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Wiedermann, Jiří
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Jiří Wiedermann

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Institute of Computer Science, Academy of Sciences of the Czech Republic
Pod vodárenskou věží 2, 182 07 Prague 8, Czech Republic
phone: (+422) 6605 3520 fax: (+422) 8585789
e-mail: wieder@uivt.cas.cz

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Abstract

A new finite computational structure — a so–called *cogitoid* — is proposed. Formally it is represented by a lattice of concepts with two basic operations — abstraction and concretization. Its computational behaviour supports a formation of new concepts and of both excitatory and inhibitory associations among them. It is proved that cogitoids are able to realize behaviour elicited by the presentation of specific stimulus–response patterns, such as retrieval by causality, learning of sequences, learning of composed concepts from partial ones, or similarity based behaviour. Also instances of Pavlovian conditioning can be acquired. Final examples give the plausible algorithmic explanation of the so–called operant conditioning that is determined by the positive or negative stimuli which occur after the responses. The case of delayed reinforcement is also handled.

Keywords

Computational Models of the Brain; Neurocomputing; Cognitive Science

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Contents

1 Introduction

The idea of thinking about the brain as being a computational device that can be mathematically formalized and explained, has attracted a lot of attention in artificial intelligence, cognitive sciences, and in computer science. Within the latter science we are nowadays witnessing rapid development especially in the related field of neurocomputing. Besides various kinds of neural nets designed for specific learning tasks, computational models of the brain, or mind, based on paradigms of neurocomputing, have also emerged (for a recent overview cf. [1] or [6]). Along these lines, the *memory surface model* by Goldschlager [5], and the *neural tabula rasa* by Valiant [7] seem to belong among the most elaborated models (for a brief overview of these, and other computational models of the brain, cf. [8]). Unfortunately, so far none of these or related models appears to be able to formally, mathematically treat more complicated cognitive tasks, for various reasons. In the case of memory surface this is because the Goldschlager's model is not precise enough in details that are necessary in any formal, or semi–formal reasoning. Also, the mechanism of inhibition that seems to be necessary to explain some cases of operant conditioning is missing in his model. On the other hand, this model offers a good conceptual framework for the explanation of basic cognitive processes, such as concept and association formation, and even offers a plausible "high–level" explanation of some higher brain functions. Contrary to this, Valiant's approach is very precise in details. It offers the notion of the so–called neuroid, a kind of a neuron that can be programmed to fullfil various atomic cognitive tasks out of which more complicated tasks ought to be assembled. At the same time this seems to introduce some limits to the potential of this model: to explain, or model more complicated cognitive tasks at the level of actions of individual neuroids, moreover in a probabilistic setting, is not easy. A similar objection holds for other "bottom–up"

approaches starting at neuronal level.

In this circumstance a sufficiently high–level model of the brain that would concentrate onto the global aspects of mental processes and would be sufficiently formal to eventually allow an exact mathematical reasoning, but at the same time would abstract from "implementation details" at the neuronal level, could be useful.

In the paper at hand a candidate for such a model — a so–called cogitoid, is presented. The basic entities which the model deals with are concepts. Formally, the cogitoid is represented by a lattice of concepts with operation of abstraction and concretization. Moreover, in the course of cogitoid's computation associations among concepts keep developing. The model is not programmable in the standard sense, but can be trained to perform certain cognitive tasks by providing it with the right sequence of inputs.

The paper is organized as follows. In section 2, main ideas behind cogitoid's definition are presented. This is followed by a semi–formal definition of a cogitoid inclusively its computational behaviour.

Next, in section 3 it is shown that the model as defined in the previous section is sound, in a certain pragmatic sense: it presents not only a suitable framework for the description and formalization of certain cognitive tasks, as described in various experiments with animals, but is able to also offer an algorithmic explanation of the respective phenomena, with sufficient accuracy. Its minimality is supported by the fact that the proofs of the respective theorems concerning the cognitive power of cogitoids make use of all of model's features.

The above mentioned properties of cogitoids are first demonstrated on simple cognitive tasks such as the acquisition of a behaviour elicited by the repeated presentation of specific stimulus–response patterns, acquisition of sequences, the emergence of composed concepts from simpler ones, or the behaviour elicited by similar circumstances. Next, examples of standard textbook instances of Pavlovian conditioning are shown. This kind of behaviour refers to the fact that animals can be conditioned in such a way that they will tend to activate a concept that is apparently unrelated to the stimulus at hand. Final examples also give the plausible algorithmic explanation of the so–called operant conditioning. This is the behaviour that is acquired by exposing an animal to circumstances in which a certain stimuli–response behaviour is consistently rewarded or punished *after* the response is obtained. The case of delayed reinforcement is considered as well.

Finally, section 4 contains some open problems and conclusions.

2 The Cogitoid

In the sequel we shall give a semi–formal definition of a cogitoid and of its computational behaviour. First, we shall review some basic notion concerning lattices (cf. [2]).

A *lattice* $\mathcal{L}(\Sigma)$ over the set Σ is the set Σ that is partially ordered by the relation $\leq$, that has the least element ℓ and the greatest element g, and any two elements a and b of Σ have a supremum $a \vee b$ called *join* and an infimum $a \wedge b$ called *meet*. For any $\Pi \subseteq \Sigma$ the algebra $\mathcal{L}(\Pi)$ is a *sublattice* of $\mathcal{L}(\Sigma)$ iff $\mathcal{L}(\Pi)$ itself is a lattice. Note than

for any $a \in \Sigma$ and for $\Pi = \{b \in \Sigma | b \leq a\}$ the system $\mathcal{L}(\Pi)$ is a sublattice (in fact, it is a principal ideal) denoted as $\mathcal{L}_a$. Its greatest element is a while its least element is ℓ.

A *lattice of concepts* is a lattice whose elements are called *concepts*. For any two elements a and b of such a lattice, with $a \leq b$, we say that a is an *abstraction* of b, while b is a *concretization* of a. Then, a supremum of any two of its elements is the smallest concretization of these elements, while their infimum is the largest abstraction of these elements. We shall say that two concepts are *non–meeting* iff their largest abstraction is equal to the least element of the respective lattice.

For any finite lattice $\mathcal{L}(\Sigma)$ the respective cogitoid $\mathcal{C}$ can be seen as a *transducer* that translates an infinite input string σ over the finite alphabet 2^Σ into an infinite output string γ, also over 2^Σ. Thus, the elements of both σ and γ are sets of concepts. The elements of σ are sometimes also called *contexts* while those of γ *behaviors*, or *actions*.

Remark: Assuming $|\Sigma| = 2^n$, each symbol in Σ can be represented by a Boolean vector of length n. Each of these vectors can be seen as a *characteristic vector* of some set w.r.t. to some fixed *universe* $\mathcal{U} = \{x_1, x_2, \ldots, x_n\}$ consisting of n elements that are called *features*. Similarly each symbol from 2^Σ can be represented as a Boolean vector of length 2^n that in turn can be seen by a characteristic vector of some subset of $2^\mathcal{U}$.

Thus, there is a natural correspondence between the symbols of Σ and 2^Σ and subsets of $\mathcal{U}$ and $2^\mathcal{U}$. Consequently, concepts can be seen as subsets of $\mathcal{U}$, i.e., as sets of features, while contexts and behaviors as sets of concepts. In this case the corresponding lattice of concepts is simply the set $2^\mathcal{U}$ of all subsets of $\mathcal{U}$. The sets in $2^\mathcal{U}$ are ordered by the relation "$\subseteq$" of set inclusion. The operation of set union corresponds to the concretization operation while that of set intersection to the operation of abstraction. Then, the empty set plays the role of the least element while the entire universum the role of the greatest element. In such a case the cogitoid can also be seen as a transducer $\{0,1\}^{2^n} \rightarrow \{0,1\}^{2^n}$. This is basically the framework in which the first ideas about cogitoids were originally presented in [9].

Any ordered pair of form (a, b) of concepts is called an *association* of a with b (or between a and b), written also $a \rightarrow b$. We also say that a and b are then correlated by *causality*. The associations appear in two forms: as excitatory ones, denoted as $(a, b)^+$, and inhibitory ones, denoted as $(a, b)^-$.

In a cogitoid, at each time t each concept or association can be present with a certain *strength* that depends on the history of previous computations. The strength is a non–negative integer. We shall say that a concept or association is *present* in $\mathcal{C}$ at time t iff its strength at time t is positive. The strength of a concept or of an association is increased each time when the respective concepts are activated.

At time t each concept that is present in $\mathcal{C}$ can be in any of the two possible states — either in an *active*, or in a *passive*, state.

In cogitoids, new concepts are formed from existing ones by principles of simultaneous appearance, and by resemblance. If two concepts a and b are activated simultaneously (what shall be sometimes called as the appearance of $\{a, b\}$), they give rise to a single concretization $a \vee b$. We also say that then a and b are correlated by *simultaneous occurrence*.

If $a \wedge b = c \neq \ell$, then we shall say that the concept a resembles the concept b (or a is similar to b) in c. Otherwise, when $r = \ell$, the concepts are considered not to be similar. When a is similar to b we also say that the former two concepts are correlated by *similarity*, or *resemblance* via the concept c. Then, under certain conditions, the activation of a may give rise to the activation of the concept b that resembles a in c (cf. Phase 2(a) in the following description of computational behaviour of cogitoids).

Besides new concepts also new associations keep emerging in cogitoids. A new association $a \rightarrow b$ will appear whenever some concept b is activated immediately after a was activated in the previous step.

In any cogitoid there are two distinguished disjoint sets A^+ and A^-, respectively, of non–meeting concepts, called positive or negative *affects*, or *operant concepts*. They correspond to pleasurable or painful feelings of animals.

The affects determine the so–called *quality* of concepts. The quality of a concept at time t is an element from the set $\{+1, -1, \pm 1\}$ and is defined only for concepts active at time t. The exact rules for assigning quality to concepts are described in Phase 3 of the following description of computational behavior of a cogitoid. The quality of a concept determines the kind of association among this concept and other concepts (cf. Phase 4(b) in the sequel).

A *configuration* c_t of $\mathcal{C}$ at time t is a complete list of all concepts and associations present in $\mathcal{C}$ at time t, inclusively their strengths, and in case of concepts also inclusively their qualities and states.

The computation of a cogitoid consists of an infinite sequence of computational steps.

Let c_t be the configuration of $\mathcal{C}$ at the beginning of the t–th computational step, let O_t be the set of all active concepts in c_t.

Let the set $I_t \in \sigma$ be the set of concepts at the input of $\mathcal{C}$ at time t, for some $t \geq 0$, let $i_t = \vee_{a \in I_t} a$.

We shall show that $\mathcal{C}$ maintains the following invariant that holds at the beginning of t–th computational step: O_t is a union of two sublattices of $\mathcal{L}(\Sigma)$ with ℓ being the least element of both of them. The first sublattice is $\mathcal{L}_{i_{t-1}}$ and the second one $\mathcal{L}_a$ for some $a \in O_t$.

At time $t = 0$ no concepts and no associations are present in $\mathcal{C}$. Thus, $O_t = \emptyset$ and the invariant holds vacuously.

For any $t > 0$, assume that the cogitoid finds itself in a configuration c_t, with O_t being the set of active concepts that fulfills the previous invariant.

We shall show that after performing the t–th computational step the above invariant will be restored.

The t-th computational step consists of 6 phases:

Phase 1: *Input/Output:*

(a) *The Input:* The t–th symbol I_t from the input string σ is read. Subsequently, all concepts from I_t are activated by external stimuli. This gives rise to the activation of the concretization $i_t = \vee_{a \in I_t} a$ by the virtue of *simultaneous occurrence*. In parallel, all concepts in $\mathcal{L}_{i_t}$ are also activated by the virtue of simultaneous occurrence; this is because the activation of i_t alone is interpreted as though all

of its abstractions were present simultaneously at the input of $\mathcal{C}$.

(b) *The Output:* The t–th symbol of the output string γ corresponding to the behaviour O_t is produced as the output at time t.

Phase 2: *Activation of New Concepts by Internal Stimuli:* Based on the set of O_t of active concepts the so–called *selection mechanism* will activate new concepts.

Let P_t be the set of all passive concepts in c_t. The selection mechanism keeps in check the number of active concepts and works as follows. It first determines the *excitation* of all concepts in P_t that are correlated with the set of active concepts O_t via the relation of resemblance or via associations:

(a) *Excitation by Resemblance:* Let $p \in O_t$, let $q \in P_t$ be a concept that resembles p in some $r \neq \ell$, i.e., $p \wedge q = r$. Then the excitation of the concept $p \vee q$ at time t is defined as $E(p \vee q, t) = \sum_{x \in O_t, x \leq p} W(x,t) + \sum_{y \in O_t, y \leq q} W(y,t) + \sum_{z \in O_t, z \leq p \vee q} W(z,t)$, where $W(x,t)$ is the strength of x at time t.

(b) *Excitation via Associations:* Let $p \in O_t$, let $q \in P_t$ be the concept to which p is associated via some excitatory association $(p,q)^+$ and/or via some inhibitory association $(p,q)^-$ (if one of the both is missing than it is treated as being present with the strength 0) Then the excitation of q at time t is defined as $E(q,t) = \sum_{p \in O_t} [W((p,q)^+, t) - W((p,q)^-, t)]$, where $W((p,q)^+, t)$ and $W((p,q)^-, t)$ are the strengths of the respective associations at time t.

The selection mechanism then activates the maximally excited concept a among all concepts excited by resemblance or via associations. If there are more concepts $a_1, a_2, \ldots, a_k$ equally excited satisfying the maximality condition, then their concretization $a = a_1 \vee a_2 \ldots \vee a_k$ get activated.

Activation by Simultaneous Occurrence: Let a be the concept activated by the selection mechanism. Then all the concepts in the sublattice $\mathcal{L}_a$ get activated by the virtue of simultaneous occurrence at time t.

Phase 3: *Determining the Quality of Concepts:* Let $A_t = \mathcal{L}_{i_t} \cup O_t \cup \mathcal{L}_a$ be the set of all active concepts at this time. The quality of each concept in A_t is determined according to the following rules. The quality of an active negative affect is -1, of an active positive affect is $+1$. From concepts that have already obtained their quality this quality propagates "upwards" to all larger concepts (concretizations) and "downwards" to all smaller concepts (abstractions). Should some concept obtain in this way both the quality $+1$ and -1, it gets the quality ± 1. The quality of all concepts to which the quality cannot be assigned by the previous rules is $+1$.

Phase 4: *Long–Term Memorization:*

(a) *Strengthening of Concepts:* The strength of all concepts in A_t is increased by some fixed positive constant c_1.

(b) *Strengthening of Associations:* The strength of associations between all subsequently occurring different active concepts is increased by some fixed value

$c_2 \geq c_1$. This is done as follows: the first concept in the pair is selected from the set O_t (since the respective concepts have been activated by the end of $(t-1)$–st step) while the second one in the pair is chosen from the set $\mathcal{L}_{i_t} \cup \mathcal{L}_a - O_t$ (since the respective concepts have been activated in the present, t–th step). If the quality of the first concepts was negative, then the inhibitory association, if it was negative, then the excitatory one is strengthened. Otherwise, when the quality of the first concept was ± 1, then both excitatory and inhibitory associations get strengthened. Moreover, if in the association at hand its first concept was activated via external stimuli in the previous step, then the amount c_3 of this association strengthening is somewhat larger then in other cases.

Phase 5: *Gradual Forgetting:* The strength of all concepts that are not active at this very moment, and the strength of all association among them, is decreased by some fixed amount $c_0 \leq c_1$.

Phase 6: *Deactivation:* The concepts in $O_t - \mathcal{L}_{i_t} - \mathcal{L}_a$ are deactivated. The set of currently active concepts remains only $\mathcal{L}_{i_t} \cup \mathcal{L}_a$, and this is going to be the set of all active concepts at the beginning of the next step.

The previous six phases uniquely determine the next configuration c_{t+1} of $\mathcal{C}$. Note that in Phase 6 the invariant has been restored.

From the previous description it is seen that a cogitoid is completely specified by the respective lattice, affects, and the constants $c_0 \leq c_1 \leq c_2 \leq c_3$. The computational mechanism of all cogitoids is the same.

3 Simple Cognitive Tasks

3.1 Behavioristic Learning

The previous principles are enough to ensure the elicitation of any desired behaviour of a cogitoid solely by purposefully externally applied stimulus–response pattern via the activation of the respective concepts. Namely, by repeating the required stimulus–response pattern enough times, strong concepts representing stimuli and the respective responses are formed with strong associations among them. Then, any later activation of a stimulating concept will invoke the respective response. This can be done with many different patterns, and patterns can be presented to a cogitoid in a random order, and some of them also in parallel, grouped arbitrarily.

The details are given in the next theorem:

Theorem 3.1 *Let $X = \{x_1, x_2, \ldots, x_k\}$ and $Y = \{y_1, y_2, \ldots, y_k\}$, $k > 0$, be two disjoint sets of non–meeting concepts. We shall say that a pair $\{x_i, y_i\}$, for any $i = 1, 2, \ldots, k$, is presented sequentially to a cogitoid $\mathcal{C}$ at time t iff x_i is at the input of $\mathcal{C}$ at time t, y_i is at the input at time $t+1$ and ℓ (the least element of the respective lattice) is at the input at time $t+2$. Let pairs be presented randomly to a cogitoid $\mathcal{C}$, in any order, with the possibility of presenting a few randomly chosen pairs at the same time.*

Then, later on, after each pair has been presented to $\mathcal{C}$ several times, whenever x_i is presented to $\mathcal{C}$ at some step, y_i will be activated in the next step, for any $1 \leq i \leq k$.

Sketch of the proof: First assume that at time t the pair $\{x_i, y_i\}$ alone is presented sequentially to $\mathcal{C}$ for the first time. Thus, at time t the concept x_i is activated from the input (in Phase 1(a), in Section 2) and as a member of A_t it will remain active also at the beginning of time $t+1$ when y_i will be activated from the input. Then, according to Phase 4(b) the association $x_i \rightarrow y_i$ will be established by setting its strength to the value c_3. This association will be in the future strengthened at any similar occasion. Thus, when after a few repetitions of the above process the successor association $x_i \rightarrow y_i$ will become the strongest one among all associations invoked by x_i, whenever x_i will be activated, y_i will get in turn activated by the virtue of Phase 2(b). The above arguments hold for any $1 \leq i \leq k$.

Note that when more randomly chosen pairs will be presented to $\mathcal{C}$ at the same time, then concepts joining all concepts appearing simultaneously will also emerge. Nevertheless, the probability that such concepts will reappear in the near future is small and therefore they will tend to be forgotten due to Phase 5.

□

In a similar manner as above also sequences of concepts can be acquired:

Theorem 3.2 *Let $(x_1, x_2, \ldots, x_k)$, $k > 1$, be a sequence of non–meeting concepts in which each concept occurs exactly once. Let the pairs $\{x_i, x_{i+1}\}$ be sequentially presented to a cogitoid $\mathcal{C}$, in any random order, with the possibility of presenting a few randomly chosen pairs at the same time, for any $k > i > 0$.*

Then, later on, after each pair has been presented to $\mathcal{C}$ several times, whenever x_i alone is presented to $\mathcal{C}$ at some step, for $1 \leq i < k$, concepts $x_{i+1}, \ldots, x_k$ will be subsequently activated in $k - i$ next steps, one concept at each step.

Sketch of the proof: Under the assumptions of the theorem, similarly as in the previous proof, associations of form $x_i \rightarrow x_{i+1}$ start to emerge in $\mathcal{C}$, for any $k > i > 0$. After these associations start to be strong enough to activate x_{i+1} whenever x_i gets activated, the cogitoid will start to behave as predicted in the theorem.

□

The next theorem shows that in a cogitoid more complex, joint concepts are spontaneously built from simple concepts that resemble each other:

Theorem 3.3 *Let $\mathcal{C}$ be any cogitoid, let a, b be any concepts that resemble each other in some $r = a \wedge b \neq \ell$. Than the repeated unrelated activation of a or b (i.e., a and b can be activated at different times) will give rise to the joint concept $a \vee b$ that will be activated whenever either a or b is activated.*

Sketch of the proof: After both a and b have already been activated a few times both concepts will get a positive strength. Then, whenever a (or b) gets activated it will by resemblance (cf. Phase 2(a) from Section 2) excite $a \vee b$. In the presence of no other stronger excitation the selection mechanism will eventually activate $a \vee b$.

□

The last theorem in this section is concerned with the "similarity based" behaviour that forms the basis of more complex behaviors:

Theorem 3.4 *Let $\mathcal{C}$ be any cogitoid, let $X = \{x_1, x_2, \ldots, x_k\}$ be a set of concepts, let $y \notin X$ be a concept. Let the pairs $\{x_i, y\}$ be sequentially presented to $\mathcal{C}$ in any random order.*

Then, later on, whenever any concept $x \notin X$, so far unrelated to y, is presented to $\mathcal{C}$ at time t, then

- *when x resembles x_i for some $1 \leq i \leq k$, then y will be activated at time $t+1$;*
- *when x does not resemble any x_i for all $1 \leq i \leq k$, y will not be activated at time $t+1$.*

Sketch of the proof: By presenting sequentially the above pairs to the cogitoid not only associations of form $x_i \rightarrow y$ will emerge. Due to the computational mechanism of a cogitoid, also associations of form $z \rightarrow y$ for all abstractions z of x_i. will appear.

Now, let later on some $x \notin X$ appear at the input at time t. By the definition, x resembles some x_i iff $x \wedge x_i \neq \ell$.

Therefore, when x resembles x_i the selection mechanism will activate the concept $x \vee x_i$ (cf. Phase 2(a) in Section 2) and also all abstractions of this concept. Among them, also abstractions of x_i will get activated which in turn, in the next step, will activate y as stated in the theorem.

However, it is obvious that when x does not resemble any x_i, the activation of x will have no effect on y since x excites no abstraction of any x_i.

□

3.2 Pavlovian Conditioning

Translated to our terminology, the Pavlovian conditioning is a phenomenon in which an animal can be conditioned to activate a concept as a response to an apparently unrelated stimulating concept (cf. [7], p. 217).

This behaviour and its variations can be precised, modeled, and explained in the framework of cogitoids in the following way:.

Theorem 3.5 *Let there be a cogitoid $\mathcal{C}$ containing non–meeting concepts a, b, r, s in which such a strong association $s \rightarrow r$ has been established that s, a stimulus, alone, and only s, elicits r, a response. Let $\mathcal{C}$ undergo the* basic training process *that consists of repeating the following two steps a few times:*

- *at the beginning of a computational cycle, s appears simultaneously with another concept a, as the set $\{a, s\}$, at the input of $\mathcal{C}$. The concept a has so far no particular associations to r;*
- *at the beginning of the next computational cycle, there in "no input" to $\mathcal{C}$ — i.e., the minimal abstraction ℓ corresponding to the empty input is activated;*

Then:

(a) during the basic training process $\mathcal{C}$ starts to activate r at every second computational step;

(b) when after the end of the basic training process the input a alone is presented to $\mathcal{C}$ in the first step, and ℓ in the second step, $\mathcal{C}$ will activate r in the third step, for some time;

(c) (extinction): *after some time, instead of answering r in the third step, $\mathcal{C}$ will start to answer ℓ;*

(d) (inhibition): *moreover, when after the end of the basic training process $\mathcal{C}$ will undergo an additional training consisting of repeatedly performing the next two steps:*

- *in the first step, an input chosen randomly from the set $\{\{a, s\}, \{a, b\}\}$ is presented to $\mathcal{C}$, where the concept b is a stimulus that does not elicit r by itself;*
- *in the next step, ℓ is presented to $\mathcal{C}$,*

then, after some time, a alone, presented in the first step, will continue to activate r in the third step, while $\{a, b\}$ presented simultaneously will activate ℓ in the third step.

Sketch of the proof: To see the first claim, note that the simultaneous occurrence of $\{a, s\}$ at the input in the first step will activate and strengthen $a \vee s$ and both a and s, according to Phase 1(a) and 4(a). In the next step, s will activate r since, according to our assumption, there is a strong association $s \to r$. Moreover, since ℓ appears at the input, in this step the associations $a \to \ell$, $s \to \ell$, and $a \vee s \to \ell$ also start to establish themselves very strongly, as associations between input concepts, according to the rules in Phase 4(b). Nevertheless, for some time, thanks to the strong initial association $s \to r$, the response r will be the most excited concept and hence the cogitoid keeps activating r in every second step due to the rule in Phase 2(b).

As far as the second claim is concerned, observe that after some time, when the concept $a \vee s$ becomes strong enough, being repeatedly at the input, the appearance of a alone, at some time, will activate, in the second step, $a \vee s$ by the relation of resemblance (Phase 2(a)) and, in turn, still in the same step, also a and s as the abstractions of $a \vee s$. Finally, in the third step, s will activate r. Here we assume that thanks to this initially strong association the excitation of r is for some time greater than that of ℓ (coming from a, s, and $a \vee s$) in this step.

The extinction of r's activation will come into power when the lastly mentioned activation of r will be overruled by the activation of ℓ, since the respective associations from a, s and $a \vee s$ to ℓ are repeatedly strengthened when empty inputs in the second step is read by $\mathcal{C}$.

The inhibition can be explained by similar mechanisms as before. Namely, during the additional training, concepts $a \vee s$ and $a \vee b$ start to establish themselves, similarly as the associations of all combinations (joins) of concepts a, b, and s to ℓ.

When a alone appears at the input, in the next step, a activates $a \vee s$ by similarity rather than $a \vee b$ since due to the basic training $a \vee s$ has a very strong presence. This is followed by immediate activation of s and a (the last rule in Phase 2). In the third step the selection mechanism of $\mathcal{C}$ eventually activates r, via the initially strongly established association $s \rightarrow r$. Here we have to assume that via this association r is excited stronger then ℓ is via the previously mentioned associations, and stronger than $a \vee b$ is excited by resemblance .

However, when $\{a, b\}$ appears at the input, a, b, and $a \vee b$ are activated in the first step from the external stimuli. In the second step, not only $a \vee s$ is activated again, but a, b, and $a \vee b$ remain active also, as members of A_t. In the third step, ℓ, rather than r, will be the most excited concept (by the joint effort of all combinations of a,b, and s) and therefore will be activated. All other concepts will be deactivated in Phase 5 of this step. □

Note that in order to explain the Pavlovian conditioning no use of negative operant concepts and of the related inhibitory associations was needed. Also observe that in the case of extinction and inhibition the cogitoid keeps activating some concepts in its second and third step, but others than r.

3.3 Operant Conditioning

Now we show that our model is also able to realize so–called *operant behaviour*. This is a behaviour acquired, shaped, and maintained by stimuli occurring *after* the responses rather than before. Thus, the invocation of a certain response concept r is confirmed as a "good one" (by invoking the positive operant concept p) or "bad one" (the negative operant concept n) only after r has been invoked. It is the reward (p), or punishment (n) that act to enhance the likelihood of r being re–invoked under the similar circumstances as before.

The real problem here is hidden in the last statement which says that r should be re–invoked (or not re–invoked) only under similar circumstances as before. Thus, inhibition, or excitation of r must not depend on s alone: in some contexts, r should be inhibited, and in others, excited.

The implementation of such a behaviour is described in the following theorem. In this theorem the context in which the conditioning should occur is called *operant context*; it is represented by a concept that appears invariantly as the part of the input of a cogitoid during the circumstances at hand.

Theorem 3.6 *Let $\mathcal{C}$ be any cogitoid containing the non–meeting concepts a, n, r, p, s and z. Let $t_1 < t_2 < \ldots$ be the distinct times, with $t_{i+1} > t_i + 3$. Let a strong association $s \rightarrow r$ be established in $\mathcal{C}$ prior to time t_1. Let at times $t_1 < t_2 < \ldots$ $\mathcal{C}$ finds itself in the operant context z for four subsequent steps (i.e., in times t_i, $t_i + 1$, $t_i + 2$, and $t_i + 3$, for $i = 1, 2, \ldots$).*

Let $\mathcal{C}$ undergo either of following two trainings:

(a) Negative Conditioning: *Let s, a stimulus, appear at the input at time $t_i + 1$ and let the activation of r, a response, at time $t_i + 2$ will be punished by the negative operant concept n appearing at the input at time $t_i + 3$.*

Then there exists a constant $k > 0$ such that for $i = 1, 2, \ldots, k$, s elicits r at time $t_i + 2$, whereas for $i = k + 1, \ldots$, s will not elicit r at time $t_i + 2$.

(b) Positive Conditioning: *Let s appear at the input at time $t_i + 1$ and let the activation of r at time $t_i + 2$ will be rewarded by the positive operant concept p appearing at the input at time $t_i + 3$.*

Then r will be activated at time $t_i + 2$, for $i = 1, 2, \ldots$

Whenever s is presented to $\mathcal{C}$ and z is not active r then r still will be activated in the next step.

Prior to giving the proof of this theorem, a few words to explain its essence are in order.

Observe that the assumptions of the theorem are fairly general. To take a lesson from behaviour confirmed by an operant concept, no particularly intensive, continuous training is needed. It is enough when a similar circumstance, modeled by the concept s and the operant context z will occur a few times (i.e., at times $t_1 < t_2 < \ldots$), possibly with long intervals between the successive occurrences. When the subsequently elicited action r will be always punished, the theorem says that after only a few punishments the cogitoid ceases to invoke r at these occasions. The "similarity" of circumstances is modeled by the requirement that the operant context z remains the same at all pertinent occasions. This is fulfilled e.g. in cases when, at all times, $\mathcal{C}$ finds itself in configurations that share a subset of sufficiently strong concepts that alone uniquely characterize the event.

Sketch of the proof: The idea of the proof is as follows: we have to achieve that whenever at each time t_i after the time t_1 the concepts z and s appear simultaneously at the input, $\mathcal{C}$ has to "recall" what it did at a similar occasion in the past, i.e., at time t_{i-1} and shorty afterwards. Then it has to accept the decision that was either approved by p (i.e., reinforce the excitation of r), or rejected by n (inhibit the excitation of r) at time $t_{i-1} + 3$. This is achieved by "coupling" the operant context z with the respective affect $a \in \{p, n\}$ and by building the association $z \vee a \rightarrow r$. Then, whenever at any future similar occasion, $z \vee a$ will be active jointly with s, depending on the quality of a, $z \vee a \rightarrow r$ will reinforce, or inhibit, the activation of r.

In a more detail this idea works as follows. Consider the state of affairs in our cogitoid at time t_1.

At this time, $\mathcal{C}$ finds itself in the operant context z and therefore z appears at the input for the first time.

At time $t_1 + 1$, z keeps appearing at the input, with s also appearing simultaneously. This will activate $s \vee z$ and reactivate z and s by the rules from Phase 1(a) in Section 2 . Subsequently, in the next step, r will be activated via $s \rightarrow r$, and the association $z \rightarrow r$ and $z \vee s \rightarrow r$ will emerge.

In turn, at time $t_1 + 3$, external activation of an operant concept a will follow, according to our assumptions. Thanks to our assumption on z, this concept will still be at the input, and thanks to its simultaneous occurrence with a, a composed concept $z \vee a$ will thus appear.

Next time, when z appears at the input, both $z \vee s$ and $z \vee a$ will be reminded (activated) by similarity in the next step, since both are equally excited (see the selection mechanism from Phase 2, Section 2). Hence, a joint concept $z \vee s \vee a$ will be established. In the next step, the association $z \vee s \vee a \rightarrow r$ will emerge for the first time.

This scenario is repeated a few times, with the latter association becoming repeatedly stronger. At some point, say at time t_k, it becomes so strong that its influence to the excitation of r is no longer negligible.

Thus, in the case of negative conditioning, from this time on this association will prevent r from being activated, since it inherits the negative strength from $a = n$. However, in the case of positive conditioning this association will reinforce the activation of r.

Assume now that z is not active, and s appears at the input. In the next step, the activation of r by causality competes with the activation of $z \vee s \vee a$ by similarity. In this case we may safely suppose that the former activation will be selected by the selection mechanism. This is because we have assumed that the strength of the association $s \rightarrow r$ was great initially and therefore the excitation of r is greater than that of $z \vee s \vee a$.

□

From the previous theorem and from its proof the following interesting fact follows. When, at time $t_1 + 2$, the response r is activated and it is a correct one, confirmed by p, the cogitoid behaves like having already acquired the right behavior. But from the proof it is seen that this is not yet the case, since r is activated by the stimulus s alone, not yet taking p into account. In the case of an incorrect answer the cogitoid takes a proper lesson very fast and after a few exercises behaves "correctly". Note, however, that due to Phase 5, in the long run the cogitoid tends to forget what it has learned in this case, since the inhibitory connections are not strengthened any longer, because r is not invoked.

In practice this means that behaviour acknowledged by positive rewards looks like it is being acquired faster than the one acknowledged by negative rewards. Moreover, in the long run, if not rewarded, the inhibited behaviour tends to be forgotten. This seems to correspond well to our everyday experience — reinforcing good habits is easier than suppressing wrong ones.

3.4 Delayed Reinforcement

It appears that by a similar mechanism that ties a certain operant concept to some temporarily prevailing operant context one can also explain a more complicated case of the so–called *delayed reinforcing* when the reinforcing stimulus — a punishment or a reward — does not necessarily appear immediately after the step to be reinforced.

Thus, we shall deal with a problem when a punishment appears only after the

cogitoid activates a previously acquired sequence of concepts in a way described in Theorem 3.2.

Theorem 3.7 *Let a sequence of non–meeting concepts $(x_1, x_2, \ldots, x_k)$, $k > 1$, in which each concept occurs exactly once, be acquired by a cogitoid $\mathcal{C}$. Let i be fixed, with $1 \leq i < k$, let z be an operant context that is active only during the activities of $x_{i+1}, \ldots, x_k$.*

Then, if each activation of x_k subsequently invoked by the initial activation of x_1 (cf. Theorem 3.2) will be punished by activating a negative operant concept n, then, after this happens a few times, activation of x_1 will activate $x_2, \ldots, x_i$, but none of x_j, for $i < j \leq k$.

Sketch of the proof: The similar mechanism as in the Theorem 3.6, viz. the repeated punishment of x_k by n in the context z, will give rise to the negatively reinforcing association $z \vee n \vee x_{k-1} \rightarrow x_k$. Thus, under the circumstances at hand after several repetitions of the previous experience $\mathcal{C}$ ceases to activate x_k, but will still activate x_{k-1} (when $i < k-1$). Nevertheless, the latter activation will then by resemblance activate the concept $z \vee n \vee x_{k-1}$ that has emerged previously. This concept will act like a punishment for the activation of x_{k-1}, and hence the negatively reinforcing association $z \vee n \vee x_{k-2} \rightarrow x_{k-1}$ will eventually emerge. After some time, it will start to inhibit the activation of x_{k-1}. The similar process repeats itself for x_{k-3}, etc., until it reaches x_i. Then, clearly, in accordance with Theorem 3.2 activation of x_1 will still activate the concepts $x_2, \ldots, x_i$. Nevertheless, unlike the preceeding ones, the latter concept, x_i, will be active simultaneously with the operant context z whose association with the negative operant concept n will cause that none of x_j, for $i < j \leq k$, will be activated. □

Based on the proof of the previous theorem it appears that cogitoids are also able to realize a strategy that is similar to the so–called *backtracking* technique used in searching for the accepting solutions in decision trees. In this case, at each potential branching point cogitoid must find itself in a specific operant context. When some sequence of concept activation presents a "blind alley" (i.e., leads to a punishment) in this context, this operant context will get associated with the negative operant concept much in the same way as in the proof of theorem 3.6. This will subsequently enable the cogitoid to return to the branching point and, eventually, to take an other path. However, we will abstain from stating the respective theorem since its proper formulation requires a lot of technical assumptions.

4 Conclusions and Open Problems

The contribution of this paper is twofold. First, a new formal finite model of mind has been introduced. It presents a deviation from similar models that are based on paradigms of neurocomputing. This is because it abstracts from brain–like implementations and instead focuses on the substance of mental processes — viz. the concept and association calculus. Secondly, the viability of this model has been demonstrated

by its capability to exactly formulate the instances of Pavlovian and operant conditioning and to give their plausible algorithmic explanation. This seems to be the first occasion when a computational justification of the related mental phenomena, in a form usual in computer science, is given.

Of course, the underlying emerging theory of cogitoids has a lot of open ends. First of all, the versatility of the model must by further verified on other cognitive tasks described in experimental psychology. This will probably lead to further tuning of the model. But already now, in its present form the model seems to offer an interesting and promising alternative for studying the computational aspects of mental processes. It opens ways for its possible computer simulation. It appears that with the help of this model algorithmic explanation of higher brain functions (such as the concept of self, consciousness, thinking, language acquisition and generation, etc.) could be also possible. Such possibilities are subject of the author's current research in this field. The work by [4] or [5] can serve as a vital source of inspiration along these lines.

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