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A Generalization of Möbius Transformation for Knowledge Bases, which Include Rules with Disjunction in Antecedent

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Abstract

Möbius transformation is an important tool for establishing weights of rules of compositional expert systems from conditional weights.

In this report, an applicability of Möbius transformation of rule bases is extended also to knowledge bases with elementary disjunctions in antecedents of rules. The paper contains an existence theorem, an algorithm of the transformation and some open problems which tend to maximal generality as well.

Keywords
Möbius Transformation, Expert system, Knowledge Base, Uncertainty, Weight of rule

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1 Introduction

The first ideas, on how to establish weights of rules of compositional expert systems from conditional weights related to real evidence, data, experience, were published in 1984 [4]. For a full description of Möbius transformation see e.g. [5], [6].

The possibility of utilization of Möbius transformation is not only restricted to MYCIN-like systems, it is important also for a common generalization of MYCIN-like systems and fuzzy expert systems which use a composition of fuzzy relations like Conorm-CADIAG-2 extended with the handling of negative knowledge, see [3]. The system is derived from the fuzzy expert system CADIAG-2 [1].

The original Möbius transformation is formulated and only used for rules of a special form. The present work generalizes it for a wider class of rules.

Necessary preliminaries are introduced and the original Möbius transformation theorem for MYCIN-like systems is stated in the second section.

Section 3 describes ideas on how to extend the field of applicability of Möbius transformation to rules with elementary disjunctions in antecedents also. Some principal problems are shown. It is suggested how they can be, more or less, overcome. The existence theorem is stated in the end of the section. In section 4, the possibilities of the simplification of Möbius transformation obtained by simple minded algorithm and tools for improvement of the algorithm are introduced. Section 5 brings an improved algorithm for founded knowledge bases, which include elementary conjunctions and disjunctions in antecedents of rules.

In section 6 is a comparison of Möbius transformation for MYCIN-like systems and of an introduced generalization. After it follows conclusions and ideas for future work.

2 Möbius transformation

2.1 Preliminaries

We shall consider, in this paper, *low knowledge bases*, i.e. there are no intermediate propositions, there are only questions (symptoms) and goals (hypotheses, diagnoses) in knowledge bases. In this section, let us suppose rules $A \Rightarrow S(w)$, where antecedent A is an elementary conjunction of questions and S is a goal. *An elementary conjunction* (of questions) is a conjunction of *literals* (of questions), i.e. questions or their negations, where every question has at most one occurrence in the elementary conjunction. Let weights be from interval $[-1, 1]$, and let contributions (effects) of different rules to the same succedents be summarized by a group operation $\oplus$ on $[-1, 1]$. Three-valued questionnaire q is a mapping of questions into the set $\{-1, 0, 1\}$, i.e. there are only answers $\{-1, 0, 1\}$ (i.e. No, I don't know, Yes). Each questionnaire of this kind can be represented by an elementary conjunction (positive literals for 1, negative ones for -1, and no literals for 0). All of the above terminology correspond with monography [5].

An *elementary disjunction* is a disjunction of literals. An *ecd knowledge base* (elementary-conjunction-disjunction) is a knowledge base such that antecedents of rules are either elementary conjunctions or elementary disjunctions.

Further we shall use the following terminology. A rule $R: \ A \Rightarrow S(w)$ is a *simple rule* if its antecedent A is a literal, R is *conjunctive/disjunctive rule* if A is a conjunction/disjunction, *maximal conjunctive/disjunctive rule* if there is no rule $B \Rightarrow S(w_B)$ in the knowledge base, so that A is a subconjunction/subdisjunction of B. A conjunction $Conj = A\&B\&...\&K$ is a conjunctive translation of a disjunction $Disj = A \vee B \vee ... \vee K$, a rule $Conj \Rightarrow H$ is a conjunctive translation of the disjunctive rule $Disj \Rightarrow H$.

An *ecd* knowledge base Θ is *founded* if it contains rules $A \Rightarrow H$ for every literal A which is involved in some disjunctive rule $A \vee Disj \Rightarrow H$ for any elementary disjunction $Disj$. An *ecd* knowledge base Θ is *weakly founded* if for every literal A from any disjunctive rule $A \vee Disj \Rightarrow H$ there is a simple rule $A \Rightarrow H$ or conjunctive rule $A\&Conj \Rightarrow H$ (for some $Conj$) included in Θ.
Notice, that every knowledge base without disjunction in antecedents is founded.

A literal A is founded for hypothesis H, if the rule $A \Rightarrow H$ is included in the knowledge base. A literal A is weakly founded for hypothesis H, if rule $A\&Conj \Rightarrow H$ is included in the knowledge base for some conjunction literals $Conj$.
A rule $A_1\&A_2\&...\&A_n \Rightarrow H$ (resp. a rule $A_1 \vee A_2 \vee ... \vee A_n \Rightarrow H$) is founded if every A_i is founded for the hypothesis H. A rule $A_1\&A_2\&...\&A_n \Rightarrow H$ (resp. $A_1 \vee A_2 \vee ... \vee A_n \Rightarrow H$) is weakly founded if for every A_i exist At such that $A_i \subseteq At \subseteq A_1\&A_2\&...\&A_n$ and rule $At \Rightarrow H$ is included in the knowledge base.

2.2 Möbius transformation theorem

Let us denote by $\beta(H|E)$ a conditional expert belief that hypothesis H is valid if just evidence E is known. For Möbius transformation, we shall suppose that given (original) weight of rule $R: A \Rightarrow S(w)$ is conditional, i.e. an expert's belief that the weight of S is w provided just A holds ($w = \beta(S|A)$).

Weights of rules are transformed within Möbius transformation; to distinguish them, we shall denote original (i.e. given, source, conditional) weight of rule R as w^0_R (or $w^0_{S,A}$), while Möbied weight i.e. weight after transformation as w_R (or $w_{S,A}$), we will also use w^0_A and w_A if the succedent of a rule is clear from context.

We say, that a set of rules is *weakly sound* if for every two rules such that $Ant_1 \subset Ant_2$ (Ant_1 is a subconjunction of Ant_2 or Ant_2 implies Ant_1) holds: if $w^0_{Ant_1} = 1$, then $w^0_{Ant_2} = 1$.

We say, that a low knowledge base is weakly sound if its set of rules is weakly sound.

Theorem 2.1 *Let β be a weakly sound set of rules such that $w^0_{H,E} = \beta(H|E)$. Then there exists a weighting of rules which forms a knowledge base Θ of MYCIN-like expert system, such that for any three-valued questionaire E_q and hypothesis H for which $\beta(H|E_q)$ is defined, it holds*

$$W_\Theta(H|E_q) = \beta(H|E_q),$$

where $W_\Theta(H|E_q)$ is a global weight of hypothesis H given by E_q,
$W_\Theta(H|E_q) = \oplus\{\Theta(H|E')|E' \subseteq E_q\}$.

The new knowledge base Θ is called *Möbius transform* of the source rule base β. For particularities see [5], [6].

Note: There is no limitation to questionnaire values (to possible answers of a user) for Möbius transform of a rule base existence. But, the equation $W_\Theta(H|E_q) = \beta(H|E_q)$ only makes sense for three-valued questionnaires.

3 Including a disjunction into Möbius transformation

3.1 The idea of Möbius transformation

First have a look at the principal idea of Möbius transformation for MYCIN-like systems. We suppose conditional rules, where their weights rely on the expert's belief that the succedent holds if the antecedent of the rule is true. Let us have the following rules

$A \Rightarrow H(w_1)$,
$B \Rightarrow H(w_2)$,
$A\&B \Rightarrow H(w_3)$.

Thus if both of A and B are true we want to infer the conditional weight w_3 (a belief that H given $A\&B$) as a result, while MYCIN-like system infers $w_1 \oplus w_2 \oplus w_3$. It is trivial that $w_1 \oplus w_2 \oplus w_3 \neq w_3$ in general, w_3 may be greater, equal or less than $w_1 \oplus w_2$. In the case $w_3 = w_1 \oplus w_2$, the third rule is redundant and so we remove it from the knowledge base. Otherwise, we make a transformation of weight w_3 of rule $A\&B \Rightarrow H$ to $w = w_3 \ominus (w_1 \oplus w_2)$, thus the resulting Möbied weight w is positive if $w_3 > w_1 \oplus w_2$ or negative if $w_3 < (w_1 \oplus w_2)$ (a positive or negative effect of the rule — support/unsupport of hypothesis), especially, $w = 0$ for $w_3 = w_1 \oplus w_2$ (the rule is redundant as before).

We can easily verify that we obtain expected results: for A we get w_1, for B we get w_3, and finally, for A, B we get $w_1 \oplus w_2 \oplus (w_3 \ominus (w_1 \oplus w_2)) = w_3$.

3.2 The first attempt to include a disjunction

Now, we shall try to apply this simple idea to *ecd* knowledge bases, i.e. knowledge bases in which antecedents can be a conjunction or a disjunction of literals (propositions or their negations), i.e. an *elementary conjunction* or an *elementary disjunction*. A very simple example follows:

$A \Rightarrow H(w_1)$, (*)
$B \Rightarrow H(w_2)$,
$A \vee B \Rightarrow H(w_3)$.

For A we want a resulting weight w_1 instead of $w_1 \oplus w_3$, thus we change w_1 with $w_A = w_1 \ominus w_3$, and analogically, $w_B = w_2 \ominus w_3$. If we know that $A \vee B$ is true and we are not able to specify whether A or B or $A\&B$, then the third rule is the only one which fires, and we keep its weight w_3.

After this transformation, we really get w_1 for A, we get w_2 for B, but for A, B, we obtain $w_1 \ominus w_3 \oplus w_2 \ominus w_3 \oplus w_3 = w_1 \oplus w_2 \ominus w_3$ which is not an assumed value $w_1 \oplus w_2$ (It is assumed because there is no rule $A\&B \Rightarrow H$. We shall discuss this assumption later on).

Let us formulate our problem more precisely, we get a set of equations, where w_A, w_B, and w are modifications of weights w_1, w_2, and w_3, respectively:
$w = w_3$
$w_A \oplus w = w_1$
$w_B \oplus w = w_2$
$w_A \oplus w_B \oplus w = w_1 \oplus w_2$.
Thus we get $(w_1 \ominus w) \oplus (w_2 \ominus w) \oplus w = w_1 \oplus w_2$, hence $w = 0$. The system of equations has the only solution $w_3 = w = 0$, which only describes and admits the situation without the rule with disjunction.

There are two possibilities of how to overcome the problem, first, to express the rule $A \vee B \Rightarrow H(w_3)$ in another way without a disjunction in the antecedent, i.e. preliminary modification of the knowledge base before application of Möbius transformation, or second, to modify our approach of understanding disjunctive rules.

3.3 Rewriting of disjunction

The easiest way of rewriting a disjunction as $A \vee B = \neg(\neg A \& \neg B)$ is inacceptable, because $\neg(\neg A \& \neg B)$ is not an elementary conjunction. Syntactically we can rewrite $A \vee B \Rightarrow H(w)$ with a couple of rules $\neg A \& \neg B \Rightarrow C(1)$ and $\neg C \Rightarrow H(w)$, but not from the semantic point of view, because a contribution of the original rule is positive for $A > 0$ or $B > 0$, while the joint contribution of the new rules is always 0. Thus we have to rewrite the original rule as three rules $A \Rightarrow D(1), B \Rightarrow D(1), D \Rightarrow H(w)$. There exists conditions under which the three new rules are equivalent to the original one, e.g. if $q(A) \geq w$ & $q(B) \geq w$ or $q(A) \leq 0$ or $q(B) \leq 0$.

The substitution is not equivalent to the original rule in general, but it is fully correct in the case of a three-valued questionnaire for which Möbius transformation was constructed.

Proof. For three-valued questionnaire we have $W(D|q) = |q(A)| \oplus |q(B)|$, $W(D|q) = 0$ for $q(A) \leq 0$ & $q(B) \leq 0$, and $W(D|q) = 1$ otherwise. Thus, for $w \geq 0$ we get $W(H|q) = min(W(A\&B|q), w) = min(max(q(A), q(B)), w) = min(W(D|q), w)$, and similarly $W(H|q) = -min(W(A\&B|q), -w) = -min(max(q(A), q(B)), -w) = -min(W(D|q), -w)$ for $w < 0$.

Hence we can rewrite the set of rules (*) as:
$A \Rightarrow H(w_1)$,
$B \Rightarrow H(w_2)$,
$A \Rightarrow D(1)$,
$B \Rightarrow D(1)$,
$D \Rightarrow H(w_3)$.
This set of rules contains an intermediate proposition D, so another modification of it is necessary before Möbius transformation, otherwise it would be necessary to formulate Möbius transformation also for knowledge bases with intermediate propositions.

As far as D is concerned as a new question, it is necessary to somehow express its connection (dependence) to A and to B not to cumulate weights too much. I.e. it would be requested another modification of knowledge base before Möbius transformation again.

Thus, non of the stated substitutions is really convenient to our purpose.

3.4 What does a disjunction rule mean?

Now, we shall turn our attention, to a better understanding of rule $A \vee B \Rightarrow H(w)$. What does the rule mean? How do we understand $A \vee B$?

If $A \vee B$ holds it means that either we want and we can distinguish one of the following possibilities: only A holds, only B holds, both A and B hold or we cannot distinguish or we don't like to distinguish them. Thus we can rewrite (*) as

$A \Rightarrow H(w_1)$,
$B \Rightarrow H(w_2)$,
$(A \vee B) \,\&\, A \Rightarrow H(w_4)$,
$(A \vee B) \,\&\, B \Rightarrow H(w_5)$,
$(A \vee B) \,\&\, (A \,\&\, B) \Rightarrow H(w_6)$,
$(A \vee B) \,\&\, (A \vee B) \Rightarrow H(w_3)$.

The 3-rd and 4-th rules are copies of the first and second ones, hence we can remove them. We can simplify the antecedent of the last two rules, thus all of the original rules remain in the knowledge base and there is only the new one $A\&B \Rightarrow H(w_6)$. To get an expected Möbius transform of the knowledge base we have to put $w_6 = w_1 \oplus w_2$, because there is no other more precise specification for $A\&B$ given by an expert. So we can rewrite our knowledge base in the following way:

$A \Rightarrow H(w_A^0)$,
$B \Rightarrow H(w_B^0)$,
$A \,\&\, B \Rightarrow H(w_A^0 \oplus w_B^0)$,
$A \vee B \Rightarrow H(w_\vee^0)$.

Antecedents of rules are elementary conjunctions or elementary disjunctions again.

Now, we can apply the idea of Möbius transformation to our modified knowledge base, hence we get

$A \Rightarrow H(w_A^0 \ominus w_\vee^0)$,
$B \Rightarrow H(w_B^0 \ominus w_\vee^0)$,
$A \,\&\, B \Rightarrow H(w_\vee^0)$,
$A \vee B \Rightarrow H(w_\vee^0)$,

($\vee$, & are used in indices as abbreviations of $A \vee B$ and $A\&B$)
($w_\& = w_\&^0 \ominus (w_A \oplus w_B \oplus w_\vee) = w_A^0 \oplus w_B^0 \ominus (w_A^0 \ominus w_\vee^0 \oplus w_B \ominus w_\vee^0 \oplus w_\vee^0) = w_\vee^0$)

We can easily verify that if only A holds, then we get w_A^0. Similarly, if only B holds, then we get w_B^0. If both A and B hold we get $w_A^0 \oplus w_B^0$. And finally, if we know only that $A \vee B$ holds, then we get $w_\vee^0$.

Similarly, if we consider an apriori weight w_0 of hypothesis H we get the following transformed knowledge base:

$\Rightarrow H(w_0)$,
$A \Rightarrow H(w_A^0 \ominus w_\vee^0)$,
$B \Rightarrow H(w_B^0 \ominus w_\vee^0)$,
$A \,\&\, B \Rightarrow H(w_\vee^0)$,

$A \vee B \Rightarrow H(w_\vee^0 \ominus w_0)$.

We have succeeded in the first trivial example of Möbius transformation for rules with a disjunction in the antecedent. Now, let us consider following a more complicated, yet still a simple example of a knowledge base:

$\Rightarrow H(w_0)$, (**)
$A \Rightarrow H(w_A^0)$,
$B \Rightarrow H(w_B^0)$,
$C \Rightarrow H(w_C^0)$,
$A \vee B \Rightarrow H(w_{A\vee B}^0)$,
$A \vee C \Rightarrow H(w_{A\vee C}^0)$,
$A \vee B \vee C \Rightarrow H(w_\vee^0)$.

From now on, $\vee$ which is used as index means an abbreviation of disjunction of all the literals used, here $A \vee B \vee C$, i.e. $w_\vee = w_{A-B-C}$.

In the present example we have disjunctions $A \vee B$, $A \vee C$, and $A \vee B \vee C$. According to our interpretation of them we obtain five new rules with antecedents $A\&B\&C$, $A\&B$, $A\&C$, $B\&C$, $B \vee C$. We are looking for a "Möbiable" knowledge base as close to the original one as possible.

We try, at first (a), to only add rule $A\&B\&C \Rightarrow H$ (the rule with a maximal conjunction in the antecedent), and second (b), also rules $A\&B \Rightarrow H$ and $A\&C \Rightarrow H$ (conjunctive translations of the original ones). Similarly as in the previous example (*), there is stated no other specification of weights for conjunctive rules, thus we analogically suppose, that missing source weights are implicitly given as follows: $w_{AB}^0 = w_A^0 \oplus w_B^0 \ominus w_0$, $w_{AC}^0 = w_A^0 \oplus w_C^0 \ominus w_0$, $w_{ABC}^0 = w_A^0 \oplus w_B^0 \oplus w_C^0 \ominus w_0 \ominus w_0$. Nevertheless, both of these attempts are unsuccessful, resulting weights obtained after inference according to transformed knowledge bases are different from our expectations, see appendix A.

In this case it is sufficient (c) to add four rules $A\&B\&C \Rightarrow H$, $A\&B \Rightarrow H$, $A\&C \Rightarrow H$, and $B\&C \Rightarrow H$, see appendix A again. It is not necessary to add a rule with antecedent $B \vee C$. We obtain the following Möbius transform of the knowledge base (**):

$\Rightarrow H(w_0)$,
$A \Rightarrow H(w_A^0 \ominus w_{A-B}^0 \ominus w_{A-C}^0 \oplus w_{A-B-C}^0)$,
$B \Rightarrow H(w_B^0) \ominus w_{A-B}^0$,
$C \Rightarrow H(w_C^0) \ominus w_{A-C}^0$,
$A \vee B \Rightarrow H(w_{A\vee B}^0 \ominus w_\vee^0)$,
$A \vee C \Rightarrow H(w_{A\vee C}^0 \ominus w_\vee^0)$,
$A \vee B \vee C \Rightarrow H(w_\vee^0 \ominus w_0)$,
$A\&B \Rightarrow H(w_{A-B}^0 \ominus w_0)$,
$A\&C \Rightarrow H(w_{A-C}^0 \ominus w_0)$,
$A\&C \Rightarrow H(w_{A-B-C}^0 \ominus w_0)$,
$A\&B\&C \Rightarrow H(w_0 \ominus w_{A-B-C}^0)$.

We can also present another example. Let us have the following simple knowledge base:

$\Rightarrow H(w_0)$, (***)
$A \Rightarrow H(w_A^0)$,

$B \Rightarrow H(w_B^0)$,
$A \vee B \vee C \Rightarrow H(w_\vee^0)$.
We get the following Möbius transform:
$\Rightarrow H(w_0)$,
$A \vee B \vee C \Rightarrow H(w_\vee^0 \ominus v_0)$,
$A \Rightarrow H(w_A^0 \ominus w_\vee^0)$,
$B \Rightarrow H(w_B^0 \ominus w_\vee^0)$,
$A \,\&\, B \Rightarrow H(w_{A-B-C}^0 \ominus w_0)$.
Here, we suppose w_C^0 to be equal to $w_\vee^0$ (we have not another more precise specification), and w_{AC}^0 to be equal to w_A^0 (we have w_A^0 and knowledge of validity of C expresses, here, nothing more, because contribution of rule $A \vee B \vee C \Rightarrow H$ should be somehow included in w_A^0). Similarly we suppose $w_{BC}^0 = w_B^0$ and $w_{ABC}^0 = w_A^0 \oplus w_B^0 \ominus w_0$.

If we add a rule $C \Rightarrow H(w_C^0)$, we get the following knowledge base and its Möbius transform:
$\Rightarrow H(w_0)$,
$A \Rightarrow H(w_A^0)$,
$B \Rightarrow H(w_B^0)$,
$C \Rightarrow H(w_C^0)$,
$A \vee B \vee C \Rightarrow H(w_\vee^0)$;

$\Rightarrow H(w_0)$,
$A \vee B \vee C \Rightarrow H(w_\vee^0 \ominus v_0)$,
$A \Rightarrow H(w_A^0 \ominus w_\vee^0)$,
$B \Rightarrow H(w_B^0 \ominus w_\vee^0)$,
$C \Rightarrow H(w_C^0 \ominus w_\vee^0)$,
$A \,\&\, B \Rightarrow H(w_\vee^0 \ominus w_0)$,
$A \,\&\, C \Rightarrow H(w_\vee^0 \ominus w_0)$,
$B \,\&\, C \Rightarrow H(w_\vee^0 \ominus w_0)$,
$A \,\&\, B \,\&\, C \Rightarrow H(w_0 \ominus w_\vee^0)$.
The addition of the only rule into the source knowledge base has caused, here, the addition of three rules into its Möbius transform.

Until now, there is no explanation of which rules should be added to the new knowledge base and which ones should not, and so we shall look for it later.

We have illustrated in the above examples that Möbius transforms of the presented knowledge bases exist and so, it makes sense to speak of Möbius transformation of rule bases with an elementary disjunction in the antecedents. Moreover, we can say the following:

Lemma 3.1 *Let Θ be a weakly sound low ecd knowledge base. If we can explicitly set or estimate implicit weights also for nonincluded combinations of literals, then Möbius transform of the knowledge base Θ exists.*

Note: weak soundness conditions in the present situation is as follows,
for every two rules such that $Ant_1 \subset Ant_2$ holds: if $w_{Ant_1}^0 = 1$, then $w_{Ant_2}^0 = 1$, where $Ant_1 \subset Ant_2$ means Ant_2 implies Ant_1, (i.e. $Conj_1$ is a subconjunction of $Conj_2$

or $Disj_2$ is a subdisjunction of $Disj_1$ or a subdisjunction of $Disj_1$ exists which is a subconjunction of $Conj_2$).

Proof. Let us show a simple idea of a construction of this Möbius transform. We have seen that maybe it is necessary to add some rules during transformation. So we have computed transformed (Möbied) weights of all possible rules.

Let us take all the elementary disjunctions from the longest to one-element ones and for every disjunction $Disj$ compute $\oplus$-combination c of all applicable rules provided just $Disj$ holds. Put $w_{Disj} = w^0_{Disj} \ominus c$. If rule $Disj \Rightarrow H$ exists and $w_{Disj} \neq w^0_{Disj}$, then rewrite the weight of the rule, if $w_{Disj} \neq 0$ and the rule does not exist, then add the rule $Disj \Rightarrow H(w_{Disj})$ into the knowledge base.

Let us go analogically through all elementary conjunctions from one-element to the longest possible one.

This construction is very simple, nevertheless, it is possible to show that the resulting transformed knowledge base is Möbius transform of the source knowledge base Θ.

At this moment, we know how to construct, in a simple yet noneffective way, Möbius transformation. So, it is logical to look for its improvement. A decision-making of whether a new possible rule will be added or not depends on the Möbied weight of the possible rule. Therefore we need a more sophisticated way of computing these weights. In the next subsection it is shown how to compute Möbied weights of rules in knowledge bases in which all possible elementary conjunctions and elementary disjunctions form antecedents of rules with the same succedent.

3.5 Formulas for computing of Möbied weights

Generally, for three questions/symptoms, we have the following knowledge base[2] :

$\Rightarrow H(w_0)$,
$A \Rightarrow H(w^0_A)$,
$B \Rightarrow H(w^0_B)$,
$C \Rightarrow H(w^0_C)$,
$A \,\&\, B \Rightarrow H(w^0_{AB})$,
$A \,\&\, C \Rightarrow H(w^0_{AC})$,
$B \,\&\, C \Rightarrow H(w^0_{BC})$,
$A \,\&\, B \,\&\, C \Rightarrow H(w^0_{ABC})$,
$A \vee B \Rightarrow H(w^0_{A-B})$,
$A \vee C \Rightarrow H(w^0_{A-C})$,
$B \vee C \Rightarrow H(w^0_{B-C})$,
$A \vee B \vee C \Rightarrow H(w^0_{\vee})$.

By the recomputation of weights keeping the original principal idea of Möbius transformation, we obtain the following Möbied weights of rules:

$w_0 = w^0_0$

$w_\vee = w^0_\vee \ominus w_0$

[2]To be precise for three questions it should be a more complicated knowledge base, the presented one corresponds to three literals of three different questions.

$$w_{A-B} = w^0_{A-B} \ominus w_\vee \ominus w_0 = w^0_{A-B} \ominus w^0_\vee$$

$$w_A = w^0_A \ominus w_{A-B} \ominus w_{A-C} \ominus w_\vee \ominus w_0 = w^0_A \ominus w^0_{A-B} \ominus w^0_{A-C} \oplus w^0_\vee$$

$$w_{AB} = w^0_{AB} \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0$$

$$= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B}$$

$$w_{ABC} = w^0_{ABC} \ominus w_{AB} \ominus w_{AC} \ominus w_{BC} \ominus w_A \ominus w_B \ominus w_C \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0$$

$$= w^0_{ABC} \ominus w^0_{AB} \ominus w^0_{AC} \ominus w^0_{BC} \oplus w^0_A \oplus w^0_B \oplus w^0_C \ominus w^0_\vee$$

Möbied weights w_{A-C}, w_{B-C}, w_B, w_C, w_{AC}, and w_{BC} are computed analogically. A complete derivation of the formulas is presented in appendix B.

By using these formulas, we can perform Möbius transformation of the source knowledge base, and we shall obtain the following Möbius transform:

$$\begin{aligned}
&\Rightarrow H(w^0_0),\\
&A \Rightarrow H(w^0_A \ominus w^0_{A-B} \ominus w^0_{A-C} \oplus w^0_\vee),\\
&B \Rightarrow H(w^0_B \ominus w^0_{A-B} \ominus w^0_{B-C} \oplus w^0_\vee),\\
&C \Rightarrow H(w^0_C \ominus w^0_{B-C} \ominus w^0_{A-C} \oplus w^0_\vee),\\
&A \,\&\, B \Rightarrow H(w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B}),\\
&A \,\&\, C \Rightarrow H(w^0_{AC} \ominus w^0_A \ominus w^0_C \oplus w^0_{A-C}),\\
&B \,\&\, C \Rightarrow H(w^0_{BC} \ominus w^0_B \ominus w^0_C \oplus w^0_{B-C}),\\
&A \,\&\, B \,\&\, C \Rightarrow H(w^0_{ABC} \ominus w^0_{AB} \ominus w^0_{AC} \ominus w^0_{BC} \oplus w^0_A \oplus w^0_B \oplus w^0_C \ominus w^0_\vee),\\
&A \vee B \Rightarrow H(w^0_{A-B} \ominus w^0_\vee),\\
&A \vee C \Rightarrow H(w^0_{A-C} \ominus w^0_\vee),\\
&B \vee C \Rightarrow H(w^0_{B-C} \ominus w^0_\vee),\\
&A \vee B \vee C \Rightarrow H(w^0_\vee \ominus w_0).
\end{aligned}$$

Similarly we can compute Möbied weights for a knowledge base with four or more questions. For formulas for computing of Möbied weights for a knowledge base with the only hypothesis H and with four questions (literals of four different questions) and their derivation, see appendix B again.

Formulas for general knowledge bases with one hypothesis and 2 (resp. 3) questions, i.e. with 4 (resp. 6) literals are shown in appendix C. We can observe, that weights of maximal disjunctive rules and of conjunctive rules (not simple ones) are the same as in the case of literals of different questions.

In general, for a knowledge base with one hypothesis H and n questions/symptoms $A, B, C, ..., N$, we can compute Möbied weights as:

$w_0 = w^0_0$

$w_\vee = w^0_\vee \ominus w_0$,

$$w_{A-B-C-...-K} = w^0_{A-B-C-...-K} \ominus \bigoplus_{|d|=k+1} w^0_d \oplus \bigoplus_{|d|=k+2} w^0_d \ominus \bigoplus_{|d|=k+3} w^0_d \oplus \;..... \; w^0_\vee$$

(antecedent A of a simple rule is here considered as one-element disjunction, thus w_A is also computed according to this formula),

$$w_{ABC...K} = w^0_{ABC...K} \ominus \bigoplus_{|c|=k-1} w^0_c \oplus \bigoplus_{|c|=k-2} w^0_c \ominus \;.....\; \bigoplus_{|c|=1} w^0_c \oplus (-1)^{|K|} w^0_{A-B-C-...-K}.$$

Hence we get:

$$w_0 = w_0^0$$

$$w_\vee = w_\vee^0 \ominus w_0$$

$$w_{A-B-C-\ldots-K} = w^0_{A-B-C-\ldots-K} \oplus \bigoplus_{i=1}^{n-k} \left((-1)^i \bigoplus_{|d|=k+i,\, A-B-\ldots-K \subset d} w_d^0 \right)$$

$$w_A = w_A^0 \oplus \bigoplus_{i=1}^{n-1} \left((-1)^i \bigoplus_{|d|=i+1,\, A\subset d} w_d^0 \right)$$

$$w_{ABC\ldots K} = w^0_{ABC\ldots K} \oplus \bigoplus_{i=1}^{k-1} \left((-1)^i \bigoplus_{|c|=k-i,\, c\subset AB\ldots K} w_c^0 \right) \;\oplus\; (-1)^k w^0_{A-B-C-\ldots-K},$$

where $w_\vee$ is an abbreviation for a weight $w_{A-B-C-\ldots-N}$ of the rule with the maximal possible disjunction in antecedent, $a \subset b$ means b implies a, $|c|$ is a length (number of conjuncts) of conjunction c, conjunction $c = ABC\ldots K$ has k elements i.e. $|c| = k$.

We can easily rewrite the formulas as:

$$w_0 = w_0^0$$

$$w_\vee = w_\vee^0 \ominus w_0$$

$$w_{A-B-C-\ldots-K} = \bigoplus_{i=0}^{n-k} \left((-1)^i \bigoplus_{|d|=k+i,\, A-B-\ldots-K \subset d} w_d^0 \right)$$

$$w_A = \bigoplus_{i=0}^{n-1} \left((-1)^i \bigoplus_{|d|=i+1,\, A\subset d} w_d^0 \right)$$

$$w_{ABC\ldots K} = \bigoplus_{i=0}^{k-1} \left((-1)^i \bigoplus_{|c|=k-i,\, c\subset AB\ldots K} w_c^0 \right) \;\oplus\; (-1)^k w^0_{A-B-C-\ldots-K},$$

If we compare the formula to compute $w_{ABC\ldots K}$ with the similar one which is used in knowledge bases without disjunction we can mention a significant similarity. From the comparison of these formulas we obtain the following one.

$$w_{ABC\ldots K} = (-1)^k (w^0_{A-B-C-\ldots-K} \ominus w_0)$$

Note, that the formulas for weights of non maximal disjunctive rules are stated in the form for knowledge bases with literals of different questions, the other ones are in general form.

Now, we have general formulas to compute Möbied weights of rules from any weakly sound low *ecd* knowledge base with one goal, where all possible elementary conjunctions or elementary disjunctions of questions are used as antecedents. (All possible conditional weights are already explicitly included in the source knowledge base. Negated questions are handled separately from the original ones like new ones.)

We can also use the formulas for deciding which types of new rules will be added into the transformed knowledge base and which ones will not, see the section called Simplifications. For this we need to know how to compute an estimation of implicit weights of possible rules which are not included in the source knowledge base.

3.6 Estimations of implicit weights of "rules" which are not included in a source knowledge base

As it was suggested in the previous subsection, we can use the formulas from there for specifying which type of rules are added into a knowledge base during Möbius transformation and which ones are not.

The formulas need all w^0's, all conditional weights which are definable on a set of questions and goals given by a source knowledge base. But, a lot of them are not given in a usual source knowledge base, i.e. not all rules with syntactically possible antecedents are included in the knowledge base. So, we have to estimate these values.

A rule $Ant \Rightarrow H$, literals of which are relevant to H, is not included in a source knowledge base Θ either if Ant is not possible or almost impossible in real situations or if *an expert thinks that Ant expresses nothing new for the hypothesis*, i.e. everything which expresses Ant has already been expressed by applicable rules which are already included in Θ. Thus we have to compute an expected value w^0_{Ant} from contributions of other rules which are applicable provided that just Ant holds, i.e. from rules which antecedents A are implied by Ant ($A \subset Ant$).

To distinguish *explicit conditional weights* w^0_{Ant} of rules from a source knowledge base from computed estimations of those which are not given (resp. which are given implicitly through other rules), we shall denote *estimated implicit weights* as w^x_{Ant}.

Let us suppose a rule $Ant \Rightarrow H$ to be added, so we have to determine an estimation w^x_{Ant} of implicit weight of the rule. It looks like, the expected implicit weight w^x_{Ant} of a rule $Ant \Rightarrow H$ should be something like a combination of Möbied weights of all the rules applicable provided that just Ant is true. But unfortunately, a generation of w^x_{Ant} is more complicated in general.

It is quite simple in the case of disjunctive rules. In the case of conjunctive rules, where w^0_A of all conjuncts of the antecedent are explicitly given or w^0_{AConj} is given for some $Conj$ ($A \subset A\&Conj \subset Ant$), the value w^x_{Ant} should be the same as in a knowledge base with only conjunctive rules. Complications start with conjunctive rules, where w^0_A of conjuncts of antecedents are not given and where a value should be transfered from disjunctive rules into conjunctive ones. We can illustrate it on a small example:

Let us suppose two questions A and B and that only w^0_{A-B} is given. Intuitively we see, that it should be $w^x_A = w^x_B = w^x_{AB} = w^x_{A-B}$. But, we cannot handle w^x_A, w^x_B as implicit value of w^0_A, w^0_B for which we suppose $w^x_{AB} = w^x_A \oplus w^x_B \ominus w_0$. Similarly if w^0_{A-B} and w^0_A are given we want to put $w^x_B = w^0_{A-B}$ and $w^x_{AB} = w^0_A$, but after it $w^0_A = w^x_{AB} \neq w^0_A \oplus w^x_B \ominus w_0 = w^0_A \oplus w^0_{A-B} \ominus w_0$ (We suppose $w^0_{A-B} \neq w_0$ to have a sense of rule $A \vee B \Rightarrow H$).

We can observe that, in this case, *" an effect of disjunction $A \vee B$ is in some sense involved in effect of its disjunct A and it is not propagated once more through the other disjunct B"*.

This is very important for generation of w^x_{Ant} of "problematic" rules, i.e. *"Not to propagate weight of disjunctive rule several times through different literals into weight of conjunctive rule"*.

In this case we have met the principal problem of transfering weight of disjunctive rules into weight of conjunctive ones.

Let us divide our task into 3 cases and singular one of w_0^x and let us sketch how w_{Ant}^x to be computed.

0) w_0^x
1) An estimation of w_{Ant}^x, where Ant is a disjunction (including single literal).
2) An estimation of w_{Ant}^x, where Ant is a conjunction of two or more literals, and where weight not to be propagated from disjunctive rules into conjunctive ones.
3) An estimation of w_{Ant}^x, where Ant is a conjunction of two or more literals, and where weight to be propagated from disjunctive rules into conjunctive ones.

Ad 0) The special singular case is w_0^x. It corresponds to a conditional belief under an empty condition, it should be an apriori weight of the hypothesis. It should be given by an expert as a value w_0 or it can be computed from a convenient database, but it is completely impossible to derive it from other rules. Hence, if there is not any additional source of knowledge, we have to put $w_0^x = w_0 = 0$.

Ad 1) In this case there is no transfer of weights from disjunctive rules into conjunctive ones, thus we can use $\oplus$-sum of Möbied weights of all applicable rules provided just Ant holds, i.e. $w_{Disj}^x = \bigoplus_{Ant \subset Disj} w_{Ant} = \bigoplus_{Ant \subset Disj} (w_{Ant}^z \ominus \bigoplus_{At \subset Ant} w_{At})$, where $z = 0$ or $z = x$, i.e. w_{Disj}^x to be computed recursively from the longest disjunctions toward shorter ones. Let us note, that if apriori weight of hypothesis H w_0 is included in the source knowledge base Θ (weight of rule $\Rightarrow H$), then it is also included as a summand in $\oplus$-sum (always fulfilled empty antecedent is implied by any disjunction $Disj$).

If we assume that all the possible disjunctive rules are included in the source knowledge base, we can use the following combinatorical formula
$w_{Disj}^x = \bigoplus_{i=1}^{n-|Disj|} ((-1)^{i-1} \bigoplus_{|d|=i} w_{Ant \vee d}^0) \oplus (-1)^{n-|Disj|} w_0$, where n is a number of (relevant) questions. To use this formula also in general case, it would be necessary to compute w^x for all missing disjunctive rules with longer antecedent ($Ant \subset Disj$) and use the formula in the form $w_{Disj}^x = \bigoplus_{i=1}^{n-|Disj|} ((-1)^{i-1} \bigoplus_{|d|=i} w_{Ant \vee d}^z) \oplus (-1)^{n-|Disj|} w_0$, where z is 0 or x.

An antecedent of a simple rule can be considered as one-element disjunction, here, because a literal A has no proper subconjunction and it is not possible to combine w_A^x from conjunctive rules with shorter antecedents.

Ad 2) In this case we take a $\oplus$-combination of Möbied weights of conjunctive rules, as well as it is considered in knowledge bases with conjunctive rules only. Thus, we consider a knowledge base Θ', which is Θ without disjunctive rules. We denote w'_{Ant} Möbied weights of rules $Ant \Rightarrow H$ from Θ'. Hence, we obtain
$w_{Ant}^x = \bigoplus_{Conj \subset Ant} w'_{Conj} \oplus w_0 = \bigoplus_{Conj \subset Ant} (w_{Conj}^z \ominus \bigoplus_{c \subset Conj} w'_c) \oplus w_0$, where $z = 0$ or $z = x$, and c is any antecedent from Θ including the empty one.

Similarly as in the previous case, we can express w^x_{Ant} as:
$w^x_{Ant} = \bigoplus_{i=1}^{k-1}((-1)^{i-1} \bigoplus_{|Conj|=k-i, Conj \subset Ant} w^0_c) \oplus (-1)^{k-1} w_0$. (Note: To use this version of the formula, $w^z_{At}{}'s$ must be given or computed for all $Conj \subset Ant$ again.) Specially, for $Ant = A\&B$ we get $w^x_{AB} = w^0_A \oplus w^0_B \ominus w_0$.

It arises the following question here. When a transfer of weights of disjunctive rules to conjunctive ones is not necessary? When are effects of all the relevant applicable disjunctive rules included in effects of conjunctive ones?

We consider rules of the following type $A_1\&A_2\&...\&A_k, \Rightarrow H$, $(Ant = A_1\&A_2\&...\&A_k$, $k \geq 2)$, in the present case. What are the rules for which is this procedure applicable?
- All literals A_i are founded, i.e. $w^0_{A_i}$ is given for $i = 1, ..., k$. (Conditional) weights of all applicable disjunctive rules are included in (conditional) weights of literals, in this case.
- $w^0_{A_i}$ is given only for $i = 1, ..., l < k$, and for every rule $A_j \vee disj \Rightarrow H$ $(j = l+1, ..., k)$ there exists some A_i which is a subdisjunction of $disj$ $(i = 1, ..., l)$, i.e. effects of all disjunctive rules $Disj \Rightarrow H$ are included in (propagated through) effects of literals $A_1, ..., A_l$, thus there is no propagation of weights from disjunctive rules into conjunctive ones.
- For every A_i $(i = 1, ..., k)$ either $w^0_{A_i}$ is given or w^0_{conj} is given, where $conj = A_i\&...$ is subconjunction of Ant. Weights of all applicable disjunctive rules are included in weights of literals and weights of subconjunctions of Ant, now.
- The previous holds only for $i = 1, ...l < k$, and for every rule $A_j \vee disj \Rightarrow H$ $(j = l + 1, ..., k)$ there exists some A_i which is a subdisjunction of $disj$ $(i = 1, ..., l)$, i.e. effects of all disjunctive rules $Disj \Rightarrow H$ are included in (propagated through) effects of literals and effects of conjunctions of literals $A_1, ..., A_l$, thus there is no propagation of weights from disjunctive rules into conjunctive ones again.

Ad 3) The last case is the most complicated, because it involves a transfer of weights of disjunctive rules to conjunctive ones. Let us consider rule $A_1\&A_2\&...\&A_k, \Rightarrow H$, $(Ant = A_1\&A_2\&...\&A_k)$. In the discussed case, there exist some A_i among $A_1, A_2, ..., A_k$ such that:
- $w^0_{A_i}$ is not given,
- w^0_{Conj} is not given for any $Conj$ $(A_i \subset Conj \subset Ant)$,
- there exists at least one disjunctive rule $DR: \; A_i \vee Disj \Rightarrow H$ such that $Disj \not\subset A_j$ for every A_j for which w^0_{Conj} is given such that $A_j \subset Conj \subseteq Ant$.

Weights of rules such as DR are not included in any conjunctive rules, so it is necessary to transfer them from disjunctive ones. On the other hand, weights of all rules $A_j \vee Disj \Rightarrow H$, such that $Conj \Rightarrow H$ for any disjunction $Disj$ and some conjunction $A_j \subseteq Conj \subseteq Ant$ are included in weights of conjunctive rules. Thus, for computing of w^x_{Ant} we use Möbied weights of conjunctive rules and Möbied weights of disjunctive rules $Disj \Rightarrow H$, which do not contain any literal Lit, such that rule $Lit\&Conj \Rightarrow H$ is included in Θ for some conjunction $Conj \subseteq Ant$. We will denote Möbied weight computed from these rules (only) as w^{Ant}_{Disj}. Hence, we get the following formula:

$$w^x_{Ant} = \bigoplus_{Conj \subset Ant} w^{Ant}_{Conj} \;\oplus \bigoplus_{\substack{Disj \subset Ant,\ Disj \not\subset Lit,\\ Lit \subset Conj \subseteq Ant,\ w^0_{Conj} \text{ is given}}} w^{Ant}_{Disj} \;\oplus\; w_0.$$

Analogically as in the previous cases, we can express w^x_{Ant} as:

$$w^x_{Ant} = \bigoplus_{i=1}^{k-1}((-1)^{i+1} \bigoplus_{\substack{|Conj|=k-i,\\ Conj \subset Ant}} w^0_{Conj}) \oplus \bigoplus_{i=2}^{n-k}((-1)^{i+k-1} \bigoplus_{\substack{|Disj|=i,\ Disj \subset Ant,\ Disj \not\subset Lit,\\ Lit \subset Conj \subseteq Ant,\ w^0_{Conj} \text{ is given}}} w^0_{Conj}) \oplus$$

$(-1)^n w_0$. (Note: To use this version of the formula, $w^z_{At}{}'s$ must be given or computed for all $Conj$, again, and also for all $Disj \subset Ant$ such that $Disj \not\subset Lit,\ Lit \subset Conj \subseteq Ant,\ w^0_{Conj}$ is given.)

We can summarize the formulas as follows:

$$w^x_0 = w_0$$

$$w^x_{Disj} = \bigoplus_{Ant \subset Disj} w_{Ant} \oplus w_0$$

$$w^x_{Ant} = \bigoplus_{Conj \subset Ant} w'_{Conj} \oplus w_0$$

$$w^x_{Ant} = \bigoplus_{Conj \subset Ant} w^{Ant}_{Conj} \;\oplus \bigoplus_{\substack{Disj \subset Ant,\ Disj \not\subset Lit,\\ Lit \subset Conj \subseteq Ant,\ w^0_{Conj} \text{ is given}}} w^{Ant}_{Disj} \;\oplus\; w_0$$

where $Ant, Conj, Disj, Lit$ is any antecedent, conjunction, disjunction or literal from Θ respectively. The third formula is applicable for conjunctive rules from the case 2).

We can notice, that the first three formulas are the special cases of the fourth one. Thus, the 4th formula is not applicable only for conjunctive rules ad 3), but is is applicable in general. Hence, in the cases either that all the possible antecedents are included in the source knowledge base or that we want to compute w^x for all the possible antecedents we can use the last formula in the following form:

$$w^x_{Ant} = \bigoplus_{i=1}^{k-1}((-1)^{i+1} \bigoplus_{\substack{|Conj|=k-i\\ Conj \subset Ant}} w^0_{Conj}) \oplus \bigoplus_{i=2}^{n-k}((-1)^{i+k-1} \bigoplus_{\substack{|Disj|=i,\ Disj \subset Ant,\\ Disj \not\subset Lit,\\ Lit \subset Conj \subseteq Ant,\\ w^0_{Conj} \text{ is given}}} w^0_{Disj}) \oplus (-1)^n w_0,$$

where z is 0 or x and $Ant, Conj, Disj$ is any possible antecedent, elementary conjunction or elementary disjunction constructed from questions the source knowledge base Θ.

To close this topic, we recapitulate that we have formulas on how to compute estimations of conditional weights for all types of rules admissible in *ecd* knowledge bases. Hence, we can close the section by a formulation of an existence theorem.

Theorem 3.2 *If Θ is a weakly sound low ecd knowledge base, then there Möbius transform of the knowledge base Θ exists.*

Idea of proof. We can perform Möbius transformation separately for every hypothesis. The rest follows from the previous text.

4 Simplifications

According to the previous section, we know how to compute Möbied weights, i.e. rule weights of Möbius transform of source knowledge base. We know, how to estimate and compute implicit weights for every rule with elementary conjunction or elementary disjunction in antecedent. Thus, we can do Möbius transformation for any *ecd* knowledge base.

Now, we are going to specify which rules are not necessary to add to Möbied knowledge base. We consider rules $Ant \Rightarrow H$ which are not included into the source knowledge base, i.e. w^0_{Ant} is not given there. By an added rule we mean such a rule that $w_{Ant} \neq 0$, while if $w_{Ant} = 0$ we say that rule is not added.

In general, we can consider all nonincluded rules to be virtually added. Whether a rule is to really be added or not, depends on its Möbied weight w_{Ant}. We can eliminate some types of rules to be added by a symbolic computation of their Möbied weight. But usually, we cannot assert that some type of rules will be added, because the actual value of its weight w_{Ant} depends on the actual values of conditional weights from the source knowledge base.

Now, we shall formulate some lemmata to describe which types of rules to be / not to be added in the knowledge base. For disjunctive rules, we easily obtain the following important lemma.

Lemma 4.1 *There are no disjunctive rules added to a knowledge base during Möbius transformation.*

Proof. Let Ant be $Ant = A_1 \vee A_2 \vee A_3 \vee ... \vee A_K$, where A_i are literals.
$w_{1-2-3-...-K} = w^0_{1-...-K} \ominus \bigoplus_{1-...-K \subset d} w^0_d$, we assume that a rule $Ant \rightarrow H$ is not included in original knowledge base, thus, we can write
$w_{1-2-3-...-K} = w^x_{1-...-K} \ominus \bigoplus_{1-...-K \subset d} w^0_d = \bigoplus_{1-...-K \subset d} w^0_d \ominus \bigoplus_{1-...-K \subset d} w^0_d = 0.$

We shall apply a similar idea to conjunctive rules; we shall investigate founded *ecd* knowledge bases, at first, and after it also general *ecd* knowledge bases.

4.1 Simplifications for founded *ecd* knowledge bases

Lemma 4.2 *If* Θ *is a founded ecd knowledge base, then there are no rules* $A_1\&A_2\&...\&A_k \Rightarrow H$ *added into the knowledge base within the process of Möbius transformation, where* $A_1 \vee A_2 \vee ... \vee A_k \vee B_1 \vee ... \vee B_l$ *is not an antecedent of some rule from the source knowledge base, for some literals* $B_1, ...B_l$*.*

Proof. Let Θ be a founded *ecd* knowledge base. Let us consider rule $R:\ A_1\&A_2\&...\&A_k\&B_1\&B_2\&...\&B_l \Rightarrow H$, where A_i are literal with a common occurrence in some disjunctive rule $A_1 \vee A_2 \vee ... \vee A_k \vee Disj \Rightarrow H$ from the source knowledge base(for some elementary disjunction $Disj$ which may be also empty), let the rule R not to be in the source knowledge base.

Let us start from $k = l = 1$, i.e. we consider a rule $A_1\&B_1 \Rightarrow H$, which is not included in the source knowledge base Θ, where moreover $A_1 \vee Disj \Rightarrow H$ is in Θ

for some disjunction $Disj$, while $A_1 \vee B_1 \vee DD \Rightarrow H$ is not in Θ for any disjunction DD (including empty one). Let us denote $Ant = A_1 \& B_1$. There it holds: $w^x_{Ant} = w^x_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1, At \not\subset A} w_{At}$; $w_{Ant} = w^x_{Ant} \ominus \bigoplus_{At \subset Ant} w_{At} = w^x_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1, At \not\subset A} w_{At} \ominus (w_0 \oplus w_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset A_1, At \not\subset B_1} w_{At} \oplus \bigoplus_{At \subset B_1, At \not\subset A_1} w_{At} \oplus \bigoplus_{At \subset A_1, At \subset B_1} w_{At}) = w^x_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1} w_{At} \ominus (w_0 \oplus w_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset A_1} w_{At} \oplus \bigoplus_{At \subset B_1} w_{At} \oplus \bigoplus_{At = A_1 \vee B_1 \vee DD} w_{At}) = w^x_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1} w_{At} \ominus (w_0 \oplus w^x_{A_1} \ominus w_0 \ominus \bigoplus_{At \subset A_1} w_{At} \oplus w_{B_1} \oplus \bigoplus_{At \subset A_1} w_{At} \oplus \bigoplus_{At \subset B_1} w_{At}) = w^x_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1} w_{At} \ominus (w^x_{A_1} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1} w_{At}) = 0$. Hence rule $Ant \Rightarrow H$ is not added into the knowledge base.

We shall continue using induction on k: Let the assertion hold for $k \leq n, l = 1$. Let us denote $A = A_1 \& A_2 \& ... \& A_{n+1}$ and $Ant = A \& B_1$. We consider a rule $A_1 \& A_2 \& ... \& A_{n+1} \& B_1 \Rightarrow H$, which is not included in the source knowledge base Θ, where moreover $A \vee Disj \Rightarrow H$ is in Θ for some disjunction $Disj$, while $A_1 \vee A_2 \vee ... \vee A_{n+1} \vee B_1 \vee DD \Rightarrow H$ is not in Θ for any disjunction DD (including empty one). There it holds:

$$w^x_{Ant} = w^x_A \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1, At \not\subset A} w_{At};$$
$$w_{Ant} = w^x_{Ant} \ominus \bigoplus_{At \subset Ant} w_{At} = w^x_A \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1, At \not\subset A} w_{At}$$
$$\ominus (w_A \oplus \bigoplus_{At \subset A, At \not\subset B_1} w_{At} \oplus w_{B_1} \oplus \bigoplus_{At \subset B_1, At \not\subset A} w_{At} \oplus \bigoplus_{At \subset A, At \subset B_1} w_{At} \oplus \bigoplus_{At \subset A \& B_1, At \not\subset A, At \not\subset B_1} w_{At})$$
$$= w^x_A \ominus (w_A \oplus \bigoplus_{At \subset A, At \not\subset B_1} w_{At} \oplus \bigoplus_{At \subset A, At \subset B_1} w_{At} \oplus \bigoplus_{At \subset A \& B_1, At \not\subset A, At \not\subset B_1} w_{At})$$
$$= w^x_A \ominus (w^x_A \ominus \bigoplus_{At \subset A} w_{At} \oplus \bigoplus_{At \subset A} w_{At} \oplus \bigoplus_{At = A_i \vee B_1 \vee DD} w_{At} \oplus \bigoplus_{At = B_1 \& Att, Att \subset A} w_{At})$$
$$= w^x_A \ominus (w^x_A \oplus \bigoplus_{At = B_1 \& Att, Att \subset A} w_{At}) = - \bigoplus_{At = B_1 \& Att, Att \subset A} w_{At}) ,$$

and it is equal to 0 according to inductional assumption. Hence rule $Ant \Rightarrow H$ is not added into the knowledge base again.

For the rest of the proof, we shall use induction on l: Let the assertion hold for any k and $l \leq m$. Let us denote $A = A_1 \& A_2 \& ... \& A_k$, $B = B_1 \& B_2 \& ... \& B_m$, and $Ant = A \& B \& B_{m+1}$. We consider a rule $A_1 \& A_2 \& ... \& A_k \& B_1 \& B_2 ... \& B_{m+1} \Rightarrow H$, which is not included in the source knowledge base Θ, where moreover $A \vee Disj \Rightarrow H$ is in Θ for some disjunction $Disj$, while $A_1 \vee A_2 \vee ... \vee A_{n+1} \vee B_i \vee DD \Rightarrow H$ is not in Θ for any $i \leq m+1$ and any disjunction DD (including empty one). There it holds:

$$w^x_{Ant} = w^x_{A\&B} \oplus w_{B_{m+1}} \bigoplus_{At \subset B_{m+1}, At \not\subset A\&B} w_{At};$$
$$w_{Ant} = w^x_{Ant} \ominus \bigoplus_{At \subset Ant} w_{At} = w^x_{A\&B} \oplus w_{B_{m+1}} \oplus \bigoplus_{At \subset B_{m+1}, At \not\subset A\&B} w_{At} \ominus (w_{A\&B} \oplus \bigoplus_{At \subset A\&B, At \not\subset B_{m+1}} w_{At}$$
$$\oplus w_{B_{m+1}} \oplus \bigoplus_{At \subset B_{m+1}, At \not\subset A\&B} w_{At} \oplus \bigoplus_{At \subset A\&B, At \subset B_{m+1}} w_{At} \oplus \bigoplus_{At \subset A\&B\&B_{m+1}, At \not\subset A\&B, At \not\subset B_{m+1}} w_{At})$$
$$= w^x_{A\&B} \ominus (w_{A\&B} \oplus (\bigoplus_{At \subset A\&B, At \not\subset B_{m+1}} w_{At} \oplus \bigoplus_{At \subset A\&B, At \subset B_{m+1}} w_{At}) \oplus \bigoplus_{\substack{At \subset A\&B\&B_{m+1},\\ At \not\subset A\&B, At \not\subset B_{m+1}}} w_{At})$$
$$= w^x_{A\&B} \ominus ((w^x_{A\&B} \ominus \bigoplus_{At \subset A\&B} w_{At}) \oplus \bigoplus_{At \subset A\&B} w_{At} \oplus \bigoplus_{At = B_{m+1}\&Att, Att \subset A\&B} w_{At})$$
$$= - \bigoplus_{At = B_{m+1}\&Att, Att \subset A\&B} w_{At}) .$$

If $At = A' \& B'$, where $A' \subseteq A, B' \subset B$, then $w_{At} = 0$ according to inductional assumption. If $At = A' \& B$, where $A' \subset A$, then it is possible to show that $w_{At} = 0$ again: it is true for k=1, further by induction on k (l is any fixed

number now), thus we assume $w_{At}=0$ for $k=n$, for $k=n+1$ we obtain similarly as above $w_{At}=-(\bigoplus_{Att\subset A\&B, Att\not\subset A, Att\not\subset B} w_{Att})$, and it is equal to 0 according to inductional assumption similarly as before
$(A=A_1\&A_2\&...\&A_{m+1}$, $B=B_1\&B_2\&...\&B_l$, $At=A=\&B$, $w^x_{At}=w^x_A\oplus w_B\oplus\bigoplus_{Att\subset B, Att\not\subset A} w_{Att}$;
$w_{At}=w^x_{At}\ominus\bigoplus_{Att\subset At} w_{Att}=$
$w^x_A\oplus w_B\oplus\bigoplus_{Att\subset B, Att\not\subset A} w_{Att}\ominus(w_A\oplus\bigoplus_{Att\subset A} w_{Att}\oplus w_B\oplus\bigoplus_{Att\subset B, Att\not\subset A} w_{Att}\oplus\bigoplus_{Att\subset A\&B, Att\not\subset A, Att\not\subset B} w_{Att})$
$=w^x_A\ominus(w_A\oplus\bigoplus_{Att\subset A} w_{Att}\oplus\bigoplus_{Att\subset A\&B, Att\not\subset A, Att\not\subset B} w_{Att})=-(\bigoplus_{Att\subset A\&B, Att\not\subset A, Att\not\subset B} w_{Att})$).

Thus rule $Ant\Rightarrow H$ is not added into the knowledge base again. Hence all the assertion of the lemma is proved.

Lemma 4.3 *If Θ is a founded ecd knowledge base, then conjunctive translations of all maximal disjunctive rules are added into the knowledge base within the process of Möbius transformation (if they are not already included in the source knowledge base Θ).*

Proof. Notice, here we have an exception and we can claim that rules are added: $w_{A_1A_2...A_k}=(-1)^{k-1}(w_{A_1-A_2-...-A_k}\ominus w_0)$, and we expect $w_{A_1-A_2-...-A_k}\neq w_0$ for maximal disjunctive rules not to be redundant.

Let $A_1\vee A_2\vee...\vee A_k\Rightarrow H$ be a maximal disjunctive rule and $A_1\&A_2\&...\&A_k\Rightarrow H$ its conjunctive translation added according to the lemma. Usually, there are also all rules added with subconjunction of $A_1\&A_2\&...\&A_k$ in antecedent. But, there are counter-examples also on a symbolic level, see appendix D, case a).

In both previous lemmata, the assumption of foundness of *ecd* knowledge base is necessary. For general *ecd* knowledge bases which are not founded, there are counter-examples against both of the lemmata presented in appendix D, cases b) and c).

Summarizing the above lemmata we see, that antecedents of rules added by Möbius transformation into *ecd* knowledge base are only all conjunctive translations of antecedents of maximal disjunctive rules and not necessarily all of their subconjunctions. Formally, we have the following:

Theorem 4.4 *Let Θ be a weakly sound low founded ecd knowledge base. During a process of Möbius transformation of Θ, rules are added into the knowledge base (if they are not already included in Θ and) if and only if they are in one of the two following types:*

- *All rules $A_1\&A_2\&...\&A_n\Rightarrow H$, where $A_1\vee A_2\vee...\vee A_n\Rightarrow H$, is a maximal disjunctive rule from Θ.*
- *Rules $A_1\&A_2\&...\&A_n\Rightarrow H$, where $A_1\vee A_2\vee...\vee A_n\vee Disj\Rightarrow H$, is a rule from Θ and $Disj$ is any disjunction. (In general, all such rules are added, but there counter-examples exist, e.g. appendix D, case a)).*

Proof. Assertion of the theorem immediately follows lemmata 4.2 and 4.3.

4.2 Simplifications for general *ecd* knowledge bases

As we have seen in the previous subsection, in the case of founded *ecd* knowledge bases, the only possibilities for antecedent added rule is either to be a conjunctive translation of antecedent of some disjunctive rule or to be a subconjunction of such a translation.

In the case of general *ecd* knowledge bases, an antecedent of added rule can be besides it also a (subconjunction of) conjunctive translation of a disjunction of several disjunctive antecedents, as it is illustrated with the following small examples, for other examples see appendix D, cases b) and c).

Example: Let us consider the following weakly founded *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_{A-B})$
$B \vee C \Rightarrow H(w^0_{B-C})$
$A \Rightarrow H(w^0_A)$
$B\&C \Rightarrow H(w^0_{BC})$

Estimations of implicit weights:
$w^x_{A-B-C} = w_0$
$w^x_{A-C} = w_0$
$w^x_B = w^0_{A-B} \oplus w^0_{B-C} \ominus w_0$
$w^x_C = w^0_{B-C}$
$w^x_{AB} = w^0_A \oplus w^0_{B-C} \ominus w_0$
$w^x_{AC} = w^0_A \oplus w^0_{B-C} \ominus w_0$
$w^x_{ABC} = w^0_A \oplus w^0_{BC} \ominus w_0$

Möbied weights:
w_0
$w_{A-B-C} = 0$
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_{A-C} = 0$
$w_{B-C} = w^0_{B-C} \ominus w_0$
$w_A = w^0_A \ominus w^0_{A-B}$
$w_B = 0$
$w_C = 0$
$w_{AB} = 0$
$w_{AC} = 0$
$w_{BC} = w^0_{BC} \ominus w^0_{A-B} \ominus w^0_{B-C} \oplus w_0$
$w_{ABC} = w^0_{A-B} \ominus w_0$

Rule $A\&B\&C \Rightarrow H(w^0_{A-B} \ominus w_0)$ is added into knowledge base even if its antecedent $A\&B\&C$ is not (subconjunction of) conjunctive translation of any disjunctive antecedent, $A \vee B \vee C$ is not (subdisjunction of) antecedent of any rule from Θ.

Example: Let us consider the following weakly founded *ecd* knowledge base Θ:

$\Rightarrow H(w_0)$
$A \vee B \Rightarrow H(w^0_\vee)$
$A \Rightarrow H(w^0_A)$
$D \Rightarrow H(w^0_D)$
$B\&D \Rightarrow H(w^0_{BD})$

Estimations of implicit weights:
$w^x_B = w^0_{A-B}$
$w^x_{AB} = w^0_A$
$w^x_{AD} = w^0_A \oplus w^0_D \ominus w_0$
$w^x_{ABD} = w^0_A \oplus w^0_{BD} \ominus w_0$

Möbied weights:
w_0
$w_{A-B} = w^0_{A-B} \ominus w_0$
$w_A = w^0_A \ominus w^0_{A-B}$
$w_D = w^0_D \ominus w_0$
$w_{BD} = w^0_{BD} \ominus w^0_{A-B} \ominus w^0_D \oplus w_0$
$w_{ABD} = w^0_{A-B} \ominus w_0$
Non stated weights are equal to zero.

Rule $A\&B\&D \Rightarrow H(w^0_{A-B} \ominus w_0)$ is added into knowledge base even if its antecedent $A\&B\&D$ is not (subconjunction of) conjunctive translation of any disjunctive antecedent, $A \vee B \vee D$ is not (subdisjunction of) antecedent of any rule from Θ.

For a presentation of a description of types of rules which are added / not added within the process of Möbius transformation of general *ecd* knowledge base, it is necessary further elaboration of the topic.

5 Algorithm of Möbius transformation

From the theorem 3.2 we have an existence of Möbius transform for weakly sound general low *ecd* knowledge bases. Using theorem 4.4, we can formulate the following algorithm of Möbius transformation of a founded knowledge base Θ.

(*) Go ahead through all hypothesis H:
and perform items (0) – (4).

(0) Construct a set Rel of literals relevant to H.
Put $w_0 = w_0^0$.
Create an empty set of maximal disjunctions $MaxD$.

(1) Go ahead through all disjunctions D in Θ relevant to H:
Put Sum equal to $\oplus$-sum of Möbied weights of all rules $D \vee D' \Rightarrow H$.
IF there is no such rule, THEN insert D into $MaxD$ and put $w_D = w_D^0 \ominus w_0$,
ELSE put $w_D = w_D^0 \ominus Sum$.
If $|D| = 1$, then sign D in Rel.

(2) Go through all unsigned literals L from Rel:
IF there is no rule $L \vee D \Rightarrow H$, THEN put $w_L = 0$,
ELSE give warning "Assumption does not hold for hypothesis H." and STOP.

(3) Go through all maximal disjunctions MD from $MaxD$:
for MD and every subdisjunction SMD of MD -
create all new rules $Ant \Rightarrow H$ which are already not included in Θ, where Ant is a conjunctive translation of MD or SMD.

(4) Go ahead through all conjunctions $|C| > 1$ in Θ relevant to H:
Put Sum equal to $\oplus$-sum of Möbied weights ($w_{C'}$) of all rules $C' \Rightarrow H$, where $C' \subset C$ (C implies C').
If w_C^0 is not given ($C \Rightarrow H$ is added rule), then put w_C^0 equal to $\oplus$-sum of Möbied weights ($w_{C'}$) of all rules $C' \Rightarrow H$, where C' is subconjunction of C (including $w_0 \sim$ empty subconjunction implied by C).
Keep w_C^0 and put $w_C = w_C^0 \ominus Sum$.
(During construction of Möbius transform it is not necessary to distinguish between w_C^0 and w_C^x, they can be represented by the same variable denoted w_C^0.)

(*) Save all rules with weights $w_{H_i,Ant_{ij}} \neq 0$ — Möbius transform of Θ.
STOP.

It is possible to show that this algorithm ends and produces Möbius transform of any weakly sound low founded *ecd* knowledge base Θ.

In the case of general *ecd* knowledge bases, we have to use simple minded algorithm suggested in proof of lemma 3.1. For generalization of the above algorithm it would be necessary to formulate analogies to lemmata 4.2 and 4.3.

6 Conclusion

Generalized Möbius transformation is a theoretical tool for the construction of more correct generalizations of expert systems both of MYCIN-like and fuzzy expert systems based on a composition of fuzzy relations.

Möbius transformation has been generalized to *ecd* knowledge bases, i.e. knowledge bases whose rules have antecedents either in the form of an elementary conjunction (as before) or in the form of an elementary disjunction (new ones) of questions.

The principal difference between original and generalized Möbius transformation consists in a complicated transfer of weights of rules with disjunctive antecedents D_i to weights of other rules with conjunctive ones C_i, where C_i implies D_i.

Original Möbius transformation is only the transformation of weights. While within the generalized one, moreover, some new rules are often added into the knowledge base.

An *estimation of implicit (expected) weights* for these added rules was shown for a class of *ecd* knowledge bases. The *existence theorem* was proved for this class of knowledge bases. Finally, an *algorithm* of the construction of this generalized Möbius transform of founded *ecd* knowledge base is described.

To generalize the presented algorithm onto all class of generall *ecd* knowledge bases, it needs a particular description and determination which rule to be added and which ones not to be added. It would be useful to have something like generalized analogies of lemmata 4.2 and 4.3 as it was suggested since in sections 4.2, 5. It is a motivation for future research.

A challenge for the future is an admission of rules with more complicated antecedents or a consideration of knowledge bases with several different conjunctions and/or disjunctions.

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1 APPENDIX A

2 APPENDIX B

Möbied weights of knowledge base with one hypothesis and with a) two b) three c) four literals of different questions (i.e. any literal is not negation of another one), provided that all possible source conditional weights $w^0_{\_}$ are given.

a) Two literals A, B :

w_0

$w_\vee = w^0_\vee \ominus w_0$

$$\begin{aligned} w_A &= w^0_A \ominus w_\vee \ominus w_0 \\ &= w^0_A \ominus w^0_\vee \\ w_B &= w^0_B \ominus w_\vee \ominus w_0 \\ &= w^0_B \ominus w^0_\vee \end{aligned}$$

$$\begin{aligned} w_{AB} &= w^0_{AB} \ominus w_A \ominus w_B \ominus w_\vee \ominus w_0 \\ &= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_\vee \end{aligned}$$

b) Three literals A, B, C :

w_0

$w_\vee = w^0_\vee \ominus w_0$

$$\begin{aligned} w_{A-B} &= w^0_{A-B} \ominus w_\vee \ominus w_0 \\ &= w^0_{A-B} \ominus w^0_\vee \\ w_{A-C} &= w^0_{A-C} \ominus w_\vee \ominus w_0 \\ &= w^0_{A-C} \ominus w^0_\vee \\ w_{B-C} &= w^0_{B-C} \ominus w_\vee \ominus w_0 \\ &= w^0_{B-C} \ominus w^0_\vee \end{aligned}$$

$$\begin{aligned} w_A &= w^0_A \ominus w_{A-B} \ominus w_{A-C} \ominus w_\vee \ominus w_0 \\ &= w^0_A \ominus w^0_{A-B} \ominus w^0_{A-C} \oplus w^0_\vee \\ w_B &= w^0_B \ominus w_{A-B} \ominus w_{B-C} \ominus w_\vee \ominus w_0 \\ &= w^0_B \ominus w^0_{A-B} \ominus w^0_{B-C} \oplus w^0_\vee \\ w_C &= w^0_C \ominus w_{B-C} \ominus w_{A-C} \ominus w_\vee \ominus w_0 \\ &= w^0_C \ominus w^0_{B-C} \ominus w^0_{A-C} \oplus w^0_\vee \end{aligned}$$

$$\begin{aligned} w_{AB} &= w^0_{AB} \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0 \\ &= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B} \\ w_{AC} &= w^0_{AC} \ominus w_A \ominus w_C \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0 \\ &= w^0_{AC} \ominus w^0_A \ominus w^0_C \oplus w^0_{A-C} \\ w_{BC} &= w^0_{BC} \ominus w_B \ominus w_C \ominus w_{A-B} \ominus w_{B-C} \ominus w_{A-C} \ominus w_\vee \ominus w_0 \\ &= w^0_{BC} \ominus w^0_B \ominus w^0_C \oplus w^0_{B-C} \end{aligned}$$

$$\begin{aligned} w_{ABC} &= w^0_{ABC} \ominus w_{AB} \ominus w_{AC} \ominus w_{BC} \ominus w_A \ominus w_B \ominus w_C \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0 \\ &= w^0_{ABC} \ominus w^0_{AB} \ominus w^0_{AC} \ominus w^0_{BC} \oplus w^0_A \oplus w^0_B \oplus w^0_C \ominus w^0_\vee \end{aligned}$$

c) Four literals A, B, C, D :

w_0

$w_\vee = w_\vee^0 \ominus w_0$

$w_{A-B-C} = w_{A-B-C}^0 \ominus w_\vee \ominus w_0$
$\quad = w_{A-B-C}^0 \ominus w_\vee^0$
$w_{A-B-D} = w_{A-B-D}^0 \ominus w_\vee^0$
$w_{A-C-D} = w_{A-C-D}^0 \ominus w_\vee^0$
$w_{B-C-D} = w_{B-C-D}^0 \ominus w_\vee^0$

$w_{A-B} = w_{A-B}^0 \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_\vee \ominus w_0$
$\quad = w_{A-B}^0 \ominus w_{A-B-C}^0 \ominus w_{A-B-D}^0 \oplus w_\vee^0$
$w_{A-C} = w_{A-C}^0 \ominus w_{A-B-C}^0 \ominus w_{A-C-D}^0 \oplus w_\vee^0$
$w_{A-D} = w_{A-D}^0 \ominus w_{A-C-D}^0 \ominus w_{A-B-D}^0 \oplus w_\vee^0$
$w_{B-C} = w_{B-C}^0 \ominus w_{A-B-C}^0 \ominus w_{B-C-D}^0 \oplus w_\vee^0$
$w_{B-D} = w_{B-D}^0 \ominus w_{B-C-D}^0 \ominus w_{A-B-D}^0 \oplus w_\vee^0$
$w_{C-D} = w_{C-D}^0 \ominus w_{A-C-D}^0 \ominus w_{B-C-D}^0 \oplus w_\vee^0$

$w_A = w_A^0 \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D} \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_\vee \ominus w_0$
$\quad = w_A^0 \ominus w_{A-B}^0 \ominus w_{A-C}^0 \ominus w_{A-D}^0 \oplus w_{A-B-C}^0 \oplus w_{A-B-D}^0 \oplus w_{A-C-D}^0 \ominus w_\vee^0$
$w_B = w_B^0 \ominus w_{A-B}^0 \ominus w_{B-C}^0 \ominus w_{B-D}^0 \oplus w_{A-B-C}^0 \oplus w_{A-B-D}^0 \oplus w_{B-C-D}^0 \ominus w_\vee^0$
$w_C = w_C^0 \ominus w_{A-C}^0 \ominus w_{B-C}^0 \ominus w_{C-D}^0 \oplus w_{A-B-C}^0 \oplus w_{A-C-D}^0 \oplus w_{B-C-D}^0 \ominus w_\vee^0$
$w_D = w_D^0 \ominus w_{A-D}^0 \ominus w_{B-D}^0 \ominus w_{C-D}^0 \oplus w_{A-B-D}^0 \oplus w_{A-C-D}^0 \oplus w_{B-C-D}^0 \ominus w_\vee^0$

$w_{AB} = w_{AB}^0 \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D} \ominus w_{B-C} \ominus w_{B-D}$
$\quad\quad \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_{B-C-D} \ominus w_\vee \ominus w_0$
$\quad = w_{AB}^0 \ominus w_A^0 \ominus w_B^0 \oplus w_{A-B}^0$
$w_{AC} = w_{AC}^0 \ominus w_A^0 \ominus w_C^0 \oplus w_{A-C}^0$
$w_{AD} = w_{AD}^0 \ominus w_A^0 \ominus w_D^0 \oplus w_{A-D}^0$
$w_{BC} = w_{BC}^0 \ominus w_B^0 \ominus w_C^0 \oplus w_{B-C}^0$
$w_{BD} = w_{BD}^0 \ominus w_B^0 \ominus w_D^0 \oplus w_{B-D}^0$
$w_{CD} = w_{CD}^0 \ominus w_C^0 \ominus w_D^0 \oplus w_{C-D}^0$

$w_{ABC} = w_{ABC}^0 \ominus w_{AB} \ominus w_{AC} \ominus w_{BC} \ominus w_A \ominus w_B \ominus w_C \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D}$
$\quad\quad \ominus w_{B-C} \ominus w_{B-D} \ominus w_{C-D} \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_{B-C-D} \ominus w_\vee \ominus w_0$
$\quad = w_{ABC}^0 \ominus w_{AB}^0 \ominus w_{AC}^0 \ominus w_{BC}^0 \oplus w_A^0 \oplus w_B^0 \oplus w_C^0 \ominus w_{A-B-C}^0$
$w_{ABD} = w_{ABD}^0 \ominus w_{AB}^0 \ominus w_{AD}^0 \ominus w_{BD}^0 \oplus w_A^0 \oplus w_B^0 \oplus w_D^0 \ominus w_{A-B-D}^0$
$w_{ACD} = w_{ACD}^0 \ominus w_{AC}^0 \ominus w_{AD}^0 \ominus w_{CD}^0 \oplus w_A^0 \oplus w_C^0 \oplus w_D^0 \ominus w_{A-C-D}^0$
$w_{BCD} = w_{BCD}^0 \ominus w_{BC}^0 \ominus w_{BD}^0 \ominus w_{CD}^0 \oplus w_B^0 \oplus w_C^0 \oplus w_D^0 \ominus w_{B-C-D}^0$

$w_{ABCD} = w_{ABCD}^0 \ominus w_{ABC} \ominus w_{ABD} \ominus w_{ACD} \ominus w_{BCD}$
$\quad\quad \ominus w_{AB} \ominus w_{AC} \ominus w_{AD} \ominus w_{BC} \ominus w_{BD} \ominus w_{CD} \ominus w_A \ominus w_B \ominus w_C \ominus w_D$
$\quad\quad \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D} \ominus w_{B-C} \ominus w_{B-D} \ominus w_{C-D}$
$\quad\quad \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_{B-C-D} \ominus w_\vee \ominus w_0$
$\quad = w_{ABCD}^0 \ominus w_{ABC}^0 \ominus w_{ABD}^0 \ominus w_{ACD}^0 \ominus w_{BCD}^0$
$\quad\quad \oplus w_{AB}^0 \oplus w_{AC}^0 \oplus w_{AD}^0 \oplus w_{BC}^0 \oplus w_{BD}^0 \oplus w_{CD}^0 \ominus w_A^0 \ominus w_B^0 \ominus w_C^0 \ominus w_D^0 \oplus w_\vee^0$

d) Complete derivations of the formulas.

ad a) A, B :

Compl.

w_0

$w_\vee = w_\vee^0 \ominus w_0$

$$\begin{aligned} w_A &= w_A^0 \ominus w_\vee \ominus w_0 = w_A^0 \ominus w_\vee^0 \oplus w_0 \ominus w_0 \\ &= w_A^0 \ominus w_\vee^0 \end{aligned}$$

$$\begin{aligned} w_{AB} &= w_{AB}^0 \ominus w_A \ominus w_B \ominus w_\vee \ominus w_0 \\ &= w_{AB}^0 \ominus w_A^0 \oplus w_\vee^0 \ominus w_B^0 \oplus w_\vee^0 \ominus w_\vee^0 \oplus w_0 \ominus w_0 \\ &= w_{AB}^0 \ominus w_A^0 \ominus w_B^0 \oplus w_\vee^0 \end{aligned}$$

ad b) A, B, C :

w_0

$w_\vee = w_\vee^0 \ominus w_0$

$$\begin{aligned} w_{A-B} &= w_{A-B}^0 \ominus w_\vee \ominus w_0 = w_{A-B}^0 \ominus w_\vee^0 \oplus w_0 \ominus w_0 \\ &= w_{A-B}^0 \ominus w_\vee^0 \end{aligned}$$

$$\begin{aligned} w_A &= w_A^0 \ominus w_{A-B} \ominus w_{A-C} \ominus w_\vee \ominus w_0 \\ &= w_A^0 \ominus w_{A-B}^0 \oplus w_\vee^0 \ominus w_{A-C}^0 \oplus w_\vee^0 \ominus w_\vee^0 \oplus w_0 \ominus w_0 \\ &= w_A^0 \ominus w_{A-B}^0 \ominus w_{A-C}^0 \oplus w_\vee^0 \end{aligned}$$

$$\begin{aligned} w_{AB} &= w_{AB}^0 \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0 \\ &= w_{AB}^0 \ominus w_A^0 \oplus w_{A-B}^0 \oplus w_{A-C}^0 \ominus w_\vee^0 \ominus w_B^0 \oplus w_{A-B}^0 \oplus w_{B-C}^0 \ominus w_\vee^0 \\ &\quad \ominus w_{A-B}^0 \oplus w_\vee^0 \ominus w_{A-C}^0 \oplus w_\vee^0 \ominus w_{B-C}^0 \oplus w_\vee^0 \ominus w_\vee^0 \oplus w_0 \ominus w_0 \\ &= w_{AB}^0 \ominus w_A^0 \ominus w_B^0 \oplus w_{A-B}^0 \end{aligned}$$

$$\begin{aligned} w_{ABC} &= w_{ABC}^0 \ominus w_{AB} \ominus w_{AC} \ominus w_{BC} \ominus w_A \ominus w_B \ominus w_C \ominus w_{A-B} \ominus w_{A-C} \ominus w_{B-C} \ominus w_\vee \ominus w_0 \\ &= w_{ABC}^0 \ominus w_{AB}^0 \oplus w_A^0 \oplus w_B^0 \ominus w_{A-B}^0 \ominus w_{AC}^0 \oplus w_A^0 \oplus w_C^0 \ominus w_{A-C}^0 \ominus w_{BC}^0 \oplus w_B^0 \oplus w_C^0 \ominus w_{B-C}^0 \\ &\quad \ominus w_A^0 \oplus w_{A-B}^0 \oplus w_{A-C}^0 \ominus w_\vee^0 \ominus w_B^0 \oplus w_{A-B}^0 \oplus w_{B-C}^0 \ominus w_\vee^0 \ominus w_C^0 \oplus w_{A-C}^0 \oplus w_{B-C}^0 \ominus w_\vee^0 \\ &\quad \ominus w_{A-B}^0 \oplus w_\vee^0 \ominus w_{A-C}^0 \oplus w_\vee^0 \ominus w_{B-C}^0 \oplus w_\vee^0 \ominus w_\vee^0 \oplus w_0 \ominus w_0 \\ &= w_{ABC}^0 \ominus w_{AB}^0 \ominus w_{AC}^0 \ominus w_{BC}^0 \oplus w_A^0 \oplus w_B^0 \oplus w_C^0 \ominus w_\vee^0 \end{aligned}$$

ad c) A, B, C, D :

w_0

$w_\vee = w^0_\vee \ominus w_0$

$$w_{A-B-C} = w^0_{A-B-C} \ominus w_\vee \ominus w_0 = w^0_{A-B-C} \ominus w^0_\vee \oplus w_0 \ominus w_0$$
$$= w^0_{A-B-C} \ominus w^0_\vee$$

$$w_{A-B} = w^0_{A-B} \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_\vee \ominus w_0$$
$$= w^0_{A-B} \ominus w^0_{A-B-C} \oplus w^0_\vee \ominus w^0_{A-B-D} \oplus w^0_\vee \ominus w^0_\vee \oplus w_0 \ominus w_0$$
$$= w^0_{A-B} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \oplus w^0_\vee$$

$$w_A = w^0_A \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D} \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_\vee \ominus w_0$$
$$= w^0_A \ominus w^0_{A-B} \oplus w^0_{A-B-C} \oplus w^0_{A-B-D} \ominus w^0_\vee$$
$$\ominus w^0_{A-C} \oplus w^0_{A-B-C} \oplus w^0_{A-C-D} \ominus w^0_\vee \ominus w^0_{A-D} \oplus w^0_{A-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$$
$$\ominus w^0_{A-B-C} \oplus w^0_\vee \ominus w^0_{A-B-D} \oplus w^0_\vee \ominus w^0_{A-C-D} \oplus w^0_\vee \ominus w^0_\vee \oplus w_0 \ominus w_0$$
$$= w^0_A \ominus w^0_{A-B} \ominus w^0_{A-C} \ominus w^0_{A-D} \oplus w^0_{A-B-C} \oplus w^0_{A-B-D} \oplus w^0_{A-C-D} \ominus w^0_\vee$$

$$w_{AB} = w^0_{AB} \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D} \ominus w_{B-C} \ominus w_{B-D}$$
$$\ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_{B-C-D}$$
$$\ominus w_\vee \ominus w_0$$
$$= w^0_{AB}$$
$$\ominus w^0_A \oplus w^0_{A-B} \oplus w^0_{A-C} \oplus w^0_{A-D} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \ominus w^0_{A-C-D}$$
$$\oplus w^0_\vee$$
$$\ominus w^0_B \oplus w^0_{A-B} \oplus w^0_{B-C} \oplus w^0_{B-D} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \ominus w^0_{B-C-D}$$
$$\oplus w^0_\vee$$
$$\ominus w^0_{A-B} \oplus w^0_{A-B-C} \oplus w^0_{A-B-D} \ominus w^0_\vee$$
$$\ominus w^0_{A-C} \oplus w^0_{A-B-C} \oplus w^0_{A-C-D} \ominus w^0_\vee$$
$$\ominus w^0_{A-D} \oplus w^0_{A-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$$
$$\ominus w^0_{B-C} \oplus w^0_{A-B-C} \oplus w^0_{B-C-D} \ominus w^0_\vee$$
$$\ominus w^0_{B-D} \oplus w^0_{B-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$$
$$\ominus w^0_{A-B-C} \oplus w^0_\vee$$
$$\ominus w^0_{A-B-D} \oplus w^0_\vee$$
$$\ominus w^0_{A-C-D} \oplus w^0_\vee$$
$$\ominus w^0_{B-C-D} \oplus w^0_\vee$$
$$\ominus w^0_\vee \oplus w_0 \ominus w_0$$
$$= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B}$$

$$w_{ABC} = w^0_{ABC} \ominus w_{AB} \ominus w_{AC} \ominus w_{BC} \ominus w_A \ominus w_B \ominus w_C \ominus w_{A-B} \ominus w_{A-C} \ominus w_{A-D}$$
$$\ominus w_{B-C} \ominus w_{B-D} \ominus w_{C-D} \ominus w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_{B-C-D} \ominus w_\vee \ominus w_0$$
$$= w^0_{ABC}$$
$$\ominus w^0_{AB} \oplus w^0_A \oplus w^0_B \ominus w^0_{A-B} \ominus w^0_{AC} \oplus w^0_A \oplus w^0_C \ominus w^0_{A-C} \ominus w^0_{BC} \oplus w^0_B \oplus w^0_C \ominus w^0_{B-C} \qquad \ominus w^0_A \oplus$$
$$w^0_{A-B} \oplus w^0_{A-C} \oplus w^0_{A-D} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \ominus w^0_{A-C-D}$$
$$\oplus w^0_\vee$$
$$\ominus w^0_B \oplus w^0_{A-B} \oplus w^0_{B-C} \oplus w^0_{B-D} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \ominus w^0_{B-C-D}$$
$$\oplus w^0_\vee$$
$$\ominus w^0_C \oplus w^0_{A-C} \oplus w^0_{B-C} \oplus w^0_{C-D} \ominus w^0_{A-B-C} \ominus w^0_{A-C-D} \ominus w^0_{B-C-D}$$

$\oplus\ w^0_\vee$

$\ominus\ w^0_{A-B} \oplus w^0_{A-B-C} \oplus w^0_{A-B-D} \ominus w^0_\vee$
$\ominus\ w^0_{A-C} \oplus w^0_{A-B-C} \oplus w^0_{A-C-D} \ominus w^0_\vee$
$\ominus\ w^0_{A-D} \oplus w^0_{A-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$
$\ominus\ w^0_{B-C} \oplus w^0_{A-B-C} \oplus w^0_{B-C-D} \ominus w^0_\vee$
$\ominus\ w^0_{B-D} \oplus w^0_{B-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$
$\ominus\ w^0_{C-D} \oplus w^0_{B-C-D} \oplus w^0_{A-C-D} \ominus w^0_\vee$
$\ominus\ w^0_{A-B-C} \oplus w^0_\vee$
$\ominus\ w^0_{A-B-D} \oplus w^0_\vee$
$\ominus\ w^0_{A-C-D} \oplus w^0_\vee$
$\ominus\ w^0_{B-C-D} \oplus w^0_\vee$
$\ominus\ w^0_\vee \oplus w_0 \ominus w_0$

$= w^0_{ABC} \ominus w^0_{AB} \ominus w^0_{AC} \ominus w^0_{BC} \oplus w^0_A \oplus w^0_B \oplus w^0_C \ominus w^0_{A-B-C}$

$w_{ABCD} = w^0_{ABCD} \ominus w_{ABC} \ominus w_{ABD} \ominus w_{ACD} \ominus w_{BCD}$
$\ominus\ w_{AB} \ominus w_{AC} \ominus w_{AD} \ominus w_{BC} \ominus w_{BD} \ominus w_{CD} \ominus w_A \ominus w_B \ominus w_C \ominus w_D$
$\ominus\ w_{A-B} \ominus w_{A-C} \ominus w_{A-D} \ominus w_{B-C} \ominus w_{B-D} \ominus w_{C-D}$
$\ominus\ w_{A-B-C} \ominus w_{A-B-D} \ominus w_{A-C-D} \ominus w_{B-C-D} \ominus w_\vee \ominus w_0$

$= w^0_{ABCD}$
$\ominus\ w^0_{ABC} \oplus w^0_{AB} \oplus w^0_{AC} \oplus w^0_{BC} \ominus w^0_A \ominus w^0_B \ominus w^0_C \oplus w^0_{A-B-C} \ominus w^0_{ABD} \oplus w^0_{AB} \oplus w^0_{AD} \oplus w^0_{BD} \ominus w^0_A \ominus w^0_B \ominus w^0_D \oplus w^0_{A-B-D} \ominus w^0_{ACD} \oplus w^0_{AC} \oplus w^0_{AD} \oplus w^0_{CD} \ominus w^0_A \ominus w^0_C \ominus w^0_D \oplus w^0_{A-C-D} \ominus w^0_{BCD} \oplus w^0_{BC} \oplus w^0_{BD} \oplus w^0_{CD} \ominus w^0_B \ominus w^0_C \ominus w^0_D \oplus w^0_{B-C-D} \ominus w^0_{AB} \oplus w^0_A \oplus w^0_B \ominus w^0_{A-B} \ominus w^0_{AC} \oplus w^0_A \oplus w^0_C \ominus w^0_{A-C} \ominus w^0_{AD} \oplus w^0_A \oplus w^0_D \ominus w^0_{A-D} \ominus w^0_{BC} \oplus w^0_B \oplus w^0_C \ominus w^0_{B-C} \ominus w^0_{BD} \oplus w^0_B \oplus w^0_D \ominus w^0_{B-D} \ominus w^0_{CD} \oplus w^0_C \oplus w^0_D \ominus w^0_{C-D} \quad \ominus w^0_A \oplus w^0_{A-B} \oplus w^0_{A-C} \oplus w^0_{A-D} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \ominus w^0_{A-C-D}$
$\oplus\ w^0_\vee$
$\ominus\ w^0_B \oplus w^0_{A-B} \oplus w^0_{B-C} \oplus w^0_{B-D} \ominus w^0_{A-B-C} \ominus w^0_{A-B-D} \ominus w^0_{B-C-D}$
$\oplus\ w^0_\vee$
$\ominus\ w^0_C \oplus w^0_{A-C} \oplus w^0_{B-C} \oplus w^0_{C-D} \ominus w^0_{A-B-C} \ominus w^0_{A-C-D} \ominus w^0_{B-C-D}$
$\oplus\ w^0_\vee$
$\ominus\ w^0_D \oplus w^0_{A-D} \oplus w^0_{B-D} \oplus w^0_{C-D} \ominus w^0_{A-B-D} \ominus w^0_{A-C-D} \ominus w^0_{B-C-D}$
$\oplus\ w^0_\vee$

$\ominus\ w^0_{A-B} \oplus w^0_{A-B-C} \oplus w^0_{A-B-D} \ominus w^0_\vee$
$\ominus\ w^0_{A-C} \oplus w^0_{A-B-C} \oplus w^0_{A-C-D} \ominus w^0_\vee$
$\ominus\ w^0_{A-D} \oplus w^0_{A-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$
$\ominus\ w^0_{B-C} \oplus w^0_{A-B-C} \oplus w^0_{B-C-D} \ominus w^0_\vee$
$\ominus\ w^0_{B-D} \oplus w^0_{B-C-D} \oplus w^0_{A-B-D} \ominus w^0_\vee$
$\ominus\ w^0_{C-D} \oplus w^0_{B-C-D} \oplus w^0_{A-C-D} \ominus w^0_\vee$
$\ominus\ w^0_{A-B-C} \oplus w^0_\vee$
$\ominus\ w^0_{A-B-D} \oplus w^0_\vee$
$\ominus\ w^0_{A-C-D} \oplus w^0_\vee$
$\ominus\ w^0_{B-C-D} \oplus w^0_\vee$
$\ominus\ w^0_\vee \oplus w_0 \ominus w_0$

$= w^0_{ABCD}$
$\ominus\ w^0_{ABC} \ominus w^0_{ABD} \ominus w^0_{ACD} \ominus w^0_{BCD} \oplus w^0_{AB} \oplus w^0_{AC} \oplus w^0_{AD} \oplus w^0_{BC} \oplus w^0_{BD} \oplus w^0_{CD} \ominus w^0_A \ominus w^0_B \ominus w^0_C \ominus w^0_D \oplus w^0_\vee$

3 APPENDIX C

Möbied weights of knowledge base with one hypothesis and with a) two b) three different questions, provided that all possible source conditional weights w^0_{-} are given.

a) Two questions A B, i.e. we have literals A, B, $\overline{A}$, and $\overline{B}$, where $\overline{A} = \neg A$ and $\overline{B} = \neg B$:

$$
\begin{aligned}
& w_0 \\
& w_{A-B} = w^0_{A-B} \ominus w_0 \\
& w_{A-\overline{B}} = w^0_{A-\overline{B}} \ominus w_0 \\
& w_{B-\overline{A}} = w^0_{B-\overline{A}} \ominus w_0 \\
& w_{\overline{A}-\overline{B}} = w^0_{\overline{A}-\overline{B}} \ominus w_0
\end{aligned}
$$

$$
\begin{aligned}
w_A &= w^0_A \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_0 \\
&= w^0_A \ominus w^0_{A-B} \ominus w^0_{A-\overline{B}} \oplus w_0 \\
w_B &= w^0_B \ominus w_{A-B} \ominus w_{B-\overline{A}} \ominus w_0 \\
&= w^0_B \ominus w^0_{A-B} \ominus w^0_{B-\overline{A}} \oplus w_0 \\
w_{\overline{A}} &= w^0_{\overline{A}} \ominus w_{B-\overline{A}} \ominus w_{\overline{A}-\overline{B}} \ominus w_0 \\
&= w^0_{\overline{A}} \ominus w^0_{B-\overline{A}} \ominus w^0_{\overline{A}-\overline{B}} \oplus w_0 \\
w_{\overline{B}} &= w^0_{\overline{B}} \ominus w_{A-\overline{B}} \ominus w_{\overline{A}-\overline{B}} \ominus w_0 \\
&= w^0_{\overline{B}} \ominus w^0_{A-\overline{B}} \ominus w^0_{\overline{A}-\overline{B}} \oplus w_0
\end{aligned}
$$

$$
\begin{aligned}
w_{AB} &= w^0_{AB} \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_{B-\overline{A}} \ominus w_0 \\
&= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B} \\
w_{A\overline{B}} &= w^0_{A\overline{B}} \ominus w_A \ominus w_{\overline{B}} \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_{\overline{A}-\overline{B}} \ominus w_0 \\
&= w^0_{A\overline{B}} \ominus w^0_A \ominus w^0_{\overline{B}} \oplus w^0_{A-\overline{B}} \\
w_{\overline{A}B} &= w^0_{\overline{A}B} \ominus w_{\overline{A}} \ominus w_B \ominus w_{B-\overline{A}} \ominus w_{A-B} \ominus w_{\overline{A}-\overline{B}} \ominus w_0 \\
&= w^0_{\overline{A}B} \ominus w^0_{\overline{A}} \ominus w^0_B \oplus w^0_{\overline{A}-B} \\
w_{\overline{AB}} &= w^0_{\overline{AB}} \ominus w_{\overline{A}} \ominus w_{\overline{B}} \ominus w_{B-\overline{A}} \ominus w_{A-\overline{B}} \ominus w_{\overline{A}-\overline{B}} \ominus w_0 \\
&= w^0_{\overline{AB}} \ominus w^0_{\overline{A}} \ominus w^0_{\overline{B}} \oplus w^0_{\overline{A}-\overline{B}}
\end{aligned}
$$

Complete derivations of the formulas.

A, B, $\overline{A}$, $\overline{B}$:

$$w_0$$

$$w_{A-B} = w^0_{A-B} \ominus w_0$$

$$
\begin{aligned}
w_A &= w^0_A \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_0 \\
&= w^0_A \ominus w^0_{A-B} \oplus w_0 \ominus w^0_{A-\overline{B}} \oplus w_0 \ominus w_0 \\
&= w^0_A \ominus w^0_{A-B} \ominus w^0_{A-\overline{B}} \oplus w_0
\end{aligned}
$$

$$
\begin{aligned}
w_{AB} &= w^0_{AB} \ominus w_A \ominus w_B \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_{B-\overline{A}} \ominus w_0 \\
&= w^0_{AB} \ominus w^0_A \oplus w^0_{A-B} \oplus w^0_{A-\overline{B}} \ominus w_0 \ominus w^0_B \oplus w^0_{A-B} \oplus w^0_{B-\overline{A}} \ominus w_0 \\
&\quad \ominus w^0_{A-B} \oplus w_0 \ominus w^0_{A-\overline{B}} \oplus w_0 \ominus w^0_{B-\overline{A}} \oplus w_0 \ominus w_0 \\
&= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B}
\end{aligned}
$$

b) Three questions A, B, C, i.e. we have literals A, B, C, $\overline{A}$, $\overline{B}$, and $\overline{C}$:

w_0

$$
\begin{aligned}
w_{A-B-C} &= w^0_{A-B-C} \ominus w_0 \\
w_{A-B-\overline{C}} &= w^0_{A-B-\overline{C}} \ominus w_0 \\
w_{A-\overline{B}-C} &= w^0_{A-\overline{B}-C} \ominus w_0 \\
w_{A-\overline{B}-\overline{C}} &= w^0_{A-\overline{B}-\overline{C}} \ominus w_0 \\
w_{\overline{A}-B-C} &= w^0_{\overline{A}-B-C} \ominus w_0 \\
w_{\overline{A}-B-\overline{C}} &= w^0_{\overline{A}-B-\overline{C}} \ominus w_0 \\
w_{\overline{A}-\overline{B}-C} &= w^0_{\overline{A}-\overline{B}-C} \ominus w_0 \\
w_{\overline{A}-\overline{B}-\overline{C}} &= w^0_{\overline{A}-\overline{B}-\overline{C}} \ominus w_0
\end{aligned}
$$

$$
\begin{aligned}
w_{A-B} &= w^0_{A-B} \ominus w_{A-B-C} \ominus w_{A-B-\overline{C}} \ominus w_0 \\
&= w^0_{A-B} \ominus w^0_{A-B-C} \ominus w^0_{A-B-\overline{C}} \oplus w_0 \\
w_{A-\overline{B}} &= w^0_{A-\overline{B}} \ominus w^0_{A-\overline{B}-C} \ominus w^0_{A-\overline{B}-\overline{C}} \oplus w_0 \\
w_{\overline{A}-B} &= w^0_{\overline{A}-B} \ominus w^0_{\overline{A}-B-C} \ominus w^0_{\overline{A}-B-\overline{C}} \oplus w_0 \\
w_{\overline{A}-\overline{B}} &= w^0_{\overline{A}-\overline{B}} \ominus w^0_{\overline{A}-\overline{B}-C} \ominus w^0_{\overline{A}-\overline{B}-\overline{C}} \oplus w_0 \\
w_{A-C} &= w^0_{A-C} \ominus w^0_{A-B-C} \ominus w^0_{A-\overline{B}-C} \oplus w_0 \\
w_{A-\overline{C}} &= w^0_{A-\overline{C}} \ominus w^0_{A-B-\overline{C}} \ominus w^0_{A-\overline{B}-\overline{C}} \oplus w_0 \\
w_{\overline{A}-C} &= w^0_{\overline{A}-C} \ominus w^0_{\overline{A}-B-C} \ominus w^0_{\overline{A}-\overline{B}-C} \oplus w_0 \\
w_{\overline{A}-\overline{C}} &= w^0_{\overline{A}-\overline{C}} \ominus w^0_{\overline{A}-B-\overline{C}} \ominus w^0_{\overline{A}-\overline{B}-\overline{C}} \oplus w_0 \\
w_{B-C} &= w^0_{B-C} \ominus w^0_{A-B-C} \ominus w^0_{\overline{A}-B-C} \oplus w_0 \\
w_{B-\overline{C}} &= w^0_{B-\overline{C}} \ominus w^0_{A-B-\overline{C}} \ominus w^0_{\overline{A}-B-\overline{C}} \oplus w_0 \\
w_{\overline{B}-C} &= w^0_{\overline{B}-C} \ominus w^0_{A-\overline{B}-C} \ominus w^0_{\overline{A}-\overline{B}-C} \oplus w_0 \\
w_{\overline{B}-\overline{C}} &= w^0_{\overline{B}-\overline{C}} \ominus w^0_{A-\overline{B}-\overline{C}} \ominus w^0_{\overline{A}-\overline{B}-\overline{C}} \oplus w_0
\end{aligned}
$$

$$
\begin{aligned}
w_A &= w^0_A \ominus w_{A-B-C} \ominus w_{A-B-\overline{C}} \ominus w_{A-\overline{B}-C} \ominus w_{A-\overline{B}-\overline{C}} \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_{A-C} \ominus w_{A-\overline{C}} \ominus w_0 \\
&= w^0_A \ominus w^0_{A-B} \ominus w^0_{A-\overline{B}} \ominus w^0_{A-C} \ominus w^0_{A-\overline{C}} \oplus w^0_{A-B-C} \oplus w^0_{A-B-\overline{C}} \oplus w^0_{A-\overline{B}-C} \oplus w^0_{A-\overline{B}-\overline{C}} \ominus w_0 \\
w_B &= w^0_B \ominus w^0_{A-B} \ominus w^0_{B-C} \ominus w^0_{\overline{A}-B} \ominus w^0_{B-\overline{C}} \oplus w^0_{A-B-C} \oplus w^0_{A-B-\overline{C}} \oplus w^0_{\overline{A}-B-C} \oplus w^0_{\overline{A}-B-\overline{C}} \ominus w_0 \\
w_C &= w^0_C \ominus w^0_{A-C} \ominus w^0_{B-C} \ominus w^0_{\overline{A}-C} \ominus w^0_{\overline{B}-C} \oplus w^0_{A-B-C} \oplus w^0_{A-\overline{B}-C} \oplus w^0_{\overline{A}-B-C} \oplus w^0_{\overline{A}-\overline{B}-C} \ominus w_0 \\
w_{\overline{A}} &= w^0_{\overline{A}} \ominus w^0_{\overline{A}-B} \ominus w^0_{\overline{A}-\overline{B}} \ominus w^0_{\overline{A}-C} \ominus w^0_{\overline{A}-\overline{C}} \oplus w^0_{\overline{A}-B-C} \oplus w^0_{\overline{A}-B-\overline{C}} \oplus w^0_{\overline{A}-\overline{B}-C} \oplus w^0_{\overline{A}-\overline{B}-\overline{C}} \ominus w_0 \\
w_{\overline{B}} &= w^0_{\overline{B}} \ominus w^0_{A-\overline{B}} \ominus w^0_{\overline{B}-C} \ominus w^0_{\overline{A}-\overline{B}} \ominus w^0_{\overline{B}-\overline{C}} \oplus w^0_{A-\overline{B}-C} \oplus w^0_{A-\overline{B}-\overline{C}} \oplus w^0_{\overline{A}-\overline{B}-C} \oplus w^0_{\overline{A}-\overline{B}-\overline{C}} \ominus w_0 \\
w_{\overline{C}} &= w^0_{\overline{C}} \ominus w^0_{A-\overline{C}} \ominus w^0_{B-\overline{C}} \ominus w^0_{\overline{A}-\overline{C}} \ominus w^0_{\overline{B}-\overline{C}} \oplus w^0_{A-B-\overline{C}} \oplus w^0_{A-\overline{B}-\overline{C}} \oplus w^0_{\overline{A}-B-\overline{C}} \oplus w^0_{\overline{A}-\overline{B}-\overline{C}} \ominus w_0
\end{aligned}
$$

$$
\begin{aligned}
w_{AB} &= w^0_{AB} \ominus w_{A-B-C} \ominus w_{A-B-\overline{C}} \ominus w_{A-\overline{B}-C} \ominus w_{A-\overline{B}-\overline{C}} \ominus w_{\overline{A}-B-C} \ominus w_{\overline{A}-B-\overline{C}} \\
&\quad \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_{A-C} \ominus w_{A-\overline{C}} \ominus w_{B-C} \ominus w_{B-\overline{A}} \ominus w_{B-\overline{C}} \ominus w_A \ominus w_B \ominus w_0 \\
&= w^0_{AB} \ominus w^0_A \ominus w^0_B \oplus w^0_{A-B} \\
w_{A\overline{B}} &= w^0_{A\overline{B}} \ominus w^0_A \ominus w^0_{\overline{B}} \oplus w^0_{A-\overline{B}} \\
w_{\overline{A}B} &= w^0_{\overline{A}B} \ominus w^0_{\overline{A}} \ominus w^0_B \oplus w^0_{\overline{A}-B} \\
w_{\overline{AB}} &= w^0_{\overline{AB}} \ominus w^0_{\overline{A}} \ominus w^0_{\overline{B}} \oplus w^0_{\overline{A}-\overline{B}} \\
w_{AC} &= w^0_{AC} \ominus w^0_A \ominus w^0_C \oplus w^0_{A-C} \\
w_{A\overline{C}} &= w^0_{A\overline{C}} \ominus w^0_{\overline{C}} \ominus w^0_C \oplus w^0_{A-\overline{C}} \\
w_{\overline{A}C} &= w^0_{\overline{A}C} \ominus w^0_{\overline{A}} \ominus w^0_C \oplus w^0_{\overline{A}-C} \\
w_{\overline{AC}} &= w^0_{\overline{AC}} \ominus w^0_{\overline{A}} \ominus w^0_{\overline{C}} \oplus w^0_{\overline{A}-\overline{C}}
\end{aligned}
$$

$$
\begin{aligned}
w_{BC} &= w^0_{BC} \ominus w^0_{B} \ominus w^0_{C} \oplus w^0_{B-C} \\
w_{B\overline{C}} &= w^0_{B\overline{C}} \ominus w^0_{B} \ominus w^0_{\overline{C}} \oplus w^0_{B-\overline{C}} \\
w_{\overline{B}C} &= w^0_{\overline{B}C} \ominus w^0_{\overline{B}} \ominus w^0_{C} \oplus w^0_{\overline{B}-C} \\
w_{\overline{BC}} &= w^0_{\overline{BC}} \ominus w^0_{\overline{B}} \ominus w^0_{\overline{C}} \oplus w^0_{\overline{B}-\overline{C}}
\end{aligned}
$$

$$
\begin{aligned}
w_{ABC} = {} & w^0_{ABC} \ominus w_{A-B-C} \ominus w_{A-B-\overline{C}} \ominus w_{A-\overline{B}-C} \ominus w_{A-\overline{B}-\overline{C}} \ominus w_{\overline{A}-B-C} \ominus w_{\overline{A}-B-\overline{C}} \ominus w_{\overline{A}-\overline{B}-C} \\
& \ominus w_{A-B} \ominus w_{A-\overline{B}} \ominus w_{A-C} \ominus w_{A-\overline{C}} \ominus w_{B-C} \ominus w_{B-\overline{A}} \ominus w_{B-\overline{C}} \ominus w_{\overline{A}-C} \ominus w_{\overline{B}-C} \\
& \ominus w_{A} \ominus w_{B} \ominus w_{C} \ominus w_{AB} \ominus w_{AC} \ominus w_{BC} \ominus w_{0} \\
= {} & w^0_{ABC} \ominus w^0_{AB} \ominus w^0_{AC} \oplus w^0_{BC} \oplus w^0_{A} \oplus w^0_{B} \oplus w^0_{C} \ominus w_{A-B-C} \\
w_{AB\overline{C}} = {} & w^0_{AB\overline{C}} \ominus w^0_{AB} \ominus w^0_{A\overline{C}} \oplus w^0_{B\overline{C}} \oplus w^0_{A} \oplus w^0_{B} \oplus w^0_{\overline{C}} \ominus w_{A-B-\overline{C}} \\
w_{A\overline{B}C} = {} & w^0_{A\overline{B}C} \ominus w^0_{A\overline{B}} \ominus w^0_{AC} \oplus w^0_{\overline{B}C} \oplus w^0_{A} \oplus w^0_{\overline{B}} \oplus w^0_{C} \ominus w_{A-\overline{B}-C} \\
w_{A\overline{BC}} = {} & w^0_{A\overline{BC}} \ominus w^0_{A\overline{B}} \ominus w^0_{A\overline{C}} \oplus w^0_{\overline{BC}} \oplus w^0_{A} \oplus w^0_{\overline{B}} \oplus w^0_{\overline{C}} \ominus w_{A-\overline{B}-\overline{C}} \\
w_{\overline{A}BC} = {} & w^0_{\overline{A}BC} \ominus w^0_{\overline{A}B} \ominus w^0_{\overline{A}C} \oplus w^0_{BC} \oplus w^0_{\overline{A}} \oplus w^0_{B} \oplus w^0_{C} \ominus w_{\overline{A}-B-C} \\
w_{\overline{A}B\overline{C}} = {} & w^0_{\overline{A}B\overline{C}} \ominus w^0_{\overline{A}B} \ominus w^0_{\overline{AC}} \oplus w^0_{B\overline{C}} \oplus w^0_{\overline{A}} \oplus w^0_{B} \oplus w^0_{\overline{C}} \ominus w_{\overline{A}-B-\overline{C}} \\
w_{\overline{AB}C} = {} & w^0_{\overline{AB}C} \ominus w^0_{\overline{AB}} \ominus w^0_{\overline{A}C} \oplus w^0_{\overline{B}C} \oplus w^0_{\overline{A}} \oplus w^0_{\overline{B}} \oplus w^0_{C} \ominus w_{\overline{A}-\overline{B}-C} \\
w_{\overline{ABC}} = {} & w^0_{\overline{ABC}} \ominus w^0_{\overline{AB}} \ominus w^0_{\overline{AC}} \oplus w^0_{\overline{BC}} \oplus w^0_{\overline{A}} \oplus w^0_{\overline{B}} \oplus w^0_{\overline{C}} \ominus w_{\overline{A}-\overline{B}-\overline{C}}
\end{aligned}
$$

We can observe that, for conjunctive rules (but not simple ones), we obtain *the same Möbied weights* as in the case of the same number of literalss of different questions (i.e. $w_{A-B-\ldots-N}$, $w_{AB\ldots K}$ and $w_{AB\ldots N}$ are the same).

4 APPENDIX D

Counter-examples, see section 4.

a) Let us consider the following founded *ecd* knowledge base:

$\Rightarrow H(w_0)$
$A \vee B \vee C \vee D \Rightarrow H(w^0_\vee)$
$A \vee C \vee D \Rightarrow H(w^0_{A-C-D})$
$B \vee C \Rightarrow H(w_0)$
$A \Rightarrow H(w^0_A)$
$B \Rightarrow H(w^0_B)$
$C \Rightarrow H(w^0_C)$
$D \Rightarrow H(w^0_D)$
$C\&D \Rightarrow H(w^0_{CD})$
$A\&B\&D \Rightarrow H(w^0_{ABD})$

We know, that no disjunctive rules are to be added, thus we have to compute Möbied weights for conjunctive rules only. So it is not necessary to estimate implicit weights of dijunctive rules.

Estimations of implicit weights:

$w^x_{AB} = w^0_A \oplus w^0_B \ominus w_0$
$w^x_{AC} = w^0_A \oplus w^0_C \ominus w_0$
$w^x_{AD} = w^0_A \oplus w^0_D \ominus w_0$
$w^x_{BC} = w^0_B \oplus w^0_C \ominus w_0$
$w^x_{BD} = w^0_B \oplus w^0_D \ominus w_0$
$w^x_{ABC} = w^0_A \oplus w^0_B \oplus w^0_C \ominus 2w_0$
$w^x_{ACD} = w^0_{CD} \oplus w^0_A \ominus w_0$
$w^x_{BCD} = w^0_{CD} \oplus w^0_B \ominus w_0$
$w^x_{ABCD} = w^0_{ABD} \oplus w^0_{CD} \ominus w^0_D$

Möbied weights:

w_0
$w_\vee = w^0_\vee \ominus w_0$
$w_{A-C-D} = w^0_{A-C-D} \ominus w^0_\vee$
$w_{B-C} = w^0_{B-C} \ominus w^0_\vee = w_0 \ominus w^0_\vee$
$w_A = w^0_A \ominus w^0_{A-C-D}$
$w_B = w^0_B \ominus w_0$
$w_C = w^0_C \ominus w_0 \ominus w^0_{A-C-D} \oplus w^0_\vee$
$w_D = w^0_D \ominus w^0_{A-C-D}$
$w_{AB} = w^0_\vee \ominus w_0$
$w_{AC} = w^0_{A-C-D} \ominus w_0$
$w_{AD} = w^0_{A-C-D} \ominus w_0$
$w_{BC} = w^0_{B-C} \ominus w_0 = 0$!!!
$w_{BD} = w^0_\vee \ominus w_0$
$w_{CD} = w^0_{CD} \ominus w^0_C \ominus w^0_D \oplus w^0_{A-C-D}$
$w_{ABC} = w^0_\vee \ominus w_0$
$w_{ABD} = w^0_{ABD} \ominus w^0_\vee \ominus w^0_A \ominus w^0_B \ominus w^0_D \oplus 3w_0$ $\quad (3w_0 = w_0 \oplus w_0 \oplus w_0)$
$w_{ACD} = w_0 \ominus w^0_{A-C-D}$
$w_{BCD} = w_0 \ominus w^0_\vee$
$w_{ABCD} = w^0_\vee \ominus w_0$

non stated weights of disjunctive rules w_{A-B-C}, w_{A-B-D}, w_{B-C-D}, w_{A-B}, w_{A-C}, w_{A-D}, w_{A-D}, w_{C-D} are equal to zero, i.e. rules are not included in Möbius transform.

We can easily verify that our present example correspond to lemmata from section 4. But, $w_{BC} = 0$ thus the rule $B\&C \Rightarrow H$ is not added into Möbius transform, even if his antecedent $B\&C$ is subconjunction of added conjunctive translation of antecedent of maximal disjunctive rule.

b) Lemmata 4.2 and 4.3 do not hold for weakly founded *ecd* knowledge bases. Let us consider the following weakly founded *ecd* knowledge base.

$\Rightarrow H(w_0)$
$A \vee B \vee C \Rightarrow H(w^0_{A-B-C})$
$A \vee D \Rightarrow H(w^0_{A-D})$
$B \Rightarrow H(w^0_B)$
$A\&C \Rightarrow H(w^0_{AC})$
$B\&D \Rightarrow H(w^0_{BD})$

Similarly as before, it is not necessary to estimate implicit weights of dijunctive rules.

Estimations of implicit weights:

$w^x_A = w^0_{A-B-C} \oplus w^0_{A-D} \ominus w_0$
$w^x_C = w^0_{A-B-C}$
$w^x_D = w^0_{A-D}$
$w^x_{AB} = w^0_B \oplus w^0_{A-D} \ominus w_0$
$w^x_{AD} = w^0_{A-D} \oplus w^0_{A-B-C} \ominus w_0$
$w^x_{BC} = w^0_B$
$w^x_{CD} = w^0_{A-D} \oplus w^0_{A-B-C} \ominus w_0$
$w^x_{ABC} = w^0_B \oplus w^0_{AC} \ominus w_0$
$w^x_{ABD} = w^0_{BD}$
$w^x_{ACD} = w^0_{AC}$
$w^x_{BCD} = w^0_{BD}$
$w^x_{ABCD} = w^0_{AC} \oplus w^0_{BD} \ominus w^0$

Möbied weights:

w_0
$w_{A-B-C} = w^0_{A-B-C} \ominus w_0$
$w_{A-D} = w^0_{A-D} \ominus w^0$
$w_A = 0$
$w_B = w^0_B \ominus w^0_{A-B-C}$
$w_C = 0$
$w_D = 0$
$w_{AB} = 0$
$w_{AC} = w^0_{AC} \ominus w^0_{A-B-C} \ominus w^0_{A-D} \oplus w_0$
$w_{AD} = 0$!!!
$w_{BC} = 0$
$w_{BD} = w^0_{BD} \ominus w^0_B \ominus w^0_{A-D} \oplus w_0$
$w_{CD} = 0$
$w_{ABC} = w^0_{A-B-C} \ominus w_0$
$w_{ABD} = 0$
$w_{ACD} = 0$
$w_{BCD} = 0$
$w_{ABCD} = w^0_{A-D} \ominus w_0$!!!

non stated weights of disjunctive rules are equal to zero again.

Rule $A\&B\&C\&D \Rightarrow H(w^0_{A-D} \ominus w_0)$ is addedd into Möbius transform even if the antecedent of the rule is neither conjunctive translation of an antecedent of any disjunctive rule nor its subconjunction. And vice-versa, conjunctive translation of maximal disjunctive rule $A \vee D \Rightarrow H$ is not added.

c) Let us consider the previous knowledge base extended with rule $A \Rightarrow H(w^0_A)$.

$\Rightarrow H(w_0)$
$A \vee B \vee C \Rightarrow H(w^0_{A-B-C})$
$A \vee D \Rightarrow H(w^0_{A-D})$
$A \Rightarrow H(w^0_A)$
$B \Rightarrow H(w^0_B)$
$A\&C \Rightarrow H(w^0_{AC})$
$B\&D \Rightarrow H(w^0_{BD})$

Similarly as before, it is not necessary to estimate implicit weights of dijunctive rules.

Estimations of implicit weights:
$w^x_C = w^0_{A-B-C}$
$w^x_D = w^0_{A-D}$
$w^x_{AB} = w^0_A \oplus w^0_B \ominus w_0$
$w^x_{AD} = w^0_A$
$w^x_{BC} = w^0_B$
$w^x_{CD} = w^0_{A-D} \oplus w^0_{A-B-C} \ominus w_0$
$w^x_{ABC} = w^0_B \oplus w^0_{AC} \ominus w_0$
$w^x_{ABD} = w^0_A \oplus w^0_{BD} \ominus w_0$
$w^x_{ACD} = w^0_{AC}$
$w^x_{BCD} = w^0_{BD}$
$w^x_{ABCD} = w^0_{AC} \oplus w^0_{BD} \ominus w^0$

Möbied weights:
w_0
$w_{A-B-C} = w^0_{A-B-C} \ominus w_0$
$w_{A-D} = w^0_{A-D} \ominus w^0$
$w_A = w^0_A \ominus w^0_{A-B-C} \ominus w^0_{A-D} \oplus w_0$
$w_B = w^0_B \ominus w^0_{A-B-C}$
$w_{AB} = w^0_{A-B-C} \ominus w_0$
$w_{AC} = w^0_{AC} \ominus w^0_A$
$w_{BD} = w^0_{BD} \ominus w^0_B \ominus w^0_{A-D} \oplus w_0$
$w_{ABC} = 0$!!!
$w_{ABD} = w^0_{A-D} \ominus w_0$!!!
$w_{ABCD} = 0$!

$w_C = w_D = w_{AD} = w_{BC} = w_{CD} = w_{ACD} = w_{BCD} = 0$,
non stated weights of disjunctive rules are also equal to zero.

Rule $A\&B\&D \Rightarrow H(w^0_{A-D} \ominus w_0)$ is addedd into Möbius transform even if the antecedent of the rule is neither conjunctive translation of an antecedent of any disjunctive rule nor its subconjunction. And vice-versa, conjunctive translation of maximal disjunctive rule $A \vee D \Rightarrow H$ is not added.