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Daniel, Milan 1997 Dostupný z http://www.nusl.cz/ntk/nusl-33712

Dílo je chráněno podle autorského zákona č. 121/2000 Sb.

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Datum stažení: 05.05.2024

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A Generalization of Möbius Transformation for Knowledge Bases, which Include Rules with Disjunction in Antecedent II (General *ecd* knowledge bases)

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Technical report No. V-712

 ${\rm May}\ 1997$

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Abstract

Möbius transformation is an important tool for establishing weights of rules of compositional expert systems from conditional weights.

In this report, an applicability of Möbius transformation of rule bases is extended also to knowledge bases with elementary disjunctions in antecedents of rules. The text is continuation of Technical Report V-706. It describes the case of general *ecd* knowledge bases including an algorithm of the Möbius transformation for this general class of knowledge base.

Keywords

Möbius Transformation, Expert system, Knowledge Base, Uncertainty, Weight of rule

¹Partial support by the grant No. 1030601 of the GA ASCR (GA AV $\check{C}R$) and by COST project OC 15.10 is acknowledged.

1 Introduction

The first ideas, on how to establish weights of rules of compositional expert systems from conditional weights related to real evidence, data, experience, were published in 1984 [5]. For a full description of Möbius transformation see e.g. [6], [7].

The possibility of utilization of Möbius transformation is not only restricted to MYCIN-like systems, it is important also for a common generalization of MYCIN-like systems and fuzzy expert systems which use a composition of fuzzy relations like Conorm-CADIAG-2 extended with the handling of negative knowledge, see [4]. The system is derived from the fuzzy expert system CADIAG-2 [1].

The original Möbius transformation is formulated and only used for rules of a special form. The present work generalizes it for a wider class of rules.

This report is a continuation of technical report V-706 [3], where are all necessary preliminaries introduced and the original Möbius transformation theorem for MYCIN-like systems is stated. In this text only new definitions from V-706 are repeated in the second section.

A sectioning of the text is the same as in V-706. In sections 3 and 4.1 are briefly reviewed main results from V-706. The main point of this report is in subsection 4.2, where are new results on Möbius transformation for general *ecd* knowledge bases. These results are used in algorithm in section 5, the algorithm for founded *ecd* knowledge bases from V-706 is presented as well.

In section 6 is a comparison of Möbius transformation for MYCIN-like systems and of an introduced generalization. After it follows conclusions and ideas for future work.

2 Möbius transformation

2.1 Preliminaries

An *elementary disjunction* is a disjunction of literals. An *ecd knowledge base* (elementary-conjunction-disjunction) is a knowledge base such that antecedents of rules are either elementary conjunctions or elementary disjunctions.

We shall use the following terminology. A rule $R : A \Rightarrow S(w)$ is a simple rule if its antecedent A is a literal, R is conjunctive/disjunctive rule if A is a conjunction/disjunction, maximal conjunctive/disjunctive rule if there is no rule $B \Rightarrow S(w_B)$ in the knowledge base, so that A is a subconjunction/subdisjunction of B. A conjunction Conj = A&B&...&K is a conjunctive translation of a disjunction $Disj = A \lor B \lor ... \lor K$, a rule $Conj \Rightarrow H$ is a conjunctive translation of the disjunctive rule $Disj \Rightarrow H$.

An ecd knowledge base Θ is founded if it contains rules $A \Rightarrow H$ for every literal A which is involved in some disjunctive rule $A \lor Disj \Rightarrow H$ for any elementary disjunction Disj. An ecd knowledge base Θ is weakly founded if for every literal A from any disjunctive rule $A \lor Disj \Rightarrow H$ there is a simple rule $A \Rightarrow H$ or conjunctive rule $A\&Conj \Rightarrow H$ (for some Conj) included in Θ .

Notice, that every knowledge base without disjunction in antecedents is founded.

A literal A is founded for hypothesis H, if the rule $A \Rightarrow H$ is included in the knowledge base. A literal A is weakly founded for hypothesis H, if rule $A\&Conj \Rightarrow H$ is included in the knowledge base for some conjunction of literals Conj (possibly empty). A rule $A_1\&A_2\&\ldots\&A_n \Rightarrow H$ (resp. a rule $A_1\lor A_2\lor\ldots\lor A_n \Rightarrow H$) is founded if every A_i is founded for the hypothesis H. A rule $A_1\&A_2\&\ldots\&A_n \Rightarrow H$ (resp. $A_1\lor A_2\lor\ldots\lor A_n \Rightarrow H$) is weakly founded² if for every A_i exist At such that $A_i \subseteq At \subseteq A_1\&A_2\&\ldots\&A_n$ and rule $At \Rightarrow H$ is included in the knowledge base.

2.2 Möbius transformation theorem

Theorem 2.1 Let β be a weakly sound set of rules such that $w_{H,E}^0 = \beta(H|E)$. Then there exists a weighting of rules which forms a knowledge base Θ of MYCIN-like expert system, such that for any three-valued questionaire E_q and hypothesis H for which $\beta(H|E_q)$ is defined, it holds

 $W_{\Theta}(H|E_q) = \beta(H|E_q),$ where $W_{\Theta}(H|E_q)$ is a global weight of hypothesis H given by E_q , $W_{\Theta}(H|E_q) = \bigoplus \{\Theta(H|E') | E' \subseteq E_q \}.$

The new knowledge base Θ is called *Möbius transform* of the source rule base β .

3 Including a disjunction into Möbius transformation

- 3.1 The idea of Möbius transformation
- 3.2 The first attempt to include a disjunction
- **3.3** Rewriting of disjunction

3.4 What does a disjunction rule mean?

Lemma 3.1 Let Θ be a weakly sound low ecd knowledge base. If we can explicitly set or estimate implicit weights also for nonincluded combinations of literals, then Möbius transform of the knowledge base Θ exists.

Note: weak soundness condition in the present situation is as follows, for every two rules such that $Ant_1 \subset Ant_2$ holds: if $w_{Ant_1}^0 = 1$, then $w_{Ant_2}^0 = 1$, where $Ant_1 \subset Ant_2$ means Ant_2 implies Ant_1 , (i.e. $Conj_1$ is a subconjunction of $Conj_2$ or $Disj_2$ is a subdisjunction of $Disj_1$ or a subdisjunction of $Disj_1$ exists which is a subconjunction of $Conj_2$).

From the proof of the lemma, we know how to construct, in a simple yet noneffective way, Möbius transformation. So, it is logical to look for its improvement. A decisionmaking of whether a new possible rule will be added or not depends on the Möbied

²Note, that every founded literal, rule, knowledge base is trivially also weakly founded.

weight of the possible rule. Therefore we need a more sophisticated way of computing these weights. In the next subsection it is shown how to compute Möbied weights of rules in knowledge bases in which all possible elementary conjunctions and elementary disjunctions form antecedents of rules with the same succedent.

3.5 Formulas for computing of Möbied weights

$$w_{0} = w_{0}^{0}$$

$$w_{\vee} = w_{\vee}^{0} \oplus w_{0}$$

$$w_{A-B-C-\dots-K} = w_{A-B-C-\dots-K}^{0} \oplus \bigoplus_{i=1}^{n-k} \left((-1)^{i} \bigoplus_{|d|=k+i, A-B-\dots-K\subset d} w_{d}^{0} \right)$$

$$w_{A} = w_{A}^{0} \oplus \bigoplus_{i=1}^{n-1} \left((-1)^{i} \bigoplus_{|d|=i+1, A\subset d} w_{d}^{0} \right)$$

$$w_{ABC\dots K} = w_{ABC\dots K}^{0} \oplus \bigoplus_{i=1}^{k-1} \left((-1)^{i} \bigoplus_{|c|=k-i, c\subset AB\dots K} w_{c}^{0} \right) \oplus (-1)^{k} w_{A-B-C-\dots-K}^{0}$$

where w_{\vee} is an abbreviation for a weight $w_{A-B-C-...-N}$ of the rule with the maximal possible disjunction in antecedent, $a \subset b$ means b implies a, |c| is a length (number of conjuncts) of conjunction c, conjunction c = ABC...K has k elements i.e. |c| = k.

,

$$w_{ABC\ldots K} = (-1)^k (w^0_{A-B-C-\ldots-K} \ominus w_0).$$

3.6 Estimations of implicit weights of "rules" which are not included in a source knowledge base

As it was suggested in the previous subsection, we can use the formulas from there for specifying which type of rules are added into a knowledge base during Möbius transformation and which ones are not.

To distinguish *explicit conditional weights* w_{Ant}^0 of rules from a source knowledge base from computed estimations of those which are not given (resp. which are given implicitly through other rules), we shall denote *estimated implicit weights* as w_{Ant}^x .

We have observed in V-706, that "an effect of disjunction $A \vee B$ is in some sense involved in effect of its disjunct A and it is not propagated once more through the other disjunct B".

This is very important for generation of w_{Ant}^x of "problematic" rules, i.e. "Not to propagate weight of disjunctive rule several times through different literals into weight of conjunctive rule".

We can summarize the formulas for estimations as follows:

$$w_0^x = w_0$$

$$w_{Disj}^{x} = \bigoplus_{Ant \in Disj} w_{Ant} \oplus w_{0}$$
$$w_{Ant}^{x} = \bigoplus_{Conj \in Ant} w_{Conj}^{\prime} \oplus w_{0}$$
$$w_{Ant}^{x} = \bigoplus_{Conj \in Ant} w_{Conj}^{Ant} \oplus \bigoplus_{\substack{Disj \in Ant, Disj \notin Lit, \\ Lit \in Conj \subseteq Ant, w_{Conj}^{0} \text{ is given}} w_{Disj}^{Ant} \oplus w_{0},$$

where Ant, Conj, Disj, Lit is any antecedent, conjunction, disjunction or literal from Θ respectively. The third formula is applicable for conjunctive rules from the case 2).

We can notice, that the first three formulas are the special cases of the fourth one. Thus, the 4th formula is not applicable only for conjunctive rules ad 3), but is is applicable in general. Hence, in the cases either that all the possible antecedents are included in the source knowledge base or that we want to compute w^x for all the possible antecedents we can use the last formula in the following form:

$$w_{Ant}^{x} = \bigoplus_{i=1}^{k-1} ((-1)^{i+1} \bigoplus_{\substack{|Conj|=k-i\\Conj \in Ant}} w_{Conj}^{0}) \oplus \bigoplus_{i=2}^{n-k} ((-1)^{i+k-1} \bigoplus_{\substack{|Disj|=i, Disj \in Ant,\\Disj \notin Lit,\\Lit \in Conj \subseteq Ant,\\w_{Conj}^{0} \text{ is given}}} w_{Disj}^{0}) \oplus (-1)^{n} w_{0},$$

where z is 0 or x and Ant, Conj, Disj is any possible antecedent, elementary conjunction or elementary disjunction constructed from questions the source knowledge base Θ .

We recapitulate, that we have formulas on how to compute estimations of conditional weights for all types of rules admissible in *ecd* knowledge bases. Hence, we have an existence theorem.

Theorem 3.2 If Θ is a weakly sound low ecd knowledge base, then there Möbius transform of the knowledge base Θ exists.

4 Simplifications

We know how to compute Möbied weights, i.e. rule weights of Möbius transform of source knowledge base. We know, how to estimate and compute implicit weights for every rule with elementary conjunction or elementary disjunction in antecedent, such that for every question and its negation Q either w_Q^0 is given (i.e. there is rule $Q \Rightarrow H(w_Q^0)$ included in the original knowledge base) or if Q is not a subdisjunction of an antecedent of any rule. Thus, we can do Möbius transformation for any knowledge base of this type.

Now, we are going to specify which rules are not necessary to add to Möbied knowledge base. We consider rules $Ant \Rightarrow H$ which are not included into the source knowledge base, i.e. w_{Ant}^0 is not given there. By an added rule we mean such a rule that $w_{Ant} \neq 0$, while if $w_{Ant} = 0$ we say that rule is not added.

In general, we can consider all nonincluded rules to be virtually added. Whether a rule is to really be added or not, depends on its Möbied weight w_{Ant} . We can eliminate some types of rules to be added by a symbolic computation of their Möbied weight. But usually, we cannot assert that some type of rules will be added, because the actual value of its weight w_{Ant} depends on the actual values of conditional weights from the source knowledge base.

In the following subsection, we shall review some lemmata (from V-706) to describe which types of rules to be / not to be added in the knowledge base. For disjunctive rules, we easily obtain the following important lemma.

Lemma 4.1 There are no disjunctive rules added to a knowledge base during Möbius transformation.

4.1 Simplifications for founded *ecd* knowledge bases

Lemma 4.2 If Θ is a founded ecd knowledge base, then there are no rules $A_1\&A_2\&\ldots\&A_k \Rightarrow H$ added into the knowledge base within the process of Möbius transformation, where $A_1 \lor A_2 \lor \ldots \lor A_k \lor B_1 \lor \ldots \lor B_l$ is not an antecedent of some rule from the source knowledge base, for some literals $B_1, \ldots B_l$.

Lemma 4.3 If Θ is a founded ecd knowledge base, then conjunctive translations of all maximal disjunctive rules are added into the knowledge base within the process of Möbius transformation (if they are not already included in the source knowledge base Θ).

Let $A_1 \lor A_2 \lor \ldots \lor A_k \Rightarrow H$ be a maximal disjunctive rule and $A_1 \& A_2 \& \ldots \& A_k \Rightarrow H$ its conjunctive translation added according to the lemma. Usually, there are also all rules added with subconjunction of $A_1 \& A_2 \& \ldots \& A_k$ in antecedent. But, there are counter-examples also on a symbolic level, see appendix D, case a).

In both previous lemmata, the assumption of foundness of ecd knowledge base is necessary. For general ecd knowledge bases which are not founded, there are counterexamples against both of the lemmata presented in appendix D, cases b) and c).

Summarizing the above lemmata we see, that antecedents of rules added by Möbius transformation into *ecd* knowledge base are only all conjunctive translations of antecedents of maximal disjunctive rules and not necessarily all of their subconjunctions. Formally, we have the following:

Theorem 4.4 Let Θ be a weakly sound low founded ecd knowledge base. During a process of Möbius transformation of Θ , rules are added into the knowledge base (if they are not already included in Θ and) if and only if they are in one of the two following types:

- All rules A₁&A₂&...&A_n ⇒ H, where A₁ ∨ A₂ ∨ ... ∨ A_n ⇒ H, is a maximal disjunctive rule from Θ.
- Rules A₁&A₂&...&A_n ⇒ H, where A₁ ∨ A₂ ∨ ... ∨ A_n ∨ Disj ⇒ H, is a rule from Θ and Disj is any disjunction. (In general, all such rules are added, but there counter-examples exist).

4.2 Simplifications for general *ecd* knowledge bases

As we have seen in the previous subsection (more particularly in V-706), in the case of founded ecd knowledge bases, the only possibilities for antecedent added rule is either to be a conjunctive translation of antecedent of some disjunctive rule or to be a subconjunction of such a translation.

In the case of general *ecd* knowledge bases, an antecedent of added rule can be besides it also a (subconjunction of) conjunctive translation of a disjunction of several disjunctive antecedents, as it is illustrated with the following small example, for other examples see appendix E.

Example: Let us consider the following weakly founded *ecd* knowledge base Θ :

 $\Rightarrow H(w_0)$ $A \lor B \Rightarrow H(w^0_{\lor})$ $A \Rightarrow H(w_A^0)$ $D \Rightarrow H(w_D^0)$ $B\&D \Rightarrow H(w_{BD}^0)$ Estimations of implicit weights: $w_{A-B-D}^x = w_0$ $w_{A-D}^x = w_0$ $w_{B-D}^x = w_0$ $w_B^x = w_{A-B}^0$ $w_{AB}^{x} = w_{A}^{0}$ $w_{AD}^{x} = w_{A}^{0} \oplus w_{D}^{0} \oplus w_{0}$ $w_{ABD}^x = w_A^0 \oplus w_{BD}^0 \oplus w_0$ Möbied weights: w_0 $w_{A-B-D} = 0$ $w_{A-B} = w_{A-B}^0 \ominus w_0$ $w_{A-D} = 0$ $w_{B-D} = 0$ $w_A = w_A^0 \ominus w_{A-B}^0$ $w_B = 0$ $w_D = w_D^0 \ominus w_0$ $w_{AB} = 0$ $w_{AD} = 0$ $w_{BD} = w_{BD}^0 \ominus w_{A-B}^0 \ominus w_D^0 \oplus w_0$ $w_{ABD} = w^0_{A-B} \ominus w_0$

Rule $A\&B\&D \Rightarrow H(w^0_{A-B} \ominus w_0)$ is added into knowledge base even if its antecedent A&B&D is not (subconjunction of) conjunctive translation of any disjunctive antecedent, $A \lor B \lor D$ is not (subdisjunction of) antecedent of any rule from Θ .

In this case, the antecedent of added rule is a conjunctive translation of disjunction of disjunctive antecedents from the source knowledge base.

Analogously to the case of founded ecd knowledge bases, we want to find restrictions for added rules. So we formulate the following hypotheses as analogy to lemmata 4.2 and 4.3. Hypotheses • If Θ is a general ecd knowledge base, then there are no rules $A_1\&A_2\&\ldots\&A_k \Rightarrow H$ added into the knowledge base within the process of Möbius transformation, where $A_1 \lor A_2 \lor \ldots \lor A_k \lor B_1 \lor \ldots \lor B_l$ is not a disjunction of antecedents of some disjunctive rules from the source knowledge base.

• If Θ is a general *ecd* knowledge base, then conjunctive translations of all maximal founded (weakly founded) disjunctive rules $A_1 \lor A_2 \lor \ldots \lor A_k \Rightarrow H$, such that antecedent of their translation $A_1 \& A_2 \& \ldots \& A_k$ is a conjunction of antecedents of some conjunctive rules (incl. simple ones) from the source knowledge base, are added into the knowledge base within the process of Möbius transformation (if they are not already included in the source knowledge base Θ).

Unfortunately neither the first nor the second of these hypotheses is true. We can demonstrate it by the following examples:

Example: Let us consider the following weakly founded *ecd* knowledge base Θ :

 $\Rightarrow H(w_0)$ $A \lor B \Rightarrow H(w_{A-B}^0)$ $A \Rightarrow H(w_A^0)$ $B\&C \Rightarrow H(w_{BC}^0)$

Möbied weights:

 w_0 $w_{A-B} = w_{A-B}^0 \ominus w_0$ $w_A = w_A^0 \ominus w_{A-B}^0$ $w_{BC} = w_{BC}^0 \ominus w_{A-B}^0$ $w_{ABC} = w_{A-B}^0 \ominus w_0$

Rule $A\&B\&C \Rightarrow H(w^0_{A-B} \ominus w_0)$ is added into knowledge base even if its disjunctive translation $A \lor B \lor C$ is neither antecedent nor disjunction of antecedents of any rules from Θ . Hence the first hypotheses does not hold.

Example: Let us consider another weakly founded *ecd* knowledge base Θ :

 $\Rightarrow H(w_0)$ $A \lor B \Rightarrow H(w_{A-B}^0)$ $B \lor C \Rightarrow H(w_{B-C}^0)$ $A \Rightarrow H(w_A^0)$ $B\&C \Rightarrow H(w_{BC}^0)$

Möbied weights:

 w_{0} $w_{A-B} = w_{A-B}^{0} \ominus w_{0}$ $w_{B-C} = w_{B-C}^{0} \ominus w_{0}$ $w_{AB} = w_{AB}^{0} \ominus w_{A-B}^{0} \ominus w_{B-C}^{0} \oplus w_{0}$ $w_{BC} = w_{BC}^{0} \ominus w_{A-B}^{0} \ominus w_{B-C}^{0} \oplus w_{0}$ $w_{ABC} = w_{A-B}^{0} \ominus w_{0} \oplus w_{B-C}^{0} \ominus w_{0}$

Whether rule $A\&B\&C \Rightarrow H(w^0_{A-B} \ominus w_0 \oplus w^0_{B-C} \ominus w_0)$ will be added into knowledge base or not, it depends on the precise given weights. In general, the the rule will be added. But in the case of $w^0_{A-B} \ominus w_0 = -(w^0_{B-C} \ominus w_0)$, the weight of the rule will be equal to 0, and the rule will not be added. Hence it is not possible to prove the second hypothesis, even if its assertion is usually fulfilled.

Let us recapitulate now, what we know on Möbius transformation of general *ecd* knowledge bases, from the previous text.

- Möbius transformation is performed for each hypothesis separately.
- It is possible perform Möbius transformation separately for every disjunctive part of knowledge base w.r.t. given hypothesis (no literal from disjunctive part of KB is connected by any rule with literals from outside)
- No disjunctive rules are added anytime.

Similarly to the third item we have the following lemma.

Lemma 4.5 There are no simple rules $Lit \Rightarrow H$ added to a knowledge base during Möbius transformation for any literal Lit.

Proof. The assertion is in fact a corollary of the way of estimation of w^x for literals and of lemma 4.1: $w_{Lit}^x = \bigoplus_{Disj \in Lit} w_Disj$, thus $w_{Lit} = w_{Lit}^x \ominus \bigoplus_{Ant \in Lit} w_{Ant} = \bigoplus_{Disj \in Lit} w_Disj \ominus \bigoplus_{Disj \in Lit} w_Disj = 0$ (or $w_{Lit} = w_{Lit}^0 \ominus \bigoplus_{Disj \in Lit} w_Disj$ if $Lit \Rightarrow H$ is already included in Θ).

Lemma 4.6 Let Θ is a general ecd knowledge base. Founded rule $A_1\&A_2\&...\&A_k \Rightarrow H$ is added into Θ within the process of Möbius transformation, only if $A_1 \lor A_2 \lor ... \lor A_k \lor Disj$ is an antecedent of some rule from Θ , for some disjunction Disj (empty disjunction is possible).

Proof. Let suppose, for contradiction, added rule $R : A_1 \& A_2 \& ... \& A_k \Rightarrow H$ such that there is not rule $A_1 \lor A_2 \lor ... \lor A_k \lor Disj$ in Θ for any disjunction Disj. Let Θ' a restriction of Θ on questions included in rule R constructed as follows: conjunctive rule is included in Θ' if and only if it is included in Θ , disjunctive rule $Disj \Rightarrow H$ is included in Θ' if and only if rule $Disj \lor Disj' \Rightarrow H$ is included in Θ for some disjunction Disj(empty disjunction is possible). If several disjunctive rules $Ant \Rightarrow H$ with the same antecedent and consequent and with weights $w_1, w_2, ..., w_k$ should be put in Θ' , then there is put into Θ' rule $Ant \Rightarrow H(w_1 \oplus w_2 \oplus ... \oplus w_k)$ instead of them.

All the expected weights w^x should are the same in both Θ and Θ' . The same rule are applicable in both the knowledge bases (resp. instead of several disjunctive rules from Θ only one rule with the same effect in Θ' . Hence, Möbied weight of rule R should be the same and the assertion follows from lemma 4.2.

Let us ask the principal question: When a rule $Ant \Rightarrow H$ is added within Möbius transformation? It is added if and only if its weight is different from zero, i.e. if and only if $w_{Ant}^x \neq \bigoplus_{At \subset Ant} w_{At}$. From the formula, it follows that the rule is never added if there are only disjunctive rules on $\{A_1, A_2, ..., A_k\}$, where $Ant = A_1 \& A_2 \& ... \& A_k$. So it must exist at least one conjunctive rule $A \Rightarrow H$, $A \subset Ant$. Weight of such a rule is used in computation of w_{Ant}^x . (Otherwise it would be $w_{Ant}^x = \bigoplus_{At \subset Ant} w_{At}$ and $w_Ant = 0$). Hence, we have: **Lemma 4.7** Every rule added into knowledge base within the process of Möbius transformation is of the form $Conj_1\&Conj_2 \Rightarrow H$, where $Conj_1$ is an antecedent of some rule from the original knowledge base.

It look like that antecedents of added rules are only conjunctions of antecedents from the original knowledge base or subconjunctions of such conjunctions. Nevertheless, an attempt to prove that added rules do not contain literals which are not weakly founded is unsuccessful but on the other hand we can prove the following.

Lemma 4.8 A literal Lit, which is not weakly founded, can be included in antecedent Lit&Conj of added rule only if it is included at least in two antecedents of disjunctive rules $Disj_2 \subset Disj_1$, such that $Disj_2 \subset Conj$ and $Disj_1 \not\subset Conj$.

Proof. Let L is literal which is not weakly founded. Rule $L\&Conj \Rightarrow H$ is added if and only if its weight $w_{L\&Conj}$ is different from zero. So we have to compute this weight. No simple rules are added and L is not weakly found, thus w_L must be equal to zero. Let us compute values w_{Conj}^{Conj} at first A) If Conj is minimal antecedent, then $w_{Conj}^{Conj} =$ $w_{Conj}^{0} \ominus \bigoplus_{D \in Conj, D \notin Conj} w_{D}^{Conj} \ominus w_{0} = w_{Conj}^{0} \ominus w_{0}$. B) For conjunctive antecedent Conj we get $w_{Conj}^{Conj} = w_{Conj}^{0} \ominus_{C \subset Conj} w_{C}^{Conj} \ominus_{D \subset Conj, D \notin Conj} w_{D}^{Conj} \ominus w_{0} = w_{Conj}^{0} \ominus_{C \subset Conj} w_{C}^{Conj} \ominus w_{0}.$ C) If conjunction Conj is not antecedent of any rule from the original knowledge base Θ , then in Θ without disjunctive rules $D \Rightarrow H$, where $D \subset C \subset Conj$, it holds $w_{Conj}^x = \bigoplus_{C \subset Conj} w_C^{Conj} \oplus \bigoplus_{D \subset Conj, D \notin C \subset Conj} w_D^{Conj} = \bigoplus_{C \subset Conj} w_C^{Conj} \oplus \bigoplus_{D \subset Conj} w_D^{Conj} = \bigoplus_{At \subset Conj} w_{At}^{Conj}$, hence $w_{Conj}^{Conj} = 0.$

hence $w_{Conj} = 0$. 1) Let Conj be a minimal conjunctive antecedent. $w_{L\&Conj}^{x} = w_{Conj}^{Conj} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \oplus w_{0} = w_{Conj}^{0} \oplus w_{0} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \oplus w_{0} = w_{Conj}^{0} \oplus w_{D} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj},$ $\bigoplus_{D \in L, D \notin Conj} \oplus w_{L} \oplus \bigoplus_{D \in L\&Conj} w_{D}^{Conj} \oplus w_{0} = w_{Conj}^{0} \oplus \bigoplus_{D \in Conj} w_{D} \oplus w_{0} \oplus w_{0} \oplus w_{D} \oplus w_{D}$ $\bigoplus_{\substack{D \subset L, D \not \in Conj \\ U \subset L, D \not \in Conj}} (w_D^{Conj} \ominus w_D).$

If there is no disjunctive antecedent D_1 in Θ such that $D_1 \subset L, D_1 \subset Conj$, then $w_D^{Conj} = w_D$ for $D \subset L$.

If there is no disjunctive antecedent D_2 in Θ such that $D_2 \subset L, D_2 \not\subset Conj$, then $\bigoplus_{D \subset L, D \not \subset Conj}$ = 0.

If $D_1 \not\subset D_2$, then $w_{D_2}^{Conj} = w_{D_2}$.

Hence, rule $L\&Conj \Rightarrow H$ is added (i.e. $w_{L\&Conj} \neq 0$) only if there exist two disjunctive antecedents D_1, D_2 relevant to H, such that $D_1 \subset D_2, D_1, D_2 \subset L, D_1 \subset Conj, D_2 \not\subset$ Conj.

2) Let *Conj* be a conjunctive antecedent, but not minimal one.

 $w_{L\&Conj}^{x} = w_{Conj}^{Conj} \oplus w_{L}^{Conj} \oplus \bigoplus_{C \in Conj} w_{C}^{Conj} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \oplus w_{0} = (w_{Conj}^{0} \oplus \bigoplus_{C \in Conj} w_{C}^{Conj} \oplus \bigoplus_{C \in Conj} w_{C}^{Conj} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \oplus w_{0} = (w_{Conj}^{0} \oplus \bigoplus_{C \in Conj} w_{C}^{Conj} \oplus \bigoplus_{C \in Conj} w_{C}^{Conj} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \oplus \bigoplus_{At \in L\& C, C \in Conj} w_{At}$ $(w_L = 0),$ $\begin{array}{l} (w_L - v_J), \\ w_{L\&Conj} = w_{L\&Conj}^x \ominus \bigoplus_{At \in L\&Conj} w_{At} = w_{Conj}^0 \oplus \bigoplus_{D \in L, D \notin Conj} w_D^{Conj} \ominus w_{Conj} \ominus \bigoplus_{At \in Conj} w_{At} \ominus \\ \bigoplus_{D \in L, D \notin Conj} w_D \ominus \bigoplus_{At = L\&C, C \in Conj} w_{At} \ominus w_0 = (w_{Conj}^0 \ominus \bigoplus_{At \in Conj} w_{At} \ominus w_0 \ominus w_{Conj}) \oplus \\ \bigoplus_{D \in L, D \notin Conj} w_D^{Conj} \ominus \bigoplus_{D \in L, D \notin Conj} w_D \ominus \bigoplus_{At = L\&C, C \in Conj} w_{At} = \\ \bigoplus_{D \in L, D \notin Conj} (w_D^{Conj} \ominus w_D) \ominus \bigoplus_{At = L\&C, C \in Conj} w_{At}. \end{array}$ 3) Let Conj not to be any antecedent in Θ .

L is not included in any conjunctive antecedent in G. $L \text{ is not included in any conjunctive antecedent from } \Theta, \text{ hence } w_{L\&Conj}^{L\&Conj} = w_{Conj}^{Conj} = w_{Conj}^{Conj} \oplus w_{C}^{Conj} \oplus w_{C}^{Co$ $(w_L = 0),$ $\begin{array}{l} \underbrace{(=L = -1)}{w_{L\&Conj}} = w_{L\&Conj}^{x} \ominus \bigoplus_{At \in L\&Conj} w_{At} = w_{Conj}^{x} \oplus \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \ominus e_{Conj} \bigoplus_{At \in Conj} w_{At} \ominus \\ \bigoplus_{D \in L, D \notin Conj} w_{D} \ominus \bigoplus_{At = L\&C, C \in Conj} w_{At} \ominus w_{0} = (w_{Conj}^{x} \ominus \bigoplus_{At \in Conj} w_{At} \ominus w_{0} \ominus w_{Conj}) \oplus \\ \bigoplus_{D \in L, D \notin Conj} w_{D}^{Conj} \ominus \bigoplus_{D \in L, D \notin Conj} w_{D} \ominus \bigoplus_{At = L\&C, C \in Conj} w_{At} = \\ \bigoplus_{D \in L, D \notin Conj} (w_{D}^{Conj} \ominus w_{D}) \ominus \bigoplus_{At = L\&C, C \in Conj} w_{At}. \end{array}$

4) The rest is induction on length of Ant = L&Conj. We know, that assertion holds for all antecedents, where Conj is minimal conjunctive antecedent from Θ . (No rule $C \Rightarrow H$ is added, where C is subconjunction of any minimal antecedent, thus assertion trivially holds also for such antecedents).

Let us suppose, that the assertion holds for $C \subset Ant$. From this assumption $\bigoplus_{At=L\&C,C\subset Conj} w_{At} \neq 0 \text{ only if there exist in } \Theta \text{ two disjunctive antecedents}$ follows, that D_1, D_2 relevant to H, such that $D_1 \subset D_2, D_1, D_2 \subset L, D_1 \subset Conj, D_2 \not\subset Conj$. $\bigoplus_{D \in L, D \notin Conj} (w_D^{Conj} \ominus w_D) \neq 0 \text{ only under the same condition, see the case 1). Thus,}$ rule $L\&Conj \Rightarrow H$ is added within Möbius transformation only if there exist in the source knowledge base Θ two disjunctive antecedents D_1, D_2 relevant to H, such that $D_1 \subset D_2, D_1, D_2 \subset L, D_1 \subset Conj, D_2 \not\subset Conj.$

By an application of the same idea we can prove also:

Lemma 4.9 A weakly founded literal Lit can be included in antecedent Ant of added rule Lit&Conj \Rightarrow H only as a part of some antecedent At, such that Lit \subseteq At \subset Lit&Conj, or if it is included at least in two antecedents of disjunctive rules $Disj_2 \subset$ $Disj_1$, such that $Disj_2 \subset Conj$ and $Disj_1 \not\subset Conj$.

Proof. For founded literal trivially always holds the first case and sometimes also the second one.

Let Lit is not founded literal.

If there exist some rule $At \to H$ in knowledge base, such that $Lit \subset At \subset L\&Conj$, then the first case holds. See, e.g. literal B from the introductive example of this subsection or all literals in added rules from appendix E.

If there is no rule $At \to H$, such that $Lit \subset At \subset L\&Conj$, then we can compute values of $w_{L\&Conj}^x$, $\bigoplus_{At \subset L\&Conj} w_{At}$, and $w_{L\&Conj}$ using the same assumptions and formulas as for literals which are not weakly founded, see the previous proof. Thus the second case holds.

Summarizing previous lemmata we obtain the following theorem.

Theorem 4.10 Let Θ be a general ecd knowledge base. Every rule, which is added into knowledge base within the process of Möbius transformation of Θ , is a conjunctive rule of the form

$$Lit_1 \& Lit_2 \& \dots \& Lit_m \& Conj_1 \& Conj_2 \& \dots \& Conj_n \Rightarrow H,$$

where $Conj_i^3$ is an antecedent of some rule from the original knowledge base Θ for $i = 1, ..., n \ge 1$, $m + n \ge 2$, and for j = 1, ..., m, Lit_j is literal, such that there exist two disjunctive antecedents $Disj_{j1}, Disj_{j2}$ in Θ for which $Disj_{j2} \subset Disj_{j1} \subset Lit_j$, $Disj_{j2} \subset Conj_1 \& ... \& Conj_n$, and $Disj_{j1} \not\subset Conj_1 \& ... \& Conj_n$.

Proof. For founded literal Lit, there is $Lit \Rightarrow H$ already included in Θ (thus such a literal may be considered as one-member conjunction — one of $Conj_i$ '). From lemma 4.7 follows $n \ge 1$, if m + n = 1 then we obtain rule from Θ , hence $m + n \ge 2$. The rest follows lemmata 4.8, 4.9.

Definition 4.11 Antecedents of two conjunctive rules $Ant_1 \Rightarrow H$, $Ant_2 \Rightarrow H$ from a knowledge base Θ are *joinable* if rule $Ant_1 \& Ant_2 \Rightarrow H$ is potentially added within the process of Möbius transformation of Θ . (If weight $w_{Ant_1\&Ant_2}$ computed according to inference net is generally non zero, if the actual zero/non-zero value of $w_{Ant_1\&Ant_2}$ depends just on particular $w^{0'}$).

A conjunctive antecedent of rule $Ant \Rightarrow H$ is *joined* if there are two rules $Ant_1 \Rightarrow H$, $Ant_2 \Rightarrow H$, such that $Ant = Ant_1 \& Ant_2$, included in knowledge base.

Let $Conj_1 \Rightarrow H$, $Conj_2 \Rightarrow H$ be two conjunctive rules from a general *ecd* knowledge base Θ , both with non joined antecedents. An antecedent of disjunctive rule $Disj \Rightarrow H$ is called *joining disjunction of* $Conj_1, Conj_2$ with respect to H, if the following holds: $Disj \subset Conj_1, Conj_2$, and for i = 1, 2 it either holds $L_{Conj_i} \subset L_{Disj}$ (all literals from $Conj_i$ are included also in Disj) or there is no rule $Conj'_1 \Rightarrow H$ in Θ , for which $Disj \subset Conj'_i \subset Conj_i$. A subdisjunction $Disj_A$ of joining disjunction Disj which contain just the literals from $Conj_1$ and $Conj_2$ (L_{Conj_1}, L_{Conj_2}) is called an active part of joining disjunction.

³Overlapping of $Conj_i$ ' is possible: e.g. for original antecedents AB, AC we can get added one ABC, but not necessarily.

Hypothesis 4.12 Let $Conj_1 \Rightarrow H$, $Conj_2 \Rightarrow H$ be two conjunctive rules from a general ecd knowledge base Θ , both with non joined antecedents. If there exists joining disjunction of antecedents $Conj_1, Conj_2 w.r.t. H$, then $Conj_1, Conj_2$ are joinable into new antecedent $Conj_1 \& Conj_2$.

If some of antecedent is joined we may get $w_{Conj_1\&Conj_2} = 0$ even on symbolic level, see appendix E b).

Lemma 4.13 Let $Conj_1 \Rightarrow H$, $Conj_2 \Rightarrow H$ be two conjunctive rules from a general ecd knowledge base Θ . The antecedents $Conj_1, Conj_2$ are joinable into new one $Conj_1\&Conj_2$ only if there exists joining disjunction of $Conj_1$ and $Conj_2$ w.r.t. H.

To be proved.

Even if it does not look like it at the first glance, this lemma is in fact a generalization of lemma 4.2, Let $Disj \subset Conj$ for rules $Disj \Rightarrow H$, $Conj \Rightarrow H$ from founded *ecd* knowledge base Θ . There are rules $Lit \Rightarrow H$ in Θ for every $Lit \subset Conj$, including literal(s) $Disj \subset Lit \subset Conj$. Hence w.r.t. the lemma, it must be $L_{Conj} \subset L_{Disj}$ for both antecedents which to be joined. We can now formulate also a generalization of lemma 4.3.

Lemma 4.14 Let Θ be a general ecd knowledge base, let Disj be an antecedent of some maximal disjunctive rule Disj \Rightarrow H from Θ . If conjunctive translation of Disj is of form $L_1 \& L_2 \& ... \& L_m \& C_1 \& C_2 \& ... \& C_n$, where $C_i \Rightarrow$ H is in Θ for i = 1, ..., n, and where L_i are literals such there exist a disjunctive rule Disj₁ \Rightarrow H, Disj \subset Disj₁ \subset $L_1 \lor ... \lor L_m$ and Disj₁ $\not\subset C_1 \& ... \& C_n$, then conjunctive translations of rule Disj \Rightarrow H, is added into the knowledge base within the process of Möbius transformation (if it is not already included in the source knowledge base Θ).

Proof. Similarly as in the proof of 4.3 it is possible to show, that $w_{Ant} = \pm (w_{Disj}^0 \ominus w_0)$ for $Ant = L_1 \& L_2 \& ... \& L_m \& C_1 \& C_2 \& ... \& C_n$, and similarly we suppose $w_{Disj}^0 \neq w_0$ for maximal disjunctive rule. Hence $w_{Ant} \neq 0$.

Lemma 4.12 holds also for n rules which satisfy the same conditions. Thus we have:

Lemma 4.15 Let us have n conjunctive rules $Conj_1 \Rightarrow H$, ..., $Conj_n \Rightarrow H$ from a general ecd knowledge base Θ , all with non joined antecedents. If there exists disjunctive rule $Disj \Rightarrow H$, such that $Disj \subset Conj_i$, and for i = 1, ..., n it either holds $L_{Conj_i} \subset L_{Disj}$ or there is no rule $Conj'_1 \Rightarrow H$ in Θ , for which $Conj'_i \subset Conj$ and $L_{Conj'_i} \subset L_{Disj}$ (all literals from $Conj'_i$ are included also in Disj). Then antecedents $Conj_1, ..., Conj_n$ are joinable into new one $Conj_1 \& ... \& Conj_n$.

The proof is analogical to the proof of lemma 4.12.

Lemma 4.16 Let us have n conjunctive rules $Conj_1 \Rightarrow H$, ..., $Conj_n \Rightarrow H$ from a general ecd knowledge base Θ . The antecedents are joinable into new one $Conj_1 \& ... \& Conj_n$ only if they are pairwise joinable, i.e. if there exists disjunctive rules $Disj_{ij} \Rightarrow H$ for every couple of antecedents $Conj_i, Conj_j$, such that $Disj_{ij} \subset Conj_i, Conj_j$, and for k = i, j it either holds $L_{Conj_k} \subset L_{Disj}$ or there is no rule $Conj'_k \Rightarrow H$ in Θ , for which $Conj'_k \subset Conj$ and $L_{Conj'_k} \subset L_{Disj}$. To be proved.

A reverse assertion does not hold, see appendix E a), moreover it holds the following lemma.

Lemma 4.17 Let us have n conjunctive rules $Conj_1 \Rightarrow H$, ..., $Conj_n \Rightarrow H$ with pairwise joinable antecedents from a general ecd knowledge base Θ . If active parts of their joining disjunctions are pairwise disjunct, then rule $Conj_1\&...\&Conj_n \Rightarrow H$ is not added into knowledge base within its Möbius transformation, i.e. $Conj_1,...,Conj_n$ are not mutually joinable.

To be proved.

Hypothesis 4.18 Let us have disjunctive rule $Disj \Rightarrow H$ and conjunctive one $Conj \Rightarrow H$ from a knowledge base Θ . If $Disj \not\subset Conj$ and if there exists disjunctive rule $Disj_2 \Rightarrow H$, such that $Disj_2 \subset Disj, Conj$, then literals from Disj and their conjunctions are joinable with conjunction Conj.

If a not founded literal Lit from disjunction Disj is joinable with conjunctions $Conj_1, Conj_2$, then it is also joinable with conjunction $Conj_1\&Conj_2$.

Lemma 4.19 Let us have disjunctive rule $Disj \Rightarrow H$ and conjunctive one $Conj \Rightarrow H$ from a knowledge base Θ . Not founded literals from Disj are joinable with Conj only if $Disj \not\subset Conj$ and if there exists disjunctive rule $Disj_2 \Rightarrow H$, such that $Disj_2 \subset$ Disj, Conj.

If rule $L_1 \& L_2 \& ... L_k \& Conj \Rightarrow H$ is added within Möbius transformation of Θ for literals L_i which are not weakly founded, then exist rules $Ant_1 \Rightarrow H$ and $Ant_2 \Rightarrow H$ in Θ , such that $Ant_i = L_1 \lor L_2 \lor ... L_k \lor Disj_i$, $Ant_1 \subset Conj$ and $Ant_1 \subset Ant_2 \not\subset Conj$, $Disj_2$ may be empty disjunction.

If rule $L_1 \& L_2 \& ... L_k \& Conj \Rightarrow H$ is added within Möbius transformation of Θ for not founded literals L_i such that there is no rule $Ant \Rightarrow H$ in Θ , where $L_i \subset Ant \subset$ $L_1 \& L_2 \& ... L_k \& Conj$, then exist rules $Ant_1 \Rightarrow H$ and $Ant_2 \Rightarrow H$ in Θ , such that $Ant_i = L_1 \lor L_2 \lor ... L_k \lor Disj_i$, $Ant_1 \subset Conj$ and $Ant_1 \subset Ant_2 \not\subset Conj$, $Disj_2$ may be empty disjunction.

If rule $L\&Conj_1\&Conj_2 \Rightarrow H$ is added within Möbius transformation of Θ for literals L such that does not exist antecedent Ant, where $L \subset Ant \subset L\&Conj_1\&Conj_2$, then L is joinable to $Conj_1, Conj_2$ and $Conj_1$ is joinable to $Conj_2$.

To be proved.

Summary

Similarly like in the case of founded *ecd* knowledge bases, it is not possible to decide, in general, whether a given rule to be added or not to be added into knowledge base within the process of Möbius transformation. The exceptions are conjunctive translations

of maximal disjunctive rules, see lemma 4.14. In another general situations we can only specify whether the rule is not added or whether it is possible that the rule is added. Even we can not state that the collection of presented lemmata bound a set of potentially added rules as close as possible. E.g. we have learnt that pairwise joinability of $Conj_i$ from (#) is necessary but not sufficient condition for mutual joinability.

But nevertheless, we can summarize what has been stated as follows:

Let Θ be a general ecd knowledge base. Every rule, which is added into knowledge base within the process of Möbius transformation of Θ , is a conjunctive rule of the form

$$Lit_1 \& Lit_2 \& \dots \& Lit_m \& Conj_1 \& Conj_2 \& \dots \& Conj_n \Rightarrow H, \tag{\#}$$

where $Conj_i$ ' are mutually joinable antecedents⁴ of some rules from the original knowledge base Θ for $i = 1, ..., n \ge 1$,

Lit_j are literals joinable to every $Conj_i$ for $j = 1, ..., m, m + n \ge 2$, and moreover there exist two disjunctive rules in Θ with antecedents $Disj_1, Disj_2$, such that $Disj_2 \subset Disj_1 \subset Lit_j$, $Disj_2 \subset Conj_i$, and $Disj_1 \notin Conj_1 \& ... \& Conj_n$.

We can use this summary for a construction of an algorithm of Möbius transformation for general ecd knowledge bases, which is presented in the next section.

5 Algorithm of Möbius transformation

5.1 Algorithm for founded *ecd* knowledge bases

From the theorem 3.2 we have an existence of Möbius transform for weakly sound low founded *ecd* knowledge bases. Using theorem 4.4, we can formulate the following algorithm of Möbius transformation of a knowledge base Θ .

- (*) Go ahead through all hypothesis H: and perform items (0) - (4).
- (0) Construct a set *Rel* of literals relevant to *H*.
 Put w₀ = w₀⁰.
 Create an empty set of maximal disjunctions *MaxD*.
- (1) Go ahead through all disjunctions D in Θ relevant to H: Put Sum equal to ⊕-sum of Möbied weights of all rules D ∨ D' ⇒ H. IF there is no such rule, THEN insert D into MaxD and put w_D = w⁰_D ⊖ w₀, ELSE put w_D = w⁰_D ⊖ Sum. If |D| = 1, then sign D in Rel.
- (2) Go through all unsigned literals L from Rel: IF there is no rule $L \vee D \Rightarrow H$, THEN put $w_L = 0$, ELSE give warning "Assumption does not hold for hypothesis H." and STOP.

⁴I.e. $Conj_i$ ' are pairwise joinable, active parts of their joining disjunctions are not pairwise disjoint (and maybe, it hold(s) some other still not precisely specified condition(s)).

Overlapping of $Conj_i$ ' is possible: e.g. for original antecedents AB, AC we can get added one ABC.

- (3) Go through all maximal disjunctions MD from MaxD: for MD and every subdisjunction SMD of MD create all new rules Ant ⇒ H which are already not included in Θ, where Ant is a conjunctive translation of MD or SMD.
- (4) Go ahead through all conjunctions |C| > 1 in Θ relevant to H: Put Sum equal to ⊕-sum of Möbied weights (w_{C'}) of all rules C' ⇒ H, where C' ⊂ C (C implies C'). If w⁰_C is not given (C ⇒ H is added rule), then put w⁰_C equal to ⊕-sum of Möbied weights (w_{C'}) of all rules C' ⇒ H, where C' is subconjunction of C (including w₀ ~ empty subconjunction implied by C). Keep w⁰_C and put w_C = w⁰_C ⊖ Sum. (During construction of Möbius transform it is not necessary to distinguish between w⁰_C and w^x_C, they can be represented by the same variable denoted w⁰_C.)
- (*) Save all rules with weights $w_{H_i,Ant_{ij}} \neq 0$ Möbius transform of Θ . STOP.

It is possible to show that this algorithm ends and produces Möbius transform of any weakly sound low founded ecd knowledge base Θ .

5.2 Algorithm for general *ecd* knowledge bases

Using theorem 3.2 and summary from the end of subsection 4.2, we can formulate an algorithm of Möbius transformation of a general ecd knowledge base Θ .

Performing the algorithm, estimations w^x of implicit weights are computed only for potentially added rules and resulting Möbied weights only for these rules and for rules from the source knowledge base. In the situation that a potential rule not to be added the algorithm compute Möbied weight equal to zero.

- (*) Go ahead through all hypothesis H from Θ : consider only rules relevant to H (i.e. $Ant \Rightarrow H$) and perform items (0) – (2).
- (0) a) create all sets of pairwise joinable conjunctive antecedents JA = {Conj₁, Conj₂, ..., Conj_k}, k ≥ 1.
 b) for every JA created in (a) create possible antecedents PA = L₁&L₂&...&L_m&Conj₁&Conj₂&...&Conj_k for every conjunction of literals L₁&L₂&...&L_m joinable to Conj₁&Conj₂&...&Conj_k, for k ≥ 1, m ≥ 0. Insert PA ⇒ H (without weight) into Θ if it is not already contained.
- (1) Go ahead through all disjunctive rules Disj ⇒ H in Θ from maximal to simple ones:
 Put Sum equal to ⊕-sum of Möbied weights of all rules Disj' ⇒ H, where Disj' ⊂ Disj.
 IF there is no such rule, THEN put w_{Disj} = w⁰_{Disj} ⊖ w₀, ELSE put w_{Disj} = w⁰_{Disj} ⊖ Sum.

- (2) Go ahead through all conjunctive rules Conj ⇒ H, |Conj| > 1 in Θ from the shortest antecedents to maximal ones: Put Sum equal to ⊕-sum of Möbied weights (w_{Ant}) of all rules Ant ⇒ H, where Ant ⊂ Conj (Conj implies Ant). IF there is no such rule, THEN put w_{Conj} = w⁰_{Conj} ⊖ w₀ ELSE If w⁰_{Conj} is not given (Conj ⇒ H is added rule), then compute w^x_{Conj} using formulas from subsection 3.6 and keep it as w⁰_{Conj}. Put w_{Conj} = w⁰_{Conj} ⊖ Sum. (During construction of Möbius transform it is not necessary to distinguish between w⁰_{Conj} and w^x_{Conj}, they can be represented by the same variable denoted w⁰_{Conj}.)
- (*) Save all rules with weights $w_{H_i,Ant_{ij}} \neq 0$ Möbius transform of Θ . STOP.

Some comments to algorithm

Note that lemma 4.14 is not employed here. It is because all the conjunctive translations which fulfill assumption of the lemma are generated among another possible antecedent of added rules.

In step 0a) it is necessary to generate different sets JA of pairwise joinable antecedents, even if conjunctions of their elements are the same and it seems, that it is possible to generate just the same rules from such sets. We need all of them because of step 0b), where antecedents are constructed from these sets and from (sets of) literals joinable to them. If literal Lit or set of literals $Lit_1, ...Lit_k$ is joinable to conjunction of antecedents $Conj = Ant_1\&...\&Ant_m = L_1\&...\&L_n$, it depends not only on literals $L_1, ...L_k$ from which Conj is composed, but also on the structure born on antecedents. On the other side, it is really possible to generate the same antecedent PA several times, but rule $PA \Rightarrow H$ is added into knowledge base, only if it is not already contained in. So, duplicity of rules does not arise.

Let us suppose as an example knowledge base Θ from appendix F a). From its antecedents AB, AC, AD, BD there are generated the following sets JA: $\{AB\}, \{AC\}, \{AD\}, \{BD\}, \{AB, AC\}, \{AB, AD\}, \{AB, BD\}, \{AC, AD\}, \{AC, BD\}, \{AD, BD\}, \{AB, AC, AD\}, \{AB, AC, BD\}, \{AC, AD, BD\}, \{AB, AC, AD, BD\}$ (for simplicity, conjunction signs & are omitted inside antecedents).

It holds AB&AD = AB&BD = AD&BD = ABD, but the only $JA = \{AB, AD\}$ enables to construct possible antecedents XABD, YABD, XYABD, because literals X, Y are not joinable to BD.

Similarly for added antecedent ABCD. ABCD is conjunction of six different sets JA. $\{AC, BD\}$ seems to be useful because it is the shortest one, but nevertheless the only set $\{AB, AC, AD\}$ enables joining of literals X and Y.

A similar situation is in knowledge base Θ' , see appendix F b). Added antecedent ABCD is generable by many ways. If it is generated from $\{AB, AC, AD\}$, $\{ABC, ABD\}$, $\{AC, ABD\}$ or $\{AD, ABC\}$ it is joinable with X, Y. The another possibilities are not joinable to X, Y ($\{ABD, BCD\}$, $\{ABC, BCD\}$, $\{AB, BCD\}$, $\{AC, BCD\}$, $\{AD, BCD\}$, $\{ABC, BCD\}$, $\{AB, BCD\}$, $\{AC, BCD\}$, $\{AD, BCD\}$, $\{ABC, ABD, BCD\}$).

All rules, which to be added into knowledge base, are added in step 0, thus steps 1 and 2 are analogies of steps 1 and 4 of the algorithm for founded ecd knowledge bases.

The algorithm can be improved by the following:

- Generation of sets JA can be sophisticated in various ways, e.g. if set $JA = \{Ant_1, ...Ant_k\}$ is generated such that $Ant_1\&...\&Ant_k = Ant_j$, then set JA is not further considerated and not used for further generation, i.e. no sets JA' with a subset JA are generated.
- Consideration of sets of literals joinable to antecedents used during generation of sets JA, but it can tend to more time-complex algorithm.
- A principal improvement of the algorithm can be based on a theoretical elaboration of a notion of mutual joinability of several antecedents. It can substantially eliminate number of sets JA which are considered.

6 Conclusion

Generalized Möbius transformation is a theoretical tool for the construction of more correct generalizations of expert systems both of MYCIN-like and fuzzy expert systems based on a composition of fuzzy relations.

Möbius transformation has been generalized to *ecd* knowledge bases, i.e. knowledge bases whose rules have antecedents either in the form of an elementary conjunction (as before) or in the form of an elementary disjunction (new ones) of questions.

The principal difference between original and generalized Möbius transformation consists in a complicated transfer of weights of rules with disjunctive antecedents D_i to weights of other rules with conjunctive ones C_i , where C_i implies D_i .

Original Möbius transformation is only the transformation of weights. While within the generalized one, moreover, some new rules are often added into the knowledge base.

An estimation of implicit (expected) weights for these added rules was shown for a class of ecd knowledge bases. The existence theorem was proved for this class of knowledge bases. Finally, an algorithm of the construction of this generalized Möbius transform of knowledge base is described. Results on founded ecd knowledge bases were described in V-706 [3], while the case of general ecd knowledge bases is presented in this text.

There is a possibility of further improvement of the algorithm using a notion of mutual joinability of several antecedents. This should be a motivation for further research.

A challenge for the future is an admission of rules with more complicated antecedents or a consideration of knowledge bases with several different conjunctions and/or disjunctions.

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7 APPENDIX D

Counter-examples, see section 4.

a) Let us consider the following founded *ecd* knowledge base:

 $\Rightarrow H(w_0)$ $A \lor B \lor C \lor D \Rightarrow H(w_{\lor}^0)$ $A \lor C \lor D \Rightarrow H(w_{A-C-D}^0)$ $B \lor C \Rightarrow H(w_0)$ $A \Rightarrow H(w_A^0)$ $B \Rightarrow H(w_B^0)$ $C \Rightarrow H(w_C^0)$ $D \Rightarrow H(w_D^0)$ $C\&D \Rightarrow H(w_{CD}^0)$ $A\&B\&D \Rightarrow H(w_{ABD}^0)$

We know, that no disjunctive rules are to be added, thus we have to compute Möbied weights for conjunctive rules only. So it is not necessary to estimate implicit weights of disjunctive rules.

Estimations of implicit weights:

 $\begin{aligned} w_{AB}^{x} &= w_{A}^{0} \oplus w_{B}^{0} \oplus w_{0} \\ w_{AC}^{x} &= w_{A}^{0} \oplus w_{C}^{0} \oplus w_{0} \\ w_{AD}^{x} &= w_{A}^{0} \oplus w_{D}^{0} \oplus w_{0} \\ w_{BC}^{x} &= w_{B}^{0} \oplus w_{C}^{0} \oplus w_{0} \\ w_{BD}^{x} &= w_{B}^{0} \oplus w_{C}^{0} \oplus w_{0} \\ \end{aligned}$ $w_{ABC}^{x} = w_{A}^{0} \oplus w_{B}^{0} \oplus w_{C}^{0} \oplus 2w_{0}$ $w_{ACD}^{x} = w_{CD}^{0} \oplus w_{A}^{0} \oplus w_{0}$ $w_{BCD}^{x} = w_{CD}^{0} \oplus w_{B}^{0} \oplus w_{0}$ $w_{BCD}^{x} = w_{CD}^{0} \oplus w_{B}^{0} \oplus w_{0}$ $w^x_{ABCD} = w^0_{ABD} \oplus w^0_{CD} \ominus w^0_D$ Möbied weights: w_0 $w_{\vee} = w_{\vee}^0 \ominus w_0$ $w_A = w_A^0 \ominus w_{A-C-D}^0$ $w_B = w_B^{\vec{0}} \ominus w_0^A$ $w_C = w_C^0 \ominus w_0 \ominus w_{A-C-D}^0 \oplus w_{\vee}^0$ $w_D = w_D^0 \ominus w_{A-C-D}^0$ $w_{AB} = \overline{w}_{\vee}^0 \ominus \overline{w}_0$ $w_{AC} = w^0_{A-C-D} \ominus w_0$ $w_{AD} = w^0_{A-C-D} \ominus w_0$ $w_{BC} = w_{B-C}^0 \oplus w_0 = 0$!!! $w_{BD} = w_{\vee}^0 \ominus w_0$ $w_{CD} = w_{CD}^0 \ominus w_C^0 \ominus w_D^0 \oplus w_{A-C-D}^0$ $w_{ABC} = w_{\vee}^0 \ominus w_0$ $w_{ABD} = w_{ABD}^{0} \stackrel{\circ}{\ominus} w_{V}^{0} \ominus w_{V}^{0} \ominus w_{A}^{0} \ominus w_{D}^{0} \oplus 3w_{0}$ $w_{ACD} = w_{0} \ominus w_{A}^{0} - C - D$ $(3w_0 = w_0 \oplus w_0 \oplus w_0)$ $w_{BCD} = w_0 \ominus w_{\vee}^{\dot{0}}$ $w_{ABCD} = w_{\vee}^0 \ominus w_0$

non stated weights of disjunctive rules w_{A-B-C} , w_{A-B-D} , w_{B-C-D} , w_{A-B} , w_{A-C} , w_{A-D} , w_{A-D} , w_{C-D} are equal to zero, i.e. rules are not included in Möbius transform.

We can easily verify that our present example correspond to lemmata from section 4. But, $w_{BC} = 0$ thus the rule $B\&C \Rightarrow H$ is not added into Möbius transform, even if his antecedent B&C is subconjunction of added conjunctive translation of antecedent of maximal disjunctive rule.

b) Lemmata 4.2 and 4.3 do not hold for weakly founded *ecd* knowledge bases. Let us consider the following weakly founded *ecd* knowledge base.

$$\Rightarrow H(w_0) A \lor B \lor C \Rightarrow H(w_{A-B-C}^0) A \lor D \Rightarrow H(w_{A-D}^0) B \Rightarrow H(w_B^0) A\&C \Rightarrow H(w_{AC}^0) B\&D \Rightarrow H(w_{BD}^0)$$

Similarly as before, it is not necessary to estimate implicit weights of disjunctive rules. Estimations of implicit weights:

$$w_{A}^{x} = w_{A-B-C}^{0} \oplus w_{A-D}^{0} \oplus w_{0}$$

$$w_{C}^{x} = w_{A-B-C}^{0}$$

$$w_{D}^{x} = w_{A-D}^{0} \oplus w_{A-D}^{0} \oplus w_{0}$$

$$w_{AD}^{x} = w_{B}^{0} \oplus w_{A-D}^{0} \oplus w_{0}$$

$$w_{AD}^{x} = w_{B}^{0} \oplus w_{A-D}^{0} \oplus w_{A-B-C}^{0} \oplus w_{0}$$

$$w_{ABC}^{x} = w_{B}^{0} \oplus w_{AC}^{0} \oplus w_{0}$$

$$w_{ABD}^{x} = w_{BD}^{0} \oplus w_{AC}^{0} \oplus w_{0}$$

$$w_{ABD}^{x} = w_{BD}^{0}$$

$$w_{ACD}^{x} = w_{AC}^{0} \oplus w_{BD}^{0} \oplus w^{0}$$
Möbied weights:

$$w_{0}$$

$$w_{A-B-C} = w_{A-D}^{0} \oplus w_{0}$$

$$w_{A-D} = w_{A-D}^{0} \oplus w^{0}$$

$$w_{A} = 0$$

non stated weights of disjunctive rules are equal to zero again.

Rule $A\&B\&C\&D \Rightarrow H(w^0_{A-D} \ominus w_0)$ is addedd into Möbius transform, even if the antecedent of the rule is neither conjunctive translation of an antecedent of any disjunctive rule nor its subconjunction. And vice-versa, conjunctive translation of maximal disjunctive rule $A \lor D \Rightarrow H$ is not added.

c) Let us consider the previous knowledge base extended with rule $A \Rightarrow H(w_A^0)$.

 $\Rightarrow H(w_0)$ $A \lor B \lor C \Rightarrow H(w_{A-B-C}^0)$ $A \lor D \Rightarrow H(w_{A-D}^0)$ $A \Rightarrow H(w_A^0)$ $B \Rightarrow H(w_B^0)$ $A\&C \Rightarrow H(w_{AC}^0)$ $B\&D \Rightarrow H(w_{BD}^0)$

Similarly as before, it is not necessary to estimate implicit weights of disjunctive rules.

Estimations of implicit weights:

non stated weights of disjunctive rules are also equal to zero.

Rule $A\&B\&D \Rightarrow H(w^0_{A-D} \ominus w_0)$ is addedd into Möbius transform even if the antecedent of the rule is neither conjunctive translation of an antecedent of any disjunctive rule nor its subconjunction. And vice-versa, conjunctive translation of maximal disjunctive rule $A \lor D \Rightarrow H$ is not added.

APPENDIX 8 E)

a) Let us consider the following weakly founded ecd knowledge base Θ :

 $\Rightarrow H(w_0)$ $A \lor B \Rightarrow H(w^0_{A-B})$ $A \lor C \Rightarrow H(w_{A-C}^{0})$ $B \lor C \Rightarrow H(w_{B-C}^{0})$ $A\&F \Rightarrow H(w^0_{AF})$ $B\&C \Rightarrow H(w_{BC}^0)$ $D\&E \Rightarrow H(w_{DE}^0)$ Estimations of implicit weights:
$$\begin{split} w^x_{ABCF} &= w^0_{AF} \oplus w^0_{BC} \ominus w_0 \\ w^x_{BCDE} &= w^0_{BC} \oplus w^0_{DE} \ominus w_0 \\ w^x_{ADEF} &= w^0_{AF} \oplus w^0_{DE} \ominus w_0 \\ w^x_{ABCDEF} &= w^0_{AF} \oplus w^0_{BC} \oplus w^0_{DE} \ominus 2w_0 \end{split}$$
Möbied weights: w_0 $w_{A-B} = w_{A-B}^0 \ominus w_0$ $w_{C-D} = w_{C-D}^0 \ominus w_0$

 $w_{C-D} = w_{C-D}^{\circ} \ominus w_{0}$ $w_{E-F} = w_{E-F}^{0} \ominus w_{0}$ $w_{AF} = w_{AF}^{0} \ominus w_{A-B}^{0} \ominus w_{E-F}^{0} \oplus w_{0}$ $w_{BC} = w_{BC}^{0} \ominus w_{A-B}^{0} \ominus w_{C-D}^{0} \oplus w_{0}$ $w_{DE} = w_{DE}^{0} \ominus w_{C-D}^{0} \ominus w_{E-F}^{0} \oplus w_{0}$ $w_{ABCF} = w_{A-B}^{0} \ominus w_{0}$ $w_{ADEF} = w_{E-F}^{0} \ominus w_{0}$ $w_{ADEF} = 0$ $w_{ABCDEF} = 0$

Other Möbied weights are equal to zero. w^x , are presented (and computed as well) only for rule with presented resulting Möbied weights.

b) Let us consider the following modification of previous ecd knowledge base Θ' :

 $\Rightarrow H(w_0)$ $A \vee B \Rightarrow H(w^0_{A-B})$ $A \lor C \Rightarrow H(w^0_{A-C})$ $B \lor C \Rightarrow H(w_{B-C}^{\vec{0}})$ $A\&F \Rightarrow H(w_{AF}^0)$ $B\&C \Rightarrow H(w_{BC}^0)$ $D\&E \Rightarrow H(w_{DE}^{\overline{0}})$ $B\&C\&D\&E \Rightarrow H(w^0_{BCDE})$

Estimations of implicit weights:

 $\begin{aligned} & w_{ABCF}^x = w_{AF}^0 \oplus w_{BC}^0 \oplus w_0 \\ & w_{ADEF}^x = w_{AF}^0 \oplus w_{DE}^0 \oplus w_0 \\ & w_{ABCDEF}^x = w_{AF}^0 \oplus w_{BCDE}^0 \oplus w_0 \end{aligned}$

Möbied weights:

Other Möbied weights are equal to zero. w^x , are presented (and computed as well) only for rule with presented resulting Möbied weights.

9 APPENDIX F

a) Let us consider the following general ecd knowledge base Θ :

 $\Rightarrow H(w_{0})$ $X \lor Y \lor A \Rightarrow H(w_{X-Y-A}^{0})$ $X \lor Y \Rightarrow H(w_{X-Y}^{0})$ $A \lor B \Rightarrow H(w_{A-B}^{0})$ $A\&B \Rightarrow H(w_{AB}^{0})$ $A\&C \Rightarrow H(w_{AC}^{0})$ $A\&D \Rightarrow H(w_{AD}^{0})$ $B\&D \Rightarrow H(w_{BD}^{0})$

Estimations of implicit weights:

$$\begin{split} & w_{ABC}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_0 \\ & w_{ABD}^x = w_{AB}^0 \oplus w_{AD}^0 \oplus w_{BD}^0 \oplus 2w_0 \\ & w_{ACD}^x = w_{AC}^0 \oplus w_{AD}^0 \oplus w_0 \\ & w_{ABCD}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AD}^0 \oplus w_{BD}^0 \oplus 3w_0 \\ & w_{XABC}^x = w_{YAB}^x = w_{XYAB}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AC}^0 \oplus w_{X-Y}^0 \oplus w_0 \\ & w_{XAC}^x = w_{YAC}^x = w_{XYAC}^x = w_{AC}^0 \oplus w_{X-Y}^0 \oplus w_0 \\ & w_{XAD}^x = w_{YAD}^x = w_{XYAD}^x = w_{AD}^0 \oplus w_{X-Y}^0 \oplus w_0 \\ & w_{XABC}^x = w_{YABC}^x = w_{XYABC}^x = w_{AD}^0 \oplus w_{X-Y}^0 \oplus w_0 \\ & w_{XABC}^x = w_{YABC}^x = w_{XYABC}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{X-Y}^0 \oplus 2w_0 \\ & w_{XABC}^x = w_{YABC}^x = w_{XYABC}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{X-Y}^0 \oplus 2w_0 \\ & w_{XABD}^x = w_{YABD}^x = w_{XYABD}^x = w_{AB}^0 \oplus w_{AD}^0 \oplus w_{X-Y}^0 \oplus 2w_0 \\ & w_{XABD}^x = w_{YABD}^x = w_{XYABD}^x = w_{AB}^0 \oplus w_{AD}^0 \oplus w_{BD}^0 \oplus w_{X-Y}^0 \oplus 3w_0 \\ & w_{XABD}^x = w_{YABD}^x = w_{XYABD}^x = w_{AB}^0 \oplus w_{AD}^0 \oplus w_{BD}^0 \oplus w_{X-Y}^0 \oplus 4w_0 \\ & w_{XABCD}^x = w_{YABCD}^x = w_{XYABCD}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AD}^A \oplus w_{BD}^0 \oplus w_{X-Y}^0 \oplus 4w_0 \\ & w_{XABCD}^x = w_{YABCD}^x = w_{XYABCD}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AD}^A \oplus w_{BD}^0 \oplus w_{X-Y}^0 \oplus 4w_0 \\ & w_{XABCD}^x = w_{YABCD}^x = w_{XYABCD}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AD}^A \oplus w_{BD}^0 \oplus w_{X-Y}^0 \oplus 4w_0 \\ & w_{XABCD}^x = w_{YABCD}^x = w_{XYABCD}^x = w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AD}^A \oplus w_{BD}^0 \oplus w_{X-Y}^0 \oplus 4w_0 \\ & w_{XABCD}^x = w_{YABCD}^x = w_{XYABCD}^x \oplus w_{AD}^0 \oplus w_{AC}^0 \oplus w_{AD}^A \oplus w_{AD}^0 \oplus w_{AC}^0 \oplus w_{AD}^0 \oplus w_{AC}^0 \oplus w_{AC}^0$$

 w^{x} , were computed only for rules which may be added within Möbius transformation of knowledge base.

Möbied weights:

$$w_{0}$$

$$w_{X-Y-A} = w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{X-Y} = w_{X-Y}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{X-Y-A}^{0}$$

$$w_{A-B} = w_{A-B}^{0} \oplus w_{0}$$

$$w_{AB} = w_{AB}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{AC} = w_{AC}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{AD} = w_{AD}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{BD} = w_{BD}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{ABC} = w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus 2w_{0}$$

$$w_{ABC} = w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus 2w_{0}$$

$$w_{ABD} = 2w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus 2w_{0}$$

$$w_{ABD} = 2w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus 2w_{0}$$

$$w_{ABC} = -w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus 2w_{0}$$

$$w_{ABC} = w_{XAB} \oplus w_{X-Y-A}^{0} \oplus 2w_{0}$$

$$w_{XAB} = w_{XAC} = w_{XAD} = w_{YAB} = w_{YAC} = w_{YAD} = w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XABC} = w_{XABD} = w_{XACD} = w_{YABC} = w_{YABD} = w_{YACD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XABC} = w_{XBD} = w_{XACD} = w_{YABC} = w_{YABD} = w_{YACD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XABC} = w_{YABD} = w_{XYACD} = w_{0}^{0}$$

$$(w_{XBCD} = w_{YBCD} = w_{XYBCD} = 0)$$

$$w_{XABCD} = w_{YABCD} = w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XYABCD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

b) Let us consider the following modification of previous ecd knowledge base Θ' : $\Rightarrow H(w_0)$

$$X \lor Y \lor A \Rightarrow H(w_{X-Y-A}^{0})$$

$$X \lor Y \lor A \Rightarrow H(w_{X-Y-A}^{0})$$

$$X \lor Y \Rightarrow H(w_{A-B}^{0})$$

$$A \lor B \Rightarrow H(w_{A-B}^{0})$$

$$A \& B \Rightarrow H(w_{AC}^{0})$$

$$A \& D \Rightarrow H(w_{AC}^{0})$$

$$B \& D \Rightarrow H(w_{BD}^{0})$$

$$A \& B \& C \Rightarrow H(w_{BD}^{0})$$

$$A \& B \& D \Rightarrow H(w_{ABC}^{0})$$

$$A \& B \& D \Rightarrow H(w_{ABD}^{0})$$

$$B \& C \& D \Rightarrow H(w_{BCD}^{0})$$

Estimations of implicit weights:

timations of implicit weights:
$$\begin{split} w_{ACD}^{x} &= w_{AC}^{0} \oplus w_{AD}^{0} \oplus w_{0} \\ w_{ABCD}^{x} &= w_{ABC}^{0} \oplus w_{ABD}^{0} \oplus w_{BCD}^{0} \oplus w_{AB}^{0} \oplus w_{0} \\ w_{ABCD}^{x} &= w_{ABC}^{0} \oplus w_{ABD}^{0} \oplus w_{BCD}^{0} \oplus w_{AB}^{0} \oplus w_{0} \\ w_{XAB}^{x} &= w_{YAB}^{x} &= w_{XYAB}^{x} &= w_{AB}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XAC}^{x} &= w_{YAC}^{x} &= w_{XYAC}^{x} &= w_{AC}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XAD}^{x} &= w_{YAD}^{x} &= w_{XYAD}^{x} &= w_{AD}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XABC}^{x} &= w_{YABC}^{x} &= w_{XYABC}^{x} &= w_{ABC}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XABC}^{x} &= w_{YABC}^{x} &= w_{XYABC}^{x} &= w_{ABC}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XABD}^{x} &= w_{YABD}^{x} &= w_{XYABD}^{x} &= w_{ABD}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XABD}^{x} &= w_{YABD}^{x} &= w_{XYABD}^{x} &= w_{ABD}^{0} \oplus w_{X-Y}^{0} \oplus w_{0} \\ w_{XABCD}^{x} &= w_{YABCD}^{x} &= w_{XYABCD}^{x} &= w_{ABD}^{0} \oplus w_{ABD}^{0} \oplus w_{BCD}^{0} \oplus w_{AB}^{0} \oplus w_{X-Y}^{0} \oplus 2w_{0} \\ w_{XABCD}^{x} &= w_{YABCD}^{x} &= w_{XYABCD}^{x} &= w_{ABC}^{0} \oplus w_{ABD}^{0} \oplus w_{BCD}^{0} \oplus w_{AB}^{0} \oplus w_{X-Y}^{0} \oplus 2w_{0} \\ w_{XABCD}^{x} &= w_{YABCD}^{x} &= w_{XYABCD}^{x} &= w_{ABC}^{0} \oplus w_{ABD}^{0} \oplus w_{BCD}^{0} \oplus w_{AB}^{0} \oplus w_{X-Y}^{0} \oplus 2w_{0} \\ w_{XABCD}^{x} &= w_{YABCD}^{x} &= w_{XYABCD}^{x} &= w_{ABC}^{0} \oplus w_{ABD}^{0} \oplus w_{BCD}^{0} \oplus w_{AB}^{0} \oplus w_{X-Y}^{0} \oplus 2w_{0} \\ w_{XABCD}^{x} &= w_{ABC}^{x} &= w_{ABC}^{0} \oplus w_{ABD}^{0} \oplus w_{ABD}^{0} \oplus w_{AB}^{0} \oplus w_{ABD}^{0} \oplus w_{AB}^{0} \oplus w$$

 w^x , were computed only for rules which may be added within Möbius transformation of knowledge base.

Möbied weights:

$$w_{0}$$

$$w_{X-Y-A} = w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{X-Y} = w_{X-Y}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{A-B} = w_{A-B}^{0} \oplus w_{0}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{AB} = w_{AB}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{AD} = w_{AD}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{BD} = w_{BD}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{ABC} = w_{ABC}^{0} \oplus w_{AB}^{0} \oplus w_{AC}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{ABC} = w_{ABC}^{0} \oplus w_{AB}^{0} \oplus w_{AC}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{ABD} = w_{BDD}^{0} \oplus w_{AB}^{0} \oplus w_{AC}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{ABD} = w_{BCD}^{0} \oplus w_{A-B}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{BCD} = w_{BCD}^{0} \oplus w_{A-B}^{0} \oplus w_{A-B}^{0} \oplus w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{ABCD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XAB} = w_{XAC} = w_{XAD} = w_{YAB} = w_{YAC} = w_{YAD} = w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XABC} = w_{XABD} = w_{XACD} = w_{YABC} = w_{YABD} = w_{YACD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XYABC} = w_{XABD} = w_{XYACD} = w_{XBD}^{0} = w_{YACD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XYABC} = w_{YBCD} = w_{XYBCD} = 0$$

$$w_{XABCD} = w_{YBCD} = w_{XYBCD} = 0$$

$$w_{XABCD} = -w_{X-Y-A}^{0} \oplus w_{0}$$

$$w_{XYABCD} = -w_{X-Y-A}^{0} \oplus w_{0}$$