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Technical report No. 711

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Potentiality of Chaotic Behavior in Single Neuron is Natural

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Abstract

It is shown analytically that the sigmoidal form of transfer function is important and sufficient condition for a single neuron to behave chaotically. Some possible consequences of this profound result upon an EEG analysis problem are shortly mentioned.

Keywords

McCulloch-Pitts like neuron model, sigmoidal transfer function, single neuron chaotic dynamics, conditions for chaotic behavior.

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1 Introduction

The ubiquitous feature of the nervous system is the wide spread occurrence of complex dynamical behaviors. Examples range from the neural spike trains of single neurons to the fluctuating potentials of thousands of neurons measured from the surface of the brain by EEG [1-3].

Experimental studies have shown that chaotic behavior can be observed experimentally even in a single neuron [1-2, 4-6]. Later on it was clarified numerically that responses of biological neuron modelled as the simplified Caianello [7], and Nagumo and Sato [8] models with an influence of the refractoriness due to a past firing decreases exponentially with time, can also be chaotic [9]. Besides, it was shown numerically with the Hodgkin-Huxley equations [10] that responses of a resting nerve membrane to periodic stimulation are not always periodic and can be understood as deterministic chaos [5-6, 9].

Recently neurodynamics with the nonmonotone response function on a single neuron has been reported by Morita [11-12]. Such neurons can behave chaotically too, as has been shown these days [13-14].

Let us mention in this connection that the dynamics of single neuron plays a very important role in recently very actively studying artificial neural networks. These are composer of simple elements of artificial neurons modeling, as mentioned above, biological neurons. Such neural networks are then called chaotic neural networks [9, 15].

But so far there are only numerical evidences for chaotic behavior in biological neuron models caused by a time delay (refractoriness) in the model [9, 16] or by the nonmonotonicity of transfer function of the model [13,14].

In this paper we will show analytically that in the basic McCulloch-Pitts like neuron model [17] with biologically confirmed sigmoidal response function [18], the potentiality of chaotic dynamics is rather natural.

2 Potentiality of dynamic chaos in a single neuron with the sigmoidal transfer function

A usual neuron model is a simple threshold element transforming a weighted summation of the inputs into the output through a nonlinear transfer function with a threshold. In what follows we will use a generalization of classical McCulloch-Pitts neuron model [17] in which instead of the unit step function the sigmoidal transfer function will be exploited. Then for an i -th neuron dynamics, holds

$$y_i = g(\xi_i(t_n)) = \frac{1}{1 + e^{-\lambda \xi_i}} \quad (2.1)$$

where

$$\xi_i(t_n) = h_i - \Theta_i = \sum_j w_{ij} x_j(t_n) - \Theta_i \quad (2.2)$$

Here $y_i(t_{n+1})$ is the output of the i th neuron at the discrete time t_{n+1} ; g is the sigmoidal transfer function with a slope λ ; Θ is the axon hillock threshold of i th neuron; w_{ij} (for $i \neq j$) is the connection weight with which the firing of the j th neuron affects the i th neuron and $h_i(t_n)$ is the local field of i th neuron at the discrete time t_n . One can also speak about the input dynamics (2.2) and the output dynamics (2.1).

Now a transition from the discrete time dynamics (2.1) and (2.2) to a continuous time dynamics can be easily made [19]. Let's denote $\Delta t = t_n - t_{n-1}$ and suppose $\Delta t \ll 1$. Then from (2.2) we have $\frac{1}{\Delta t} (\xi_i(t_n) - \xi_i(t_{n-1})) = -\frac{1}{\Delta t} (\xi_i(t_{n-1}) - \xi_i(t_n))$ and in the limit, where $\Delta t \rightarrow 0$, we get for the input dynamics

$$\frac{d\xi_i}{dt} = -\alpha \left(\xi_i(t_{n-1}) - \frac{\partial E}{\partial y_i} \right) \quad (2.3)$$

where $\alpha = \frac{1}{\Delta t}$ is a parameter and E is the Hopfield-like energy function

$$E = \frac{1}{2} \sum_{i,j} (w_{i,j} x_j y_i - \Theta_i y_i) \quad (2.4)$$

Analogously one gets of (2.1) the time continuous output dynamics (2.5).

$$\frac{dy_i}{dt} = -\alpha (y_i - g(\xi_i)) \quad (2.5)$$

Here we are interesting especially in the output dynamics as we are looking for a potentiality of chaotic response of such neurons. And to this end very interesting properties of sigmoidal transfer function can be exploited now. Namely, from (2.1) one gets directly

$$\frac{dy_i}{d\xi_i} = \lambda y_i (1 - y_i) \quad (2.6)$$

what is the Ricatti-like equation.

To analyse the output dynamics of the i th neuron one performs the Taylor expansion of the output variable y_i in time, namely

$$\begin{aligned} y_i(t_{n+1}) &= y_i(t_n) + \frac{dy_i}{d\xi_i} d\xi_i + \dots = \\ &= y_i(t_n) + \frac{dy_i}{d\xi_i} \frac{d\xi_i}{dt} \Delta t + \dots \end{aligned} \quad (2.7)$$

where again we suppose $\Delta t = t_{n+1} - t_n \ll 1$. After substituting from (2.3) and (2.6) into (2.7), neglecting terms with $(\Delta t)^k$, $k \geq 2$, one gets after some elementary calculations, suprisingly

$$y(t_{n+1}) = y(t_n) + 4ay(t_n)(1 - y(t_n)) \quad (2.8)$$

where

$$a = \frac{\lambda}{4} (\xi(t_{n+1}) - \xi(t_n)) \quad (2.9)$$

For the sake of simplicity we have omitted the index i , as the analysis is performed for the i th neuron. For the same reason we can put $y(t_n) \equiv y_n$ and $\xi(t_n) \equiv \xi_n$. Then (2.8) can be prescribed in the form

$$y_{n+1} = y_n + 4ay_n(1 - y_n) \quad (2.10)$$

where

$$a = \frac{\lambda}{4} (\xi_{n+1} - \xi_n) \quad (2.11)$$

Introducing a substitution

$$z_n = 4ay_n - \frac{1 + 4a}{2} \quad (2.12)$$

into (2.10) one gets an equivalent form of logistic equation for z 's, namely

$$z_{n+1} = c - z_n^2 \quad (2.13)$$

where $c = 4a^2 - \frac{1}{4}$ is the parameter and a is determined by (2.11).

Let's note that the logistic map (2.13), on the interval $\langle -1, 1 \rangle$, where c varies in the interval $(0, 2)$, coincides with the more accustomed logistic map (2.14)

$$x_{n+1} = bx_n(1 - x_n) \quad (2.14)$$

mapping the interval $\langle 1 - \frac{b}{4}, \frac{b}{4} \rangle$ into itself when $2 < b \leq 4$. Really, under the coordinate change $x = \left(\frac{b}{4} - \frac{1}{2}\right)z + \frac{1}{2}$ with the parameter identification $c = \frac{b^2}{4} - \frac{b}{2}$, both families (2.13) and (2.14) coincide [20].

But it is well known that both (2.13) and (2.14) possess a chaotic behavior. It means there exists some critical value of parameter c_{crit} in (2.13) or b_{crit} in (2.14) such that for $c \geq c_{crit}$ or $b \geq b_{crit}$ solutions of (2.13) or (2.14) are chaotic. For (2.13) then one has $c_{crit} = 1.8284$, or for (2.14) then $b_{crit} = 3.8284$ [20].

So far we have been able to show analytically that the output dynamics, or firing patterns, of the generalized McCulloch-Pitts like model of single neuron with the sigmoidal transfer function contains inherently deterministic chaos. By other words, chaotic behavior is natural in neuronal dynamics.

3 Consequences of single neuron complex dynamics

The output dynamics described as the first approximation by the logistic equation (2.13) is much more complex as it can seem at the first sight. Namely the parameter c , as can be seen from (2.2) and (2.9), is in general the function

$$c = c(\lambda, \Theta, n) \quad (3.1)$$

That means the firing pattern of neuron depends crucially upon basic characteristics of sigmoidal transfer function, namely the threshold Θ and the slope λ . But as we already mentioned Θ represents the axon hillock threshold (bias) and from the biological point of view is a measure of sensitivity (attention). So if Θ increases then the sensitivity

decreases and vice versa. On the other hand the parameter λ presents the steepness of sigmoidal transfer function and from the biological point of view this is connected with the excitability (irritation) [21]. We will not go to details here but only mention case of onset of chaos in (2.13). For this special case it follows immediately from (2.2) and (2.9)

$$\lambda_{crit} = \frac{4a_{crit}}{h_{crit} - \Theta_{crit}} \quad (3.2)$$

where $a_{crit} = \frac{1}{2}\sqrt{c_{crit} + \frac{1}{4}}$ and $c_{crit} = 1.8284$ [20]. Surprisingly enough at the onset of potential chaotic behavior we have a functional dependence between basic characteristics of neuronal dynamics, namely $\lambda = \lambda(\Theta)$ [21-22].

4 Conclusions

We have demonstrated for the first time analytically the potentiality of chaotic behavior in the single McCulloch-Pitts like neuron model with the sigmoidal transfer function. It means such neurons can generate cyclic firing patterns of all possible periods, bursting patterns, as well as aperiodic patterns, and all possible combinations of these.

The results obtained here can shed light on important and so far open problems in today neuroscience, when simulation studies are required that use more-sophisticated single neuron models [1-2,23-24]. E.g., let's mention the role of the cortical neuron in the famous problem if such a neuron is the integrator or coincidence detector [23]. Our results are in accordance with an idea that has been the basis for leading experimental paradigm in neurophysiology from the 1950's up to present days, namely that of representation of relevant propositions about the outside world in single neurons [25-26].

On the other hand from the chaodynamics [27] we know that the behavior of system consisted of nonlinear units with chaotic behavior, can be very complex in a wide spectrum of qualitatively different types of behavior going from a regular behavior to spatiotemporal high dimensional chaos and to more complex structures. Alas, so far we do not know rules for classifying such kinds of behavior. In spite of this there have been some attempts to apply, ad hoc, methods of theory of chaos to EEG signals analysis to show that human and animal EEGs are "deterministic chaotic processes" [28-30]. As EEGs are suited to measure average properties of fairly large populations of neurons, in the spirit of what have been said above as well as of our results presented here concerning the potentiality of chaos in single neurons one must be very careful in applying such methods directly to macroscopic EEG signals analysis.

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