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Datum stažení: 09.04.2024

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Algorithms**

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Technical report No. 817

April 2000

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**Computational Experiments with Linear ABS
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Abstract

In this paper we present results of testing codes for linear systems based upon the ABS algorithms versus classical codes (from packages ABSPACK and LAPACK). Classes of problems are found where codes from ABSPACK perform significantly better in terms of speed and accuracy.

Keywords

¹Work partly supported by grant GV ČR 201/00/0080 and by MURST 1999 Programma di Cofinanziamento, Numerical Methods for Evolutionary Problems

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COMPUTATIONAL EXPERIMENTS WITH LINEAR ABS ALGORITHMS¹

Elena Bodon², Ladislav Lukšan³, Emilio Spedicato⁴

1 Introduction

In this report, we present numerical experiments with three particular ABS algorithms:

- (1) The Huang or modified Huang algorithm,
- (2) The implicit LU algorithm,
- (3) The implicit QR algorithm,

which were used for solving the following linear systems:

- (a) Finding a solution of a compatible determined linear system $Ax = b$, where $A \in R^{n,n}$, $b \in R^n$, $x \in R^n$.
- (b) Finding the minimum-norm solution of a compatible underdetermined linear system $Ax = b$, where $A \in R^{m,n}$, $b \in R^m$, $x \in R^n$ and $m \leq n$.
- (c) Finding the least-squares solution of an overdetermined linear system $Ax = b$, where $A \in R^{m,n}$, $b \in R^m$, $x \in R^n$ and $m \geq n$.
- (d) Finding a solution of the linear KKT system

$$\begin{bmatrix} B & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix},$$

where $A \in R^{m,n}$, $B \in R^{n,n}$ is symmetric, $b \in R^n$, $c \in R^m$, $x \in R^n$, $y \in R^m$ and $m \leq n$.

The scaled class of ABS algorithms is defined by the following scheme:

- (A) Let $x_1 \in R^n$ be arbitrary and $H_1 \in R^{n,n}$ be nonsingular arbitrary. Set $i = 1$.
- (B) Compute the residual $r_i = Ax_i - b$. If $r_i = 0$, then stop, x_i solves the problem. Otherwise compute $s_i = H_i A^T v_i$, where $v_i \in R^n$ is arbitrary, save that v_1, \dots, v_i are linearly independent. If $s_i \neq 0$, then go to (C). If $s_i = 0$ and $r_i^T v_i = 0$, then set $x_{i+1} = x_i$, $H_{i+1} = H_i$ and go to (F). If $s_i = 0$ and $r_i^T v_i \neq 0$, then stop, the system is incompatible.
- (C) Compute the search vector p_i by

$$p_i = H_i^T z_i,$$

where $z_i \in R^n$ is arbitrary, save that $z_i^T H_i A^T v_i \neq 0$.

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(D) Update the estimate of the solution by

$$x_{i+1} = x_i - \alpha_i p_i,$$

where the stepsize α_i is given by

$$\alpha_i = r_i^T v_i / p_i^T A^T v_i.$$

(E) Update the Abaffian matrix by

$$H_{i+1} = H_i - H_i A^T v_i w_i^T H_i / w_i^T H_i A^T v_i,$$

where $w_i \in R^n$ is arbitrary, save that $w_i^T H_i A^T v_i \neq 0$.

(F) If $i = m$, then stop, x_{i+1} solves the problem. Otherwise increment the index i by one and go to (B).

From (E), it follows by induction that $H_{i+1} A^T V_i = 0$ and $H_{i+1}^T W_i = 0$, where $V_i = [v_1, \dots, v_i]$ and $W_i = [w_1, \dots, w_i]$. Therefore, the implicit factorization $V_i^T A P_i = L_i$ holds, where $P_i = [p_1, \dots, p_i]$ and L_i is nonsingular lower triangular. Moreover the general solution of the scaled subsystem $V_i^T A x = V_i^T b$ can be expressed in the form

$$x = x_i + H_i^T q, \quad (1)$$

where $q \in R^n$ is arbitrary (see [2] for the proof).

The basic ABS class is the subclass of the scaled ABS class obtained by taking $v_i = e_i$, e_i being the i -th unitary vector (i -th column of the unit matrix). In this case, residual $r_i = Ax_i - b$ need not be computed in (B), i -th element $r_i^T e_i = a_i^T x_i - b_i$ suffices.

This report is organized as follows. In Section 2, a short description of individual algorithms is given. Section 3 contains some details concerning test matrices and numerical experiments. Section 4 reports results of numerical experiments and the Appendix contains listing of source codes. We used theoretical results proposed in [1] - [11].

2 Description of algorithms

2.1 Basic algorithms for linear systems

In this report, we deal with three basic algorithms, two belonging to the basic ABS class and one belonging to the scaled ABS class. To simplify description, we will assume that $A \in R^{m,n}$, $m \leq n$, has full row rank so that $s_i \neq 0$ in (B).

The Huang algorithm is obtained by the parameter choices $H_1 = I$, $v_i = e_i$, $z_i = a_i$, $w_i = a_i$. Therefore

$$p_i = H_i a_i \quad (2)$$

and

$$H_{i+1} = H_i - p_i p_i^T / a_i^T p_i, \quad (3)$$

From (2) and (3), it follows by induction that $p_i \in Range(A_i)$ and

$$H_{i+1} = I - P_i D_i^{-1} P_i^T. \quad (4)$$

where $A_i = [a_1, \dots, a_i]$, $P_i = [p_1, \dots, p_i]$ and $D_i = diag(a_1^T p_1, \dots, a_i^T p_i)$. Moreover H_i is symmetric, positive semidefinite and idempotent (it is the orthogonal projection matrix into

$\text{Null}(A_{i-1})$). Since the requirement $p_i \in \text{Null}(A_{i-1})$ is crucial, we can improve orthogonality by iterative refinement $p_i = H_i^j a_i$, $j > 1$ (usually $j = 2$), obtaining the modified Huang algorithm.

The Huang algorithm can be used for finding the minimum-norm solution to the compatible underdetermined system $Ax = b$, i.e. for minimizing $\|x\|$ s.t. $Ax = b$. To see this, we use the Lagrangian function and convexity of $\|x\|$. Then x is a required solution if and only if $x = A^T u$ for some $u \in R^m$ or $x \in \text{Range}(A^T)$. But

$$x_{m+1} = x_1 - \sum_{i=1}^m \alpha_i p_i$$

by (D) and $p_i \in \text{Range}(A_i) \subset \text{Range}(A^T)$, so that if $x_1 = 0$, then $x_{m+1} \in \text{Range}(A^T)$.

A short description of two versions of the Huang and modified Huang algorithms follows.

Algorithm 1

(Huang and modified Huang, formula (3)).

Set $x_1 = 0$ and $H_1 = I$.

For $i = 1$ **to** m **do**

Set $p_i = H_i a_i$ (Huang) or

$p_i = H_i(H_i a_i)$ (modified Huang),

$d_i = a_i^T p_i$ and $x_{i+1} = x_i - ((a_i^T x_i - b_i)/d_i)p_i$.

If $i < m$, then set $H_{i+1} = H_i - p_i p_i^T/d_i$.

end do

Algorithm 2

(Huang and modified Huang, formula (4)).

Set $x_1 = 0$ and P_0 empty.

For $i = 1$ **to** m **do**

Set $p_i = (I - P_{i-1} D_{i-1}^{-1} P_{i-1}^T) a_i$ (Huang) or

$p_i = (I - P_{i-1} D_{i-1}^{-1} P_{i-1}^T)(I - P_{i-1} D_{i-1}^{-1} P_{i-1}^T) a_i$ (modified Huang),

$d_i = a_i^T p_i$ and $x_{i+1} = x_i - ((a_i^T x_i - b_i)/d_i)p_i$.

If $i < m$, then set $P_i = [P_{i-1}, p_i]$.

end do

The implicit LU algorithm is obtained by the parameter choices $H_1 = I$, $v_i = e_i$, $z_i = e_i$, $w_i = e_i$. Since $W_i^T H_{i+1} = [I_i, 0] H_{i+1} = 0$, the first i rows of the Abaffian matrix H_{i+1} must be zero. More precisely, the Abaffian matrix has the following structure, with $K_i \in R^{n-i,i}$

$$H_{i+1} = \begin{bmatrix} 0 & 0 \\ K_i & I_{n-i} \end{bmatrix}. \quad (5)$$

Only the matrix K_i has to be updated. Expression (5) implies that only the first i components of the vector $p_i = H_i^T e_i$ can be nonzero and the i -th component is unity. Hence the matrix P_i is unit upper triangular so that the implicit factorization $A = LP^{-1}$ is of the LU type with units on the diagonal.

A short description of two versions of the implicit LU algorithm follows (the second version is also called the implicit LX algorithm).

Algorithm 3

(Implicit LU with explicit column interchanges).

Set $x_1 = 0$ and $H_1 = I$.**For** $i = 1$ **to** m **do**Set $s_i = H_i a_i$ (only $(i-1)(n-i+1)$ nonzero elements of H_i are used).Determine $d_i = |s_{k_i}| = \max(|s_j|, j = 1, \dots, n)$.(only $n-i+1$ nonzero elements of s_i are used).If $k_i \neq i$, then swap columns of A and elements of x and s with these indices.Set $x_{i+1} = x_i - ((a_i^T x_i - b_i)/d_i) H_i^T e_i$ (only i nonzero elements of x_i are updated).If $i < m$, then set $H_{i+1} = H_i - s_i e_i^T H_i^T / d_i$ (only $i(n-i)$ nonzero elements of H_{i+1} are updated).**end do****Algorithm 4**

(Implicit LU with implicit column interchanges).

Set $x_1 = 0$, $H_1 = I$.**For** $i = 1$ **to** m **do**Set $s_i = H_i a_i$ (only $(i-1)(n-i+1)$ nonzero elements of H_i are used).Determine $d_{k_i} = |s_{k_i}| = \max(|s_j|, j = 1, \dots, n)$.(only $n-i+1$ nonzero elements of s_i are used).Set $x_{i+1} = x_i - ((a_i^T x_i - b_i)/d_{k_i}) H_i^T e_{k_i}$ (only i nonzero elements of x_i are updated).If $i < m$, then set $H_{i+1} = H_i - s_i e_{k_i}^T H_i^T / d_{k_i}$ (only $i(n-i)$ nonzero elements of H_{i+1} are updated).**end do**

The implicit QR algorithm is obtained by the parameter choices $H_1 = I$, $v_i = Ap_i$, $z_i = e_i$, $w_i = e_i$. Since $z_i = e_i$ and $w_i = e_i$, the matrices H_i and P_i have the same structure as that in the implicit LU algorithm. Using the implicit factorization property $V_i^T AP_i = L_i$, we can see that $V_{i-1}^T v_i = V_{i-1}^T Ap_i = 0$ so that the vectors v_j , $j = 1, \dots, i$, are mutually orthogonal.

The implicit QR algorithm can be used for finding the least-squares solution of overdetermined linear system $Ax = b$, where $A \in R^{m,n}$, $b \in R^m$, $x \in R^n$ and $m \geq n$, which is obtained after at most n steps. To see this, we recall that algorithms from the scaled ABS class solve the scaled system $V_n^T Ax = V_n^T b$. Since $V_n = AP_n$, where P_n is square nonsingular, the condition $P_n^T A^T Ax = P_n^T A^T b$ implies $A^T Ax = A^T b$, which defines the least-squares solution of the overdetermined system $Ax = b$.

A short description of the implicit QR algorithm follows.

Algorithm 5

(Implicit QR).

Set $x_1 = 0$, $r_1 = -b$ and $H_1 = I$.**For** $i = 1$ **to** m **do**Set $p_i = H_i^T e_i$, $v_i = Ap_i$ (only $(i-1)(n-i+1)$ nonzero elements of H_i is used), $\alpha_i = r_i^T v_i / v_i^T v_i$, $x_{i+1} = x_i - \alpha_i p_i$ and $r_{i+1} = r_i - \alpha_i v_i$.

(only i nonzero elements of x_i are updated).
Set $x_{i+1} = x_i - ((a_i^T x_i - b_i)/d_i) H_i^T e_i$
(only i nonzero elements of x_i are updated).
If $i < m$, then set $s_i = H_i^T A^T v_i$ and $H_{i+1} = H_i - s_i p_i^T / v_i^T v_i$
(only $i(n - i)$ nonzero elements of H_{i+1} are updated).
end do

2.2 Algorithms for least-squares solution of overdetermined linear systems

Consider the linear system $Ax = b$, where $A \in R^{m,n}$, $b \in R^m$, $x \in R^n$ and $m \geq n$. This system can be solved in n steps by the implicit QR algorithm as was shown above. Another possibility is based on the application of the Huang algorithm. Since the least-squares solution has to satisfy the normal equation

$$A^T Ax = A^T b,$$

it can be obtained from the solution of the following augmented system .

$$Ax = y, \quad (6)$$

$$A^T y = A^T b. \quad (7)$$

Since $y \in Range(A)$ by (6), it is a minimum-norm solution of (7) and it can be obtained by the Huang algorithm. Having y , the compatible system (6) can be solved by any ABS algorithm. Moreover, using the implicit factorization property $A^T P_n = L_n$, we can write

$$L_n^T x = P_n^T Ax = P_n^T b = \tilde{b}$$

which as a system with a lower triangular matrix can be easily solved by the back substitution. The lower triangular matrix L_n can be obtained as a by-product of the Huang algorithm. If we use formula (4), then d_i is i -th diagonal element of L_n and $P_{i-1}^T \tilde{a}_i$ contains the other i -th-column elements of L_n (\tilde{a}_i is i -th column of the matrix A).

Matrix L_n has not to be stored since it can be reconstructed from columns of the matrix P_n . However, the back substitution has to be realized in a slightly different way in this case.

A short description of two versions the Huang algorithm for least-squares solution of over-determined systems follows.

Algorithm 6

(Huang and modified Huang for least-squares with stored L_n).

Set $x_1 = 0$ and P_0 empty.

For $i = 1$ **to** n **do**

If $i > 1$, then compute $g_i = P_{i-1}^T \tilde{a}_i$ and copy it into the lower triangular matrix L_n .

Set $p_i = \tilde{a}_i - P_{i-1} D_{i-1}^{-1} g_i$ (Huang) or

$p_i = (I - P_{i-1} D_{i-1}^{-1} P_{i-1}^T)(\tilde{a}_i - P_{i-1} D_{i-1}^{-1} g_i)$ (modified Huang).

Set $d_i = \tilde{a}_i^T p_i$ and copy it into the lower triangular matrix L_n . Set $\tilde{b}_i = d_i^T p_i$.

If $i < n$, then set $P_i = [P_{i-1}, p_i]$.

end do

Solve the triangular system $L_n^T x = \tilde{b}$.

Algorithm 7

(Huang and modified Huang for least-squares without stored L_n).

Set $x_1 = 0$ and P_0 empty.

For $i = 1$ **to** n **do**

Set $p_i = (I - P_{i-1}D_{i-1}^{-1}P_{i-1}^T)\tilde{a}_i$ (Huang) or

$p_i = (I - P_{i-1}D_{i-1}^{-1}P_{i-1}^T)(I - P_{i-1}D_{i-1}^{-1}P_{i-1}^T)\tilde{a}_i$ (modified Huang).

Set $d_i = \tilde{a}_i^T p_i$.

If $i < n$, then set $P_i = [P_{i-1}, p_i]$.

end do

Set $f_n = b$.

For $i = n$ **to** 1 **do**

Set $x_i = p_i^T f_i / d_i$.

If $i > 1$, then set $f_{i-1} = f_i - x_i \tilde{a}_i$

end do

2.3 Algorithms for linear KKT systems

Consider the linear KKT system

$$Bx + A^T y = b, \quad (8)$$

$$Ax = c. \quad (9)$$

The underdetermined equation (9) can be solved by an arbitrary ABS method obtaining the particular solution x_{m+1} . Since $H_{m+1}A^T = 0$, we get

$$H_{m+1}Bx = H_{m+1}b, \quad (10)$$

by multiplying (8) by H_{m+1} . The coupled system (9)-(10) is overdetermined but compatible so that its unique solution can be found by an arbitrary ABS method. Since $\text{rank}(H_{m+1}) = n - m$, m equations will be eliminated. Notice that solution of (9)-(10) is obtained in two steps. First we solve (9) to obtain x_{m+1} and H_{m+1} . Then we construct $H_{m+1}B$ and $H_{m+1}b$ and starting with x_{m+1} and H_{m+1} we continue with the ABS method to solve (10).

If we use the implicit LU algorithm, then matrix H_{m+1} has the special form (5) and the first m equations of (10) are trivially satisfied, leaving a determined system consisting of (9) and the equation.

$$S_m Bx = S_m b, \quad (11)$$

where $S_m = [K_m, I_{n-m}]$. Moreover, substituting the general solution (see (1))

$$x = x_{m+1} + H_{m+1}^T q,$$

with $q \in R^n$ arbitrary into (8) and using the special form of H_{m+1} , we obtain

$$\begin{bmatrix} 0 & 0 \\ 0 & S_m B S_m^T \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ S_m(b - Bx_{m+1}) \end{bmatrix}. \quad (12)$$

Thus q_1 can be chosen arbitrarily, e.g. $q_1 = 0$, and q_2 is a solution to $n - m$ dimensional system

$$S_m B S_m^T q_2 = S_m(b - Bx_{m+1}), \quad (13)$$

which can be solved by any ABS method.

Once x is determined, we obtain y by solving the compatible overdetermined system $A^T y = b - Bx$. Since the equation $Ax = c$ was solved beforehand, we are in the same position as in the case of the least-squares solution of an overdetermined linear system. Therefore, using P_m , we can construct the lower triangular matrix $L_m = AP_m$ and get the solution from the equation $L_m^T y = P_m^T(b - Bx)$.

3 Description of computational experiments

Performance of ABS algorithms has been tested by using several types of ill-conditioned matrices. These matrices can be classified in the following way. The first letter distinguishes matrices with integer 'I' and real 'R' elements, both actually stored as reals in double precision arithmetic. The second letter denotes randomly generated matrices 'R' or matrices determined by an explicit formula 'D'. For randomly generated matrices, a number specifying the interval for the random number generator follows, while the name of matrices determined by the explicit formula contains the formula number (F1, F2, F3). The last letter of the name denotes a way for obtaining ill-conditioned matrices: 'R' - matrices with nearly dependent rows, 'C' - matrices with nearly dependent columns, 'S' - nearly singular symmetric matrices, 'B' - both matrices in KKT system ill-conditioned. More specifically:

- IR500 Randomly generated matrices with integer elements uniformly distributed in the interval [-500,500].
- IR500R Randomly generated matrices with integer elements uniformly distributed in the interval [-500,500] perturbed in addition to have two rows nearly dependent.
- IR500C Randomly generated matrices with integer elements uniformly distributed in the interval [-500,500] perturbed in addition to have two columns nearly dependent.
- RR100 Randomly generated matrices with real elements uniformly distributed in the interval [-100,100]
- IDF1 Matrices with elements $a_{ij} = |i - j|$, $1 \leq i \leq m$, $1 \leq j \leq n$ (Michelli-Fiedler matrix).
- IDF2 Matrices with elements $a_{ij} = |i - j|^2$, $1 \leq i \leq m$, $1 \leq j \leq n$.
- IDF3 Matrices with elements $a_{ij} = |i + j - (m + n)/2|$, $1 \leq i \leq m$, $1 \leq j \leq n$.
- IR50 Randomly generated matrices with integer elements uniformly distributed in the interval [-50,50].

Matrices with linearly dependent rows were obtained in the following way. The input data contain four integers which specify two row indices i_1, i_2 , one column index i_3 and one exponent i_4 . Then the row a_{i_1} is copied into a_{i_2} . Furthermore $a_{i_1 i_3}$ is set to zero and $a_{i_2 i_3}$ to 2^{-i_4} . Similar procedures are used for columns and symmetric matrices.

Besides above types of matrices, several (usually ill-conditioned) matrices from the Harwell-Boeing collection were used for testing algorithms for determined linear systems.

Right hand sides were determined from the given solution vectors x^* by the formula $b = Ax^*$. Solution vectors were usually generated randomly with integer or real elements uniformly distributed in the interval [-10,10]. Solution vectors for underdetermined systems were chosen as $x^* = a_k$ with k specified in the input data. Then $x^* \in Range(A^T)$ so that it is the minimum norm solution. Right hand sides for overdetermined systems were obtained by slightly more complicated way. First, vector \tilde{b} was generated randomly with integer or real elements uniformly distributed in the interval [-10,10]. Then its first element together with elements in the first row of the matrix A were redefined by the formulas $\tilde{b}_1 = -1$ and $a_{1j} = \sum_{i=2}^n a_{ij} \tilde{b}_j$ so that $A^T \tilde{b} = 0$. The right-hand side vector was determined by the formula $b = \tilde{b} + Ax^*$. Since $A^T Ax^* = A^T(b - \tilde{b}) = A^T b$, the normal equation is satisfied and x^* is a least squares solution of

the system $Ax = b$. Vector \tilde{b} was generated nonzero, so that the system $Ax = b$ is incompatible.

We have tested the following particular algorithms:

huang1	The Huang algorithm: Algorithm 1 (subroutine alg1d.f).
mod.huang1	The modified Huang algorithm: Algorithm 1 (subroutine alg1d.f).
huang2	The Huang algorithm: Algorithm 2 (subroutine alg2d.f).
mod.huang2	The modified Huang algorithm: Algorithm 2 (subroutine alg2d.f).
impl.lu3	The implicit LU algorithm: Algorithm 3 (subroutine alg3d.f).
impl.lu4	The implicit LU algorithm: Algorithm 4 (subroutine alg4d.f).
impl.qr5	The implicit QR algorithm: Algorithm 5 (subroutine alg5d.f).
huang6	The Huang algorithm for overdetermined systems: Algorithm 6 (subroutine alg6d.f).
mod.huang6	The modified Huang algorithm for overdetermined systems: Algorithm 6 (subroutine alg6d.f).
huang7	The Huang algorithm for overdetermined systems: Algorithm 7 (subroutine alg7d.f).
mod.huang7	The modified Huang algorithm for overdetermined systems: Algorithm 7 (subroutine alg7d.f).
impl.lu8	The implicit LU algorithm for KKT systems (subroutine algkt1d.f).
impl.lu9	The implicit LU algorithm for KKT systems (subroutine algkt2d.f).
mod.huang	The modified Huang algorithm for KKT systems (subroutine alg2d.f).
expl.qr	The explicit QR factorization using Householder reflections (subroutines housefactd.f and housesold.f, based upon a NAG source code).
expl.lu	The explicit LU factorization using Gaussian elimination (subroutines lufactd.f and lusold.f, based upon a NAG source code).
lu lapack	Subroutine DGESV from the LAPACK package.
qr lapack	Subroutine DGELS from the LAPACK package.
svd lapack	Subroutine DGELSS from the LAPACK package.
gqr lapack	Subroutine DGELSX from the LAPACK package.
lu linpack	Subroutines DGEFA and DGESL from the LINPACK package.

Notice that ABS algorithms were implemented in its basic form without partitioning into block or other special adjustements serving for speed increase as used in the LAPACK software. Therefore, they should be preferentially compared with the algorithms exp.lu, exp.qr or the LINPACK routines. More details and conclusions concerning computational experiments are given in special subsections below.

Results of computational experiments are presented in Section 4. For each selected problem, the type of matrix and the dimension is given. Furthermore, both the solution and the residual errors together with the detected rank and the computational time are given for each tested algorithm. Computational experiments were performed on a Digital Unix Workstation in the double precision arithmetic (machine epsilon equal to about 10^{-16}).

3.1 Computational experiments with algorithms for determined linear systems

Algorithms 1-5 were used for solving ill-conditioned full linear systems with 1000 and 2000 equations and also for solving linear systems with selected Harwell-Boeing matrices. The results are presented in Subsection 4.1. These results imply several conclusions.

- (1) The LAPACK routine DGESV is the best but it should not be compared with other algorithms because of a special implementation as was mentioned above.

- (2) The implicit LU algorithm with implicit column interchanges (i.e. the implicit LX algorithm) is very fast. For large systems, it outperforms in speed all other algorithms with exception of the LAPACK routine DGESV. For condition numbers less than 10^{-15} , say, it gives sufficiently accurate results, but it can fail for extremely ill-conditioned systems. Nevertheless, explicit LU algorithms also give bad results or break down for such systems.
- (3) The Huang and, especially, the modified Huang algorithms are substantially slower. On the other hand, they give most accurate results for extremely ill-conditioned systems. Moreover, they are able to detect nearly dependent equations as the LAPACK routine DGELSX based on the QR decomposition. Since such equations are eliminated, the computational time can be unexpectedly low. For the IDF2 and IDF3 matrices, the computational time was 50 times lower then that used by the LAPACK routine DGESV.
- (4) The implicit QR algorithm is unsuitable for solving determined linear systems because of its lower accuracy. Nevertheless, it can be very fast for systems with many nearly dependent equations as was observed by using matrices IDF2 and IDF3.

3.2 Computational experiments with algorithms for overdetermined linear systems

We have tested overdetermined systems with $n >> m/2$, $n = m/2$ and $n << m/2$. These systems were solved by using Algorithms 5 - Algorithm 7 together with explicit QR and SVD decomposition based methods taken largely from the LAPACK package. The results, which are presented in Subsection 4.2, imply the following conclusions.

- (1) When solving well-conditioned overdetermined systems, the explicit QR algorithms based on the Householder orthogonalization are usually faster than ABS methods tested (especially if $n >> m/2$). Therefore, we cannot recommend the later for solving standard problems.
- (2) Interesting results were obtained for extremely ill-conditioned problems. The modified Huang and the implicit QR algorithms failed to solve systems with the matrix IDF2. Such systems was not solved by simple explicit QR methods as well. On the other hand, the modified Huang and the implicit QR algorithms found solutions of systems with the matrix IDF3 extremely fast and, moreover, they were able to determine the numerical rank correctly. Performance of these algorithms in that particular case is remarkable, they are at least 20 times faster then the best LAPACK routines.
- (3) The LAPACK routine DGELSS based on the SVD decomposition is extremely slow (especially if $n >> m/2$), not suitable for solving our problems (it can be substituted by the much faster routine DGELSX).

3.3 Computational experiments with algorithms for underdetermined linear systems

We have tested underdetermined systems with $m >> n/2$, $m = n/2$ and $m << n/2$. These systems were solved by using the Huang and the modified Huang algorithms together with explicit QR and SVD decomposition methods taken from the LAPACK package. The results, which are presented in Subsection 4.3, imply the following conclusions.

- (1) When solving well-conditioned underdetermined systems, the explicit QR algorithms based on the Householder orthogonalization are usually faster than the Huang and the modified Huang algorithms. On the other hand, the last method usually gives more accurate results.
- (2) For underdetermined systems, Algorithm 2 is more efficient than Algorithm 1 (especially if $m \ll n/2$).
- (3) The modified Huang method (Algorithm 2) is extremely successful in solving ill-conditioned systems with matrices IDF2 and IDF3. Moreover, it is able to find the numerical rank correctly.
- (4) The LAPACK routine DGELSS based on the SVD decomposition is extremely slow (especially if $m \gg n/2$), not suitable for solving our problems (it can be supplied by the much faster routine DGELSX).

3.4 Computational experiments with algorithms for KKT linear systems

We have tested KKT systems with $m \gg n/2$, $m = n/2$ and $m \ll n/2$. These systems were solved by using the modified Huang algorithm applied to the coupled system (9)-(10) and by two variants of the implicit LU algorithm with explicit column interchanges. The first variant is intended for solving the coupled system (9), (11) while the second one uses (9) together with the transformed system (13). For comparison, we have tested three additional methods implemented by using the LAPACK routines.

The first method, denoted by lu lapack, is in fact a direct solution of a complete KKT system by using the Bunch-Parlett decomposition. This is carried-out by the LAPACK routine DSPSV. Notice that we deal with an $n + m$ dimensional indefinite system in this case.

Another approach, a range-space method, is based on the Bunch-Parlett decomposition of the (possibly indefinite) matrix B . Using the LAPACK routine DSPSV, we obtain the solution of the matrix equation $B[\tilde{A}^T, \tilde{b}] = [A^T, b]$ together with the decomposition $B = LDL^T$, where L is n -dimensional lower unit triangular and D is n -dimensional block diagonal with the blocksize 1 or 2. Then the symmetric matrix $C = A\tilde{A}^T = AB^{-1}A^T$ is built and the solution y to the equation $Cy = \tilde{b} - c$ is found, again by using the LAPACK routine DSPSV. Finally, we solve the system $LDL^Tx = b - A^Ty$ by using the LAPACK routine DSPTRS.

The last approach, a null-space method, is based on the RQ decomposition

$$A = [0, R]Q, \quad (14)$$

where R is a m -dimensional upper triangular matrix and Q is an n -dimensional orthogonal matrix. This RQ decomposition is obtained by using the LAPACK routine DGERQF. Premultiplying equation (8) by Q , we get $QBQ^TQx + QA^Ty = Qb$ or

$$\begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} \\ \tilde{B}_{21} & \tilde{B}_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ R^T \end{bmatrix} y = \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{bmatrix}. \quad (15)$$

Where $\tilde{B} = QBQ^T$, $\tilde{b} = Qb$ and $\tilde{x} = Qx$. Matrices \tilde{B} and \tilde{b} are obtained by using the LAPACK routine DORMRQ. Now $R\tilde{x}_2 = c$ by (9), (14) and $\tilde{B}_{11}\tilde{x}_1 = \tilde{b}_1 - \tilde{B}_{12}\tilde{x}_2$ by (15). The last equation is solved by using the LAPACK routine DSYSV. Finally $R^Ty = \tilde{b}_2 - \tilde{B}_{21}\tilde{x}_1 - \tilde{B}_{22}\tilde{x}_2$ and $x = Q^T\tilde{x}$ can be obtained by the LAPACK routine DORMRQ.

The results, which are presented in Subsection 4.4, imply the following conclusions.

- (1) Implicit LU methods described in Subsection 2.3 are extremely suitable for solving KKT systems with $m \sim n$. If $m \ll n$, then a direct solution of a complete KKT system by using the Bunch-Parlett decomposition is faster.
- (2) The method based on equation (11) is faster if $m \sim n$ while the method based on equation (13) is more efficient if $m \ll n$.
- (3) The modified Huang method is usually very slow. But in the extremely ill-conditioned cases it gives the best accuracy. Moreover, the modified Huang method is able to detect the numerical rank correctly, which can lead to the substantial decrease of the computational time as can be observed from experiments with the matrix IDF2.

4 Test results

4.1 Test results for linear systems

matrix	dimension	method	solution error	residual error	rank	time
	n					
<hr/>						
IR500	1000	huang1	0.41D-04	0.38D-09	1000	44.00
IR500	1000	mod.huang1	0.99D-10	0.22D-14	1000	64.00
IR500	1000	huang2	0.36D-02	0.82D-08	1000	36.00
IR500	1000	mod.huang2	0.19D-09	0.22D-14	1000	71.00
IR500	1000	impl.lu3	0.70D-08	0.14D-13	1000	17.00
IR500	1000	impl.lu4	0.70D-08	0.14D-13	1000	11.00
IR500	1000	impl.qr5	0.18D-08	0.28D-14	1000	64.00
IR500	1000	expl.qr	0.21D-09	0.27D-14	1000	20.00
IR500	1000	expl.lu	0.18D-09	0.28D-14	1000	17.00
IR500	1000	lu lapack	0.12D-10	0.24D-14	1000	7.00
IR500	1000	qr lapack	0.59D-10	0.20D-14	1000	17.00
IR500	1000	gqr lapack	0.17D-09	0.28D-14	1000	27.00
IR500	1000	lu linpack	0.12D-10	0.24D-14	1000	16.00
condition number: 0.15D+07						
IR500	2000	huang1	0.63D-05	0.21D-08	2000	362.00
IR500	2000	mod.huang1	0.15D-09	0.25D-13	2000	518.00
IR500	2000	huang2	0.19D-03	0.18D-06	2000	262.00
IR500	2000	mod.huang2	0.26D-09	0.39D-13	2000	520.00
IR500	2000	impl.lu3	0.10D-08	0.34D-12	2000	233.00
IR500	2000	impl.lu4	0.10D-08	0.34D-12	2000	109.00
IR500	2000	impl.qr5	0.51D-09	0.17D-12	2000	492.00
IR500	2000	expl.qr	0.34D-10	0.45D-13	2000	167.00
IR500	2000	expl.lu	0.88D-10	0.20D-13	2000	220.00
IR500	2000	lu lapack	0.51D-11	0.38D-14	2000	53.00
IR500	2000	qr lapack	0.63D-10	0.31D-13	2000	138.00
IR500	2000	gqr lapack	0.25D-10	0.16D-13	2000	252.00
IR500	2000	lu linpack	0.51D-11	0.38D-14	2000	136.00
condition number: 0.55D+06						
IR500R	1000	huang1	0.30D-03	0.13D-12	1000	44.00
IR500R	1000	mod.huang1	0.30D-03	0.22D-14	999	64.00
IR500R	1000	huang2	0.30D-03	0.86D-11	1000	36.00
IR500R	1000	mod.huang2	0.30D-03	0.15D-14	999	73.00
IR500R	1000	impl.lu3	0.30D-06	0.12D-13	1000	17.00
IR500R	1000	impl.lu4	0.30D-06	0.12D-13	1000	11.00
IR500R	1000	impl.qr5	0.31D+04	0.61D-10	999	65.00
IR500R	1000	expl.qr	0.30D-03	0.21D-14	999	20.00
IR500R	1000	expl.lu	0.28D-06	0.15D-14	1000	18.00
IR500R	1000	lu lapack	0.17D-10	0.12D-14	1000	6.00
IR500R	1000	qr lapack	0.10D-04	0.35D-14	1000	17.00
IR500R	1000	gqr lapack	0.30D-03	0.28D-14	999	27.00
IR500R	1000	lu linpack	0.17D-10	0.12D-14	1000	16.00
condition number: 0.44D+12						

Test results for linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
IR500R	2000	huang1	0.72D-04	0.26D-11	2000	362.00
IR500R	2000	mod.huang1	0.72D-04	0.21D-14	1999	519.00
IR500R	2000	huang2	0.72D-04	0.52D-10	2000	262.00
IR500R	2000	mod.huang2	0.72D-04	0.21D-14	1999	520.00
IR500R	2000	impl.lu3	0.42D-05	0.18D-13	2000	233.00
IR500R	2000	impl.lu4	0.42D-05	0.18D-13	2000	108.00
IR500R	2000	impl.qr5	0.12D+05	0.52D-10	1999	492.00
IR500R	2000	expl.qr	0.72D-04	0.25D-14	1999	167.00
IR500R	2000	expl.lu	0.53D-05	0.21D-14	2000	219.00
IR500R	2000	lu lapack	0.41D-08	0.18D-14	2000	53.00
IR500R	2000	qr lapack	0.18D-04	0.40D-14	2000	138.00
IR500R	2000	gqr lapack	0.72D-04	0.22D-14	1999	245.00
IR500R	2000	lu linpack	0.41D-08	0.18D-14	2000	152.00
condition number: 0.12D+13						
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IR500C	1000	huang1	0.71D+00	0.45D-13	1000	44.00
IR500C	1000	mod.huang1	0.71D+00	0.45D-10	999	65.00
IR500C	1000	huang2	0.71D+00	0.36D-11	1000	36.00
IR500C	1000	mod.huang2	0.71D+00	0.40D-10	999	72.00
IR500C	1000	impl.lu3	0.16D+00	0.57D-14	1000	17.00
IR500C	1000	impl.lu4	0.16D+00	0.57D-14	1000	11.00
IR500C	1000	impl.qr5	0.14D+01	0.89D-12	999	65.00
IR500C	1000	expl.qr	0.71D+00	0.47D-10	999	20.00
IR500C	1000	expl.lu	0.19D-01	0.15D-14	1000	17.00
IR500C	1000	lu lapack	0.45D-02	0.13D-14	1000	7.00
IR500C	1000	qr lapack	0.21D-01	0.19D-14	1000	17.00
IR500C	1000	gqr lapack	0.71D+00	0.15D-14	999	27.00
IR500C	1000	lu linpack	0.45D-02	0.13D-14	1000	16.00
condition number: 0.93D+15						
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IR500C	2000	huang1	0.71D+00	0.12D-11	2000	399.00
IR500C	2000	mod.huang1	0.71D+00	0.30D-10	1999	567.00
IR500C	2000	huang2	0.71D+00	0.24D-10	2000	284.00
IR500C	2000	mod.huang2	0.71D+00	0.30D-10	1999	530.00
IR500C	2000	impl.lu3	0.62D-02	0.29D-13	2000	233.00
IR500C	2000	impl.lu4	0.62D-02	0.29D-13	2000	109.00
IR500C	2000	impl.qr5	0.14D+01	0.28D-12	1999	491.00
IR500C	2000	expl.qr	0.71D+00	0.30D-10	1999	167.00
IR500C	2000	expl.lu	0.36D-03	0.23D-14	2000	219.00
IR500C	2000	lu lapack	0.29D-03	0.18D-14	2000	53.00
IR500C	2000	qr lapack	0.78D-03	0.23D-14	2000	137.00
IR500C	2000	gqr lapack	0.71D+00	0.17D-13	1999	226.00
IR500C	2000	lu linpack	0.29D-03	0.18D-14	2000	136.00
condition number: 0.34D+14						

Test results for linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
RR100	1000	mod.huang1	0.57D-13	0.12D-14	1000	64.00
RR100	1000	huang2	0.00D+00	0.12D-14	1000	36.00
RR100	1000	mod.huang2	0.00D+00	0.12D-14	1000	72.00
RR100	1000	impl.lu3	0.31D-12	0.73D-14	1000	17.00
RR100	1000	impl.lu4	0.31D-12	0.73D-14	1000	11.00
RR100	1000	impl.qr5	0.18D-12	0.24D-14	1000	65.00
RR100	1000	expl.qr	0.17D-13	0.14D-14	1000	21.00
RR100	1000	expl.lu	0.40D-12	0.46D-13	1000	18.00
RR100	1000	lu lapack	0.42D-12	0.48D-13	1000	6.00
RR100	1000	qr lapack	0.20D-12	0.21D-14	1000	17.00
RR100	1000	gqr lapack	0.42D-13	0.29D-14	1000	29.00
RR100	1000	lu linpack	0.42D-12	0.48D-13	1000	16.00
condition number: 0.23D+03						
RR100	2000	huang1	0.11D-12	0.25D-14	2000	362.00
RR100	2000	mod.huang1	0.11D-12	0.25D-14	2000	518.00
RR100	2000	huang2	0.78D-13	0.21D-14	2000	261.00
RR100	2000	mod.huang2	0.78D-13	0.21D-14	2000	520.00
RR100	2000	impl.lu3	0.49D-12	0.13D-13	2000	234.00
RR100	2000	impl.lu4	0.49D-12	0.13D-13	2000	109.00
RR100	2000	impl.qr5	0.50D-13	0.36D-14	2000	491.00
RR100	2000	expl.qr	0.15D-12	0.20D-14	2000	167.00
RR100	2000	expl.lu	0.65D-12	0.10D-12	2000	219.00
RR100	2000	lu lapack	0.98D-12	0.11D-12	2000	52.00
RR100	2000	qr lapack	0.28D-12	0.31D-14	2000	138.00
RR100	2000	gqr lapack	0.32D-12	0.49D-14	2000	226.00
RR100	2000	lu linpack	0.98D-12	0.11D-12	2000	136.00
condition number: 0.20D+03						
IDF1	1000	huang1	0.98D-09	0.10D-11	1000	45.00
IDF1	1000	mod.huang1	0.26D-10	0.13D-14	1000	64.00
IDF1	1000	huang2	0.62D-07	0.59D-10	1000	36.00
IDF1	1000	mod.huang2	0.25D-10	0.13D-14	1000	72.00
IDF1	1000	impl.lu3	0.26D-10	0.13D-14	1000	17.00
IDF1	1000	impl.lu	0.26D-10	0.13D-14	1000	10.00
IDF1	1000	impl.qr5	0.26D-07	0.99D-12	1000	66.00
IDF1	1000	expl.qr	0.37D-10	0.16D-14	1000	20.00
IDF1	1000	expl.lu	0.29D-10	0.16D-14	1000	18.00
IDF1	1000	lu lapack	0.29D-10	0.13D-14	1000	6.00
IDF1	1000	qr lapack	0.25D-09	0.76D-14	1000	18.00
IDF1	1000	gqr lapack	0.17D-10	0.25D-14	1000	27.00
IDF1	1000	lu linpack	0.29D-10	0.13D-14	1000	17.00
condition number: 0.20D+04						

Test results for linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
IDF1	2000	huang1	0.86D-08	0.78D-12	2000	362.00
IDF1	2000	mod.huang1	0.15D-09	0.18D-14	2000	519.00
IDF1	2000	huang2	0.67D-06	0.73D-10	2000	262.00
IDF1	2000	mod.huang2	0.16D-09	0.19D-14	2000	518.00
IDF1	2000	impl.lu3	0.15D-09	0.21D-14	2000	227.00
IDF1	2000	impl.lu	0.15D-09	0.21D-14	2000	103.00
IDF1	2000	impl.qr5	0.37D-06	0.41D-11	2000	491.00
IDF1	2000	expl.qr	0.20D-09	0.21D-14	2000	167.00
IDF1	2000	expl.lu	0.17D-09	0.26D-14	2000	219.00
IDF1	2000	lu lapack	0.16D-09	0.28D-14	2000	52.00
IDF1	2000	qr lapack	0.18D-08	0.11D-13	2000	138.00
IDF1	2000	gqr lapack	0.40D-10	0.18D-14	2000	227.00
IDF1	2000	lu linpack	0.16D-09	0.28D-14	2000	136.00
condition number: 0.40D+04						
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IDF2	1000	huang1	0.10D+01	0.57D-12	1000	44.00
IDF2	1000	mod.huang1	0.10D+01	0.26D-12	4	41.00
IDF2	1000	huang2	0.10D+01	0.61D-12	999	36.00
IDF2	1000	mod.huang2	0.10D+01	0.26D-09	3	1.00
IDF2	1000	impl.lu3	0.57D+09	0.68D-05	973	18.00
IDF2	1000	impl.lu4	0.57D+09	0.68D-05	973	11.00
IDF2	1000	impl.qr5	0.30D+06	0.33D-10	498	28.00
IDF2	1000	expl.qr	0.10D+01	0.15D-12	4	0.00
IDF2	1000	expl.lu	0.26D+04	0.32D-12	1000	17.00
IDF2	1000	lu lapack	0.23D+04	0.35D-12	1000	6.00
IDF2	1000	qr lapack	0.11D+05	0.13D-11	1000	18.00
IDF2	1000	gqr lapack	0.10D+01	0.16D-14	3	26.00
IDF2	1000	lu linpack	0.23D+04	0.35D-12	1000	17.00
condition number: 0.28D+20						
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IDF2	2000	huang1	0.10D+01	0.34D-10	2000	360.00
IDF2	2000	mod.huang1	0.10D+01	0.11D-10	4	316.00
IDF2	2000	huang2	0.10D+01	0.69D-11	2000	262.00
IDF2	2000	mod.huang2	0.14D+01	0.96D-12	4	7.00
IDF2	2000	impl.lu3	0.52D+18	0.12D+05	2000	233.00
IDF2	2000	impl.lu4	0.52D+18	0.12D+05	2000	109.00
IDF2	2000	impl.qr5	0.61D+05	0.17D-09	29	6.00
IDF2	2000	expl.qr	0.10D+01	0.77D-11	4	1.00
IDF2	2000	expl.lu	0.10D+04	0.19D-12	2000	219.00
IDF2	2000	lu lapack	0.67D+04	0.18D-11	2000	53.00
IDF2	2000	qr lapack	0.34D+04	0.92D-12	2000	137.00
IDF2	2000	gqr lapack	0.10D+01	0.20D-14	3	226.00
IDF2	2000	lu linpack	0.67D+04	0.18D-11	2000	136.00
condition number: 0.21D+20						

Test results for linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
IDF3	1000	huang1	0.10D+01	0.29D-12	1000	43.00
IDF3	1000	mod.huang1	0.10D+01	0.46D-12	2	40.00
IDF3	1000	huang2	0.10D+01	0.11D-11	1000	36.00
IDF3	1000	mod.huang2	0.10D+01	0.22D-12	2	1.00
IDF3	1000	impl.lu3	0.11D+08	0.30D-07	979	18.00
IDF3	1000	impl.lu4	0.11D+08	0.30D-07	979	11.00
IDF3	1000	impl.qr5	0.16D+04	0.10D-13	2	0.00
IDF3	1000	expl.qr	0.10D+01	0.44D-14	2	1.00
IDF3	1000	expl.lu	0.28D+03	0.44D-13	1000	17.00
IDF3	1000	lu lapack	0.45D+02	0.65D-14	1000	6.00
IDF3	1000	qr lapack	0.41D+02	0.13D-13	1000	17.00
IDF3	1000	gqr lapack	0.10D+01	0.22D-14	2	27.00
IDF3	1000	lu linpack	0.45D+02	0.65D-14	1000	16.00
condition number: 0.91D+19						
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IDF3	2000	huang1	0.10D+01	0.38D-12	2000	359.00
IDF3	2000	mod.huang1	0.10D+01	0.88D-12	2	316.00
IDF3	2000	huang2	0.10D+01	0.16D-10	2000	261.00
IDF3	2000	mod.huang2	0.10D+01	0.90D-12	2	5.00
IDF3	2000	impl.lu3	0.96D+15	0.36D+01	1922	223.00
IDF3	2000	impl.lu4	0.96D+15	0.36D+01	1922	99.00
IDF3	2000	impl.qr5	0.46D+04	0.30D-12	2	1.00
IDF3	2000	expl.qr	0.10D+01	0.12D-13	2	1.00
IDF3	2000	expl.lu	0.19D+04	0.29D-12	2000	218.00
IDF3	2000	lu lapack	0.13D+04	0.31D-12	2000	52.00
IDF3	2000	qr lapack	0.14D+03	0.31D-13	2000	138.00
IDF3	2000	gqr lapack	0.10D+01	0.14D-14	2	227.00
IDF3	2000	lu linpack	0.13D+04	0.31D-12	2000	136.00
condition number: 0.49D+20						
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IR50	1000	huang1	0.46D+00	0.14D-11	997	44.00
IR50	1000	mod.huang1	0.46D+00	0.22D-14	772	58.00
IR50	1000	huang2	0.46D+00	0.33D-09	1000	36.00
IR50	1000	mod.huang2	0.46D+00	0.27D-14	772	61.00
IR50	1000	impl.lu3	0.15D+02	0.40D-13	962	18.00
IR50	1000	impl.lu4	0.15D+02	0.40D-13	962	11.00
IR50	1000	impl.qr5	0.22D+02	0.30D-13	772	53.00
IR50	1000	expl.qr	0.46D+00	0.24D-14	772	19.00
IR50	1000	expl.lu	0.17D+02	0.13D-13	1000	17.00
IR50	1000	lu lapack	0.12D+04	0.12D+04	972	7.00
IR50	1000	qr lapack	0.63D+02	0.17D-12	1000	17.00
IR50	1000	gqr lapack	0.46D+00	0.42D-14	772	29.00
IR50	1000	lu linpack	--- break-down ---			
condition number: 0.10D+61						

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
IR50	2000	huang1	0.40D+00	0.11D-09	2000	362.00
IR50	2000	mod.huang1	0.40D+00	0.28D-13	1702	489.00
IR50	2000	huang2	0.40D+00	0.38D-08	2000	261.00
IR50	2000	mod.huang2	0.40D+00	0.27D-13	1702	471.00
IR50	2000	impl.lu3	0.22D+02	0.15D-11	1992	233.00
IR50	2000	impl.lu4	0.22D+02	0.15D-11	1992	109.00
IR50	2000	impl.qr5	0.28D+02	0.15D-11	1702	437.00
IR50	2000	expl.qr	0.40D+00	0.11D-13	1702	161.00
IR50	2000	expl.lu	0.30D+03	0.66D-11	2000	219.00
IR50	2000	lu lapack	0.93D+02	0.76D+03	1993	52.00
IR50	2000	qr lapack	0.46D+02	0.13D-11	2000	137.00
IR50	2000	gqr lapack	0.40D+00	0.40D-13	1702	258.00
IR50	1000	lu linpack	--- break-down ---			
condition number: 0.10D+61						

4.2 Test results for overdetermined linear systems

matrix	dimension	method	solution	residual	rank	time	
			m	n			
<hr/>							
IR500	1050	950	huang6	0.17D-01	0.23D-14	950	32.00
IR500	1050	950	mod.huang6	0.78D-10	0.20D-15	950	63.00
IR500	1050	950	huang7	0.17D-01	0.16D-14	950	30.00
IR500	1050	950	mod.huang7	0.79D-10	0.85D-15	950	60.00
IR500	1050	950	impl.qr5	0.51D-09	0.12D-13	950	57.00
IR500	1050	950	expl.qr	0.80D-10	0.52D-15	950	20.00
IR500	1050	950	qr lapack	0.84D-09	0.83D-14	950	16.00
IR500	1050	950	svd lapack	0.31D-08	0.80D-14	950	119.00
IR500	1050	950	gqr lapack	0.28D-08	0.28D-14	950	26.00
condition number: 0.42D+09							
IR500	1400	700	huang6	0.13D-02	0.51D-14	700	23.00
IR500	1400	700	mod.huang6	0.41D-11	0.12D-13	700	49.00
IR500	1400	700	huang7	0.13D-02	0.82D-14	700	24.00
IR500	1400	700	mod.huang7	0.39D-11	0.15D-14	700	47.00
IR500	1400	700	impl.qr5	0.33D-10	0.31D-13	700	40.00
IR500	1400	700	expl.qr	0.66D-11	0.80D-14	700	17.00
IR500	1400	700	qr lapack	0.54D-10	0.56D-14	700	15.00
IR500	1400	700	svd lapack	0.54D-10	0.18D-13	700	67.00
IR500	1400	700	gqr lapack	0.23D-09	0.77D-14	700	23.00
condition number: 0.65D+08							

Test results for overdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time	
			m	n			
<hr/>							
IR500	2000	400	huang6	0.35D-03	0.95D-15	400	12.00
IR500	2000	400	mod.huang6	0.96D-12	0.78D-15	400	21.00
IR500	2000	400	huang7	0.35D-03	0.41D-15	400	11.00
IR500	2000	400	mod.huang7	0.87D-12	0.33D-15	400	21.00
IR500	2000	400	impl.qr5	0.55D-11	0.11D-13	400	18.00
IR500	2000	400	expl.qr	0.11D-11	0.91D-15	400	9.00
IR500	2000	400	qr lapack	0.18D-10	0.46D-13	400	7.00
IR500	2000	400	svd lapack	0.18D-10	0.54D-13	400	20.00
IR500	2000	400	gqr lapack	0.65D-10	0.10D-12	400	12.00
condition number:			0.24D+08				
IR500C	1050	950	huang6	0.23D-01	0.15D-14	950	33.00
IR500C	1050	950	mod.huang6	0.11D-01	0.19D-14	950	63.00
IR500C	1050	950	huang7	0.23D-01	0.36D-16	950	30.00
IR500C	1050	950	mod.huang7	0.11D-01	0.11D-14	950	60.00
IR500C	1050	950	impl.qr5	0.12D-01	0.40D-12	950	57.00
IR500C	1050	950	expl.qr	0.64D-02	0.19D-15	950	19.00
IR500C	1050	950	qr lapack	0.45D-02	0.36D-14	950	17.00
IR500C	1050	950	svd lapack	0.69D+00	0.41D-14	949	114.00
IR500C	1050	950	gqr lapack	0.69D+00	0.19D-14	949	26.00
condition number:			0.25D+14				
IR500C	1400	700	huang6	0.27D+00	0.11D-14	700	24.00
IR500C	1400	700	mod.huang6	0.21D-05	0.34D-15	700	49.00
IR500C	1400	700	huang7	0.27D+00	0.62D-15	700	24.00
IR500C	1400	700	mod.huang7	0.21D-05	0.23D-15	700	47.00
IR500C	1400	700	impl.qr5	0.12D+01	0.65D-14	700	29.00
IR500C	1400	700	expl.qr	0.40D-05	0.17D-14	700	17.00
IR500C	1400	700	qr lapack	0.35D-06	0.11D-15	700	15.00
IR500C	1400	700	svd lapack	0.35D-06	0.80D-15	700	66.00
IR500C	1400	700	gqr lapack	0.11D-05	0.32D-14	700	22.00
condition number:			0.51D+12				
IR500C	2000	400	huang6	0.11D-01	0.31D-14	400	11.00
IR500C	2000	400	mod.huang6	0.33D-03	0.18D-14	400	21.00
IR500C	2000	400	huang7	0.11D-01	0.50D-15	400	11.00
IR500C	2000	400	mod.huang7	0.33D-03	0.37D-15	400	21.00
IR500C	2000	400	impl.qr5	0.80D-03	0.23D-13	400	18.00
IR500C	2000	400	expl.qr	0.74D-03	0.55D-15	400	9.00
IR500C	2000	400	qr lapack	0.10D-02	0.37D-15	400	7.00
IR500C	2000	400	svd lapack	0.70D+00	0.55D-15	399	19.00
IR500C	2000	400	gqr lapack	0.70D+00	0.19D-13	399	11.00
condition number:			0.52D+13				

Test results for overdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time	
			error	error			
<hr/>							
RR100	1050	950	huang6	0.59D-10	0.21D-14	950	32.00
RR100	1050	950	mod.huang6	0.96D-14	0.25D-14	950	63.00
RR100	1050	950	huang7	0.59D-10	0.93D-15	950	30.00
RR100	1050	950	mod.huang7	0.85D-14	0.58D-15	950	61.00
RR100	1050	950	impl.qr5	0.82D-14	0.31D-14	950	57.00
RR100	1050	950	expl.qr	0.75D-14	0.10D-15	950	20.00
RR100	1050	950	qr lapack	0.79D-14	0.19D-15	950	16.00
RR100	1050	950	svd lapack	0.20D-13	0.30D-14	950	158.00
RR100	1050	950	gqr lapack	0.80D-14	0.26D-14	950	26.00
condition number:		0.35D+04					
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RR100	1400	700	huang6	0.25D-11	0.47D-12	700	24.00
RR100	1400	700	mod.huang6	0.44D-14	0.99D-12	700	48.00
RR100	1400	700	huang7	0.25D-11	0.99D-12	700	24.00
RR100	1400	700	mod.huang7	0.24D-14	0.85D-13	700	47.00
RR100	1400	700	impl.qr5	0.35D-14	0.54D-12	700	40.00
RR100	1400	700	expl.qr	0.31D-14	0.57D-13	700	18.00
RR100	1400	700	qr lapack	0.28D-14	0.49D-12	700	15.00
RR100	1400	700	svd lapack	0.86D-14	0.91D-12	700	84.00
RR100	1400	700	gqr lapack	0.36D-14	0.16D-11	700	22.00
condition number:		0.49D+03					
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RR100	2000	400	huang6	0.18D-11	0.40D-14	400	11.00
RR100	2000	400	mod.huang6	0.55D-14	0.49D-15	400	22.00
RR100	2000	400	huang7	0.18D-11	0.41D-14	400	11.00
RR100	2000	400	mod.huang7	0.21D-14	0.12D-14	400	21.00
RR100	2000	400	impl.qr5	0.60D-14	0.33D-15	400	18.00
RR100	2000	400	expl.qr	0.27D-14	0.33D-14	400	9.00
RR100	2000	400	qr lapack	0.30D-14	0.59D-14	400	8.00
RR100	2000	400	svd lapack	0.84D-14	0.74D-14	400	22.00
RR100	2000	400	gqr lapack	0.27D-14	0.15D-13	400	11.00
conition number:		0.26D+03					
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IDF1	1050	950	huang6	0.17D-03	0.25D-14	950	32.00
IDF1	1050	950	mod.huang6	0.44D-10	0.23D-16	950	63.00
IDF1	1050	950	huang7	0.17D-03	0.98D-15	950	30.00
IDF1	1050	950	mod.huang7	0.24D-10	0.23D-17	950	60.00
IDF1	1050	950	impl.qr5	0.32D-07	0.42D-12	950	58.00
IDF1	1050	950	expl.qr	0.63D-10	0.15D-14	950	19.00
IDF1	1050	950	qr lapack	0.23D-09	0.14D-14	950	17.00
IDF1	1050	950	svd lapack	0.22D-09	0.16D-14	950	128.00
IDF1	1050	950	gqr lapack	0.18D-09	0.52D-14	950	29.00
condition number:		0.15D+08					

Test results for overdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			error	error		
<hr/>						
IDF1	1400	700	huang6	0.53D-03	0.21D-14	700 23.00
IDF1	1400	700	mod.huang6	0.61D-10	0.11D-14	700 47.00
IDF1	1400	700	huang7	0.53D-03	0.11D-14	700 24.00
IDF1	1400	700	mod.huang7	0.23D-10	0.45D-15	700 47.00
IDF1	1400	700	impl.qr5	0.28D-07	0.30D-12	700 43.00
IDF1	1400	700	expl.qr	0.84D-10	0.77D-15	700 19.00
IDF1	1400	700	qr lapack	0.24D-09	0.23D-14	700 15.00
IDF1	1400	700	svd lapack	0.23D-09	0.89D-15	700 63.00
IDF1	1400	700	gqr lapack	0.23D-09	0.33D-14	700 23.00
condition number:			0.18D+08			
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IDF1	2000	400	huang6	0.12D-02	0.17D-14	400 11.00
IDF1	2000	400	mod.huang6	0.14D-09	0.90D-15	400 22.00
IDF1	2000	400	huang7	0.12D-02	0.21D-15	400 11.00
IDF1	2000	400	mod.huang7	0.31D-10	0.98D-15	400 21.00
IDF1	2000	400	impl.qr5	0.20D-07	0.18D-12	400 17.00
IDF1	2000	400	expl.qr	0.42D-10	0.22D-15	400 9.00
IDF1	2000	400	qr lapack	0.36D-09	0.97D-15	400 8.00
IDF1	2000	400	svd lapack	0.36D-09	0.87D-15	400 17.00
IDF1	2000	400	gqr lapack	0.52D-09	0.82D-16	400 12.00
condition number:			0.33D+08			
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IDF2	1050	950	huang6	0.65D+03	0.18D-12	950 29.00
IDF2	1050	950	mod.huang6	0.83D+10	0.86D-05	4 1.00
IDF2	1050	950	huang7	0.65D+03	0.76D-13	950 30.00
IDF2	1050	950	mod.huang7	0.83D+10	0.17D-05	4 0.00
IDF2	1050	700	impl.qr5	---	break-down ---	
IDF2	1050	950	expl.qr	0.33D+12	0.76D+11	4 0.00
IDF2	1050	950	qr lapack	0.11D+14	0.11D-05	950 16.00
IDF2	1050	950	svd lapack	0.10D+01	0.36D-15	3 138.00
IDF2	1050	950	gqr lapack	0.10D+01	0.31D-15	3 25.00
condition number:			0.11D+21			
<hr/>						
IDF2	1400	700	huang6	0.18D+04	0.97D-12	700 20.00
IDF2	1400	700	mod.huan	0.23D+11	0.50D-04	4 1.00
IDF2	1400	700	huang7	0.18D+04	0.38D-12	700 20.00
IDF2	1400	700	mod.huang7	0.23D+11	0.35D-04	4 0.00
IDF2	1050	700	impl.qr5	---	break-down ---	
IDF2	1400	700	expl.qr	0.11D+13	0.36D+12	4 0.00
IDF2	1400	700	qr lapack	0.32D+13	0.41D-06	700 14.00
IDF2	1400	700	svd lapack	0.10D+01	0.29D-15	3 74.00
IDF2	1400	700	gqr lapack	0.10D+01	0.68D-15	3 21.00
condition number:			0.45D+20			

Test results for overdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time	
			error	error			
<hr/>							
IDF2	2000	400	huang6	0.10D+04	0.51D-12	400	9.00
IDF2	2000	400	mod.huang6	0.76D+10	0.33D-05	4	0.00
IDF2	2000	400	huang7	0.10D+04	0.50D-12	400	9.00
IDF2	2000	400	mod.huang7	0.76D+10	0.45D-05	4	0.00
IDF2	1050	700	impl.qr5	--- break-down ---			
IDF2	2000	400	expl.qr	0.96D+13	0.30D+13	4	1.00
IDF2	2000	400	qr lapack	0.91D+12	0.22D-07	400	7.00
IDF2	2000	400	svd lapack	0.10D+01	0.68D-15	3	18.00
IDF2	2000	400	gqr lapack	0.10D+01	0.29D-14	3	11.00
condition number:			0.17D+20				
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IDF3	1050	950	huang6	0.32D+04	0.80D-13	950	30.00
IDF3	1050	950	mod.huang6	0.14D+04	0.14D-11	2	1.00
IDF3	1050	950	huang7	0.32D+04	0.52D-13	950	31.00
IDF3	1050	950	mod.huang7	0.14D+04	0.20D-09	2	0.00
IDF3	1050	950	impl.qr	0.14D+04	0.99D-15	2	0.00
IDF3	1050	950	expl.qr	0.10D+01	0.19D-03	2	0.00
IDF3	1050	950	qr lapack	0.37D+13	0.83D-02	950	17.00
IDF3	1050	950	svd lapack	0.10D+01	0.24D-14	2	145.00
IDF3	1050	950	gqr lapack	0.10D+01	0.22D-14	2	27.00
condition number:			0.16D+21				
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IDF3	1400	700	huang6	0.72D+03	0.24D-12	700	21.00
IDF3	1400	700	mod.huang6	0.89D+03	0.17D-11	2	0.00
IDF3	1400	700	huang7	0.72D+03	0.51D-13	700	21.00
IDF3	1400	700	mod.huang7	0.89D+03	0.30D-09	2	0.00
IDF3	1400	700	impl.qr5	0.89D+03	0.17D-11	2	0.00
IDF3	1400	700	expl.qr	0.10D+01	0.53D-03	2	0.00
IDF3	1400	700	qr lapack	0.10D+13	0.17D-02	700	15.00
IDF3	1400	700	svd lapack	0.10D+01	0.31D-13	2	76.00
IDF3	1400	700	gqr lapack	0.10D+01	0.29D-13	2	24.00
condition number:			0.27D+20				
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IDF3	2000	400	huang6	0.38D+04	0.58D-12	400	10.00
IDF3	2000	400	mod.huang6	0.44D+03	0.94D-16	2	0.00
IDF3	2000	400	huang7	0.38D+04	0.35D-12	400	9.00
IDF3	2000	400	mod.huang7	0.44D+03	0.67D-12	2	0.00
IDF3	2000	400	impl.qr5	0.44D+03	0.62D-16	2	0.00
IDF3	2000	400	expl.qr	0.10D+01	0.62D-03	2	0.00
IDF3	2000	400	qr lapack	0.45D+12	0.24D-02	400	8.00
IDF3	2000	400	svd lapack	0.10D+01	0.65D-15	2	17.00
IDF3	2000	400	gqr lapack	0.10D+01	0.19D-14	2	12.00
condition number:			0.63D+19				

Test results for overdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time	
			error	error			
<hr/>							
IR50	1050	950	huang6	0.10D+02	0.34D-15	950	33.00
IR50	1050	950	mod.huang6	0.92D+01	0.35D-13	773	61.00
IR50	1050	950	huang7	0.10D+02	0.66D-14	950	32.00
IR50	1050	950	mod.huang7	0.92D+01	0.48D-13	773	58.00
IR50	1050	950	impl.qr5	0.92D+01	0.68D-13	773	52.00
IR50	1050	950	expl.qr	0.68D+05	0.30D+06	773	37.00
IR50	1050	950	qr lapack	0.44D+12	0.16D-01	950	17.00
IR50	1050	950	svd lapack	0.41D+00	0.13D-13	773	96.00
IR50	1050	950	gqr lapack	0.41D+00	0.17D-13	773	28.00
condition number:			0.65D+21				
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IR50	1400	700	huang6	0.22D+02	0.29D-13	700	24.00
IR50	1400	700	mod.huang6	0.64D+01	0.12D-12	618	48.00
IR50	1400	700	huang7	0.22D+02	0.26D-13	700	24.00
IR50	1400	700	mod.huang7	0.64D+01	0.18D-13	618	48.00
IR50	1400	700	impl.qr5	0.64D+01	0.57D-13	618	37.00
IR50	1400	700	expl.qr	0.10D+05	0.25D+05	618	23.00
IR50	1400	700	qr lapack	0.34D+12	0.87D-01	700	15.00
IR50	1400	700	svd lapack	0.36D+00	0.62D-13	618	50.00
IR50	1400	700	gqr lapack	0.36D+00	0.11D-12	618	22.00
condition number:			0.43D+21				
<hr/>							
IR50	2000	400	huang6	0.41D+04	0.30D-11	400	10.00
IR50	2000	400	mod.huang6	0.45D+00	0.12D-14	374	20.00
IR50	2000	400	huang7	0.41D+04	0.71D-12	400	11.00
IR50	2000	400	mod.huang7	0.45D+00	0.24D-14	374	21.00
IR50	2000	400	impl.qr5	0.45D+00	0.43D-14	374	17.00
IR50	2000	400	expl.qr	0.19D+04	0.65D+04	374	9.00
IR50	2000	400	qr lapack	0.80D+12	0.19D+00	400	7.00
IR50	2000	400	svd lapack	0.33D+00	0.14D-13	374	18.00
IR50	2000	400	gqr lapack	0.33D+00	0.11D-12	374	11.00
condition number:			0.11D+21				

4.3 Test results for underdetermined linear systems

matrix	dimension	method	solution	residual	rank	time	
			error	error			
<hr/>							
IR500	950	1050	huang1	0.00D+00	0.00D+00	950	46.00
IR500	950	1050	mod.huang1	0.00D+00	0.00D+00	950	66.00
IR500	950	1050	huang2	0.00D+00	0.00D+00	950	33.00
IR500	950	1050	mod.huang2	0.00D+00	0.00D+00	950	65.00
IR500	950	1050	qr lapack	0.30D-09	0.14D-14	950	17.00
IR500	950	1050	svd lapack	0.34D-09	0.18D-14	950	145.00
IR500	950	1050	gqr lapack	0.20D-09	0.14D-14	950	28.00
condition number:			0.19D+07				

Test results for underdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			error	error		
<hr/>						
IR500	700 1400	huang1	0.00D+00	0.00D+00	700	61.00
IR500	700 1400	mod.huang1	0.00D+00	0.00D+00	700	88.00
IR500	700 1400	huang2	0.00D+00	0.00D+00	700	25.00
IR500	700 1400	mod.huang2	0.00D+00	0.00D+00	700	50.00
IR500	700 1400	qr lapack	0.12D-09	0.19D-14	700	16.00
IR500	700 1400	svd lapack	0.15D-09	0.22D-14	700	63.00
IR500	700 1400	gqr lapack	0.94D-11	0.11D-14	700	35.00
condition number:		0.27D+06				
IR500	400 2000	huang1	0.00D+00	0.00D+00	400	72.00
IR500	400 2000	mod.huang1	0.00D+00	0.00D+00	400	105.00
IR500	400 2000	huang2	0.00D+00	0.00D+00	400	12.00
IR500	400 2000	mod.huang2	0.00D+00	0.00D+00	400	23.00
IR500	400 2000	qr lapack	0.43D-10	0.22D-14	400	10.00
IR500	400 2000	svd lapack	0.11D-09	0.30D-14	400	19.00
IR500	400 2000	gqr lapack	0.30D-10	0.14D-14	400	24.00
condition number:		0.11D+06				
IR500R	950 1050	huang1	0.23D-09	0.12D-14	950	49.00
IR500R	950 1050	mod.huang1	0.28D-09	0.12D-14	949	66.00
IR500R	950 1050	huang2	0.26D-09	0.35D-14	950	33.00
IR500R	950 1050	mod.huang2	0.27D-09	0.12D-14	949	65.00
IR500R	950 1050	qr lapack	0.46D-03	0.14D-14	950	17.00
IR500R	950 1050	svd lapack	0.34D-09	0.18D-14	949	143.00
IR500R	950 1050	gqr lapack	0.13D-09	0.11D-14	949	28.00
condition number:		0.12D+15				
IR500R	700 1400	huang1	0.12D-09	0.43D-14	700	62.00
IR500R	700 1400	mod.huang1	0.11D-09	0.15D-14	699	88.00
IR500R	700 1400	huang2	0.97D-10	0.18D-14	700	25.00
IR500R	700 1400	mod.huang2	0.11D-09	0.15D-14	699	50.00
IR500R	700 1400	qr lapack	0.12D-09	0.16D-14	700	16.00
IR500R	700 1400	svd lapack	0.24D-09	0.30D-14	699	63.00
IR500R	700 1400	gqr lapack	0.47D-10	0.15D-14	699	34.00
condition number:		0.64D+14				
IR500R	400 2000	huang1	0.48D-10	0.20D-14	400	72.00
IR500R	400 2000	mod.huang1	0.37D-10	0.16D-14	399	104.00
IR500R	400 2000	huang2	0.44D-10	0.34D-14	400	12.00
IR500R	400 2000	mod.huang2	0.38D-10	0.16D-14	399	23.00
IR500R	400 2000	qr lapack	0.27D-03	0.20D-14	400	10.00
IR500R	400 2000	svd lapack	0.67D-10	0.34D-14	399	19.00
IR500R	400 2000	gqr lapack	0.18D-10	0.13D-14	399	24.00
condition number:		0.45D+14				

Test results for underdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			error	error		
<hr/>						
RR100	950 1050	huang1	0.36D-14	0.12D-14	950	46.00
RR100	950 1050	mod.huang1	0.44D-14	0.13D-14	950	67.00
RR100	950 1050	huang2	0.31D-14	0.11D-14	950	33.00
RR100	950 1050	mod.huang2	0.46D-14	0.14D-14	950	65.00
RR100	950 1050	qr lapack	0.66D-14	0.15D-14	950	17.00
RR100	950 1050	svd lapack	0.15D-13	0.80D-14	950	201.00
RR100	950 1050	gqr lapack	0.10D-13	0.26D-14	950	28.00
condition number:		0.39D+02				
RR100	700 1400	huang1	0.24D-14	0.15D-14	700	62.00
RR100	700 1400	mod.huang1	0.33D-14	0.18D-14	700	88.00
RR100	700 1400	huang2	0.24D-14	0.15D-14	700	25.00
RR100	700 1400	mod.huang2	0.32D-14	0.16D-14	700	50.00
RR100	700 1400	qr lapack	0.27D-14	0.16D-14	700	17.00
RR100	700 1400	svd lapack	0.12D-13	0.86D-14	700	86.00
RR100	700 1400	gqr lapack	0.48D-14	0.26D-14	700	35.00
condition number:		0.56D+01				
RR100	400 2000	huang1	0.44D-15	0.10D-14	400	72.00
RR100	400 2000	mod.huang1	0.19D-14	0.10D-14	400	103.00
RR100	400 2000	huang2	0.24D-15	0.11D-14	400	12.00
RR100	400 2000	mod.huang2	0.20D-14	0.11D-14	400	23.00
RR100	400 2000	qr lapack	0.17D-14	0.19D-14	400	9.00
RR100	400 2000	svd lapack	0.71D-14	0.62D-14	400	20.00
RR100	400 2000	gqr lapack	0.30D-14	0.20D-14	400	24.00
condition number:		0.26D+01				
IDF1	950 1050	huang1	0.00D+00	0.00D+00	950	46.00
IDF1	950 1050	mod.huang1	0.00D+00	0.00D+00	950	66.00
IDF1	950 1050	huang2	0.00D+00	0.00D+00	950	33.00
IDF1	950 1050	mod.huang2	0.00D+00	0.00D+00	950	65.00
IDF1	950 1050	qr lapack	0.26D-09	0.52D-14	950	17.00
IDF1	950 1050	svd lapack	0.18D-08	0.50D-14	950	157.00
IDF1	950 1050	gqr lapack	0.11D-09	0.14D-14	950	29.00
condition number:		0.71D+06				
IDF1	700 1400	huang1	0.00D+00	0.00D+00	700	61.00
IDF1	700 1400	mod.huang1	0.00D+00	0.00D+00	700	90.00
IDF1	700 1400	huang2	0.13D-08	0.37D-14	700	25.00
IDF1	700 1400	mod.huang2	0.13D-08	0.31D-14	700	51.00
IDF1	700 1400	qr lapack	0.40D-09	0.12D-13	700	16.00
IDF1	700 1400	svd lapack	0.76D-09	0.24D-14	700	65.00
IDF1	700 1400	gqr lapack	0.32D-09	0.35D-14	700	34.00
condition number:		0.11D+07				

Test results for underdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			error	error		
<hr/>						
IDF1	400 2000	huang1	0.00D+00	0.00D+00	400	72.00
IDF1	400 2000	mod.huang1	0.00D+00	0.00D+00	400	103.00
IDF1	400 2000	huang2	0.00D+00	0.00D+00	400	12.00
IDF1	400 2000	mod.huang2	0.00D+00	0.00D+00	400	23.00
IDF1	400 2000	qr lapack	0.76D-09	0.18D-13	400	10.00
IDF1	400 2000	svd lapack	0.15D-08	0.40D-14	400	16.00
IDF1	400 2000	gqr lapack	0.30D-09	0.30D-14	400	24.00
condition number:			0.18D+07			
IDF2	950 1050	huang1	0.93D-11	0.41D-12	950	46.00
IDF2	950 1050	mod.huang1	0.12D+01	0.72D-12	4	41.00
IDF2	950 1050	huang2	0.22D-11	0.13D-12	950	33.00
IDF2	950 1050	mod.huang2	0.69D+01	0.24D-12	4	1.00
IDF2	950 1050	qr lapack	0.10D+04	0.15D-13	950	17.00
IDF2	950 1050	svd lapack	0.29D-14	0.27D-14	3	173.00
IDF2	950 1050	gqr lapack	0.14D-14	0.16D-14	3	26.00
condition number:			0.48D+19			
IDF2	700 1400	huang1	0.73D-10	0.19D-11	700	61.00
IDF2	700 1400	mod.huang1	0.18D+01	0.10D-12	4	55.00
IDF2	700 1400	huang2	0.29D-11	0.54D-12	700	26.00
IDF2	700 1400	mod.huang2	0.20D-08	0.29D-09	3	1.00
IDF2	700 1400	qr lapack	0.42D+03	0.65D-14	700	16.00
IDF2	700 1400	svd lapack	0.37D-14	0.26D-14	3	126.00
IDF2	700 1400	gqr lapack	0.28D-14	0.19D-14	3	23.00
condition number:			0.14D+19			
IDF2	400 2000	huang1	0.17D-10	0.19D-12	400	72.00
IDF2	400 2000	mod.huang1	0.81D+00	0.79D-13	4	63.00
IDF2	400 2000	huang2	0.12D-10	0.10D-12	400	12.00
IDF2	400 2000	mod.huang2	0.36D-08	0.61D-10	3	1.00
IDF2	400 2000	qr lapack	0.29D+03	0.37D-14	400	9.00
IDF2	400 2000	svd lapack	0.43D-13	0.22D-14	3	68.00
IDF2	400 2000	gqr lapack	0.18D-13	0.24D-14	3	12.00
condition number:			0.29D+18			
IDF3	950 1050	huang1	0.00D+00	0.00D+00	950	46.00
IDF3	950 1050	mod.huang1	0.00D+00	0.00D+00	2	41.00
IDF3	950 1050	huang2	0.00D+00	0.00D+00	950	33.00
IDF3	950 1050	mod.huang2	0.00D+00	0.00D+00	2	1.00
IDF3	950 1050	qr lapack	0.24D+03	0.56D-14	950	17.00
IDF3	950 1050	svd lapack	0.17D-14	0.92D-16	2	178.00
IDF3	950 1050	gqr lapack	0.21D-14	0.55D-15	2	26.00
condition number:			0.24D+19			

Test results for underdetermined linear systems - continued

matrix	dimension	method	solution	residual	rank	time
			error	error		
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IDF3	700 1400	huang1	0.00D+00	0.00D+00	700	61.00
IDF3	700 1400	mod.huang1	0.00D+00	0.00D+00	2	56.00
IDF3	700 1400	huang2	0.00D+00	0.00D+00	700	24.00
IDF3	700 1400	mod.huang2	0.00D+00	0.00D+00	2	1.00
IDF3	700 1400	qr lapack	0.62D+02	0.20D-14	700	15.00
IDF3	700 1400	svd lapack	0.27D-15	0.73D-18	2	139.00
IDF3	700 1400	gqr lapack	0.12D-14	0.29D-16	2	23.00
condition number:						
0.46D+18						
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IDF3	400 2000	huang1	0.00D+00	0.00D+00	400	71.00
IDF3	400 2000	mod.huang1	0.00D+00	0.00D+00	2	63.00
IDF3	400 2000	huang2	0.19D-10	0.18D-11	400	12.00
IDF3	400 2000	mod.huang2	0.10D-12	0.58D-13	2	1.00
IDF3	400 2000	qr lapack	0.38D+02	0.31D-14	400	8.00
IDF3	400 2000	svd lapack	0.14D-14	0.71D-16	2	67.00
IDF3	400 2000	gqr lapack	0.21D-14	0.85D-17	2	13.00
condition number:						
0.23D+18						
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IR50	950 1050	huang1	0.00D+00	0.00D+00	948	47.00
IR50	950 1050	mod.huang1	0.00D+00	0.00D+00	744	61.00
IR50	950 1050	huang2	0.00D+00	0.00D+00	950	33.00
IR50	950 1050	mod.huang2	0.00D+00	0.00D+00	744	57.00
IR50	950 1050	qr lapack	0.87D+02	0.55D-14	950	17.00
IR50	950 1050	svd lapack	0.65D-10	0.23D-14	744	115.00
IR50	950 1050	gqr lapack	0.39D-10	0.22D-14	744	31.00
condition number:						
0.70D+19						
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IR50	700 1400	huang1	0.00D+00	0.00D+00	688	61.00
IR50	700 1400	mod.huang1	0.00D+00	0.00D+00	567	83.00
IR50	700 1400	huang2	0.00D+00	0.00D+00	700	25.00
IR50	700 1400	mod.huang2	0.00D+00	0.00D+00	567	44.00
IR50	700 1400	qr lapack	0.19D+02	0.78D-15	700	16.00
IR50	700 1400	svd lapack	0.46D-10	0.37D-14	567	85.00
IR50	700 1400	gqr lapack	0.64D-11	0.19D-14	567	32.00
condition number:						
0.13D+18						
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IR50	400 2000	huang1	0.00D+00	0.00D+00	398	71.00
IR50	400 2000	mod.huang1	0.00D+00	0.00D+00	385	102.00
IR50	400 2000	huang2	0.00D+00	0.00D+00	400	11.00
IR50	400 2000	mod.huang2	0.00D+00	0.00D+00	385	21.00
IR50	400 2000	qr lapack	0.12D+01	0.15D-14	400	9.00
IR50	400 2000	svd lapack	0.76D-11	0.41D-14	385	64.00
IR50	400 2000	gqr lapack	0.21D-11	0.15D-14	385	22.00
condition number:						
0.53D+17						

4.4 Test results for KKT linear systems

matrix	dimension	method	solution	residual	rank	time
			error	error		
<hr/>						
IR500	1000	900	mod.huang	0.50D-07	0.36D-14	1900 124.00
IR500	1000	900	impl.lu8	0.70D-06	0.16D-13	1900 17.00
IR500	1000	900	impl.lu9	0.71D-06	0.18D-13	1900 19.00
IR500	1000	900	lu lapack	0.22D-05	0.96D-14	1900 56.00
IR500	1000	900	range space	0.42D-01	0.52D-11	1900 79.00
IR500	1000	900	null space	0.19D-06	0.20D-14	1900 85.00
condition number:			0.22D+10			
IR500	1200	600	mod.huang	0.32D-07	0.90D-14	1800 188.00
IR500	1200	600	impl.lu8	0.83D-07	0.11D-13	1800 44.00
IR500	1200	600	impl.lu9	0.89D-07	0.11D-13	1800 33.00
IR500	1200	600	lu lapack	0.49D-07	0.41D-14	1800 47.00
IR500	1200	600	range space	0.23D-02	0.55D-11	1800 64.00
IR500	1200	600	null space	0.55D-08	0.26D-14	1800 105.00
condition number:			0.15D+09			
IR500	1500	200	mod.huang	0.15D-07	0.75D-14	1700 303.00
IR500	1500	200	impl.lu8	0.76D-08	0.63D-14	1700 78.00
IR500	1500	200	impl.lu9	0.92D-08	0.14D-13	1700 59.00
IR500	1500	200	lu lapack	0.31D-08	0.91D-14	1700 40.00
IR500	1500	200	range space	0.24D-05	0.16D-12	1700 41.00
IR500	1500	200	null space	0.19D-08	0.25D-14	1700 78.00
condition number:			0.10D+08			
IR500R	1000	900	mod.huang	0.30D+01	0.49D-14	1900 124.00
IR500R	1000	900	impl.lu8	0.80D-01	0.14D-13	1900 17.00
IR500R	1000	900	impl.lu9	0.80D-01	0.10D-13	1900 19.00
IR500R	1000	900	lu lapack	0.22D+01	0.10D-13	1900 55.00
IR500R	1000	900	range space	0.74D+00	0.75D-11	1900 79.00
IR500R	1000	900	null space	0.64D+01	0.28D-14	1900 85.00
condition number:			0.18D+17			
IR500R	1200	600	mod.huang	0.15D+00	0.81D-14	1800 187.00
IR500R	1200	600	impl.lu8	0.35D-02	0.12D-13	1800 44.00
IR500R	1200	600	impl.lu9	0.35D-02	0.14D-13	1800 34.00
IR500R	1200	600	lu lapack	0.57D+00	0.64D-14	1800 47.00
IR500R	1200	600	range space	0.16D+01	0.91D-11	1800 63.00
IR500R	1200	600	null space	0.46D+00	0.20D-14	1800 105.00
condition number:			0.15D+16			
IR500R	1500	200	mod.huang	0.50D+00	0.98D-14	1700 302.00
IR500R	1500	200	impl.lu8	0.81D-01	0.61D-14	1700 78.00
IR500R	1500	200	impl.lu9	0.81D-01	0.83D-14	1700 59.00
IR500R	1500	200	lu lapack	0.13D+01	0.16D-13	1700 39.00
IR500R	1500	200	range space	0.27D+01	0.36D-12	1700 41.00
IR500R	1500	200	null space	0.19D+00	0.31D-14	1700 78.00
condition number:			0.17D+16			

Test results for KKT linear systems - continued

matrix	dimension		method	solution	residual	rank	time
	n	m		error			
<hr/>							
IR500S	1000	900	mod.huang	0.31D-06	0.36D-14	1900	125.00
IR500S	1000	900	impl.lu8	0.41D-04	0.14D-13	1900	17.00
IR500S	1000	900	impl.lu9	0.41D-04	0.30D-13	1900	21.00
IR500S	1000	900	lu lapack	0.75D-05	0.19D-14	1900	56.00
IR500S	1000	900	range space	0.34D+02	0.37D+01	1900	80.00
IR500S	1000	900	null space	0.11D-04	0.19D-14	1900	89.00
condition number:		0.14D+12					
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IR500S	1200	600	mod.huang	0.57D-08	0.37D-13	1800	189.00
IR500S	1200	600	impl.lu8	0.12D-06	0.43D-12	1800	42.00
IR500S	1200	600	impl.lu9	0.15D-06	0.16D-11	1800	34.00
IR500S	1200	600	lu lapack	0.18D-08	0.19D-13	1800	49.00
IR500S	1200	600	range space	0.18D+02	0.22D+01	1800	65.00
IR500S	1200	600	null space	0.23D-07	0.19D-13	1800	110.00
condition number:		0.23D+09					
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IR500S	1500	200	mod.huang	0.23D-08	0.76D-14	1700	304.00
IR500S	1500	200	impl.lu8	0.13D-07	0.91D-14	1700	78.00
IR500S	1500	200	impl.lu9	0.12D-07	0.14D-14	1700	60.00
IR500S	1500	200	lu lapack	0.15D-08	0.11D-14	1700	41.00
IR500S	1500	200	range space	0.14D+03	0.53D+02	1700	43.00
IR500S	1500	200	null space	0.95D-08	0.19D-14	1700	77.00
condition number:		0.30D+08					
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IR500B	1000	900	mod.huang	0.12D-01	0.83D-14	1900	125.00
IR500B	1000	900	impl.lu8	0.14D-02	0.13D-13	1900	17.00
IR500B	1000	900	impl.lu9	0.14D-02	0.14D-13	1900	20.00
IR500B	1000	900	lu lapack	0.17D-02	0.45D-14	1900	58.00
IR500B	1000	900	range space	0.26D+03	0.13D-02	1900	78.00
IR500B	1000	900	null space	0.17D-01	0.23D-14	1900	88.00
condition number:		0.76D+14					
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IR500B	1200	600	mod.huang	0.69D-03	0.60D-14	1800	189.00
IR500B	1200	600	impl.lu8	0.15D-03	0.10D-13	1800	43.00
IR500B	1200	600	impl.lu9	0.15D-03	0.11D-13	1800	33.00
IR500B	1200	600	lu lapack	0.38D-02	0.20D-13	1800	47.00
IR500B	1200	600	range space	0.66D+03	0.22D-03	1800	62.00
IR500B	1200	600	null space	0.11D-01	0.21D-14	1800	108.00
condition number:		0.24D+14					
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IR500B	1500	200	mod.huang	0.89D-03	0.11D-13	1700	309.00
IR500B	1500	200	impl.lu8	0.23D-05	0.11D-13	1700	77.00
IR500B	1500	200	impl.lu9	0.16D-05	0.40D-13	1700	57.00
IR500B	1500	200	lu lapack	0.15D-03	0.42D-14	1700	39.00
IR500B	1500	200	range space	0.22D+04	0.12D+00	1700	39.00
IR500B	1500	200	null space	0.18D-02	0.34D-14	1700	77.00
condition number:		0.19D+13					

Test results for KKT linear systems - continued

matrix	dimension		method	solution error	residual error	rank	time
	n	m					
<hr/>							
RR100	1000	900	mod.huang	0.57D-10	0.18D-12	1900	136.00
RR100	1000	900	impl.lu8	0.64D-12	0.13D-13	1900	18.00
RR100	1000	900	impl.lu9	0.50D-12	0.23D-13	1900	21.00
RR100	1000	900	lu lapack	0.23D-11	0.11D-12	1900	60.00
RR100	1000	900	range space	0.18D-08	0.10D-10	1900	87.00
RR100	1000	900	null space	0.23D-12	0.26D-14	1900	91.00
condition number:		0.13D+04					
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RR100	1200	600	mod.huang	0.92D-10	0.83D-12	1800	187.00
RR100	1200	600	impl.lu8	0.13D-11	0.26D-13	1800	44.00
RR100	1200	600	impl.lu9	0.16D-11	0.70D-13	1800	34.00
RR100	1200	600	lu lapack	0.15D-11	0.11D-12	1800	47.00
RR100	1200	600	range space	0.25D-09	0.19D-11	1800	64.00
RR100	1200	600	null space	0.88D-13	0.24D-14	1800	105.00
condition number:		0.50D+03					
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RR100	1500	200	mod.huang	0.23D-08	0.36D-10	1700	301.00
RR100	1500	200	impl.lu8	0.10D-11	0.42D-13	1700	78.00
RR100	1500	200	impl.lu9	0.33D-11	0.10D-12	1700	59.00
RR100	1500	200	lu lapack	0.62D-12	0.11D-12	1700	39.00
RR100	1500	200	range space	0.28D-10	0.87D-12	1700	41.00
RR100	1500	200	null space	0.96D-13	0.23D-14	1700	78.00
condition number:		0.15D+03					
<hr/>							
IDF1	1000	900	mod.huang	0.85D-10	0.18D-14	1900	133.00
IDF1	1000	900	impl.lu8	0.38D-10	0.13D-14	1900	17.00
IDF1	1000	900	impl.lu9	0.36D-10	0.14D-14	1900	20.00
IDF1	1000	900	lu lapack	0.48D-10	0.35D-14	1900	59.00
IDF1	1000	900	range space	0.79D-10	0.18D-14	1900	85.00
IDF1	1000	900	null space	0.66D-10	0.21D-14	1900	92.00
condition number:		0.80D+04					
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IDF1	1200	600	mod.huang	0.16D-08	0.27D-13	1800	182.00
IDF1	1200	600	impl.lu8	0.55D-10	0.15D-14	1800	43.00
IDF1	1200	600	impl.lu9	0.49D-10	0.15D-14	1800	33.00
IDF1	1200	600	lu lapack	0.28D-10	0.19D-14	1800	47.00
IDF1	1200	600	range space	0.87D-10	0.16D-14	1800	63.00
IDF1	1200	600	null space	0.64D-10	0.25D-14	1800	105.00
condition number:		0.96D+04					
<hr/>							
IDF1	1500	200	mod.huang	0.33D-07	0.42D-12	1700	303.00
IDF1	1500	200	impl.lu8	0.52D-10	0.19D-14	1700	77.00
IDF1	1500	200	impl.lu9	0.63D-10	0.15D-14	1700	58.00
IDF1	1500	200	lu lapack	0.28D-10	0.15D-14	1700	39.00
IDF1	1500	200	range space	0.11D-09	0.18D-14	1700	41.00
IDF1	1500	200	null space	0.15D-09	0.33D-14	1700	78.00
condition number:		0.12D+05					

Test results for KKT linear systems - continued

matrix	dimension		method	solution	residual	rank	time
	n	m		error			
<hr/>							
IDF2	1000	900	mod.huang	0.55D+01	0.23D-14	16	24.00
IDF2	1000	900	impl.lu8	0.44D+13	0.21D-03	1900	18.00
IDF2	1000	900	impl.lu9	0.12D+15	0.80D-02	1900	21.00
IDF2	1000	900	lu lapack	0.25D+03	0.31D-13	1900	62.00
IDF2	1000	900	range space	0.16D+05	0.14D-11	1900	87.00
IDF2	1000	900	null space	0.89D+03	0.15D-12	1900	93.00
condition number:	0.26D+21						
<hr/>							
IDF2	1200	600	mod.huang	0.62D+01	0.20D-14	17	36.00
IDF2	1200	600	impl.lu8	0.22D+07	0.10D-08	1800	44.00
IDF2	1200	600	impl.lu9	0.21D+06	0.56D-09	1800	33.00
IDF2	1200	600	lu lapack	0.10D+03	0.79D-14	1800	47.00
IDF2	1200	600	range space	0.11D+05	0.15D-11	1800	63.00
IDF2	1200	600	null space	0.38D+04	0.13D-12	1800	105.00
condition number:	0.70D+20						
<hr/>							
IDF2	1500	200	mod.huang	0.38D+01	0.22D-14	17	68.00
IDF2	1500	200	impl.lu8	0.31D+04	0.76D-11	1700	79.00
IDF2	1500	200	impl.lu9	0.58D+03	0.57D-12	1700	59.00
IDF2	1500	200	lu lapack	0.28D+03	0.19D-13	1700	39.00
IDF2	1500	200	range space	0.99D+05	0.16D-10	1700	41.00
IDF2	1500	200	null space	0.25D+03	0.36D-13	1700	78.00
condition number:	0.28D+20						
<hr/>							
IR50	1000	900	mod.huang	0.76D+00	0.48D-14	1702	143.00
IR50	1000	900	impl.lu8	0.29D+16	0.28D+00	1900	18.00
IR50	1000	900	impl.lu9	0.13D+16	0.15D+00	1900	21.00
IR50	1000	900	lu lapack	0.10D+04	0.59D-13	1900	60.00
IR50	1000	900	range space	0.24D+03	0.17D-10	1900	84.00
IR50	1000	900	null space	0.18D+18	0.11D+02	1900	91.00
condition number:	0.27D+19						
<hr/>							
IR50	1200	600	mod.huang	0.30D+00	0.84D-14	1751	186.00
IR50	1200	600	impl.lu8	0.44D+14	0.53D-02	1800	44.00
IR50	1200	600	impl.lu9	0.23D+14	0.33D-02	1800	34.00
IR50	1200	600	lu lapack	0.63D+02	0.15D-13	1800	47.00
IR50	1200	600	range space	0.10D+03	0.28D-09	1800	63.00
IR50	1200	600	null space	0.82D+15	0.66D-01	1800	105.00
condition number:	0.14D+18						
<hr/>							
IR50	1500	200	mod.huang	0.33D-07	0.68D-14	1700	302.00
IR50	1500	200	impl.lu8	0.73D-08	0.76D-14	1700	78.00
IR50	1500	200	impl.lu9	0.63D-07	0.99D-14	1700	59.00
IR50	1500	200	lu lapack	0.58D-09	0.63D-14	1700	39.00
IR50	1500	200	range space	0.20D-05	0.36D-13	1700	41.00
IR50	1500	200	null space	0.11D-07	0.36D-14	1700	78.00
condition number:	0.12D+08						

4.5 Test results for Harwell-Boeing matrices

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
arc130	130	huang1	0.22D-11	0.59D-16	130	0.00
arc130	130	mod.huang1	0.22D-11	0.59D-16	130	0.00
arc130	130	huang2	0.26D-11	0.64D-16	130	0.00
arc130	130	mod.huang2	0.22D-11	0.59D-16	130	0.00
arc130	130	impl.lu3	0.21D-11	0.67D-16	130	0.00
arc130	130	impl.lu4	0.21D-11	0.67D-16	130	0.00
arc130	130	impl.qr5	0.99D-11	0.12D-14	130	0.00
arc130	130	expl.qr	0.54D-11	0.14D-15	130	0.00
arc130	130	expl.lu	0.61D-11	0.18D-15	130	0.00
arc130	130	lu lapack	0.61D-11	0.18D-15	130	1.00
arc130	130	qr lapack	0.48D-05	0.96D-15	130	0.00
arc130	130	gqr lapack	0.24D-10	0.65D-15	130	0.00
arc130	130	lu linpack	0.61D-11	0.18D-15	130	0.00
condition number:			0.11D+11			
<hr/>						
bp1200	822	huang1	0.12D-07	0.39D-12	822	13.00
bp1200	822	mod.huang1	0.29D-11	0.10D-14	822	21.00
bp1200	822	huang2	0.63D-06	0.68D-11	822	9.00
bp1200	822	mod.huang2	0.52D-11	0.96D-15	822	24.00
bp1200	822	impl.lu3	0.24D-12	0.42D-15	822	5.00
bp1200	822	impl.lu4	0.24D-12	0.42D-15	822	3.00
bp1200	822	impl.qr5	0.75D-11	0.30D-14	822	32.00
bp1200	822	expl.qr	0.65D-11	0.71D-15	822	8.00
bp1200	822	expl.lu	0.33D-10	0.78D-15	822	5.00
bp1200	822	lu lapack	0.44D-10	0.48D-15	822	0.00
bp1200	822	qr lapack	0.14D-08	0.16D-14	822	8.00
bp1200	822	gqr lapack	0.25D-10	0.11D-14	822	11.00
bp1200	822	lu linpack	0.44D-10	0.48D-15	822	0.00
condition number:			0.15D+09			
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bp1400	822	huang1	0.25D-09	0.51D-13	822	13.00
bp1400	822	mod.huang1	0.19D-12	0.11D-14	822	20.00
bp1400	822	huang2	0.11D-08	0.81D-12	822	10.00
bp1400	822	mod.huang2	0.96D-13	0.87D-15	822	23.00
bp1400	822	impl.lu3	0.16D-12	0.42D-15	822	6.00
bp1400	822	impl.lu4	0.16D-12	0.42D-15	822	2.00
bp1400	822	impl.qr5	0.60D-11	0.32D-14	822	32.00
bp1400	822	expl.qr	0.48D-12	0.63D-15	822	8.00
bp1400	822	expl.lu	0.12D-11	0.54D-15	822	5.00
bp1400	822	lu lapack	0.14D-11	0.58D-15	822	1.00
bp1400	822	qr lapack	0.39D-10	0.21D-14	822	8.00
bp1400	822	gqr lapack	0.54D-10	0.64D-15	822	11.00
bp1400	822	lu linpack	0.14D-11	0.58D-15	822	1.00
condition number:			0.23D+08			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
fs1833	183	huang1	0.12D-11	0.33D-16	183	0.00
fs1833	183	mod.huang1	0.37D-13	0.69D-15	183	0.00
fs1833	183	huang2	0.29D-11	0.49D-15	183	0.00
fs1833	183	mod.huang2	0.25D-13	0.10D-14	183	0.00
fs1833	183	impl.lu3	0.46D-13	0.65D-15	183	0.00
fs1833	183	impl.lu4	0.46D-13	0.65D-15	183	0.00
fs1833	183	impl.qr5	0.32D-05	0.36D-15	183	0.00
fs1833	183	expl.qr	0.44D-13	0.20D-15	183	1.00
fs1833	183	expl.lu	0.35D-12	0.16D-15	183	0.00
fs1833	183	lu lapack	0.31D-12	0.65D-16	183	0.00
fs1833	183	qr lapack	0.17D-03	0.47D-15	183	0.00
fs1833	183	gqr lapack	0.24D+00	0.18D-11	177	0.00
fs1833	183	lu linpack	0.31D-12	0.65D-16	183	0.00
condition number:			0.13D+14			
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fs5413	541	huang1	0.63D-03	0.80D-11	541	4.00
fs5413	541	mod.huang1	0.11D-09	0.77D-15	541	6.00
fs5413	541	huang2	0.22D-02	0.31D-09	541	1.00
fs5413	541	mod.huang2	0.11D-09	0.10D-14	541	4.00
fs5413	541	impl.lu3	0.23D-09	0.44D-15	541	2.00
fs5413	541	impl.lu4	0.23D-09	0.44D-15	541	1.00
fs5413	541	impl.qr5	0.22D-09	0.14D-13	541	7.00
fs5413	541	expl.qr	0.69D-11	0.30D-14	541	1.00
fs5413	541	expl.lu	0.15D-09	0.48D-15	541	2.00
fs5413	541	lu lapack	0.16D-09	0.24D-14	541	0.00
fs5413	541	qr lapack	0.22D-05	0.45D-14	541	2.00
fs5413	541	gqr lapack	0.89D-06	0.60D-14	541	2.00
fs5413	541	lu linpack	0.16D-09	0.24D-14	541	1.00
condition number:			0.98D+12			
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fs6803	680	huang1	0.11D-13	0.17D-14	680	7.00
fs6803	680	mod.huang1	0.68D-14	0.16D-14	680	10.00
fs6803	680	huang2	0.17D-13	0.15D-14	680	4.00
fs6803	680	mod.huang2	0.43D-14	0.17D-14	680	10.00
fs6803	680	impl.lu3	0.17D-14	0.17D-14	680	3.00
fs6803	680	impl.lu4	0.17D-14	0.17D-14	680	1.00
fs6803	680	impl.qr5	0.78D-14	0.63D-14	680	15.00
fs6803	680	expl.qr	0.10D-13	0.11D-14	680	4.00
fs6803	680	expl.lu	0.40D-14	0.46D-15	680	3.00
fs6803	680	lu lapack	0.36D-14	0.45D-15	680	1.00
fs6803	680	qr lapack	0.18D-13	0.92D-15	680	3.00
fs6803	680	gqr lapack	0.25D-11	0.49D-15	680	5.00
fs6803	680	lu linpack	0.36D-14	0.45D-15	680	2.00
condition number:			0.21D+07			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
fs7603	760	huang1	0.38D+00	0.13D-12	645	11.00
fs7603	760	mod.huang1	0.38D+00	0.17D-13	645	16.00
fs7603	760	huang2	0.38D+00	0.16D-12	645	6.00
fs7603	760	mod.huang2	0.38D+00	0.17D-13	645	18.00
fs7603	760	impl.lu3	0.24D-08	0.15D-14	760	6.00
fs7603	760	impl.lu4	0.24D-08	0.15D-14	760	2.00
fs7603	760	impl.qr5	0.39D+01	0.15D-13	664	22.00
fs7603	760	expl.qr	0.38D+00	0.17D-13	645	6.00
fs7603	760	expl.lu	0.18D-07	0.11D-15	760	4.00
fs7603	760	lu lapack	0.17D-07	0.99D-16	760	1.00
fs7603	760	qr lapack	0.22D-02	0.40D-15	760	6.00
fs7603	760	gqr lapack	0.73D+00	0.31D-11	330	10.00
fs7603	760	lu linpack	0.17D-07	0.99D-16	760	1.00
condition number:			0.50D+20			
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gre216b	216	huang1	0.18D+00	0.10D-09	207	0.00
gre216b	216	mod.huang1	0.18D+00	0.11D-09	207	0.00
gre216b	216	huang2	0.18D+00	0.11D-09	207	0.00
gre216b	216	mod.huang2	0.18D+00	0.11D-09	207	1.00
gre216b	216	impl.lu3	0.97D-01	0.36D-13	214	0.00
gre216b	216	impl.lu4	0.97D-01	0.36D-13	214	0.00
gre216b	216	impl.qr5	0.63D+08	0.33D-04	216	0.00
gre216b	216	expl.qr	0.18D+00	0.11D-09	207	0.00
gre216b	216	expl.lu	0.18D-02	0.31D-15	216	0.00
gre216b	216	lu lapack	0.26D-03	0.33D-15	216	0.00
gre216b	216	qr lapack	0.60D-02	0.81D-15	216	0.00
gre216b	216	gqr lapack	0.13D+00	0.49D-13	213	1.00
gre216b	216	lu linpack	0.26D-03	0.33D-15	216	0.00
condition number:			0.80D+15			
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gre1107	1107	huang1	0.44D-03	0.84D-10	1107	50.00
gre1107	1107	mod.huang1	0.29D-09	0.11D-14	1107	76.00
gre1107	1107	huang2	0.55D-03	0.17D-09	1107	23.00
gre1107	1107	mod.huang2	0.34D-09	0.11D-14	1107	64.00
gre1107	1107	impl.lu3	0.16D-09	0.15D-14	1107	23.00
gre1107	1107	impl.lu4	0.16D-09	0.15D-14	1107	8.00
gre1107	1107	impl.qr5	0.83D-10	0.17D-13	1107	84.00
gre1107	1107	expl.qr	0.62D-10	0.73D-15	1107	19.00
gre1107	1107	expl.lu	0.17D-09	0.91D-15	1107	22.00
gre1107	1107	lu lapack	0.57D-10	0.86D-15	1107	1.00
gre1107	1107	qr lapack	0.27D-09	0.13D-14	1107	18.00
gre1107	1107	gqr lapack	0.63D-09	0.20D-14	1107	29.00
gre1107	1107	lu linpack	0.57D-10	0.86D-15	1107	1.00
condition number:			0.89D+08			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
impcola	207	huang1	0.11D-04	0.17D-10	207	1.00
impcola	207	mod.huang1	0.42D-10	0.34D-15	207	0.00
impcola	207	huang2	0.13D-04	0.69D-10	207	0.00
impcola	207	mod.huang2	0.32D-10	0.22D-15	207	0.00
impcola	207	impl.lu3	0.13D-10	0.17D-15	207	0.00
impcola	207	impl.lu4	0.13D-10	0.17D-15	207	0.00
impcola	207	impl.qr5	0.11D-10	0.14D-13	207	1.00
impcola	207	expl.qr	0.61D-11	0.66D-15	207	0.00
impcola	207	expl.lu	0.19D-11	0.76D-16	207	0.00
impcola	207	lu lapack	0.45D-11	0.79D-16	207	0.00
impcola	207	qr lapack	0.12D-09	0.70D-15	207	0.00
impcola	207	gqr lapack	0.28D-10	0.32D-15	207	0.00
impcola	207	lu linpack	0.45D-11	0.79D-16	207	0.00
condition number:			0.43D+08			
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jphwh991	991	huang1	0.14D-14	0.12D-14	991	33.00
jphwh991	991	mod.huang1	0.17D-14	0.12D-14	991	52.00
jphwh991	991	huang2	0.21D-14	0.12D-14	991	16.00
jphwh991	991	mod.huang2	0.15D-14	0.11D-14	991	45.00
jphwh991	991	impl.lu3	0.13D-14	0.93D-15	991	12.00
jphwh991	991	impl.lu4	0.13D-14	0.93D-15	991	4.00
jphwh991	991	impl.qr5	0.21D-14	0.14D-14	991	57.00
jphwh991	991	expl.qr	0.25D-14	0.70D-15	991	12.00
jphwh991	991	expl.lu	0.12D-14	0.66D-15	991	12.00
jphwh991	991	lu lapack	0.13D-14	0.66D-15	991	1.00
jphwh991	991	qr lapack	0.64D-14	0.10D-14	991	11.00
jphwh991	991	gqr lapack	0.77D-14	0.28D-14	991	23.00
jphwh991	991	lu linpack	0.13D-14	0.66D-15	991	1.00
condition number:			0.36D+03			
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lns511	511	huang1	0.11D+01	0.15D-08	511	3.00
lns511	511	mod.huang1	0.18D-04	0.56D-15	511	5.00
lns511	511	huang2	0.45D+00	0.31D-08	511	1.00
lns511	511	mod.huang2	0.18D-04	0.56D-15	511	4.00
lns511	511	impl.lu3	0.19D-04	0.43D-15	511	1.00
lns511	511	impl.lu4	0.19D-04	0.43D-15	511	0.00
lns511	511	impl.qr5	0.30D-01	0.19D-11	511	5.00
lns511	511	expl.qr	0.18D-04	0.40D-15	511	1.00
lns511	511	expl.lu	0.19D-02	0.46D-15	511	1.00
lns511	511	lu lapack	0.19D-02	0.54D-15	511	0.00
lns511	511	qr lapack	0.52D-01	0.11D-14	511	1.00
lns511	511	gqr lapack	0.50D+00	0.23D-11	394	1.00
lns511	511	lu linpack	0.19D-02	0.54D-15	511	1.00
condition number:			0.49D+16			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
lnsp511	511	huang1	0.34D+00	0.11D-08	511	3.00
lnsp511	511	mod.huang1	0.77D-04	0.52D-15	511	5.00
lnsp511	511	huang2	0.29D+04	0.44D-05	511	1.00
lnsp511	511	mod.huang2	0.77D-04	0.53D-15	511	4.00
lnsp511	511	impl.lu3	0.93D-04	0.27D-15	511	0.00
lnsp511	511	impl.lu4	0.93D-04	0.27D-15	511	0.00
lnsp511	511	impl.qr5	0.17D-03	0.14D-12	511	5.00
lnsp511	511	expl.qr	0.40D-03	0.41D-15	511	1.00
lnsp511	511	expl.lu	0.10D-02	0.19D-15	511	1.00
lnsp511	511	lu lapack	0.50D-03	0.22D-15	511	0.00
lnsp511	511	qr lapack	0.78D-02	0.55D-15	511	1.00
lnsp511	511	gqr lapack	0.47D+00	0.16D-11	394	2.00
lnsp511	511	lu linpack	0.50D-03	0.22D-15	511	0.00
condition number:			0.49D+16			
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mcfe	765	huang1	0.73D-06	0.32D-10	765	10.00
mcfe	765	mod.huang1	0.24D-11	0.32D-15	765	15.00
mcfe	765	huang2	0.66D-06	0.21D-10	765	6.00
mcfe	765	mod.huang2	0.65D-11	0.37D-15	765	18.00
mcfe	765	impl.lu3	0.62D-11	0.20D-15	765	4.00
mcfe	765	impl.lu4	0.62D-11	0.20D-15	765	2.00
mcfe	765	impl.qr5	0.11D-10	0.51D-15	765	25.00
mcfe	765	expl.qr	0.70D-11	0.34D-15	765	5.00
mcfe	765	expl.lu	0.14D-10	0.31D-15	765	4.00
mcfe	765	lu lapack	0.14D-10	0.32D-15	765	0.00
mcfe	765	qr lapack	0.11D-10	0.74D-15	765	5.00
mcfe	765	gqr lapack	0.11D+00	0.76D-12	753	8.00
mcfe	765	lu linpack	0.14D-10	0.32D-15	765	0.00
condition number:			0.96D+14			
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nnc261	261	huang1	0.39D+00	0.25D-10	216	1.00
nnc261	261	mod.huang1	0.39D+00	0.24D-10	216	0.00
nnc261	261	huang2	0.39D+00	0.20D-09	216	0.00
nnc261	261	mod.huang2	0.39D+00	0.24D-10	216	1.00
nnc261	261	impl.lu3	0.19D-05	0.65D-15	261	0.00
nnc261	261	impl.lu4	0.19D-05	0.65D-15	261	0.00
nnc261	261	impl.qr5	0.67D+00	0.91D-08	235	1.00
nnc261	261	expl.qr	0.39D+00	0.24D-10	216	0.00
nnc261	261	expl.lu	0.11D-04	0.14D-15	261	0.00
nnc261	261	lu lapack	0.62D-05	0.18D-15	261	0.00
nnc261	261	qr lapack	0.35D-02	0.59D-15	261	0.00
nnc261	261	gqr lapack	0.28D-01	0.99D-15	260	0.00
nnc261	261	lu linpack	0.62D-05	0.18D-15	261	0.00
condition number:			0.75D+14			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
nnc666	666	huang1	0.23D-01	0.48D-09	666	6.00
nnc666	666	mod.huang1	0.35D-08	0.10D-14	666	10.00
nnc666	666	huang2	0.18D-01	0.35D-08	666	4.00
nnc666	666	mod.huang2	0.44D-08	0.10D-14	666	11.00
nnc666	666	impl.lu3	0.18D-08	0.86D-15	666	2.00
nnc666	666	impl.lu4	0.18D-08	0.86D-15	666	1.00
nnc666	666	impl.qr5	0.47D-01	0.32D-08	665	15.00
nnc666	666	expl.qr	0.27D-08	0.54D-15	666	3.00
nnc666	666	expl.lu	0.12D-07	0.19D-15	666	2.00
nnc666	666	lu lapack	0.12D-07	0.20D-15	666	0.00
nnc666	666	qr lapack	0.28D-06	0.57D-15	666	2.00
nnc666	666	gqr lapack	0.20D-06	0.12D-14	666	6.00
nnc666	666	lu linpack	0.12D-07	0.20D-15	666	0.00
condition number:			0.11D+11			
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nnc1374	1374	huang1	0.11D+01	0.63D-08	1084	84.00
nnc1374	1374	mod.huang1	0.46D+00	0.62D-10	1097	135.00
nnc1374	1374	huang2	0.59D+00	0.11D-07	1094	35.00
nnc1374	1374	mod.huang2	0.46D+00	0.62D-10	1097	96.00
nnc1374	1374	impl.lu3	0.78D-06	0.12D-14	1374	33.00
nnc1374	1374	impl.lu4	0.78D-06	0.12D-14	1374	14.00
nnc1374	1374	impl.qr5	0.77D+00	0.75D-10	957	102.00
nnc1374	1374	expl.qr	0.46D+00	0.62D-10	1097	26.00
nnc1374	1374	expl.lu	0.77D-05	0.22D-15	1374	35.00
nnc1374	1374	lu lapack	0.84D-05	0.22D-15	1374	1.00
nnc1374	1374	qr lapack	0.27D-02	0.66D-15	1374	30.00
nnc1374	1374	gqr lapack	0.23D+00	0.21D-12	1308	69.00
nnc1374	1374	lu linpack	0.84D-05	0.22D-15	1374	1.00
condition number:			0.27D+15			
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orsirr1	1030	huang1	0.34D-10	0.95D-13	1030	38.00
orsirr1	1030	mod.huang1	0.19D-12	0.92D-15	1030	59.00
orsirr1	1030	huang2	0.61D-10	0.18D-12	1030	18.00
orsirr1	1030	mod.huang2	0.14D-12	0.88D-15	1030	49.00
orsirr1	1030	impl.lu3	0.15D-12	0.80D-15	1030	13.00
orsirr1	1030	impl.lu4	0.15D-12	0.80D-15	1030	6.00
orsirr1	1030	impl.qr5	0.21D-11	0.28D-13	1030	63.00
orsirr1	1030	expl.qr	0.23D-12	0.75D-15	1030	13.00
orsirr1	1030	expl.lu	0.28D-12	0.67D-15	1030	13.00
orsirr1	1030	lu lapack	0.31D-12	0.68D-15	1030	1.00
orsirr1	1030	qr lapack	0.63D-12	0.79D-15	1030	13.00
orsirr1	1030	gqr lapack	0.44D-12	0.74D-15	1030	24.00
orsirr1	1030	lu linpack	0.31D-12	0.68D-15	1030	1.00
condition number:			0.79D+05			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
orsirr2	886	huang1	0.24D-10	0.62D-13	886	21.00
orsirr2	886	mod.huang1	0.22D-12	0.87D-15	886	31.00
orsirr2	886	huang2	0.56D-10	0.14D-12	886	12.00
orsirr2	886	mod.huang2	0.18D-12	0.79D-15	886	32.00
orsirr2	886	impl.lu3	0.11D-12	0.90D-15	886	13.00
orsirr2	886	impl.lu4	0.11D-12	0.90D-15	886	4.00
orsirr2	886	impl.qr5	0.29D-12	0.96D-14	886	39.00
orsirr2	886	expl.qr	0.35D-12	0.93D-15	886	8.00
orsirr2	886	expl.lu	0.25D-12	0.48D-15	886	12.00
orsirr2	886	lu lapack	0.21D-12	0.62D-15	886	0.00
orsirr2	886	qr lapack	0.48D-12	0.97D-15	886	9.00
orsirr2	886	gqr lapack	0.27D-12	0.65D-15	886	15.00
orsirr2	886	lu linpack	0.21D-12	0.62D-15	886	0.00
condition number:			0.79D+05			
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pde9511	961	huang1	0.15D-14	0.11D-14	961	29.00
pde9511	961	mod.huang1	0.14D-14	0.11D-14	961	44.00
pde9511	961	huang2	0.18D-14	0.12D-14	961	15.00
pde9511	961	mod.huang2	0.15D-14	0.11D-14	961	41.00
pde9511	961	impl.lu3	0.12D-14	0.11D-14	961	16.00
pde9511	961	impl.lu4	0.12D-14	0.11D-14	961	4.00
pde9511	961	impl.qr5	0.23D-14	0.13D-14	961	51.00
pde9511	961	expl.qr	0.24D-14	0.51D-15	961	9.00
pde9511	961	expl.lu	0.13D-14	0.40D-15	961	16.00
pde9511	961	lu lapack	0.16D-14	0.41D-15	961	1.00
pde9511	961	qr lapack	0.81D-14	0.84D-15	961	10.00
pde9511	961	gqr lapack	0.82D-14	0.17D-14	961	15.00
pde9511	961	lu linpack	0.16D-14	0.41D-15	961	1.00
condition number:			0.11D+03			
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pores2	1224	huang1	0.85D-05	0.37D-10	1224	71.00
pores2	1224	mod.huang1	0.81D-10	0.23D-14	1224	104.00
pores2	1224	huang2	0.68D-04	0.19D-09	1224	31.00
pores2	1224	mod.huang2	0.50D-10	0.22D-14	1224	91.00
pores2	1224	impl.lu3	0.11D-09	0.79D-15	1224	24.00
pores2	1224	impl.lu4	0.11D-09	0.79D-15	1224	11.00
pores2	1224	impl.qr5	0.59D-09	0.30D-14	1224	115.00
pores2	1224	expl.qr	0.18D-09	0.44D-15	1224	24.00
pores2	1224	expl.lu	0.17D-10	0.32D-15	1224	24.00
pores2	1224	lu lapack	0.74D-11	0.36D-15	1224	2.00
pores2	1224	qr lapack	0.20D-09	0.66D-15	1224	24.00
pores2	1224	gqr lapack	0.13D-09	0.86D-15	1224	43.00
pores2	1224	lu linpack	0.74D-11	0.36D-15	1224	4.00
condition number:			0.17D+09			

Test results for Harwell-Boeing matrices - continued

matrix	dimension	method	solution	residual	rank	time
			n	error		
<hr/>						
pores3	532	huang1	0.78D-09	0.16D-12	532	3.00
pores3	532	mod.huang1	0.97D-13	0.78D-15	532	5.00
pores3	532	huang2	0.70D-09	0.40D-12	532	1.00
pores3	532	mod.huang2	0.51D-12	0.85D-15	532	4.00
pores3	532	impl.lu3	0.48D-12	0.11D-14	532	1.00
pores3	532	impl.lu4	0.48D-12	0.11D-14	532	1.00
pores3	532	impl.qr5	0.64D-10	0.31D-13	532	5.00
pores3	532	expl.qr	0.87D-12	0.40D-15	532	1.00
pores3	532	expl.lu	0.14D-11	0.18D-15	532	1.00
pores3	532	lu lapack	0.14D-11	0.20D-15	532	0.00
pores3	532	qr lapack	0.26D-11	0.45D-15	532	1.00
pores3	532	gqr lapack	0.82D-12	0.42D-15	532	2.00
pores3	532	lu linpack	0.14D-11	0.20D-15	532	0.00
condition number:			0.33D+06			
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shl0	663	huang1	0.00D+00	0.00D+00	663	3.00
shl0	663	mod.huang1	0.00D+00	0.00D+00	663	7.00
shl0	663	huang2	0.00D+00	0.00D+00	663	4.00
shl0	663	mod.huang2	0.00D+00	0.00D+00	663	7.00
shl0	663	impl.lu3	0.00D+00	0.00D+00	663	2.00
shl0	663	impl.lu4	0.00D+00	0.00D+00	663	0.00
shl0	663	impl.qr5	0.12D-12	0.76D-15	663	14.00
shl0	663	expl.qr	0.56D-16	0.12D-18	663	3.00
shl0	663	expl.lu	0.30D-12	0.62D-16	663	2.00
shl0	663	lu lapack	0.30D-12	0.62D-16	663	0.00
shl0	663	qr lapack	0.48D-07	0.12D-14	663	4.00
shl0	663	gqr lapack	0.48D-07	0.21D-14	663	5.00
shl0	663	lu linpack	0.30D-12	0.62D-16	663	0.00
condition number:			0.22D+08			
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shl400	663	huang1	0.50D-14	0.13D-16	663	4.00
shl400	663	mod.huang1	0.30D-14	0.17D-16	663	7.00
shl400	663	huang2	0.82D-14	0.17D-16	663	4.00
shl400	663	mod.huang2	0.33D-14	0.17D-16	663	8.00
shl400	663	impl.lu3	0.00D+00	0.00D+00	663	3.00
shl400	663	impl.lu4	0.00D+00	0.00D+00	663	1.00
shl400	663	impl.qr5	0.53D-12	0.26D-15	663	14.00
shl400	663	expl.qr	0.79D-14	0.41D-16	663	2.00
shl400	663	expl.lu	0.11D-12	0.14D-15	663	2.00
shl400	663	lu lapack	0.11D-12	0.14D-15	663	1.00
shl400	663	qr lapack	0.84D-08	0.27D-14	663	3.00
shl400	663	gqr lapack	0.73D-08	0.13D-14	663	5.00
shl400	663	lu linpack	0.11D-12	0.14D-15	663	0.00
condition number:			0.19D+08			

References

- [1] J.Abaffy, C.G.Broyden, E.Spedicato: A class of direct methods for linear systems. Numerische Mathematik 45 (1984) pp. 361-376.
- [2] J.Abaffy, E.Spedicato: ABS Projection Algorithms, Mathematical Techniques for solving Linear and Nonlinear Equations. Ellis Horwood, Chichester 1989.
- [3] E.Bodon: Numerical results on the ABS algorithms for linear systems of equations. Rept. No. DMSIA 93/13, University of Bergamo, 1993.
- [4] H.Y.Huang: A direct method for the general solution of a system of linear equations. JOTA 16 (1975) pp. 429-445.
- [5] E.Spedicato: ABS algorithms for linear least squares: Theoretical results and computational performance. Rept. No. DMSIA 94/2, University of Bergamo, 1994.
- [6] E.Spedicato, E.Bodon: Numerical behaviour of the implicit QR algorithm in the ABS class for linear least squares. Ricerca Operativa 22 (1992) pp.45-55.
- [7] E.Spedicato, E.Bodon: Solving linear least squares by orthogonal factorization and pseudoinverse computation via the modified Huang algorithm in the ABS class. Computing 42 (1989) pp. 195-205.
- [8] E.Spedicato, E.Bodon: Solution of linear least squares via the ABS algorithms. Mathematical Programming 58 (1993) pp. 111-136.
- [9] E.Spedicato, Z.Chen, E.Bodon: ABS methods for KKT systems. In: Nonlinear Optimization and Applications (G.Di Pillo and F.Giannessi, eds), Plenum Press, New York 1996.
- [10] E.Spedicato, H.Esmaeili, Z.Xia: A review of ABS algorithms for linear real and Diophantine equations and optimization. In: Proc. of the VI CMA Congresso de Matematica Aplicada (R.Montenegro, G.Montero, G.Winter, eds.), Las Palmas, Gran Canaria, September 1999.
- [11] E.Spedicato, M.T.Vespuci: Variations on the Gramm-Schmidt and the Huang algorithms for linear systems: a numerical study. Aplications of Mathematics 2 (1223), 81-100.