A Generalization of Möbius Transformation for Knowledge Bases, which Include Rules with Disjunction in Antecedent II. General ecd knowledge bases

Daniel, Milan

INSTITUTE OF COMPUTER SCIENCE

ACADEMY OF SCIENCES OF THE CZECH REPUBLIC

A Generalization of Möbius Transformation for Knowledge Bases, which Include Rules with Disjunction in Antecedent II (General *ecd* knowledge bases)

Milan Daniel

Technical report No. V-712

May 1997

Institute of Computer Science, Academy of Sciences of the Czech Republic
Pod vodárenskou věží 2, 182 07 Prague 8, Czech Republic
phone: (+4202) 6884244 fax: (+4202) 8585789
e-mail: milan@uivt.cas.cz

INSTITUTE OF COMPUTER SCIENCE

ACADEMY OF SCIENCES OF THE CZECH REPUBLIC

A Generalization of Möbius Transformation for Knowledge Bases, which Include Rules with Disjunction in Antecedent II (General *ecd* knowledge bases)

Milan Daniel [1]

Technical report No. V-712
May 1997

Abstract

Möbius transformation is an important tool for establishing weights of rules of compositional expert systems from conditional weights.

In this report, an applicability of Möbius transformation of rule bases is extended also to knowledge bases with elementary disjunctions in antecedents of rules. The text is continuation of Technical Report V-706. It describes the case of general *ecd* knowledge bases including an algorithm of the Möbius transformation for this general class of knowledge base.

Keywords

Möbius Transformation, Expert system, Knowledge Base, Uncertainty, Weight of rule

[1]Partial support by the grant No. 1030601 of the GA ASCR (GA AV ČR) and by COST project OC 15.10 is acknowledged.

1 Introduction

The first ideas, on how to establish weights of rules of compositional expert systems from conditional weights related to real evidence, data, experience, were published in 1984 [5]. For a full description of Möbius transformation see e.g. [6], [7].

The possibility of utilization of Möbius transformation is not only restricted to MYCIN-like systems, it is important also for a common generalization of MYCIN-like systems and fuzzy expert systems which use a composition of fuzzy relations like Conorm-CADIAG-2 extended with the handling of negative knowledge, see [4]. The system is derived from the fuzzy expert system CADIAG-2 [1].

The original Möbius transformation is formulated and only used for rules of a special form. The present work generalizes it for a wider class of rules.

This report is a continuation of technical report V-706 [3], where are all necessary preliminaries introduced and the original Möbius transformation theorem for MYCIN-like systems is stated. In this text only new definitions from V-706 are repeated in the second section.

A sectioning of the text is the same as in V-706. In sections 3 and 4.1 are briefly reviewed main results from V-706. The main point of this report is in subsection 4.2, where are new results on Möbius transformation for general *ecd* knowledge bases. These results are used in algorithm in section 5, the algorithm for founded *ecd* knowledge bases from V-706 is presented as well.

In section 6 is a comparison of Möbius transformation for MYCIN-like systems and of an introduced generalization. After it follows conclusions and ideas for future work.

2 Möbius transformation

2.1 Preliminaries

An *elementary disjunction* is a disjunction of literals. An *ecd knowledge base* (elementary-conjunction-disjunction) is a knowledge base such that antecedents of rules are either elementary conjunctions or elementary disjunctions.

We shall use the following terminology. A rule $R: \ A \Rightarrow S(w)$ is a *simple rule* if its antecedent A is a literal, R is *conjunctive/disjunctive rule* if A is a conjunction/disjunction, *maximal conjunctive/disjunctive rule* if there is no rule $B \Rightarrow S(w_B)$ in the knowledge base, so that A is a subconjunction/subdisjunction of B. A conjunction $Conj = A\&B\&...\&K$ is a conjunctive translation of a disjunction $Disj = A \vee B \vee ... \vee K$, a rule $Conj \Rightarrow H$ is a conjunctive translation of the disjunctive rule $Disj \Rightarrow H$.

An *ecd* knowledge base Θ is *founded* if it contains rules $A \Rightarrow H$ for every literal A which is involved in some disjunctive rule $A \vee Disj \Rightarrow H$ for any elementary disjunction $Disj$. An *ecd* knowledge base Θ is *weakly founded* if for every literal A from any disjunctive rule $A \vee Disj \Rightarrow H$ there is a simple rule $A \Rightarrow H$ or conjunctive rule $A\&Conj \Rightarrow H$ (for some $Conj$) included in Θ.
Notice, that every knowledge base without disjunction in antecedents is founded.

A literal A is founded for hypothesis H, if the rule $A \Rightarrow H$ is included in the knowledge base. A literal A is weakly founded for hypothesis H, if rule $A \& Conj \Rightarrow H$ is included in the knowledge base for some conjunction of literals $Conj$ (possibly empty). A rule $A_1 \& A_2 \& ... \& A_n \Rightarrow H$ (resp. a rule $A_1 \vee A_2 \vee ... \vee A_n \Rightarrow H$) is founded if every A_i is founded for the hypothesis H. A rule $A_1 \& A_2 \& ... \& A_n \Rightarrow H$ (resp. $A_1 \vee A_2 \vee ... \vee A_n \Rightarrow H$) is weakly founded[2] if for every A_i exist At such that $A_i \subseteq At \subseteq A_1 \& A_2 \& ... \& A_n$ and rule $At \Rightarrow H$ is included in the knowledge base.

2.2 Möbius transformation theorem

Theorem 2.1 *Let β be a weakly sound set of rules such that $w^0_{H,E} = \beta(H|E)$. Then there exists a weighting of rules which forms a knowledge base Θ of MYCIN-like expert system, such that for any three-valued questionaire E_q and hypothesis H for which $\beta(H|E_q)$ is defined, it holds*

$$W_\Theta(H|E_q) = \beta(H|E_q),$$

where $W_\Theta(H|E_q)$ is a global weight of hypothesis H given by E_q, $W_\Theta(H|E_q) = \oplus\{\Theta(H|E')|E' \subseteq E_q\}$.

The new knowledge base Θ is called *Möbius transform* of the source rule base β.

3 Including a disjunction into Möbius transformation

3.1 The idea of Möbius transformation

3.2 The first attempt to include a disjunction

3.3 Rewriting of disjunction

3.4 What does a disjunction rule mean?

Lemma 3.1 *Let Θ be a weakly sound low ecd knowledge base. If we can explicitly set or estimate implicit weights also for nonincluded combinations of literals, then Möbius transform of the knowledge base Θ exists.*

Note: weak soundness condition in the present situation is as follows, for every two rules such that $Ant_1 \subset Ant_2$ holds: if $w^0_{Ant_1} = 1$, then $w^0_{Ant_2} = 1$, where $Ant_1 \subset Ant_2$ means Ant_2 implies Ant_1, (i.e. $Conj_1$ is a subconjunction of $Conj_2$ or $Disj_2$ is a subdisjunction of $Disj_1$ or a subdisjunction of $Disj_1$ exists which is a subconjunction of $Conj_2$).

From the proof of the lemma, we know how to construct, in a simple yet noneffective way, Möbius transformation. So, it is logical to look for its improvement. A decision-making of whether a new possible rule will be added or not depends on the Möbied

[2]Note, that every founded literal, rule, knowledge base is trivially also weakly founded.

weight of the possible rule. Therefore we need a more sophisticated way of computing these weights. In the next subsection it is shown how to compute Möbied weights of rules in knowledge bases in which all possible elementary conjunctions and elementary disjunctions form antecedents of rules with the same succedent.

3.5 Formulas for computing of Möbied weights

$$w_0 = w_0^0$$

$$w_\vee = w_\vee^0 \ominus w_0$$

$$w_{A-B-C-...-K} = w^0_{A-B-C-...-K} \oplus \bigoplus_{i=1}^{n-k} ((-1)^i \bigoplus_{|d|=k+i,\, A-B-...-K \subset d} w_d^0)$$

$$w_A = w_A^0 \oplus \bigoplus_{i=1}^{n-1} ((-1)^i \bigoplus_{|d|=i+1,\, A\subset d} w_d^0)$$

$$w_{ABC...K} = w^0_{ABC...K} \oplus \bigoplus_{i=1}^{k-1} ((-1)^i \bigoplus_{|c|=k-i,\, c\subset AB...K} w_c^0) \;\oplus\; (-1)^k w^0_{A-B-C-...-K},$$

where $w_\vee$ is an abbreviation for a weight $w_{A-B-C-...-N}$ of the rule with the maximal possible disjunction in antecedent, $a \subset b$ means b implies a, $|c|$ is a length (number of conjuncts) of conjunction c, conjunction $c = ABC...K$ has k elements i.e. $|c| = k$.

$$w_{ABC...K} = (-1)^k (w^0_{A-B-C-...-K} \ominus w_0).$$

3.6 Estimations of implicit weights of "rules" which are not included in a source knowledge base

As it was suggested in the previous subsection, we can use the formulas from there for specifying which type of rules are added into a knowledge base during Möbius transformation and which ones are not.

To distinguish *explicit conditional weights* w^0_{Ant} of rules from a source knowledge base from computed estimations of those which are not given (resp. which are given implicitly through other rules), we shall denote *estimated implicit weights* as w^x_{Ant}.

We have observed in V-706, that *" an effect of disjunction $A \vee B$ is in some sense involved in effect of its disjunct A and it is not propagated once more through the other disjunct B"*.

This is very important for generation of w^x_{Ant} of "problematic" rules, i.e. *"Not to propagate weight of disjunctive rule several times through different literals into weight of conjunctive rule"*.

We can summarize the formulas for estimations as follows:

$$w_0^x = w_0$$

$$w^x_{Disj} = \bigoplus_{Ant \subset Disj} w_{Ant} \oplus w_0$$

$$w^x_{Ant} = \bigoplus_{Conj \subset Ant} w'_{Conj} \oplus w_0$$

$$w^x_{Ant} = \bigoplus_{Conj \subset Ant} w^{Ant}_{Conj} \;\oplus \bigoplus_{\substack{Disj \subset Ant,\ Disj \not\subset Lit,\\ Lit \subset Conj \subseteq Ant,\ w^0_{Conj} \text{ is given}}} w^{Ant}_{Disj} \;\oplus\; w_0,$$

where $Ant, Conj, Disj, Lit$ is any antecedent, conjunction, disjunction or literal from Θ respectively. The third formula is applicable for conjunctive rules from the case 2).

We can notice, that the first three formulas are the special cases of the fourth one. Thus, the 4th formula is not applicable only for conjunctive rules ad 3), but is is applicable in general. Hence, in the cases either that all the possible antecedents are included in the source knowledge base or that we want to compute w^x for all the possible antecedents we can use the last formula in the following form:

$$w^x_{Ant} = \bigoplus_{i=1}^{k-1}((-1)^{i+1} \bigoplus_{\substack{|Conj|=k-i\\ Conj \subset Ant}} w^0_{Conj}) \oplus \bigoplus_{i=2}^{n-k}((-1)^{i+k-1} \bigoplus_{\substack{|Disj|=i,\ Disj \subset Ant,\\ Disj \not\subset Lit,\\ Lit \subset Conj \subseteq Ant,\\ w^0_{Conj} \text{ is given}}} w^0_{Disj}) \oplus (-1)^n w_0,$$

where z is 0 or x and $Ant, Conj, Disj$ is any possible antecedent, elementary conjunction or elementary disjunction constructed from questions the source knowledge base Θ.

We recapitulate, that we have formulas on how to compute estimations of conditional weights for all types of rules admissible in *ecd* knowledge bases. Hence, we have an existence theorem.

Theorem 3.2 *If Θ is a weakly sound low ecd knowledge base, then there Möbius transform of the knowledge base Θ exists.*

4 Simplifications

We know how to compute Möbied weights, i.e. rule weights of Möbius transform of source knowledge base. We know, how to estimate and compute implicit weights for every rule with elementary conjunction or elementary disjunction in antecedent, such that for every question and its negation Q either w^0_Q is given (i.e. there is rule $Q \Rightarrow H(w^0_Q)$ included in the original knowledge base) or if Q is not a subdisjunction of an antecedent of any rule. Thus, we can do Möbius transformation for any knowledge base of this type.

Now, we are going to specify which rules are not necessary to add to Möbied knowledge base. We consider rules $Ant \Rightarrow H$ which are not included into the source knowledge base, i.e. w^0_{Ant} is not given there. By an added rule we mean such a rule that $w_{Ant} \neq 0$, while if $w_{Ant} = 0$ we say that rule is not added.

In general, we can consider all nonincluded rules to be virtually added. Whether a rule is to really be added or not, depends on its Möbied weight w_{Ant}. We can eliminate some types of rules to be added by a symbolic computation of their Möbied weight. But usually, we cannot assert that some type of rules will be added, because the actual value of its weight w_{Ant} depends on the actual values of conditional weights from the source knowledge base.

In the following subsection, we shall review some lemmata (from V-706) to describe which types of rules to be / not to be added in the knowledge base. For disjunctive rules, we easily obtain the following important lemma.

Lemma 4.1 *There are no disjunctive rules added to a knowledge base during Möbius transformation.*

4.1 Simplifications for founded *ecd* knowledge bases

Lemma 4.2 *If Θ is a founded ecd knowledge base, then there are no rules $A_1\&A_2\&...\&A_k \Rightarrow H$ added into the knowledge base within the process of Möbius transformation, where $A_1 \vee A_2 \vee ... \vee A_k \vee B_1 \vee ... \vee B_l$ is not an antecedent of some rule from the source knowledge base, for some literals $B_1, ...B_l$.*

Lemma 4.3 *If Θ is a founded ecd knowledge base, then conjunctive translations of all maximal disjunctive rules are added into the knowledge base within the process of Möbius transformation (if they are not already included in the source knowledge base Θ).*

Let $A_1 \vee A_2 \vee ... \vee A_k \Rightarrow H$ be a maximal disjunctive rule and $A_1\&A_2\&...\&A_k \Rightarrow H$ its conjunctive translation added according to the lemma. Usually, there are also all rules added with subconjunction of $A_1\&A_2\&...\&A_k$ in antecedent. But, there are counter-examples also on a symbolic level, see appendix D, case a).

In both previous lemmata, the assumption of foundness of *ecd* knowledge base is necessary. For general *ecd* knowledge bases which are not founded, there are counter-examples against both of the lemmata presented in appendix D, cases b) and c).

Summarizing the above lemmata we see, that antecedents of rules added by Möbius transformation into *ecd* knowledge base are only all conjunctive translations of antecedents of maximal disjunctive rules and not necessarily all of their subconjunctions. Formally, we have the following:

Theorem 4.4 *Let Θ be a weakly sound low founded ecd knowledge base. During a process of Möbius transformation of Θ, rules are added into the knowledge base (if they are not already included in Θ and) if and only if they are in one of the two following types:*

- *All rules $A_1\&A_2\&...\&A_n \Rightarrow H$, where $A_1 \vee A_2 \vee ... \vee A_n \Rightarrow H$, is a maximal disjunctive rule from Θ.*
- *Rules $A_1\&A_2\&...\&A_n \Rightarrow H$, where $A_1 \vee A_2 \vee ... \vee A_n \vee Disj \Rightarrow H$, is a rule from Θ and $Disj$ is any disjunction. (In general, all such rules are added, but there counter-examples exist).*

4.2 Simplifications for general *ecd* knowledge bases

As we have seen in the previous subsection (more particularly in V-706), in the case of founded *ecd* knowledge bases, the only possibilities for antecedent added rule is either to be a conjunctive translation of antecedent of some disjunctive rule or to be a subconjunction of such a translation.

In the case of general *ecd* knowledge bases, an antecedent of added rule can be besides it also a (subconjunction of) conjunctive translation of a disjunction of several disjunctive antecedents, as it is illustrated with the following small example, for other examples see appendix E.

Example: Let us consider the following weakly founded *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_\vee)$
$A \Rightarrow H(w^0_A)$
$D \Rightarrow H(w^0_D)$
$B\&D \Rightarrow H(w^0_{BD})$

Estimations of implicit weights:

$w^x_{A-B-D} = w_0$
$w^x_{A-D} = w_0$
$w^x_{B-D} = w_0$
$w^x_B = w^0_{A-B}$
$w^x_{AB} = w^0_A$
$w^x_{AD} = w^0_A \oplus w^0_D \ominus w_0$
$w^x_{ABD} = w^0_A \oplus w^0_{BD} \ominus w_0$

Möbied weights:

w_0
$w_{A-B-D} = 0$
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{A-D} = 0$
$w_{B-D} = 0$
$w_A = w^0_A \ominus w^0_{A-B}$
$w_B = 0$
$w_D = w^0_D \ominus w_0$
$w_{AB} = 0$
$w_{AD} = 0$
$w_{BD} = w^0_{BD} \ominus w^0_{A-B} \ominus w^0_D \oplus w_0$
$w_{ABD} = w^0_{A-B} \ominus w_0$

Rule $A\&B\&D \Rightarrow H(w^0_{A-B} \ominus w_0)$ is added into knowledge base even if its antecedent $A\&B\&D$ is not (subconjunction of) conjunctive translation of any disjunctive antecedent, $A \vee B \vee D$ is not (subdisjunction of) antecedent of any rule from Θ.

In this case, the antecedent of added rule is a conjunctive translation of disjunction of disjunctive antecedents from the source knowledge base.

Analogously to the case of founded *ecd* knowledge bases, we want to find restrictions for added rules. So we formulate the following hypotheses as analogy to lemmata 4.2 and 4.3.

Hypotheses • If Θ is a general *ecd* knowledge base, then there are no rules $A_1 \& A_2 \& ... \& A_k \Rightarrow H$ added into the knowledge base within the process of Möbius transformation, where $A_1 \vee A_2 \vee ... \vee A_k \vee B_1 \vee ... \vee B_l$ is not a disjunction of antecedents of some disjunctive rules from the source knowledge base.

• If Θ is a general *ecd* knowledge base, then conjunctive translations of all maximal founded (weakly founded) disjunctive rules $A_1 \vee A_2 \vee ... \vee A_k \Rightarrow H$, such that antecedent of their translation $A_1 \& A_2 \& ... \& A_k$ is a conjunction of antecedents of some conjunctive rules (incl. simple ones) from the source knowledge base, are added into the knowledge base within the process of Möbius transformation (if they are not already included in the source knowledge base Θ).

Unfortunately neither the first nor the second of these hypotheses is true. We can demonstrate it by the following examples:

Example: Let us consider the following weakly founded *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_{A-B})$
$A \Rightarrow H(w^0_A)$
$B \& C \Rightarrow H(w^0_{BC})$

Möbied weights:

w_0
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_A = w^0_A \ominus w^0_{A-B}$
$w_{BC} = w^0_{BC} \ominus w^0_{A-B}$
$w_{ABC} = w^0_{A-B} \ominus w_0$

Rule $A \& B \& C \Rightarrow H(w^0_{A-B} \ominus w_0)$ is added into knowledge base even if its disjunctive translation $A \vee B \vee C$ is neither antecedent nor disjunction of antecedents of any rules from Θ. Hence the first hypotheses does not hold.

Example: Let us consider another weakly founded *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_{A-B})$
$B \vee C \Rightarrow H(w^0_{B-C})$
$A \Rightarrow H(w^0_A)$
$B \& C \Rightarrow H(w^0_{BC})$

Möbied weights:

w_0
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{B-C} = w^0_{B-C} \ominus w_0$
$w_{AB} = w^0_{AB} \ominus w^0_{A-B} \ominus w^0_{B-C} \oplus w_0$
$w_{BC} = w^0_{BC} \ominus w^0_{A-B} \ominus w^0_{B-C} \oplus w_0$
$w_{ABC} = w^0_{A-B} \ominus w_0 \oplus w^0_{B-C} \ominus w_0$

Whether rule $A \& B \& C \Rightarrow H(w^0_{A-B} \ominus w_0 \oplus w^0_{B-C} \ominus w_0)$ will be added into knowledge base or not, it depends on the precise given weights. In general, the the rule will be added. But in the case of $w^0_{A-B} \ominus w_0 = -(w^0_{B-C} \ominus w_0)$, the weight of the rule will be equal to 0, and the rule will not be added. Hence it is not possible to prove the second hypothesis, even if its assertion is usually fulfilled.

Let us recapitulate now, what we know on Möbius transformation of general *ecd* knowledge bases, from the previous text.

- Möbius transformation is performed for each hypothesis separately.
- It is possible perform Möbius transformation separately for every disjunctive part of knowledge base w.r.t. given hypothesis (no literal from disjunctive part of KB is connected by any rule with literals from outside)
- No disjunctive rules are added anytime.

Similarly to the third item we have the following lemma.

Lemma 4.5 *There are no simple rules $Lit \Rightarrow H$ added to a knowledge base during Möbius transformation for any literal Lit.*

Proof. The assertion is in fact a corollary of the way of estimation of w^x for literals and of lemma 4.1: $w^x_{Lit} = \bigoplus_{Disj \subset Lit} w_D isj$, thus $w_{Lit} = w^x_{Lit} \ominus \bigoplus_{Ant \subset Lit} w_{Ant} = \bigoplus_{Disj \subset Lit} w_{Disj} \ominus \bigoplus_{Disj \subset Lit} w_{Disj} = 0$ (or $w_{Lit} = w^0_{Lit} \ominus \bigoplus_{Disj \subset Lit} w_{Disj}$ if $Lit \Rightarrow H$ is already included in Θ).

Lemma 4.6 *Let Θ is a general ecd knowledge base. Founded rule $A_1 \& A_2 \& ... \& A_k \Rightarrow H$ is added into Θ within the process of Möbius transformation, only if $A_1 \vee A_2 \vee ... \vee A_k \vee Disj$ is an antecedent of some rule from Θ, for some disjunction $Disj$ (empty disjunction is possible).*

Proof. Let suppose, for contradiction, added rule $R : A_1 \& A_2 \& ... \& A_k \Rightarrow H$ such that there is not rule $A_1 \vee A_2 \vee ... \vee A_k \vee Disj$ in Θ for any disjunction $Disj$. Let Θ' a restriction of Θ on questions included in rule R constructed as follows: conjunctive rule is included in Θ' if and only if it is included in Θ, disjunctive rule $Disj \Rightarrow H$ is included in Θ' if and only if rule $Disj \vee Disj' \Rightarrow H$ is included in Θ for some disjunction $Disj$ (empty disjunction is possible). If several disjunctive rules $Ant \Rightarrow H$with the same antecedent and consequent and with weights $w_1, w_2, ... w_k$ should be put in Θ', then there is put into Θ' rule $Ant \Rightarrow H(w_1 \oplus w_2 \oplus ... \oplus w_k)$ instead of them.

All the expected weights w^x should are the same in both Θ and Θ'. The same rule are applicable in both the knowledge bases (resp. instead of several disjunctive rules from Θ only one rule with the same effect in Θ'. Hence, Möbied weight of rule R should be the same and the assertion follows from lemma 4.2.

Let us ask the principal question: When a rule $Ant \Rightarrow H$ is added within Möbius transformation? It is added if and only if its weight is different from zero, i.e. if and only if $w^x_{Ant} \neq \bigoplus_{At \subset Ant} w_{At}$. From the formula, it follows that the rule is never added if there are only disjunctive rules on $\{A_1, A_2, ..., A_k\}$, where $Ant = A_1 \& A_2 \& ... \& A_k$. So it must exist at least one conjunctive rule $A \Rightarrow H$, $A \subset Ant$. Weight of such a rule is used in computation of w^x_{Ant}. (Otherwise it would be $w^x_{Ant} = \bigoplus_{At \subset Ant} w_{At}$ and $w_A nt = 0$). Hence, we have:

Lemma 4.7 *Every rule added into knowledge base within the process of Möbius transformation is of the form $Conj_1 \& Conj_2 \Rightarrow H$, where $Conj_1$ is an antecedent of some rule from the original knowledge base.*

It look like that antecedents of added rules are only conjunctions of antecedents from the original knowledge base or subconjunctions of such conjunctions. Nevertheless, an attempt to prove that added rules do not contain literals which are not weakly founded is unsuccessful but on the other hand we can prove the following.

Lemma 4.8 *A literal Lit, which is not weakly founded, can be included in antecedent $Lit \& Conj$ of added rule only if it is included at least in two antecedents of disjunctive rules $Disj_2 \subset Disj_1$, such that $Disj_2 \subset Conj$ and $Disj_1 \not\subset Conj$.*

Proof. Let L is literal which is not weakly founded. Rule $L \& Conj \Rightarrow H$ is added if and only if its weight $w_{L\&Conj}$ is different from zero. So we have to compute this weight. No simple rules are added and L is not weakly found, thus w_L must be equal to zero.

Let us compute values w_{Conj}^{Conj} at first A) If $Conj$ is minimal antecedent, then $w_{Conj}^{Conj} = w_{Conj}^0 \ominus \bigoplus_{D \subset Conj, D \not\subset Conj} w_D^{Conj} \ominus w_0 = w_{Conj}^0 \ominus w_0$. B) For conjunctive antecedent $Conj$ we get $w_{Conj}^{Conj} = w_{Conj}^0 \ominus \bigoplus_{C \subset Conj} w_C^{Conj} \ominus \bigoplus_{D \subset Conj, D \not\subset Conj} w_D^{Conj} \ominus w_0 = w_{Conj}^0 \ominus \bigoplus_{C \subset Conj} w_C^{Conj} \ominus w_0$. C) If conjunction $Conj$ is not antecedent of any rule from the original knowledge base Θ , then in Θ without disjunctive rules $D \Rightarrow H$, where $D \subset C \subset Conj$, it holds $w_{Conj}^x = \bigoplus_{C \subset Conj} w_C^{Conj} \oplus \bigoplus_{D \subset Conj, D \not\subset C \subset Conj} w_D^{Conj} = \bigoplus_{C \subset Conj} w_C^{Conj} \oplus \bigoplus_{D \subset Conj} w_D^{Conj} = \bigoplus_{At \subset Conj} w_{At}^{Conj}$, hence $w_{Conj}^{Conj} = 0$.

1) Let $Conj$ be a minimal conjunctive antecedent.

$w_{L\&Conj}^x = w_{Conj}^{Conj} \oplus \bigoplus_{D \subset L, D \not\subset Conj} w_D^{Conj} \oplus w_0 = w_{Conj}^0 \ominus w_0 \oplus \bigoplus_{D \subset L, D \not\subset Conj} w_D^{Conj} \oplus w_0 = w_{Conj}^0 \oplus \bigoplus_{D \subset L, D \not\subset Conj} w_D^{Conj}$,

$\bigoplus_{At \subset L\&Conj} w_{At} = w_{Conj} \oplus w_L \oplus \bigoplus_{D \subset L\&Conj} w_D^{Conj} \oplus w_0 = w_{Conj}^0 \ominus \bigoplus_{D \subset Conj} w_D \ominus w_0 \oplus \bigoplus_{D \subset Conj} w_D \oplus \bigoplus_{D \subset L, D \not\subset Conj} w_D \oplus w_0 = w_{Conj}^0 \oplus \bigoplus_{D \subset L, D \not\subset Conj} w_D$,

$w_{L\&Conj} = w_{Conj}^x \ominus \bigoplus_{At \subset L\&Conj} w_{At} = w_{Conj}^0 \oplus \bigoplus_{D \subset L, D \not\subset Conj} w_D^{Conj} \ominus w_{Conj}^0 \ominus \bigoplus_{D \subset L, D \not\subset Conj} w_D = \bigoplus_{D \subset L, D \not\subset Conj} (w_D^{Conj} \ominus w_D)$.

If there is no disjunctive antecedent D_1 in Θ such that $D_1 \subset L, D_1 \subset Conj$, then $w_D^{Conj} = w_D$ for $D \subset L$.

If there is no disjunctive antecedent D_2 in Θ such that $D_2 \subset L, D_2 \not\subset Conj$, then $\bigoplus_{D \subset L, D \not\subset Conj} = 0$.

If $D_1 \not\subset D_2$, then $w_{D_2}^{Conj} = w_{D_2}$.

Hence, rule $L \& Conj \Rightarrow H$ is added (i.e. $w_{L\&Conj} \neq 0$) only if there exist two disjunctive antecedents D_1, D_2 relevant to H, such that $D_1 \subset D_2, D_1, D_2 \subset L, D_1 \subset Conj, D_2 \not\subset Conj$.

2) Let $Conj$ be a conjunctive antecedent, but not minimal one.
$w^x_{L\&Conj} = w^{Conj}_{Conj} \oplus w^{Conj}_L \oplus \bigoplus_{C\subset Conj} w^{Conj}_C \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \oplus w_0 = (w^0_{Conj} \ominus \bigoplus_{C\subset Conj} w^{Conj}_C \ominus w_0) \oplus \bigoplus_{C\subset Conj} w^{Conj}_C \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \oplus w_0 = w^0_{Conj} \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \quad (w^{Conj}_L = 0)$,
$\bigoplus_{At\subset L\&Conj} w_{At} = w_{Conj} \oplus w_L \oplus \bigoplus_{At\subset Conj} w_{At} \oplus \bigoplus_{D\subset L, D\not\subset Conj} w_D \oplus w_0 \oplus \bigoplus_{At=L\&C, C\subset Conj} w_{At}$
$(w_L = 0)$,
$w_{L\&Conj} = w^x_{L\&Conj} \ominus \bigoplus_{At\subset L\&Conj} w_{At} = w^0_{Conj} \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \ominus w_{Conj} \ominus \bigoplus_{At\subset Conj} w_{At} \ominus \bigoplus_{D\subset L, D\not\subset Conj} w_D \ominus \bigoplus_{At=L\&C, C\subset Conj} w_{At} \ominus w_0 = (w^0_{Conj} \ominus \bigoplus_{At\subset Conj} w_{At} \ominus w_0 \ominus w_{Conj}) \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \ominus \bigoplus_{D\subset L, D\not\subset Conj} w_D \ominus \bigoplus_{At=L\&C, C\subset Conj} w_{At} =$
$\bigoplus_{D\subset L, D\not\subset Conj} (w^{Conj}_D \ominus w_D) \ominus \bigoplus_{At=L\&C, C\subset Conj} w_{At}$.

3) Let $Conj$ not to be any antecedent in Θ.
L is not included in any conjunctive antecedent from Θ, hence $w^{L\&Conj} = w^{Conj}$.
$w^x_{L\&Conj} = \bigoplus_{C\subset Conj} w^{Conj}_C \oplus w_0 \oplus \bigoplus_{D\subset Conj, D\not\subset C\subset L\&Conj} w^{Conj}_D = w^{Conj}_{Conj} \oplus w^{Conj}_L \oplus \bigoplus_{C\subset Conj} w^{Conj}_C \oplus \bigoplus_{D\subset Conj, D\not\subset C\subset Conj} w^{Conj}_D \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \oplus w_0 = w^x_{Conj} \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D$,
$\bigoplus_{At\subset L\&Conj} w_{At} = w_{Conj} \oplus w_L \oplus \bigoplus_{At\subset Conj} w_{At} \oplus \bigoplus_{D\subset L, D\not\subset Conj} w_D \oplus w_0 \oplus \bigoplus_{At=L\&C, C\subset Conj} w_{At}$
$(w_L = 0)$,
$w_{L\&Conj} = w^x_{L\&Conj} \ominus \bigoplus_{At\subset L\&Conj} w_{At} = w^x_{Conj} \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \ominus e_{Conj} \bigoplus_{At\subset Conj} w_{At} \ominus \bigoplus_{D\subset L, D\not\subset Conj} w_D \ominus \bigoplus_{At=L\&C, C\subset Conj} w_{At} \ominus w_0 = (w^x_{Conj} \ominus \bigoplus_{At\subset Conj} w_{At} \ominus w_0 \ominus w_{Conj}) \oplus \bigoplus_{D\subset L, D\not\subset Conj} w^{Conj}_D \ominus \bigoplus_{D\subset L, D\not\subset Conj} w_D \ominus \bigoplus_{At=L\&C, C\subset Conj} w_{At} =$
$\bigoplus_{D\subset L, D\not\subset Conj} (w^{Conj}_D \ominus w_D) \ominus \bigoplus_{At=L\&C, C\subset Conj} w_{At}$.

4) The rest is induction on length of $Ant = L\&Conj$. We know, that assertion holds for all antecedents, where $Conj$ is minimal conjunctive antecedent from Θ. (No rule $C \Rightarrow H$ is added, where C is subconjunction of any minimal antecedent, thus assertion trivially holds also for such antecedents).

Let us suppose, that the assertion holds for $C \subset Ant$. From this assumption follows, that $\bigoplus_{At=L\&C, C\subset Conj} w_{At} \neq 0$ only if there exist in Θ two disjunctive antecedents D_1, D_2 relevant to H, such that $D_1 \subset D_2, D_1, D_2 \subset L, D_1 \subset Conj, D_2 \not\subset Conj$. $\bigoplus_{D\subset L, D\not\subset Conj} (w^{Conj}_D \ominus w_D) \neq 0$ only under the same condition, see the case 1). Thus, rule $L\&Conj \Rightarrow H$ is added within Möbius transformation only if there exist in the source knowledge base Θ two disjunctive antecedents D_1, D_2 relevant to H, such that $D_1 \subset D_2, D_1, D_2 \subset L, D_1 \subset Conj, D_2 \not\subset Conj$.

By an application of the same idea we can prove also:

Lemma 4.9 *A weakly founded literal Lit can be included in antecedent Ant of added rule $Lit\&Conj \Rightarrow H$ only as a part of some antecedent At, such that $Lit \subseteq At \subset Lit\&Conj$, or if it is included at least in two antecedents of disjunctive rules $Disj_2 \subset Disj_1$, such that $Disj_2 \subset Conj$ and $Disj_1 \not\subset Conj$.*

Proof. For founded literal trivially always holds the first case and sometimes also the second one.

Let Lit is not founded literal.

If there exist some rule $At \rightarrow H$ in knowledge base, such that $Lit \subset At \subset L\&Conj$, then the first case holds. See, e.g. literal B from the introductive example of this subsection or all literals in added rules from appendix E.

If there is no rule $At \rightarrow H$, such that $Lit \subset At \subset L\&Conj$, then we can compute values of $w^x_{L\&Conj}$, $\bigoplus_{At \subset L\&Conj} w_{At}$, and $w_{L\&Conj}$ using the same assumptions and formulas as for literals which are not weakly founded, see the previous proof. Thus the second case holds.

Summarizing previous lemmata we obtain the following theorem.

Theorem 4.10 *Let Θ be a general ecd knowledge base. Every rule, which is added into knowledge base within the process of Möbius transformation of Θ, is a conjunctive rule of the form*

$$Lit_1 \,\&\, Lit_2 \,\&\, ... \,\&\, Lit_m \,\&\, Conj_1 \,\&\, Conj_2 \,\&\, ... \,\&\, Conj_n \Rightarrow H,$$

where $Conj_i$[3] is an antecedent of some rule from the original knowledge base Θ for $i = 1,..,n \geq 1$, $m + n \geq 2$, and for $j = 1,...,m$, Lit_j is literal, such that there exist two disjunctive antecedents $Disj_{j1}, Disj_{j2}$ in Θ for which $Disj_{j2} \subset Disj_{j1} \subset Lit_j$, $Disj_{j2} \subset Conj_1\&...\&Conj_n$, and $Disj_{j1} \not\subset Conj_1\&...\&Conj_n$.

Proof. For founded literal Lit, there is $Lit \Rightarrow H$ already included in Θ (thus such a literal may be considered as one-member conjunction — one of $Conj_i$'). From lemma 4.7 follows $n \geq 1$, if $m + n = 1$ then we obtain rule from Θ, hence $m + n \geq 2$. The rest follows lemmata 4.8, 4.9.

Definition 4.11 Antecedents of two conjunctive rules $Ant_1 \Rightarrow H$, $Ant_2 \Rightarrow H$ from a knowledge base Θ are *joinable* if rule $Ant_1\&Ant_2 \Rightarrow H$ is potentially added within the process of Möbius transformation of Θ. (If weight $w_{Ant_1\&Ant_2}$ computed according to inference net is generally non zero, if the actual zero/non-zero value of $w_{Ant_1\&Ant_2}$ depends just on particular w^0').

A conjunctive antecedent of rule $Ant \Rightarrow H$ is *joined* if there are two rules $Ant_1 \Rightarrow H$, $Ant_2 \Rightarrow H$, such that $Ant = Ant_1\&Ant_2$, included in knowledge base.

Let $Conj_1 \Rightarrow H$, $Conj_2 \Rightarrow H$ be two conjunctive rules from a general *ecd* knowledge base Θ, both with non joined antecedents. An antecedent of disjunctive rule $Disj \Rightarrow H$ is called *joining disjunction of* $Conj_1, Conj_2$ *with respect to* H, if the following holds: $Disj \subset Conj_1, Conj_2$, and for $i = 1, 2$ it either holds $L_{Conj_i} \subset L_{Disj}$ (all literals from $Conj_i$ are included also in $Disj$) or there is no rule $Conj'_1 \Rightarrow H$ in Θ, for which $Disj \subset Conj'_i \subset Conj_i$. A subdisjunction $Disj_A$ of joining disjunction $Disj$ which contain just the literals from $Conj_1$ and $Conj_2$ (L_{Conj_1}, L_{Conj_2}) is called an active part of joining disjunction.

[3]Overlapping of $Conj_i$' is possible: e.g. for original antecedents AB, AC we can get added one ABC, but not necessarily.

Hypothesis 4.12 *Let $Conj_1 \Rightarrow H$, $Conj_2 \Rightarrow H$ be two conjunctive rules from a general ecd knowledge base Θ, both with non joined antecedents.*
If there exists joining disjunction of antecedents $Conj_1, Conj_2$ w.r.t. H, then $Conj_1, Conj_2$ are joinable into new antecedent $Conj_1 \& Conj_2$.

If some of antecedent is joined we may get $w_{Conj_1 \& Conj_2} = 0$ even on symbolic level, see appendix E b).

Lemma 4.13 *Let $Conj_1 \Rightarrow H$, $Conj_2 \Rightarrow H$ be two conjunctive rules from a general ecd knowledge base Θ. The antecedents $Conj_1, Conj_2$ are joinable into new one $Conj_1 \& Conj_2$ only if there exists joining disjunction of $Conj_1$ and $Conj_2$ w.r.t. H.*

To be proved.

Even if it does not look like it at the first glance, this lemma is in fact a generalization of lemma 4.2, Let $Disj \subset Conj$ for rules $Disj \Rightarrow H$, $Conj \Rightarrow H$ from founded *ecd* knowledge base Θ. There are rules $Lit \Rightarrow H$ in Θ for every $Lit \subset Conj$, including literal(s) $Disj \subset Lit \subset Conj$. Hence w.r.t. the lemma, it must be $L_{Conj} \subset L_{Disj}$ for both antecedents which to be joined. We can now formulate also a generalization of lemma 4.3.

Lemma 4.14 *Let Θ be a general ecd knowledge base, let $Disj$ be an antecedent of some maximal disjunctive rule $Disj \Rightarrow H$ from Θ. If conjunctive translation of $Disj$ is of form $L_1 \& L_2 \& ... \& L_m \& C_1 \& C_2 \& ... \& C_n$, where $C_i \Rightarrow H$ is in Θ for $i = 1, ..., n$, and where L_i are literals such there exist a disjunctive rule $Disj_1 \Rightarrow H$, $Disj \subset Disj_1 \subset L_1 \vee ... \vee L_m$ and $Disj_1 \not\subset C_1 \& ... \& C_n$, then conjunctive translations of rule $Disj \Rightarrow H$, is added into the knowledge base within the process of Möbius transformation (if it is not already included in the source knowledge base Θ).*

Proof. Similarly as in the proof of 4.3 it is possible to show, that $w_{Ant} = \pm(w^0_{Disj} \ominus w_0)$ for $Ant = L_1 \& L_2 \& ... \& L_m \& C_1 \& C_2 \& ... \& C_n$, and similarly we suppose $w^0_{Disj} \neq w_0$ for maximal disjunctive rule. Hence $w_{Ant} \neq 0$.

Lemma 4.12 holds also for n rules which satisfy the same conditions. Thus we have:

Lemma 4.15 *Let us have n conjunctive rules $Conj_1 \Rightarrow H, \ldots, Conj_n \Rightarrow H$ from a general ecd knowledge base Θ, all with non joined antecedents. If there exists disjunctive rule $Disj \Rightarrow H$, such that $Disj \subset Conj_i$, and for $i = 1, ..., n$ it either holds $L_{Conj_i} \subset L_{Disj}$ or there is no rule $Conj'_1 \Rightarrow H$ in Θ, for which $Conj'_i \subset Conj$ and $L_{Conj'_i} \subset L_{Disj}$ (all literals from $Conj'_i$ are included also in $Disj$). Then antecedents $Conj_1, ..., Conj_n$ are joinable into new one $Conj_1 \& ... \& Conj_n$.*

The proof is analogical to the proof of lemma 4.12.

Lemma 4.16 *Let us have n conjunctive rules $Conj_1 \Rightarrow H, \ldots, Conj_n \Rightarrow H$ from a general ecd knowledge base Θ. The antecedents are joinable into new one $Conj_1 \& ... \& Conj_n$ only if they are pairwise joinable, i.e. if there exists disjunctive rules $Disj_{ij} \Rightarrow H$ for every couple of antecedents $Conj_i, Conj_j$, such that $Disj_{ij} \subset Conj_i, Conj_j$, and for $k = i, j$ it either holds $L_{Conj_k} \subset L_{Disj}$ or there is no rule $Conj'_k \Rightarrow H$ in Θ, for which $Conj'_k \subset Conj$ and $L_{Conj'_k} \subset L_{Disj}$.*

To be proved.

A reverse assertion does not hold, see appendix E a), moreover it holds the following lemma.

Lemma 4.17 *Let us have n conjunctive rules $Conj_1 \Rightarrow H$, ..., $Conj_n \Rightarrow H$ with pairwise joinable antecedents from a general ecd knowledge base Θ. If active parts of their joining disjunctions are pairwise disjunct, then rule $Conj_1\&...\&Conj_n \Rightarrow H$ is not added into knowledge base within its Möbius transformation, i.e. $Conj_1, ..., Conj_n$ are not mutually joinable.*

To be proved.

Hypothesis 4.18 *Let us have disjunctive rule $Disj \Rightarrow H$ and conjunctive one $Conj \Rightarrow H$ from a knowledge base Θ. If $Disj \not\subset Conj$ and if there exists disjunctive rule $Disj_2 \Rightarrow H$, such that $Disj_2 \subset Disj, Conj$, then literals from $Disj$ and their conjunctions are joinable with conjunction $Conj$.*

If a not founded literal Lit from disjunction $Disj$ is joinable with conjunctions $Conj_1, Conj_2$, then it is also joinable with conjunction $Conj_1\&Conj_2$.

Lemma 4.19 *Let us have disjunctive rule $Disj \Rightarrow H$ and conjunctive one $Conj \Rightarrow H$ from a knowledge base Θ. Not founded literals from $Disj$ are joinable with $Conj$ only if $Disj \not\subset Conj$ and if there exists disjunctive rule $Disj_2 \Rightarrow H$, such that $Disj_2 \subset Disj, Conj$.*

If rule $L_1\&L_2\&...L_k\&Conj \Rightarrow H$ is added within Möbius transformation of Θ for literals L_i which are not weakly founded, then exist rules $Ant_1 \Rightarrow H$ and $Ant_2 \Rightarrow H$ in Θ, such that $Ant_i = L_1 \vee L_2 \vee ...L_k \vee Disj_i$, $Ant_1 \subset Conj$ and $Ant_1 \subset Ant_2 \not\subset Conj$, $Disj_2$ may be empty disjunction.

If rule $L_1\&L_2\&...L_k\&Conj \Rightarrow H$ is added within Möbius transformation of Θ for not founded literals L_i such that there is no rule $Ant \Rightarrow H$ in Θ, where $L_i \subset Ant \subset L_1\&L_2\&...L_k\&Conj$, then exist rules $Ant_1 \Rightarrow H$ and $Ant_2 \Rightarrow H$ in Θ, such that $Ant_i = L_1 \vee L_2 \vee ...L_k \vee Disj_i$, $Ant_1 \subset Conj$ and $Ant_1 \subset Ant_2 \not\subset Conj$, $Disj_2$ may be empty disjunction.

If rule $L\&Conj_1\&Conj_2 \Rightarrow H$ is added within Möbius transformation of Θ for literals L such that does not exist antecedent Ant, where $L \subset Ant \subset L\&Conj_1\&Conj_2$, then L is joinable to $Conj_1, Conj_2$ and $Conj_1$ is joinable to $Conj_2$.

To be proved.

Summary

Similarly like in the case of founded *ecd* knowledge bases, it is not possible to decide, in general, whether a given rule to be added or not to be added into knowledge base within the process of Möbius transformation. The exceptions are conjunctive translations of maximal disjunctive rules, see lemma 4.14. In another general situations we can only specify whether the rule is not added or whether it is possible that the rule is added. Even we can not state that the collection of presented lemmata bound a set of potentially added rules as close as possible. E.g. we have learnt that pairwise joinability of $Conj_i$' from (#) is necessary but not sufficient condition for mutual joinability.

But nevertheless, we can summarize what has been stated as follows:

Let Θ be a general ecd knowledge base. Every rule, which is added into knowledge base within the process of Möbius transformation of Θ, is a conjunctive rule of the form

$$Lit_1 \ \& \ Lit_2 \ \& \ ... \ \& \ Lit_m \ \& \ Conj_1 \ \& \ Conj_2 \ \& \ ... \ \& \ Conj_n \Rightarrow H, \qquad (\#)$$

where $Conj_i$' are mutually joinable antecedents[4] *of some rules from the original knowledge base Θ for $i = 1, .., n \geq 1$,*
Lit_j are literals joinable to every $Conj_i$ for $j = 1, ..., m$, $m + n \geq 2$,
and moreover there exist two disjunctive rules in Θ with antecedents $Disj_1, Disj_2$, such that $Disj_2 \subset Disj_1 \subset Lit_j$, $Disj_2 \subset Conj_i$, and $Disj_1 \not\subset Conj_1\&...\&Conj_n$.

We can use this summary for a construction of an algorithm of Möbius transformationfor general *ecd* knowledge bases, which is presented in the next section.

5 Algorithm of Möbius transformation

5.1 Algorithm for founded *ecd* knowledge bases

From the theorem 3.2 we have an existence of Möbius transform for weakly sound low founded *ecd* knowledge bases. Using theorem 4.4, we can formulate the following algorithm of Möbius transformation of a knowledge base Θ.

(*) Go ahead through all hypothesis H:
and perform items (0) – (4).

(0) Construct a set Rel of literals relevant to H.
Put $w_0 = w_0^0$.
Create an empty set of maximal disjunctions $MaxD$.

(1) Go ahead through all disjunctions D in Θ relevant to H:
Put Sum equal to $\oplus$-sum of Möbied weights of all rules $D \vee D' \Rightarrow H$.
IF there is no such rule, THEN insert D into $MaxD$ and put $w_D = w_D^0 \ominus w_0$,
ELSE put $w_D = w_D^0 \ominus Sum$.
If $|D| = 1$, then sign D in Rel.

(2) Go through all unsigned literals L from Rel:
IF there is no rule $L \vee D \Rightarrow H$, THEN put $w_L = 0$,
ELSE give warning "Assumption does not hold for hypothesis H." and STOP.

[4] I.e. $Conj_i$' are pairwise joinable, active parts of their joining disjunctions are not pairwise disjoint (and maybe, it hold(s) some other still not precisely specified condition(s)).
Overlapping of $Conj_i$' is possible: e.g. for original antecedents AB, AC we can get added one ABC.

(3) Go through all maximal disjunctions MD from $MaxD$:
for MD and every subdisjunction SMD of MD -
create all new rules $Ant \Rightarrow H$ which are already not included in Θ, where Ant is a conjunctive translation of MD or SMD.

(4) Go ahead through all conjunctions $|C| > 1$ in Θ relevant to H:
Put Sum equal to $\oplus$-sum of Möbied weights ($w_{C'}$) of all rules $C' \Rightarrow H$, where $C' \subset C$ (C implies C').
If w_C^0 is not given ($C \Rightarrow H$ is added rule), then put w_C^0 equal to $\oplus$-sum of Möbied weights ($w_{C'}$) of all rules $C' \Rightarrow H$, where C' is subconjunction of C (including $w_0 \sim$ empty subconjunction implied by C).
Keep w_C^0 and put $w_C = w_C^0 \ominus Sum$.
(During construction of Möbius transform it is not necessary to distinguish between w_C^0 and w_C^x, they can be represented by the same variable denoted w_C^0.)

(*) Save all rules with weights $w_{H_i,Ant_{ij}} \neq 0$ — Möbius transform of Θ.
STOP.

It is possible to show that this algorithm ends and produces Möbius transform of any weakly sound low founded *ecd* knowledge base Θ.

5.2 Algorithm for general *ecd* knowledge bases

Using theorem 3.2 and summary from the end of subsection 4.2, we can formulate an algorithm of Möbius transformation of a general *ecd* knowledge base Θ.

Performing the algorithm, estimations w^x of implicit weights are computed only for potentially added rules and resulting Möbied weights only for these rules and for rules from the source knowledge base. In the situation that a potential rule not to be added the algorithm compute Möbied weight equal to zero.

(*) Go ahead through all hypothesis H from Θ :
consider only rules relevant to H (i.e. $Ant \Rightarrow H$) and perform items (0) – (2).

(0) a) create all sets of pairwise joinable conjunctive antecedents
$JA = \{Conj_1, Conj_2, ..., Conj_k\}$, $k \geq 1$.
b) for every JA created in (a) create possible antecedents
$PA = L_1 \& L_2 \& ... \& L_m \& Conj_1 \& Conj_2 \& ... \& Conj_k$ for every conjunction of literals $L_1 \& L_2 \& ... \& L_m$ joinable to $Conj_1 \& Conj_2 \& ... \& Conj_k$, for $k \geq 1, m \geq 0$.
Insert $PA \Rightarrow H$ (without weight) into Θ if it is not already contained.

(1) Go ahead through all disjunctive rules $Disj \Rightarrow H$ in Θ from maximal to simple ones:
Put Sum equal to $\oplus$-sum of Möbied weights of all rules $Disj' \Rightarrow H$, where $Disj' \subset Disj$.
IF there is no such rule, THEN put $w_{Disj} = w_{Disj}^0 \ominus w_0$,
ELSE put $w_{Disj} = w_{Disj}^0 \ominus Sum$.

(2) Go ahead through all conjunctive rules $Conj \Rightarrow H$, $|Conj| > 1$ in Θ from the shortest antecedents to maximal ones:
Put Sum equal to $\oplus$-sum of Möbied weights (w_{Ant}) of all rules $Ant \Rightarrow H$, where $Ant \subset Conj$ ($Conj$ implies Ant).
IF there is no such rule, THEN put $w_{Conj} = w^0_{Conj} \ominus w_0$
ELSE If w^0_{Conj} is not given ($Conj \Rightarrow H$ is added rule), then compute w^x_{Conj} using formulas from subsection 3.6 and keep it as w^0_{Conj}. Put $w_{Conj} = w^0_{Conj} \ominus Sum$.
(During construction of Möbius transform it is not necessary to distinguish between w^0_{Conj} and w^x_{Conj}, they can be represented by the same variable denoted w^0_{Conj}.)

(*) Save all rules with weights $w_{H_i,Ant_{ij}} \neq 0$ — Möbius transform of Θ.
STOP.

Some comments to algorithm

Note that lemma 4.14 is not employed here. It is because all the conjunctive translations which fulfill assumption of the lemma are generated among another possible antecedent of added rules.

In step 0a) it is necessary to generate different sets JA of pairwise joinable antecedents, even if conjunctions of their elements are the same and it seems, that it is possible to generate just the same rules from such sets. We need all of them because of step 0b), where antecedents are constructed from these sets and from (sets of) literals joinable to them. If literal Lit or set of literals $Lit_1, ...Lit_k$ is joinable to conjunction of antecedents $Conj = Ant_1\&...\&Ant_m = L_1\&...\&L_n$, it depends not only on literals $L_1, ...L_k$ from which $Conj$ is composed, but also on the structure born on antecedents. On the other side, it is really possible to generate the same antecedent PA several times, but rule $PA \Rightarrow H$ is added into knowledge base, only if it is not already contained in. So, duplicity of rules does not arise.

Let us suppose as an example knowledge base Θ from appendix F a). From its antecedents AB, AC, AD, BD there are generated the following sets JA: $\{AB\}, \{AC\}$, $\{AD\}, \{BD\}, \{AB, AC\}, \{AB, AD\}, \{AB, BD\}, \{AC, AD\}, \{AC, BD\}, \{AD, BD\}$, $\{AB, AC, AD\}, \{AB, AC, BD\}, \{AB, AD, BD\}, \{AC, AD, BD\}, \{AB, AC, AD, BD\}$ (for simplicity, conjunction signs & are omitted inside antecedents).

It holds $AB\&AD = AB\&BD = AD\&BD = ABD$, but the only $JA = \{AB, AD\}$ enables to construct possible antecedents $XABD, YABD, XYABD$, because literals X, Y are not joinable to BD.

Similarly for added antecedent $ABCD$. $ABCD$ is conjunction of six different sets JA. $\{AC, BD\}$ seems to be useful because it is the shortest one, but nevertheless the only set {AB, AC, AD} enables joining of literals X and Y.

A similar situation is in knowledge base Θ', see appendix F b). Added antecedent $ABCD$ is generable by many ways. If it is generated from $\{AB, AC, AD\}, \{ABC, ABD\}$, $\{AC, ABD\}$ or $\{AD, ABC\}$ it is joinable with X, Y. The another possibilities are not joinable to X, Y ($\{ABD, BCD\}, \{ABC, BCD\}, \{AB, BCD\}, \{AC, BCD\}, \{AD, BCD\}$, $\{ABC, ABD, BCD\}$).

All rules, which to be added into knowledge base, are added in step 0, thus steps 1 and 2 are analogies of steps 1 and 4 of the algorithm for founded *ecd* knowledge bases.

The algorithm can be improved by the following:

- Generation of sets JA can be sophisticated in various ways, e.g. if set $JA = \{Ant_1, ...Ant_k\}$ is generated such that $Ant_1\&...\&Ant_k = Ant_j$, then set JA is not further considerated and not used for further generation, i.e. no sets JA' with a subset JA are generated.

- Consideration of sets of literals joinable to antecedents used during generation of sets JA, but it can tend to more time-complex algorithm.

- A principal improvement of the algorithm can be based on a theoretical elaboration of a notion of mutual joinability of several antecedents. It can substantially eliminate number of sets JA which are considered.

6 Conclusion

Generalized Möbius transformation is a theoretical tool for the construction of more correct generalizations of expert systems both of MYCIN-like and fuzzy expert systems based on a composition of fuzzy relations.

Möbius transformation has been generalized to *ecd* knowledge bases, i.e. knowledge bases whose rules have antecedents either in the form of an elementary conjunction (as before) or in the form of an elementary disjunction (new ones) of questions.

The principal difference between original and generalized Möbius transformation consists in a complicated transfer of weights of rules with disjunctive antecedents D_i to weights of other rules with conjunctive ones C_i, where C_i implies D_i.

Original Möbius transformation is only the transformation of weights. While within the generalized one, moreover, some new rules are often added into the knowledge base.

An *estimation of implicit (expected) weights* for these added rules was shown for a class of *ecd* knowledge bases. The *existence theorem* was proved for this class of knowledge bases. Finally, an *algorithm* of the construction of this generalized Möbius transform of knowledge base is described. Results on founded *ecd* knowledge bases were described in V-706 [3], while the case of general *ecd* knowledge bases is presented in this text.

There is a possibility of further improvement of the algorithm using a notion of mutual joinability of several antecedents. This should be a motivation for further research.

A challenge for the future is an admission of rules with more complicated antecedents or a consideration of knowledge bases with several different conjunctions and/or disjunctions.

Bibliography

[1] Klaus-Peter Adlassnig. Fuzzy set theory in medical diagnosis. *IEEE Transaction on Systems, Man, and Cybernetics*, SMC-16(2):260–265, 1986.

[2] Milan Daniel. A Generalization of Möbius transformation. In R. Jiroušek, and G. D. Kleiter, editors, *4th Workshop on Uncertainty Processing*, Prague, 1997, 31–42.

[3] Milan Daniel. A Generalization of Möbius transformation for knowledge bases, which include rules with disjunction in antecedent. Technical Report V-706, ICS ASCR, Prague, 1997.

[4] Milan Daniel, Petr Hájek, and Phuong Hoang Nguyen. CADIAG-2 and MYCIN-like Systems. *Artificial Intelligence in Medicine*, 9:241–259, 1997.

[5] Petr Hájek. Combining functions in consulting systems and dependence of premisses. In I. Plander, editor, *Artificial Intelligence and Information-Control Systems of Robots*, North-Holland, 1984, 163–166.

[6] Petr Hájek, Tomáš Havránek, and Radim Jiroušek. *Uncertain Information Processing in Expert Systems.* CRC Press, Boca Raton, Florida, 1992.

[7] Petr Hájek and Julio J. Valdés. An analysis of MYCIN-like expert systems. *Mathware & Soft Computing*, 1:45–68, 1994.

7 APPENDIX D

Counter-examples, see section 4.

a) Let us consider the following founded *ecd* knowledge base:

$\Rightarrow H(w_0)$
$A \vee B \vee C \vee D \Rightarrow H(w^0_\vee)$
$A \vee C \vee D \Rightarrow H(w^0_{A-C-D})$
$B \vee C \Rightarrow H(w_0)$
$A \Rightarrow H(w^0_A)$
$B \Rightarrow H(w^0_B)$
$C \Rightarrow H(w^0_C)$
$D \Rightarrow H(w^0_D)$
$C\&D \Rightarrow H(w^0_{CD})$
$A\&B\&D \Rightarrow H(w^0_{ABD})$

We know, that no disjunctive rules are to be added, thus we have to compute Möbied weights for conjunctive rules only. So it is not necessary to estimate implicit weights of disjunctive rules.

Estimations of implicit weights:

$w^x_{AB} = w^0_A \oplus w^0_B \ominus w_0$
$w^x_{AC} = w^0_A \oplus w^0_C \ominus w_0$
$w^x_{AD} = w^0_A \oplus w^0_D \ominus w_0$
$w^x_{BC} = w^0_B \oplus w^0_C \ominus w_0$
$w^x_{BD} = w^0_B \oplus w^0_D \ominus w_0$
$w^x_{ABC} = w^0_A \oplus w^0_B \oplus w^0_C \ominus 2w_0$
$w^x_{ACD} = w^0_{CD} \oplus w^0_A \ominus w_0$
$w^x_{BCD} = w^0_{CD} \oplus w^0_B \ominus w_0$
$w^x_{ABCD} = w^0_{ABD} \oplus w^0_{CD} \ominus w^0_D$

Möbied weights:

w_0
$w_\vee = w^0_\vee \ominus w_0$
$w_{A-C-D} = w^0_{A-C-D} \ominus w^0_\vee$
$w_{B-C} = w^0_{B-C} \ominus w^0_\vee = w_0 \ominus w^0_\vee$
$w_A = w^0_A \ominus w^0_{A-C-D}$
$w_B = w^0_B \ominus w_0$
$w_C = w^0_C \ominus w_0 \ominus w^0_{A-C-D} \oplus w^0_\vee$
$w_D = w^0_D \ominus w^0_{A-C-D}$
$w_{AB} = w^0_\vee \ominus w_0$
$w_{AC} = w^0_{A-C-D} \ominus w_0$
$w_{AD} = w^0_{A-C-D} \ominus w_0$
$w_{BC} = w^0_{B-C} \ominus w_0 = 0$!!!
$w_{BD} = w^0_\vee \ominus w_0$
$w_{CD} = w^0_{CD} \ominus w^0_C \ominus w^0_D \oplus w^0_{A-C-D}$
$w_{ABC} = w^0_\vee \ominus w_0$
$w_{ABD} = w^0_{ABD} \ominus w^0_\vee \ominus w^0_A \ominus w^0_B \ominus w^0_D \oplus 3w_0$ $\qquad (3w_0 = w_0 \oplus w_0 \oplus w_0)$
$w_{ACD} = w_0 \ominus w^0_{A-C-D}$
$w_{BCD} = w_0 \ominus w^0_\vee$
$w_{ABCD} = w^0_\vee \ominus w_0$

non stated weights of disjunctive rules w_{A-B-C}, w_{A-B-D}, w_{B-C-D}, w_{A-B}, w_{A-C}, w_{A-D}, w_{A-D}, w_{C-D} are equal to zero, i.e. rules are not included in Möbius transform.

We can easily verify that our present example correspond to lemmata from section 4. But, $w_{BC} = 0$ thus the rule $B\&C \Rightarrow H$ is not added into Möbius transform, even if his antecedent $B\&C$ is subconjunction of added conjunctive translation of antecedent of maximal disjunctive rule.

b) Lemmata 4.2 and 4.3 do not hold for weakly founded *ecd* knowledge bases. Let us consider the following weakly founded *ecd* knowledge base.

$\Rightarrow H(w_0)$
$A \vee B \vee C \Rightarrow H(w^0_{A-B-C})$
$A \vee D \Rightarrow H(w^0_{A-D})$
$B \Rightarrow H(w^0_B)$
$A\&C \Rightarrow H(w^0_{AC})$
$B\&D \Rightarrow H(w^0_{BD})$

Similarly as before, it is not necessary to estimate implicit weights of disjunctive rules.

Estimations of implicit weights:
$w^x_A = w^0_{A-B-C} \oplus w^0_{A-D} \ominus w_0$
$w^x_C = w^0_{A-B-C}$
$w^x_D = w^0_{A-D}$
$w^x_{AB} = w^0_B \oplus w^0_{A-D} \ominus w_0$
$w^x_{AD} = w^0_{A-D} \oplus w^0_{A-B-C} \ominus w_0$
$w^x_{BC} = w^0_B$
$w^x_{CD} = w^0_{A-D} \oplus w^0_{A-B-C} \ominus w_0$
$w^x_{ABC} = w^0_B \oplus w^0_{AC} \ominus w_0$
$w^x_{ABD} = w^0_{BD}$
$w^x_{ACD} = w^0_{AC}$
$w^x_{BCD} = w^0_{BD}$
$w^x_{ABCD} = w^0_{AC} \oplus w^0_{BD} \ominus w^0$

Möbied weights:
w_0
$w_{A-B-C} = w^0_{A-B-C} \ominus w_0$
$w_{A-D} = w^0_{A-D} \ominus w^0$
$w_A = 0$
$w_B = w^0_B \ominus w^0_{A-B-C}$
$w_C = 0$
$w_D = 0$
$w_{AB} = 0$
$w_{AC} = w^0_{AC} \ominus w^0_{A-B-C} \ominus w^0_{A-D} \oplus w_0$
$w_{AD} = 0$!!!
$w_{BC} = 0$
$w_{BD} = w^0_{BD} \ominus w^0_B \ominus w^0_{A-D} \oplus w_0$
$w_{CD} = 0$
$w_{ABC} = w^0_{A-B-C} \ominus w_0$
$w_{ABD} = 0$
$w_{ACD} = 0$
$w_{BCD} = 0$
$w_{ABCD} = w^0_{A-D} \ominus w_0$!!!

non stated weights of disjunctive rules are equal to zero again.

Rule $A\&B\&C\&D \Rightarrow H(w^0_{A-D} \ominus w_0)$ is addedd into Möbius transform, even if the antecedent of the rule is neither conjunctive translation of an antecedent of any disjunctive rule nor its subconjunction. And vice-versa, conjunctive translation of maximal disjunctive rule $A \vee D \Rightarrow H$ is not added.

c) Let us consider the previous knowledge base extended with rule $A \Rightarrow H(w^0_A)$.

$\Rightarrow H(w_0)$
$A \vee B \vee C \Rightarrow H(w^0_{A-B-C})$
$A \vee D \Rightarrow H(w^0_{A-D})$
$A \Rightarrow H(w^0_A)$
$B \Rightarrow H(w^0_B)$
$A\&C \Rightarrow H(w^0_{AC})$
$B\&D \Rightarrow H(w^0_{BD})$

Similarly as before, it is not necessary to estimate implicit weights of disjunctive rules.

Estimations of implicit weights:
$w^x_C = w^0_{A-B-C}$
$w^x_D = w^0_{A-D}$
$w^x_{AB} = w^0_A \oplus w^0_B \ominus w_0$
$w^x_{AD} = w^0_A$
$w^x_{BC} = w^0_B$
$w^x_{CD} = w^0_{A-D} \oplus w^0_{A-B-C} \ominus w_0$
$w^x_{ABC} = w^0_B \oplus w^0_{AC} \ominus w_0$
$w^x_{ABD} = w^0_A \oplus w^0_{BD} \ominus w_0$
$w^x_{ACD} = w^0_{AC}$
$w^x_{BCD} = w^0_{BD}$
$w^x_{ABCD} = w^0_{AC} \oplus w^0_{BD} \ominus w^0$

Möbied weights:
w_0
$w_{A-B-C} = w^0_{A-B-C} \ominus w_0$
$w_{A-D} = w^0_{A-D} \ominus w^0$
$w_A = w^0_A \ominus w^0_{A-B-C} \ominus w^0_{A-D} \oplus w_0$
$w_B = w^0_B \ominus w^0_{A-B-C}$
$w_{AB} = w^0_{A-B-C} \ominus w_0$
$w_{AC} = w^0_{AC} \ominus w^0_A$
$w_{BD} = w^0_{BD} \ominus w^0_B \ominus w^0_{A-D} \oplus w_0$
$w_{ABC} = 0$!!!
$w_{ABD} = w^0_{A-D} \ominus w_0$!!!
$w_{ABCD} = 0$!

$w_C = w_D = w_{AD} = w_{BC} = w_{CD} = w_{ACD} = w_{BCD} = 0$,
non stated weights of disjunctive rules are also equal to zero.

Rule $A\&B\&D \Rightarrow H(w^0_{A-D} \ominus w_0)$ is addedd into Möbius transform even if the antecedent of the rule is neither conjunctive translation of an antecedent of any disjunctive rule nor its subconjunction. And vice-versa, conjunctive translation of maximal disjunctive rule $A \vee D \Rightarrow H$ is not added.

8 APPENDIX E

a) Let us consider the following weakly founded *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_{A-B})$
$A \vee C \Rightarrow H(w^0_{A-C})$
$B \vee C \Rightarrow H(w^0_{B-C})$
$A \& F \Rightarrow H(w^0_{AF})$
$B \& C \Rightarrow H(w^0_{BC})$
$D \& E \Rightarrow H(w^0_{DE})$

Estimations of implicit weights:

$w^x_{ABCF} = w^0_{AF} \oplus w^0_{BC} \ominus w_0$
$w^x_{BCDE} = w^0_{BC} \oplus w^0_{DE} \ominus w_0$
$w^x_{ADEF} = w^0_{AF} \oplus w^0_{DE} \ominus w_0$
$w^x_{ABCDEF} = w^0_{AF} \oplus w^0_{BC} \oplus w^0_{DE} \ominus 2w_0$

Möbied weights:

w_0
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{C-D} = w^0_{C-D} \ominus w_0$
$w_{E-F} = w^0_{E-F} \ominus w_0$
$w_{AF} = w^0_{AF} \ominus w^0_{A-B} \ominus w^0_{E-F} \oplus w_0$
$w_{BC} = w^0_{BC} \ominus w^0_{A-B} \ominus w^0_{C-D} \oplus w_0$
$w_{DE} = w^0_{DE} \ominus w^0_{C-D} \ominus w^0_{E-F} \oplus w_0$
$w_{ABCF} = w^0_{A-B} \ominus w_0$
$w_{BCDE} = w^0_{C-D} \ominus w_0$
$w_{ADEF} = w^0_{E-F} \ominus w_0$
$w_{ABCDEF} = 0$

Other Möbied weights are equal to zero. w^{x}' are presented (and computed as well) only for rule with presented resulting Möbied weights.

b) Let us consider the following modification of previous *ecd* knowledge base Θ':

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_{A-B})$
$A \vee C \Rightarrow H(w^0_{A-C})$
$B \vee C \Rightarrow H(w^0_{B-C})$
$A \& F \Rightarrow H(w^0_{AF})$
$B \& C \Rightarrow H(w^0_{BC})$
$D \& E \Rightarrow H(w^0_{DE})$
$B \& C \& D \& E \Rightarrow H(w^0_{BCDE})$

Estimations of implicit weights:

$w^x_{ABCF} = w^0_{AF} \oplus w^0_{BC} \ominus w_0$
$w^x_{ADEF} = w^0_{AF} \oplus w^0_{DE} \ominus w_0$
$w^x_{ABCDEF} = w^0_{AF} \oplus w^0_{BCDE} \ominus w_0$

Möbied weights:

w_0
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{C-D} = w^0_{C-D} \ominus w_0$
$w_{E-F} = w^0_{E-F} \ominus w_0$
$w_{AF} = w^0_{AF} \ominus w^0_{A-B} \ominus w^0_{E-F} \oplus w_0$
$w_{BC} = w^0_{BC} \ominus w^0_{A-B} \ominus w^0_{C-D} \oplus w_0$
$w_{DE} = w^0_{DE} \ominus w^0_{C-D} \ominus w^0_{E-F} \oplus w_0$
$w_{ABCF} = w^0_{A-B} \ominus w_0$
$w_{BCDE} = w^0_{BCDE} \ominus w^0_{BC} \ominus w^{DE} \oplus w^0_{C-D}$
$w_{ADEF} = w^0_{E-F} \ominus w_0$
$w_{ABCDEF} = 0$

Other Möbied weights are equal to zero. w^{x}' are presented (and computed as well) only for rule with presented resulting Möbied weights.

9 APPENDIX F

a) Let us consider the following general *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$X \vee Y \vee A \Rightarrow H(w^0_{X-Y-A})$
$X \vee Y \Rightarrow H(w^0_{X-Y})$
$A \vee B \Rightarrow H(w^0_{A-B})$
$A\&B \Rightarrow H(w^0_{AB})$
$A\&C \Rightarrow H(w^0_{AC})$
$A\&D \Rightarrow H(w^0_{AD})$
$B\&D \Rightarrow H(w^0_{BD})$

Estimations of implicit weights:

$w^x_{ABC} = w^0_{AB} \oplus w^0_{AC} \ominus w_0$
$w^x_{ABD} = w^0_{AB} \oplus w^0_{AD} \oplus w^0_{BD} \ominus 2w_0$
$w^x_{ACD} = w^0_{AC} \oplus w^0_{AD} \ominus w_0$
$w^x_{ABCD} = w^0_{AB} \oplus w^0_{AC} \oplus w^0_{AD} \oplus w^0_{BD} \ominus 3w_0$
$w^x_{XAB} = w^x_{YAB} = w^x_{XYAB} = w^0_{AB} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XAC} = w^x_{YAC} = w^x_{XYAC} = w^0_{AC} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XAD} = w^x_{YAD} = w^x_{XYAD} = w^0_{AD} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XABC} = w^x_{YABC} = w^x_{XYABC} = w^0_{AB} \oplus w^0_{AC} \oplus w^0_{X-Y} \ominus 2w_0$
$w^x_{XACD} = w^x_{YACD} = w^x_{XYACD} = w^0_{AC} \oplus w^{AD} \oplus w^0_{X-Y} \ominus 2w_0$
$w^x_{XABD} = w^x_{YABD} = w^x_{XYABD} = w^0_{AB} \oplus w^0_{AD} \oplus w^0_{BD} \oplus w^0_{X-Y} \ominus 3w_0$
$w^x_{XABCD} = w^x_{YABCD} = w^x_{XYABCD} = w^0_{AB} \oplus w^0_{AC} \oplus w^{AD} \oplus w^0_{BD} \oplus w^0_{X-Y} \ominus 4w_0$

w^{x}' were computed only for rules which may be added within Möbius transformation of knowledge base.

Möbied weights:

w_0
$w_{X-Y-A} = w^0_{X-Y-A} \ominus w_0$
$w_{X-Y} = w^0_{X-Y} \ominus w^0_{X-Y-A}$
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{AB} = w^0_{AB} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{AC} = w^0_{AC} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{AD} = w^0_{AD} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{BD} = w^0_{BD} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{ABC} = w^0_{A-B} \oplus w^0_{X-Y-A} \ominus 2w_0$
$w_{ACD} = w^0_{A-B} \oplus w^0_{X-Y-A} \ominus 2w_0$
$w_{ABD} = 2w^0_{A-B} \oplus w^0_{X-Y-A} \ominus 3w_0$
$w_{ABCD} = -w^0_{A-B} \ominus w^0_{X-Y-A} \oplus 2w_0$
$w_{XAB} = w_{XAC} = w_{XAD} = w_{YAB} = w_{YAC} = w_{YAD} = w^0_{X-Y-A} \ominus w_0$
$w_{XYAB} = w_{XYAC} = w_{XYAD} = -w^0_{X-Y-A} \oplus w_0$
$w_{XABC} = w_{XABD} = w_{XACD} = w_{YABC} = w_{YABD} = w_{YACD} = -w^0_{X-Y-A} \oplus w_0$
$w_{XYABC} = w_{XYABD} = w_{XYACD} = w^0_{X-Y-A} \ominus w_0$
$(w_{XBCD} = w_{YBCD} = w_{XYBCD} = 0)$
$w_{XABCD} = w_{YABCD} = w^0_{X-Y-A} \ominus w_0$
$w_{XYABCD} = -w^0_{X-Y-A} \oplus w_0$

b) Let us consider the following modification of previous *ecd* knowledge base Θ':

$\Rightarrow H(w_0)$
$X \vee Y \vee A \Rightarrow H(w^0_{X-Y-A})$
$X \vee Y \Rightarrow H(w^0_{X-Y})$
$A \vee B \Rightarrow H(w^0_{A-B})$
$A \& B \Rightarrow H(w^0_{AB})$
$A \& C \Rightarrow H(w^0_{AC})$
$A \& D \Rightarrow H(w^0_{AD})$
$B \& D \Rightarrow H(w^0_{BD})$
$A \& B \& C \Rightarrow H(w^0_{ABC})$
$A \& B \& D \Rightarrow H(w^0_{ABD})$
$B \& C \& D \Rightarrow H(w^0_{BCD})$

Estimations of implicit weights:

$w^x_{ACD} = w^0_{AC} \oplus w^0_{AD} \ominus w_0$
$w^x_{ABCD} = w^0_{ABC} \oplus w^0_{ABD} \oplus w^0_{BCD} \ominus w^0_{AB} \ominus w_0$
$w^x_{XAB} = w^x_{YAB} = w^x_{XYAB} = w^0_{AB} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XAC} = w^x_{YAC} = w^x_{XYAC} = w^0_{AC} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XAD} = w^x_{YAD} = w^x_{XYAD} = w^0_{AD} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XABC} = w^x_{YABC} = w^x_{XYABC} = w^0_{ABC} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XACD} = w^x_{YACD} = w^x_{XYACD} = w^0_{AC} \oplus w^{AD} \oplus w^0_{X-Y} \ominus 2w_0$
$w^x_{XABD} = w^x_{YABD} = w^x_{XYABD} = w^0_{ABD} \oplus w^0_{X-Y} \ominus w_0$
$w^x_{XABCD} = w^x_{YABCD} = w^x_{XYABCD} = w^0_{ABC} \oplus w^0_{ABD} \oplus w^0_{BCD} \ominus w^0_{AB} \oplus w^0_{X-Y} \ominus 2w_0$

w^{x}' were computed only for rules which may be added within Möbius transformation of knowledge base.

Möbied weights:

w_0
$w_{X-Y-A} = w^0_{X-Y-A} \ominus w_0$
$w_{X-Y} = w^0_{X-Y} \ominus w^0_{X-Y-A}$
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{AB} = w^0_{AB} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{AC} = w^0_{AC} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{AD} = w^0_{AD} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{BD} = w^0_{BD} \ominus w^0_{A-B} \ominus w^0_{X-Y-A} \oplus w_0$
$w_{ABC} = w^0_{ABC} \ominus w^0_{AB} \ominus w^0_{AC} \oplus w^0_{A-B} \oplus w^0_{X-Y-A} \ominus w_0$
$w_{ABD} = w^0_{ABD} \ominus w^0_{AB} \ominus w^0_{AD} \oplus w^0_{A-B} \oplus w^0_{X-Y-A} \ominus w_0$
$w_{BCD} = w^0_{BCD} \ominus w^0_{A-B}$
$w_{ACD} = w^0_{A-B} \oplus w^0_{X-Y-A} \ominus 2w_0$
$w_{ABCD} = -w^0_{X-Y-A} \oplus w_0$
$w_{XAB} = w_{XAC} = w_{XAD} = w_{YAB} = w_{YAC} = w_{YAD} = w^0_{X-Y-A} \ominus w_0$
$w_{XYAB} = w_{XYAC} = w_{XYAD} = -w^0_{X-Y-A} \oplus w_0$
$w_{XABC} = w_{XABD} = w_{XACD} = w_{YABC} = w_{YABD} = w_{YACD} = -w^0_{X-Y-A} \oplus w_0$
$w_{XYABC} = w_{XYABD} = w_{XYACD} = w^0_{X-Y-A} \ominus w_0$
$(w_{XBCD} = w_{YBCD} = w_{XYBCD} = 0)$
$w_{XABCD} = w_{YABCD} = w^0_{X-Y-A} \ominus w_0$
$w_{XYABCD} = -w^0_{X-Y-A} \oplus w_0$