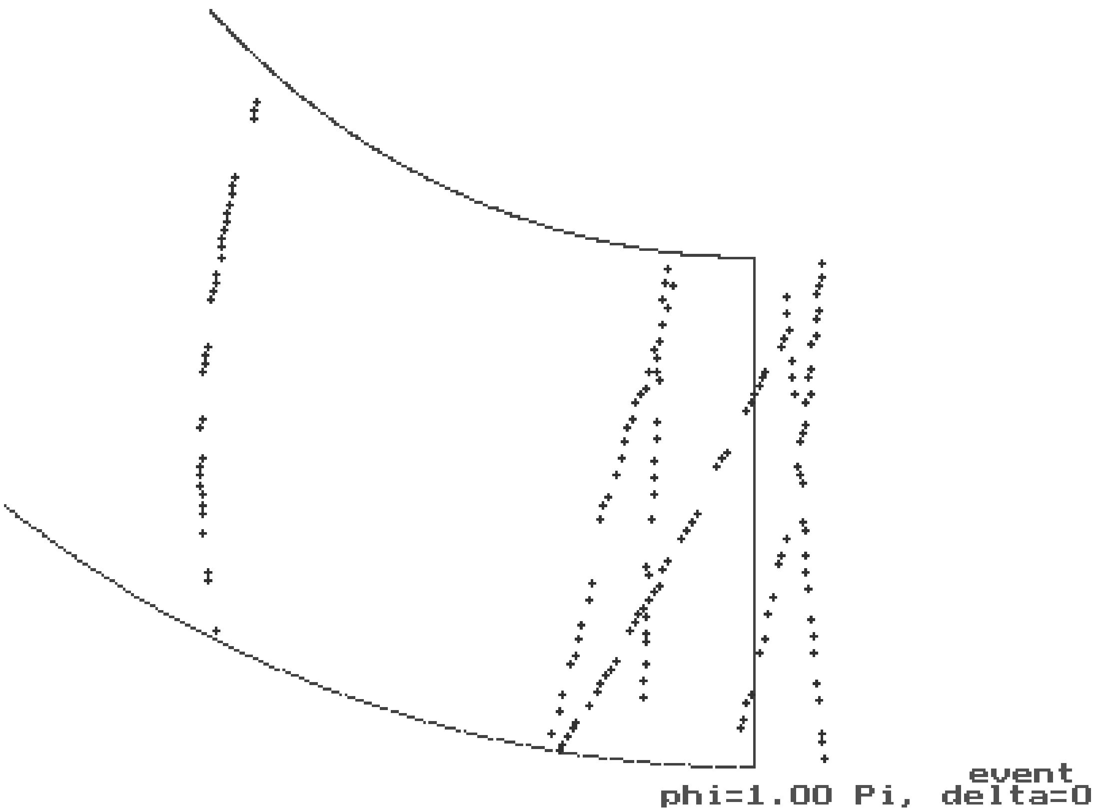

Figure 8. The same part of TRT as in Fig. 7 with six individual tracks found.

The statistics of ϕ and radius of the tracks found:

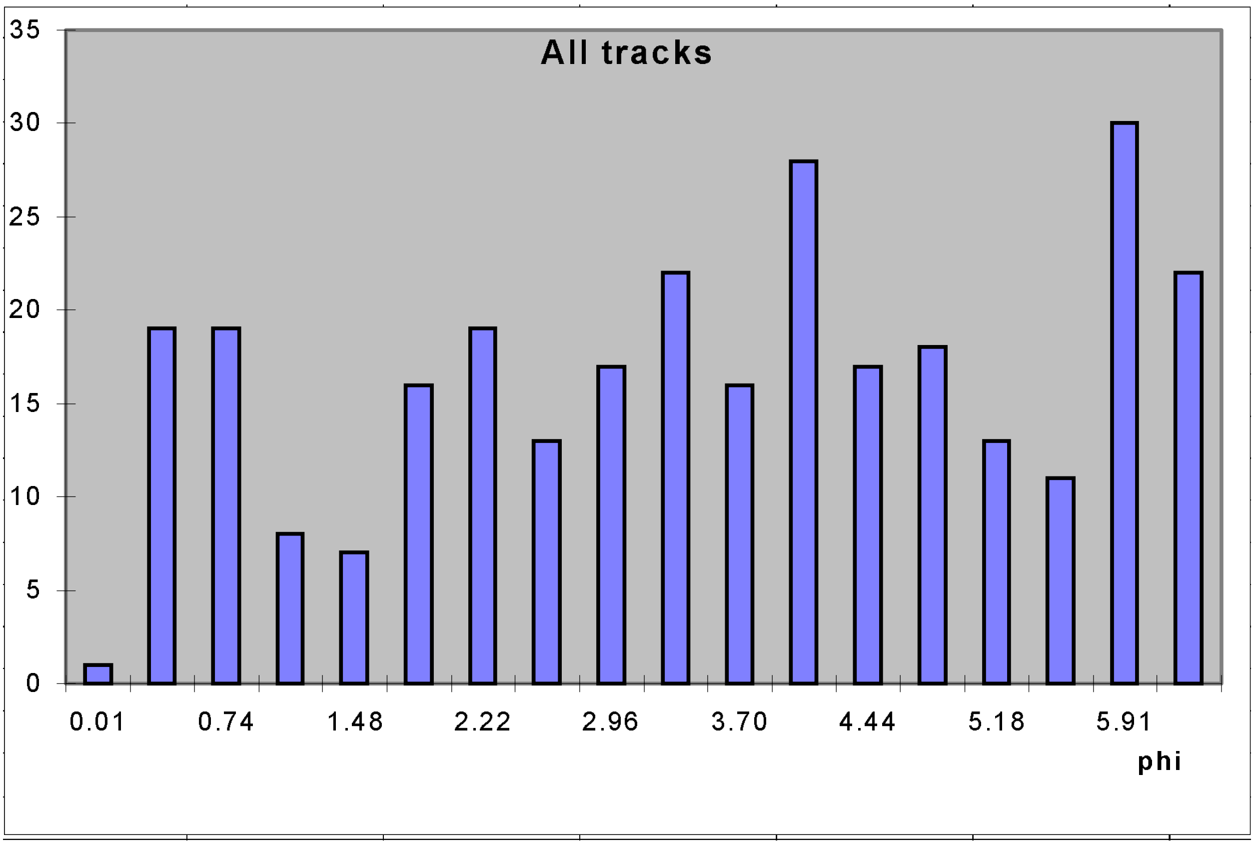

Figure 9. The statistics of ϕ of the tracks found

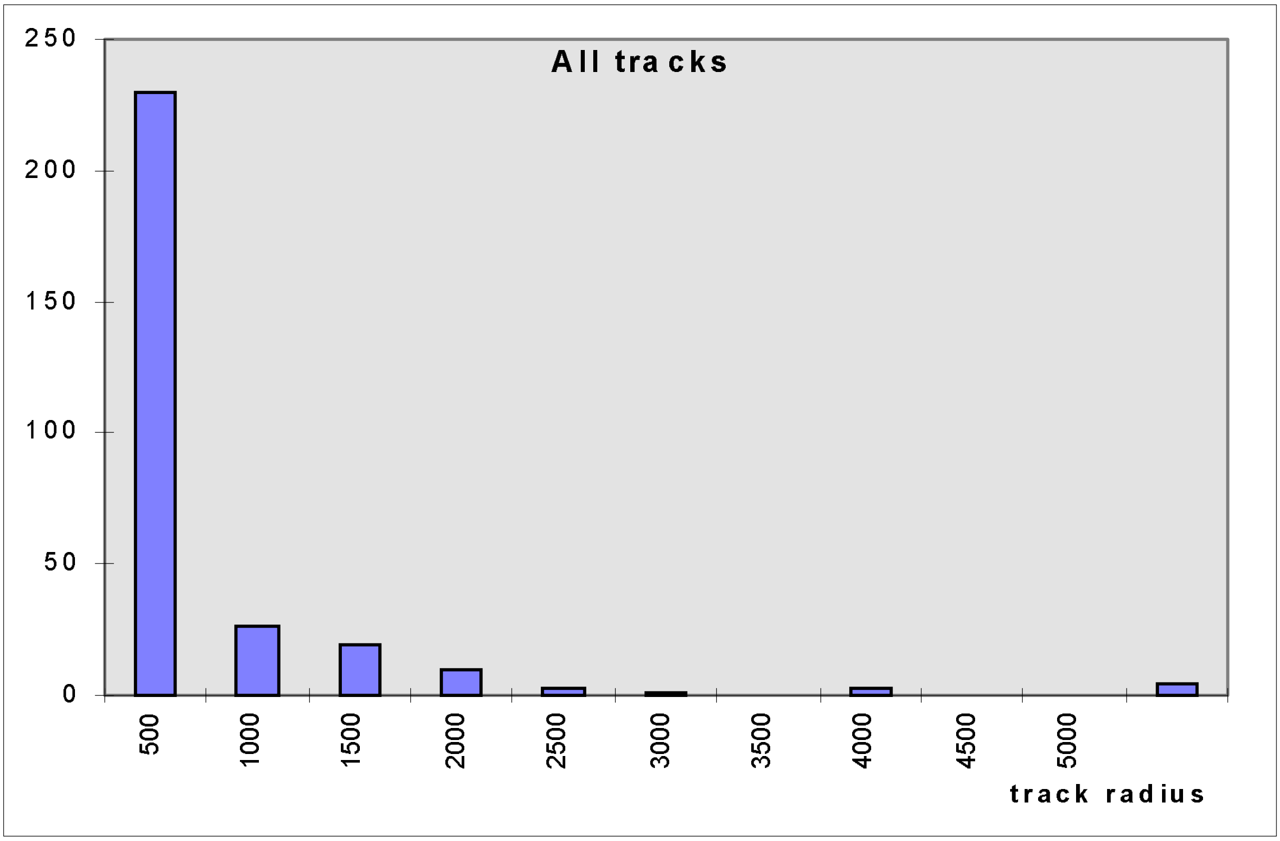

Figure 10. The statistics of radius of the tracks found

Tracks accepted for acceptance criterion $\sigma / \overline{\rho} < 0.02$.:

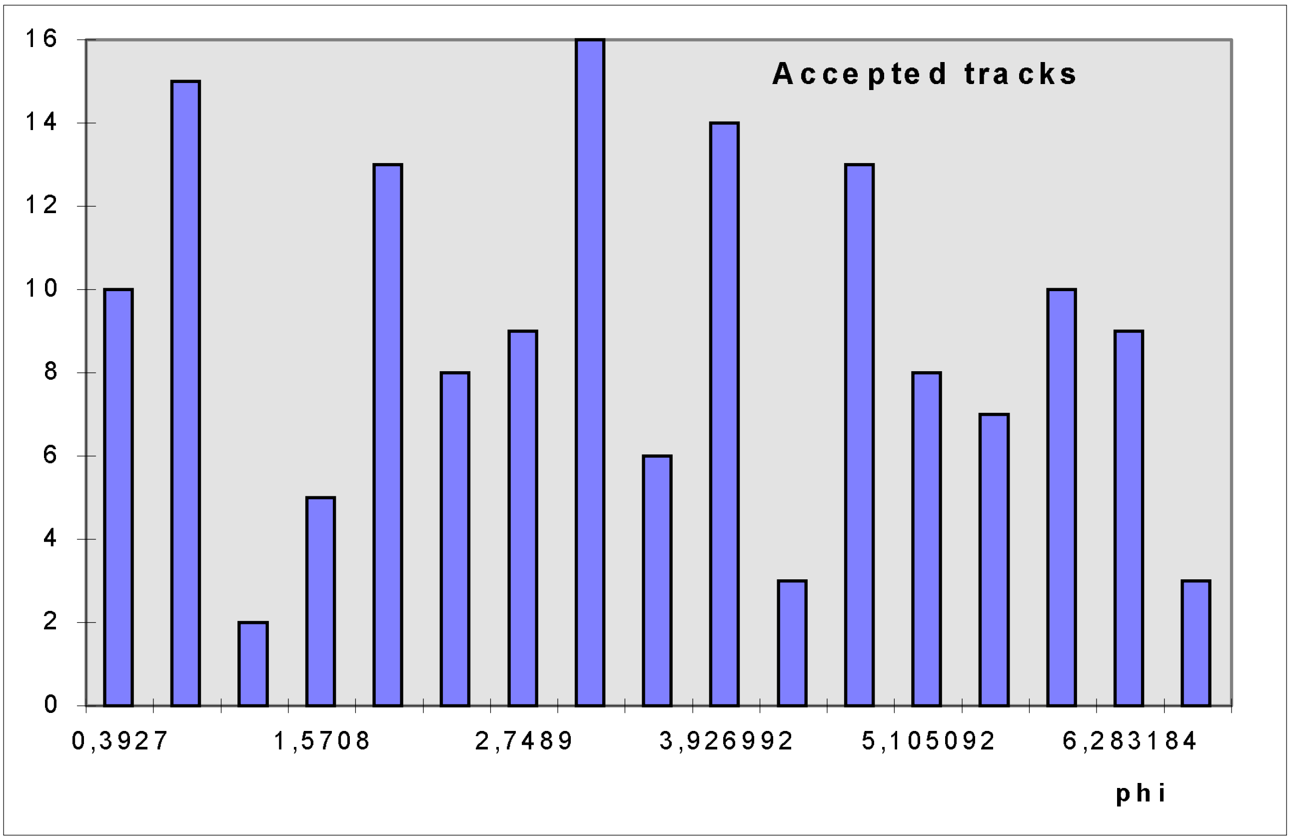

Figure 11. Histogram for track ϕ of the accepted tracks for acceptance criterion $\sigma / \overline{\rho} < 0.02$.

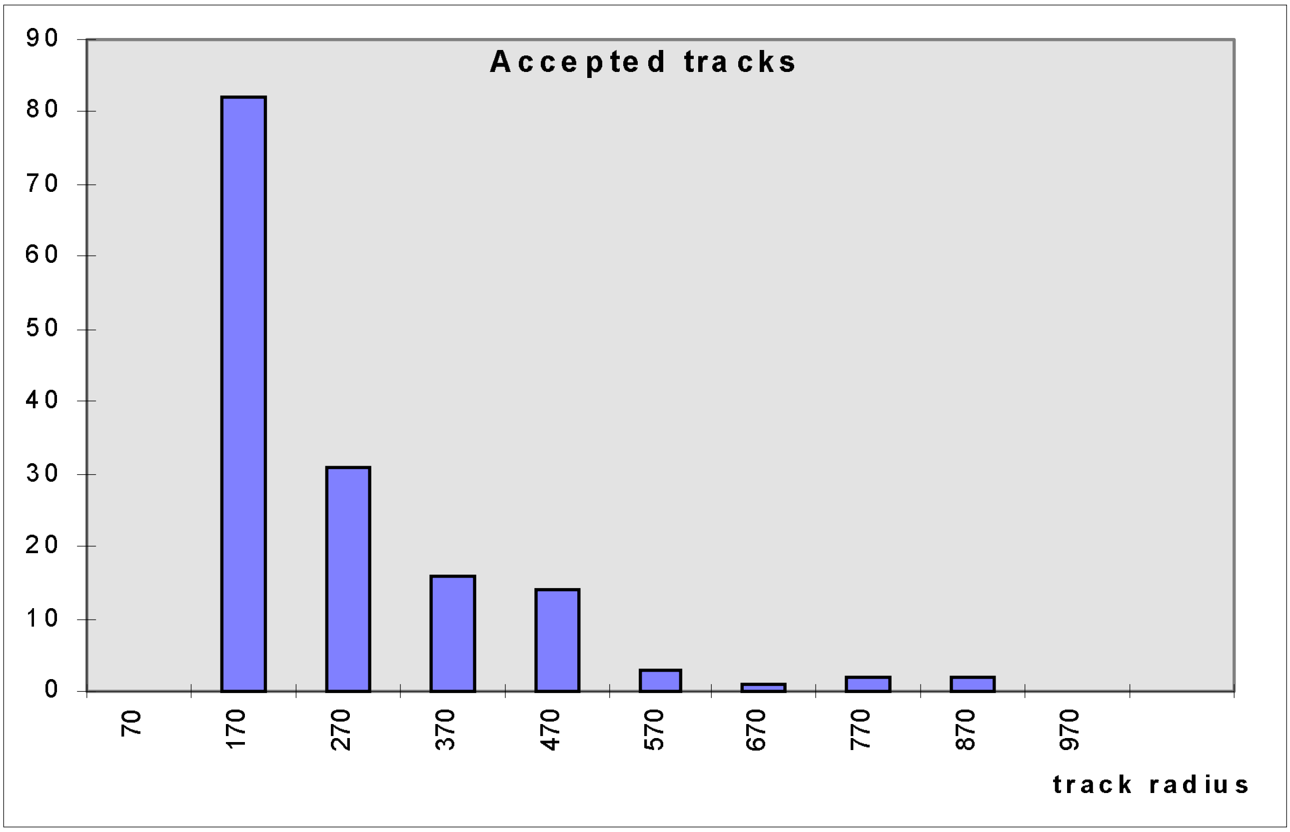

Figure 12. Histogram for track radius of the accepted tracks for acceptance criterion $\sigma / \overline{\rho} < 0.02$.

Accepted tracks for acceptance criterion $70\sigma / \overline{\rho}^2 < 0.004$.

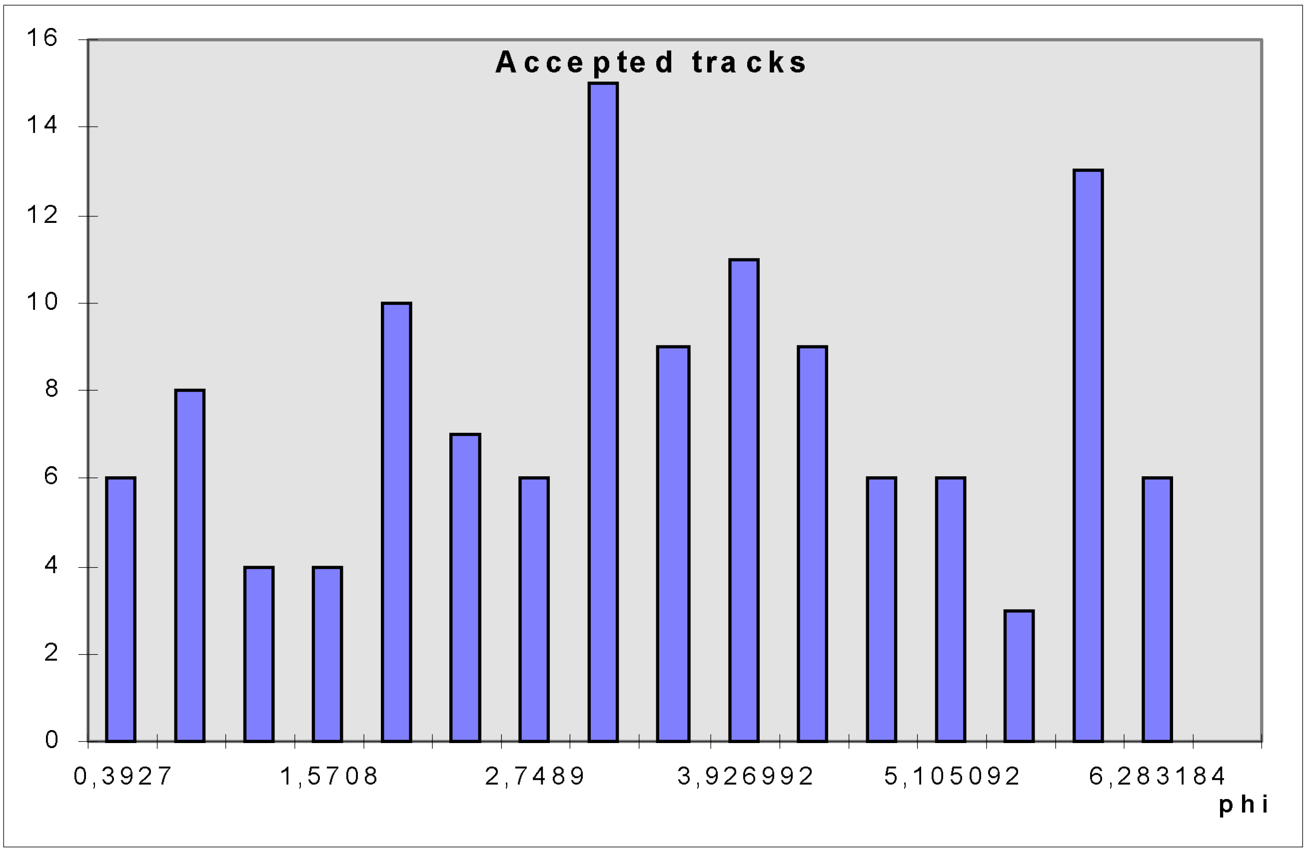

Figure 13. Histogram for track ϕ of the accepted tracks for acceptance criterion $70\sigma / \overline{\rho}^2 < 0.004$.

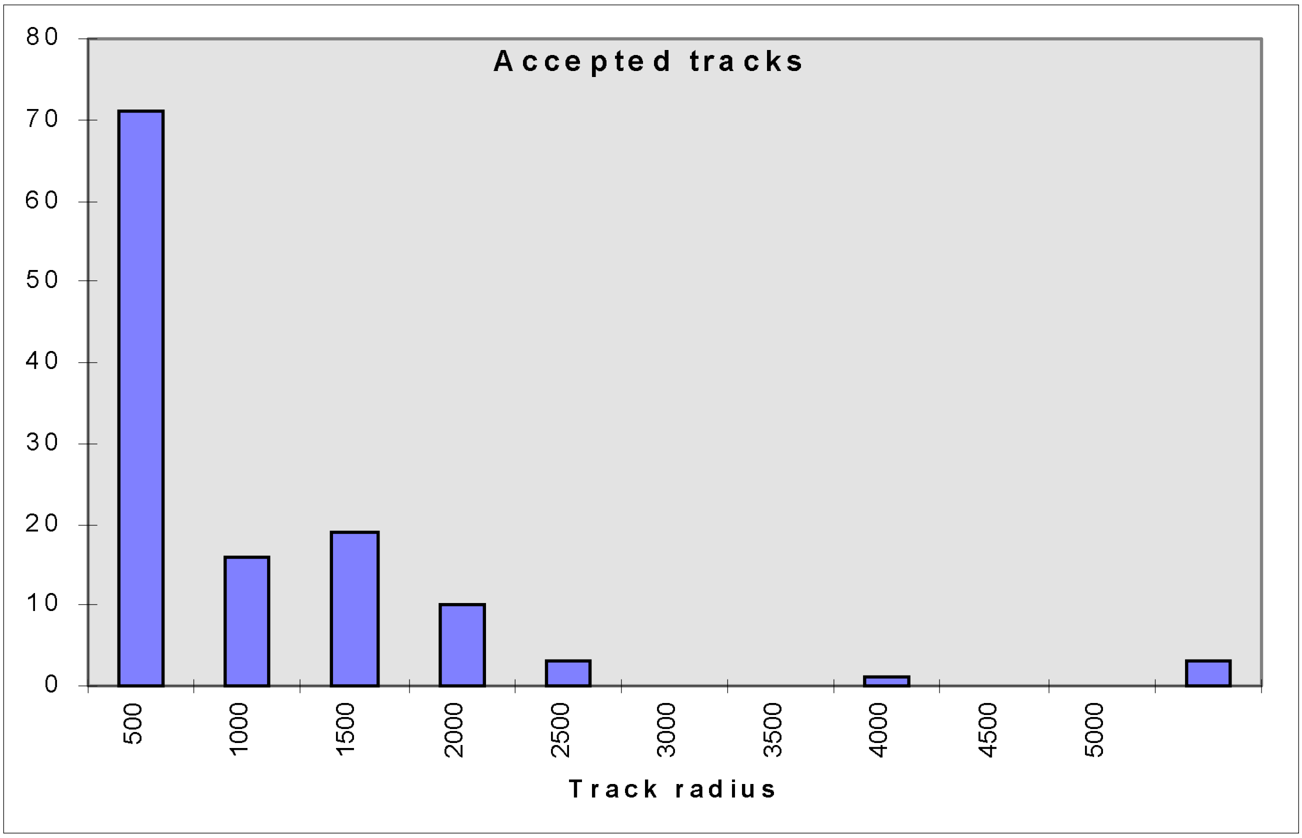

Figure 14. Histogram for track radius of the accepted tracks for acceptance criterion $70\sigma / \overline{\rho}^2 < 0.004$.

All these pictures show that the results correspond to real contents of data given. The ϕ should have constant distribution, because the direction of electron or positron movement is totally random. On the other hand, the distribution of particle pT, i. e. the track radius is (and should be) exponential. It was found that the error criterion $\sigma / \overline{\rho} > 0.02$ supresses acceptance of tracks with large track radius as seen in Fig. 12 compared with Fig. 10. Then the criterion was changed to $70\sigma / \overline{\rho}^2 > 0.004$, which gives to the large radius track the same opportunity to be accepted, see Fig. 14. More detailed investigation is under procedure.

IV.Efficiency Estimations

The Table 1 gives some efficiency measures. For statistics the following data for each track found were collected:

- RH - real number of hits for the track; this nuber was known in advance. For hits found as members of the track the numbers of corresponding tracks were found and prevailing number was used as an original track number. The total number of hits having this original track number is RH,
- HF - hits of the track found by the algorithm,
- WH - wrong hits, i.e. hits found but belonging to another track
- GH - good hits, the hits found and really belonging to the track.

Tracks considered are the tracks fulfilling the acceptance criterion and having RH less than number of layers.

From Table 1 it is seen that number of tracks considered as well as the number of tracks accepted depends on thresholds in acceptance criteria. It is interesting that approx 50 % of tracks accepted have no wrong hits.

Table 1.

Some efficiency data from 37 events.

Acceptance criterion	$\sigma / \overline{\rho} < 0.02$	$70\sigma / \overline{\rho}^2 < 0.004$
WH/HF	4,51%	3,29%
HF/RH	88,06%	83,73%
WH/GH	5,83%	4,13%
Tracks total	296	296
Tracks considered	130	101
Tracks accepted	151	123
No. of tracks with WH/GH = 0	74	65

In Table 1 WH/HF denotes the ratio of wrong hits (not belonging to the track) to hits found as members of the track. HF/RH denotes the ratio of hits found to real hits of the track. WH/GH denotes the ratio of wrong hits to good hits, i.e. hits found and belonging to the track. Only the tracks having RH less than number of layers were considered.

V. Conclusions

The algorithm for searching tracks of electrons and positrons in transition radiation detector (TRT) has been described and its behavior was shown. The algorithm is designed for triggering, i.e. for on-line processing of b-physics events in ATLAS detector on LHC in CERN. Standard algorithms use a transform of circular TRT to polar coordinates, which changes the circular tracks going through the origin to straight lines, and searches these straight lines systematically in all possible directions and places.

After the track candidate is found, several criteria are tested, especially whether the track is really circular and goes through the origin - the conditions for the tracks which may arise from b-physics events. In the tests the error of pT was computed and only the tracks with estimation error less than 2 % were accepted. In accepted tracks there were 94.4 % of hits really belonging to the track and the track was formed by 79.9 % of hits of the real track.

VI. Acknowledgment

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VII. References

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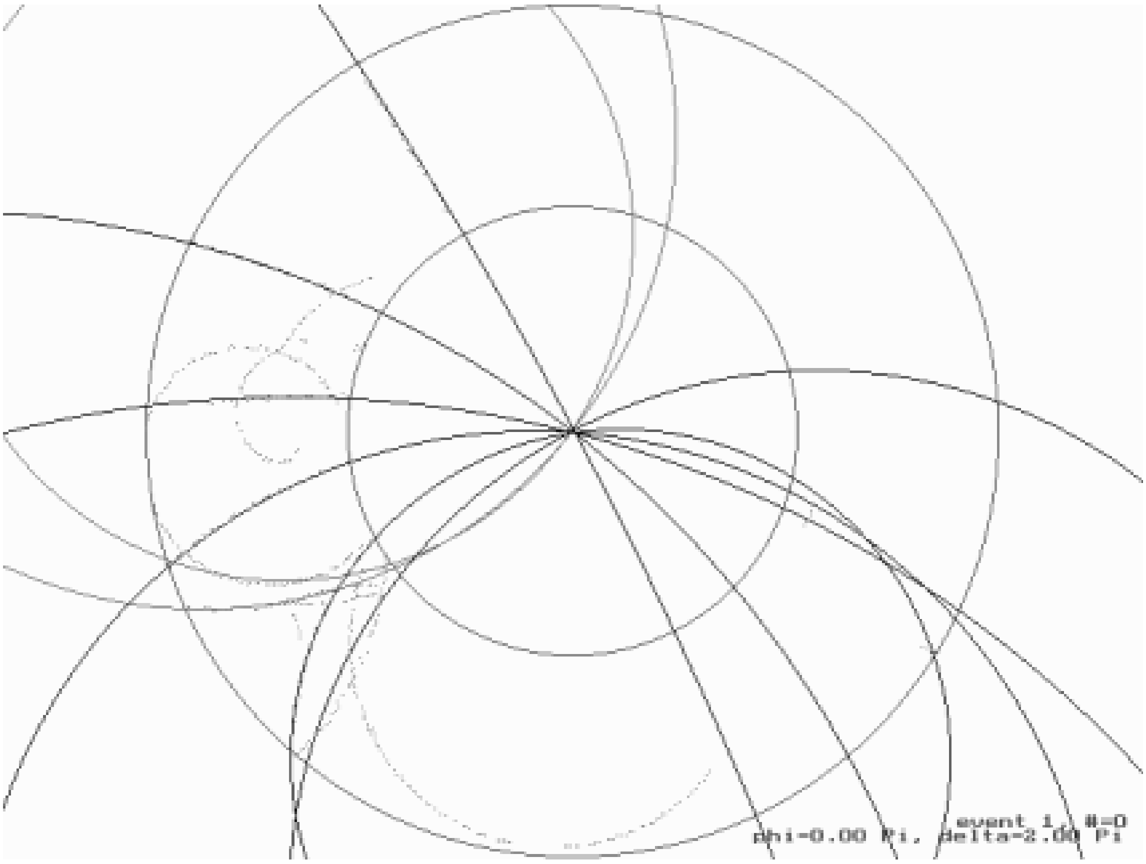
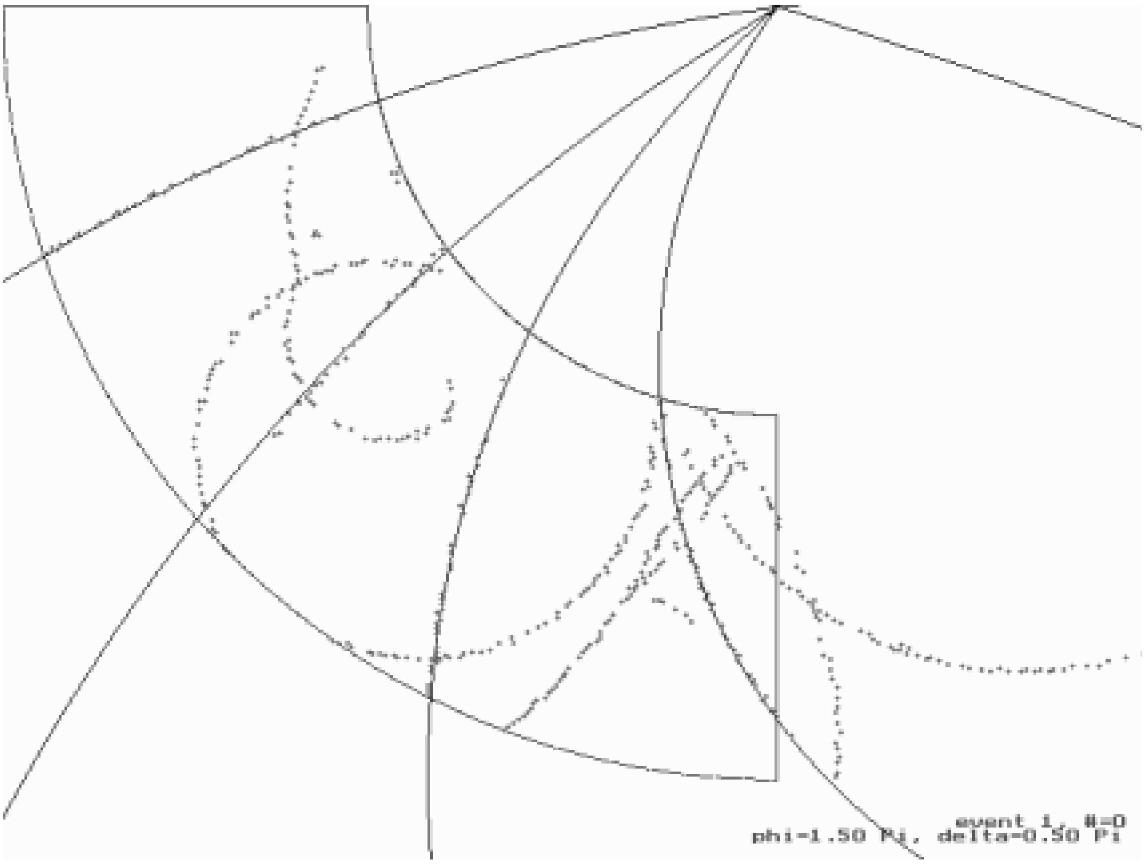
APPENDIX

Some examples of track reconstruction

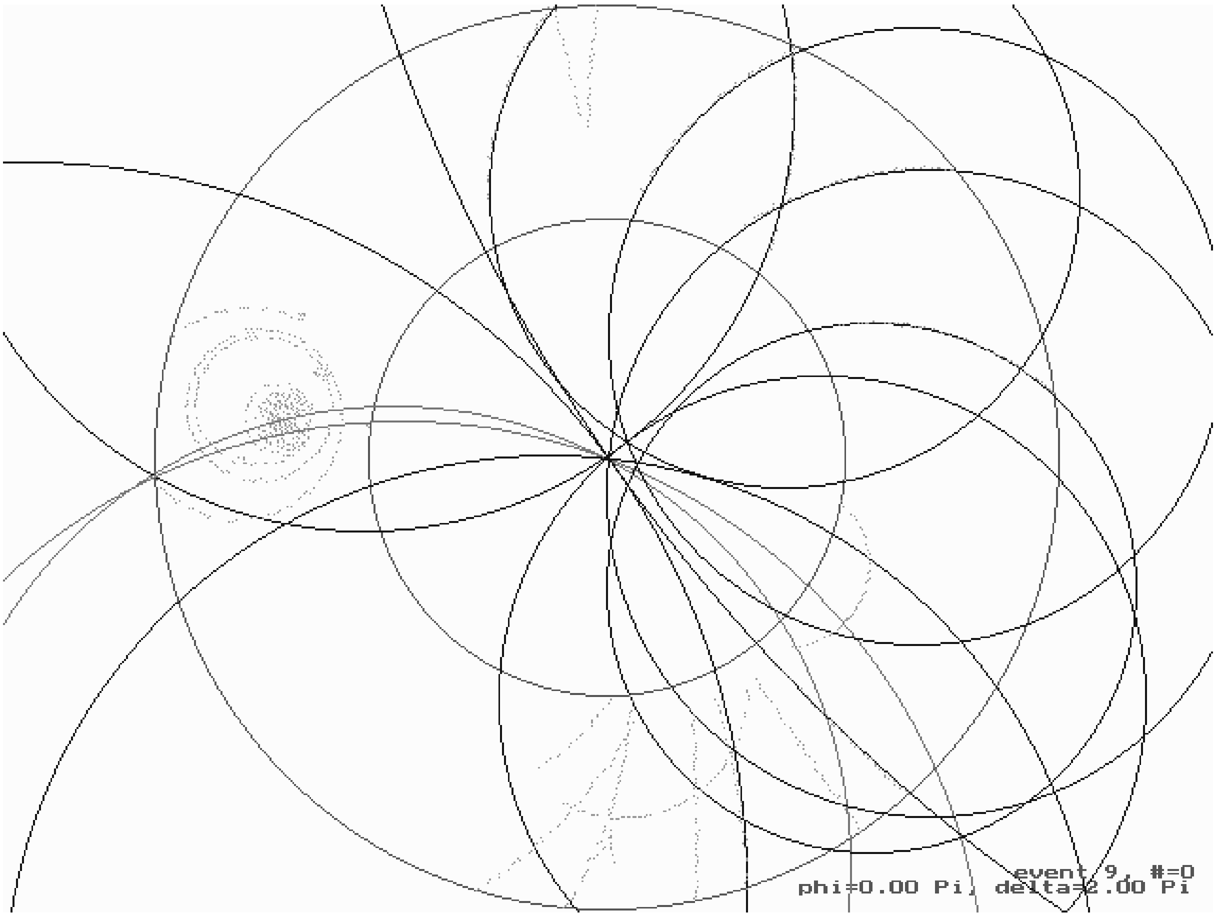
c.91



c.91A



c.92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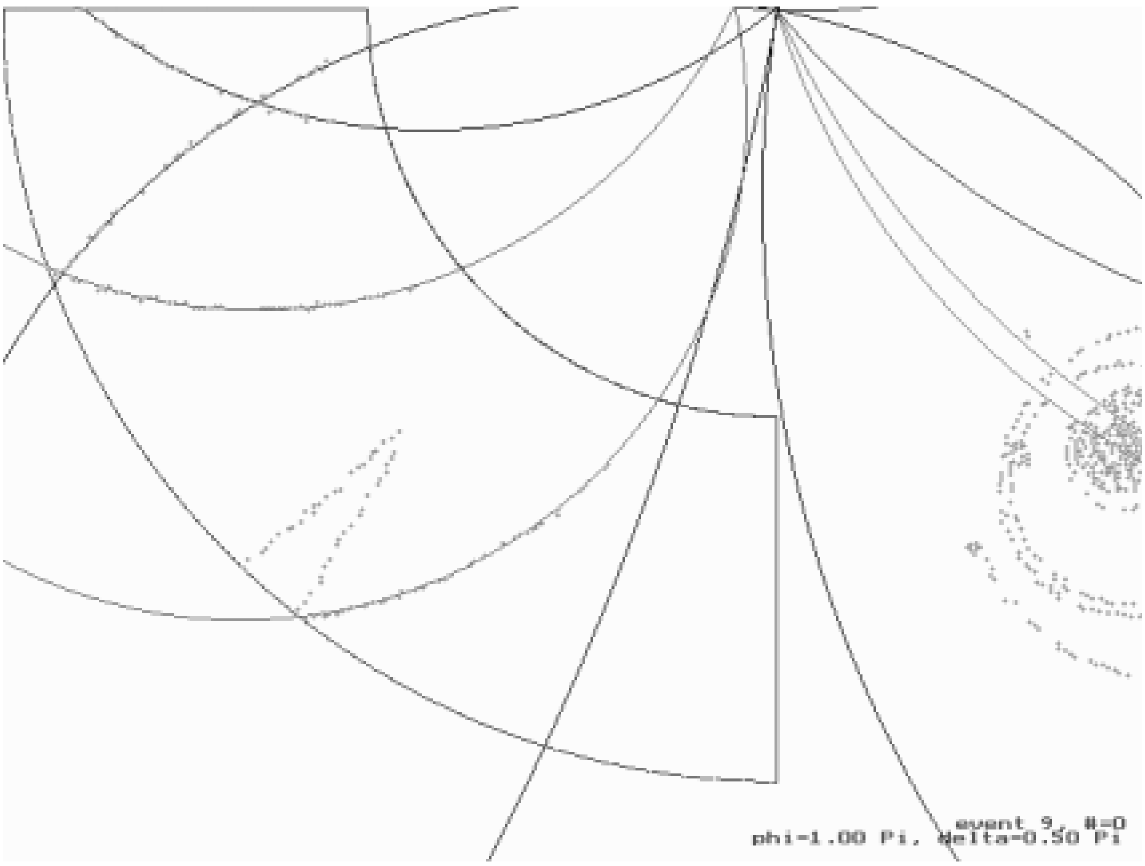
phi=0.00 Pi, delta=2.00 Pi

c92A


event 9, #=0
phi-1.00 Pi, delta=0.50 Pi

c.92B