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Abstract

This study presents how the Möbius transform can be used for Max-Min compositions of rules of the CADIAG-2. The algorithm for construction of Möbius transform to find new weights of rules for CADIAG-2 is proposed. This method is tested for different examples and some remarks are indicated.

Keywords

MaxMin inference, Möbius transform, CADIAG-2

Preface (by P. Hájek)

This report contains Mr. Nguyen's elaboration of my suggestion to extend Möbius transform (in the sense of MYCIN-like systems, (Hájek, Valdes, 1994)) to CADIAG-like fuzzy expert systems, extended by negative weights. The new and slightly surprising result is that non-invertibility of the maximum operation does not make the transform impossible provided we carefully combine positive and negative weights.
This contributes to our observation that CADIAG-like systems are very close to MYCIN-like systems, even if we keep maximum as the combining operation for positive weights. I want to stress that this means that CADIAG-like systems have both similar advantages as MYCIN-like systems (ease of inference) and similar *disadvantages*, namely the fact that truth-functionality (use of combining functions) prevents consequent understanding of weights as degrees of belief. Methods like Möbius transform or guarded use give only partial correctness, as discussed at large in (Hájek, Havránek, Jiroušek, 1992, Chap. VI-VIII). The main question remains:
If things as relative frequencies are used as weights of implications (rules) and fuzzy inference is applied, what meaning have the results obtained ? (see Hájek, Harmancová, 1995).
It is hoped that the present report brings a partial contribution to a future answer to this question.

1. Introduction

CADIAG-2 is a medical diagnostic expert system based on Max-Min inference. The rule base of CADIAG-2 consists of rules with the form IF(antecedent) THEN(succedent). Degrees of truth of rules in CADIAG-2 may be used as relative frequencies or their fuzzifications (Adlassnig, 1986; Adlassnig et al., 1986). In (Hájek, Nguyen, 1995), we have studied how CADIAG-2 is embedded into MYCIN-like systems if we replace Max of MaxMin composition of CADIAG-2 by a suitable t-cornom and we propose including negative knowledge for CADIAG-2 and it indicates that CADIAG-2 with confirmation and exclusion gives the same results at the corresponding MYCIN-like system. In (Hájek, Havránek, Jiroušek, 1992) an algorithm of Möbius transform for MYCIN-like systems which allows to determine the weight of a rule from the corresponding expert's belief was proposed. The new rule base produces global weights compatible with the expert's beliefs. In this study, the question is that how much the Möbius transform can be used for Max-Min compositions of rules of CADIAG-2. The answer is that it is possible, but only if negative weights are introduced. The paper is organized as follows: Section 2 presents an algorithm for construction of Möbius transform for MaxMin inference of CADIAG-2 allowing to find new weights such that the values of composition of rules satisfying to expert's beliefs. Section 3 verifies several examples by the above described algorithm and finally, some conclusions are reported.

2. Construction of Möbius transform for CADIAG-2

For construction of Möbius transform algorithm for CADIAG-2, we need add some definitions extending CADIAG-2 by negative knowledge

Definition 1:
A fuzzy patient data for patient P_q consists of values $\mu^+_{R_{PS}}(P_q, S_i)$ - degree of confirmation and $\mu^-_{R_{PS}}(P_q, S_i)$ - degree of exclusion for $i = 1, ..., m$. Assume that, at least, $\mu^+_{R_{PS}}(P_q, S_i)$ or $\mu^-_{R_{PS}}(P_q, S_i) = 0$ and let
- $\mu^+_{R_{PS}}(P_q, S_i) = 0$ and $\mu^-_{R_{PS}}(P_q, S_i) = 0$ mean symptoms S_i - unknown for patient P_q
- $\mu^+_{R_{PS}}(P_q, S_i) = 1$ means symptoms S_i - surely present for patient P_q.
- $\mu^-_{R_{PS}}(P_q, S_i) = 1$ means symptoms S_i - surely absent for patient P_q.

Definition 2:
The patient data $\mu^+_{R_{PS}}(P_q, S_i)$ and $\mu^-_{R_{PS}}(P_q, S_i)$ (for $i = 1, ..., m$) are three-valued for patient P_q, if for all S_i, $\mu^+_{R_{PS}}(P_q, S_i)$ and $\mu^-_{R_{PS}}(P_q, S_i)$ take value 0 or 1. Then $\mu^+_{R_{PS}}(P_q, S_i)$ and $\mu^-_{R_{PS}}(P_q, S_i)$ determine an elementary conjunction E_q of symptoms S_i such that S_i occurs in E_q positively if $\mu^+_{R_{PS}}(P_q, S_i) = 1$ and negatively if $\mu^-_{R_{PS}}(P_q, S_i) = 1$.

For example, given a fuzzy patient data in Table 1.

P_q	S_1	S_2	S_3	S_4
$\mu^+_{R_{PS}}(P_q, S_i)$	1	0	0	0
$\mu^-_{R_{PS}}(P_q, S_i)$	0	0	1	0

Table 1: A patient data

where, S_1, S_2, S_3, S_4 - Symptoms
P_q - Patient q
$\mu^+_{R_{PS}}(P_q, S_i)$, $\mu^-_{R_{PS}}(P_q, S_i)$ are values of the patient data

From Table 1, the following elementary conjunction of symptoms S_i for patient P_q is constructed:

$$E_q = S_1 \& \neg S_3$$

Definition 3:
The values $\mu^+_{R_{PS}}(P_q, \neg S_i)$, $\mu^-_{R_{PS}}(P_q, \neg S_i)$ of patient data for patient P_q are defined as follows

$$\mu^+_{R_{PS}}(P_q, \neg S_i) = \mu^-_{R_{PS}}(P_q, S_i)$$

$$\mu^-_{R_{PS}}(P_q, \neg S_i) = \mu^+_{R_{PS}}(P_q, S_i)$$

Definition 4:
An elementary conjunction E_q of symptoms S_i is defined by

$$E_q = (\varepsilon_1)S_1 \&, ..., \&(\varepsilon_m)S_m$$

(recall the notion $(0)S_i = \neg S_i$, $(1)S_i = S_i$)
If for each i, $i = 1, .., m$, $\mu^-_{R_{PS}}(P_q, (\varepsilon_i)S_i) = 0$ then

$$\mu^+_{R_{PS}}(P_q, E_q) = min_{S_i \in E_q}(\mu^+_{R_{PS}}(P_q, (\varepsilon_i)S_i))$$

$$\mu^-_{R_{PS}}(P_q, E_q) = 0$$

If there is i, $\mu^-_{R_{PS}}(P_q,(\varepsilon_i)S_i) > 0$ then

$$\mu^-_{R_{PS}}(P_q,E_q) = max_{S_i \in E_q}(\mu^-_{R_{PS}}(P_q,(\varepsilon_i)S_i))$$

$$\mu^+_{R_{PS}}(P_q,E_q) = 0$$

The value of an elementary conjunction E_q of symptoms S_i is defined

$$\mu_{R^{tot}_{PS}}(P_q,E_q) = \mu^+_{R_{PS}}(P_q,E_q) - \mu^-_{R_{PS}}(P_q,E_q) \tag{2.0}$$

Recall that a value $\mu_{R^+_{SD}}(E_i,D_j)$ in [0,1] used for confirmation of diagnosis, where the value $\mu_{R^+_{SD}}(E_i,D_j)$ indicates degree in which a symptom (or elementary conjunction of symptoms) E_i confirms a diagnosis D_j. The MaxMin composition of rules for confirmation of diagnosis is

$$R^+_{PD} = R_{PS} \circ R^+_{SD} \tag{2.1}$$

defined by

$$\mu_{R^+_{PD}}(P_q,D_j) = Max_{E_i \in Sys} Min(\mu^+_{R_{PS}}(P_q,E_i);\mu_{R^+_{SD}}(E_i,D_j)) \tag{2.2}$$

We extend CADIAG-2 by a relation R^-_{SD} defined by $\mu_{R^-_{SD}}(E_i,D_j)$ (E_i is a symptom or elementary conjunction of symptoms) in [0,1], where the value $\mu_{R^-_{SD}}(E_i,D_j)$ indicates degree in which a symptom (or elementary conjunction of symptoms) E_i excludes a diagnosis D_j. Thus, the following MaxMin composition of rules proposed and used to deduce the degree of exclusion of the disease D_j for the patient P_q from the observed symptoms E_i is follows:

$$R^-_{PD} = R_{PS} \circ R^-_{SD} \tag{2.3}$$

defined by

$$\mu_{R^-_{PD}}(P_q,D_j) = Max_{E_i \in Sys} Min(\mu^+_{R_{PS}}(P_q,E_i);\mu_{R^-_{SD}}(E_i,D_j)) \tag{2.4}$$

where Sys - a set of symptoms E_i

Definition 5:
A rule base Θ given by $\mu_{R^+_{SD}}(E_i,D_j)$ and $\mu_{R^-_{SD}}(E_i,D_j)$ consists of rules:

$$E_i \longrightarrow D_j(\mu_{R^+_{SD}}(E_i,D_j)) \tag{2.5}$$

$$E_i \longrightarrow \neg D_j(\mu_{R^-_{SD}}(E_i,D_j)) \tag{2.6}$$

Assume that $\mu_{R^+_{SD}}(E_i,D_j) = 0$ or $\mu_{R^-_{SD}}(E_i,D_j) = 0$ where $\mu_{R^+_{SD}}(E_i,D_j)$, $\mu_{R^-_{SD}}(E_i,D_j)$ are weights of fuzzy rules in [0,1].

Now we are going to define the total degree of confirmation and exclusion of a diagnosis as a combination of degree of confirmation and degree of exclusion. We shall see that it is more convenient to use their difference in the sense of a group operation on (-1,1) than just their difference as reals.

Definition 6:
Given a patient data, the total degree for confirmation and exclusion of diagnosis D_j by patient P_q from observed symptom S_i is:

$$\mu_{R_{PD}^{tot}}(P_q, D_j) = \mu_{R_{PD}^{+}}(P_q, D_j) \ominus \mu_{R_{PD}^{-}}(P_q, D_j) \tag{2.7}$$

in [-1,1]

where
$\mu_{R_{PD}^{+}}(P_q, D_j) = Max_{E_q'} Min[\mu_{R_{PS}}^{+}(P_q, E_q'); \mu_{R_{SD}^{+}}(E_q', D_j)]$
$\mu_{R_{PD}^{-}}(P_q, D_j) = Max_{E_q'} Min[\mu_{R_{PS}}^{+}(P_q, E_q'); \mu_{R_{SD}^{-}}(E_q', D_j)]$
where E_q' varies over all elementary conjunctions of symptoms for which $\mu_{R_{SD}^{+}}(E_q', D_j)$ or $\mu_{R_{SD}^{-}}(E_q', D_j)$ is positive.

Remark: Note that of the patient data are three-valued, i. e. given by an elementary conjunction E_q, then this reduces to $\mu_{R_{PD}^{+}}(P_q, D_j) = Max_{E_q' \subseteq E_q}(\mu_{R_{SD}^{+}}(E_q', D_j))$ and it is similar for $\mu_{R_{PD}^{-}}(P_q, D_j)$

Let us recall some notions on $\oplus$ and $\ominus$ on (-1,1) (Hájek et al.; 1992, 1994)
- Operation $\oplus$ is an odered Abelian group, extended to extremals:

$$1 \oplus x = 1, -1 \oplus x = -1$$

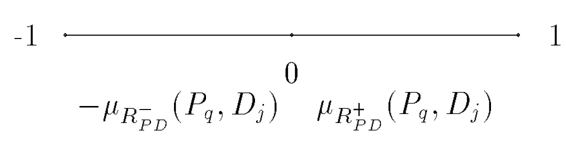
- The PROSPECTOR group operation $\oplus$ on (-1,1) is defined as follows:

$$x \oplus y = \frac{x + y}{1 + xy} \tag{2.8}$$

- Operation $\ominus$ is a group operation defined by

$$x \ominus y = x \oplus -y \tag{2.9}$$

Remark: Let recall that we compare the degree of confirmation $\mu_{R_{PD}^{+}}(P_q, D_j)$ and the degree of exclusion $\mu_{R_{PD}^{-}}(P_q, D_j)$ in [0,1] of diagnosis D_j for patient P_q. One can see the representation of these degrees in [-1,1] in Graph 1.



Graph 1: Representation of $\mu_{R_{PD}^{+}}(P_q, D_j)$ and $\mu_{R_{PD}^{-}}(P_q, D_j)$

To this end we represent the exclusion as negative confirmation, so we take $-\mu_{R^-_{PD}}(P_q, D_j)$ in [-1,1] instead of $\mu_{R^-_{PD}}(P_q, D_j)$ in [0,1].

Definition 7:
A conditional weight system β consists of $\beta^+_{SD}(D_j|E_q)$ and $\beta^-_{SD}(D_j|E_q)$ in [0,1] for a set of pairs (D_j, E_q). Assume that $\beta^+_{SD}(D_j|E_q) = 0$ or $\beta^-_{SD}(D_j|E_q) = 0$, where E_q: elementary conjunction of symptoms S_i

Definition 8:
A total conditional weight system $\beta^{tot}_{SD}(D_j|E_q)$ for a set of pairs $D_j \in Dise$ (Dise: a set of Diseases D_j), $E_q \in EC(Sym)$ (Elementary Conjunction of Symptoms) is defined as follows:

$$\beta^{tot}_{SD}(D_j|E_q) = \beta^+_{SD}(D_j|E_q) - \beta^-_{SD}(D_j|E_q) \tag{2.10}$$

Definition 9:
A conditional weight system β is weakly sound if the following holds for each $E'_q \subseteq E_q \in EC(Sym)$ and $D_j \in Dise$: if $\beta^+_{SD}(D_j|E_q)$, $\beta^-_{SD}(D_j|E_q)$, $\beta^+_{SD}(D_j|E'_q)$, $\beta^-_{SD}(D_j|E'_q)$ are defined and $\beta^+_{SD}(D_j|E'_q)$, $\beta^-_{SD}(D_j|E'_q)$ is extremal (i.e. = 1) (one of them takes value 0), then

$$\beta^+_{SD}(D_j|E'_q) = \beta^+_{SD}(D_j|E_q) \tag{2.11}$$

$$\beta^-_{SD}(D_j|E'_q) = \beta^-_{SD}(D_j|E_q) \tag{2.12}$$

Theorem:
Let β be a weakly sound conditional weight system. Then there is a rule base Θ with new weights $\mu_{R^+_{SD}}(S_i, D_j)$ and $\mu_{R^-_{SD}}(S_i, D_j)$ of fuzzy rules such that for each patient P_q and each three-valued patient data $\mu^+_{R_{PS}}(P_q, S_i)$; $\mu^-_{R_{PS}}(P_q, S_i)$ (therefore E_q exists)

$$\mu_{R^{tot}_{PD}(P_q,D_j)=\beta^{tot}_{SD}}(D_j|E_q) \tag{2.13}$$

whenever the right hand side is defined

Proof:
Fix D_j, we define $\mu_{R^+_{SD}}(E_q, D_j)$ and $\mu_{R^-_{SD}}(E_q, D_j)$ for pairs (E_q, D_j) such that $\beta^+_{SD}(D_j|E_q)$, $\beta^-_{SD}(D_j|E_q)$ are defined.
We proceed by induction on length of E_q.
Case 1:
For each E_q such that $\beta^+_{SD}(D_j|E_q)$, $\beta^-_{SD}(D_j|E_q)$ are defined but $\beta^+_{SD}(D_j|E'_q)$, $\beta^-_{SD}(D_j|E'_q)$ are undefined for each proper subconjunction E'_q of E_q, we put

$$\mu_{R^+_{SD}}(E_q, D_j) = \beta^+_{SD}(D_j|E_q) \tag{2.14}$$

$$\mu_{R^-_{SD}}(E_q, D_j) = \beta^-_{SD}(D_j|E_q) \tag{2.15}$$

Case 2:
If $\beta^+_{SD}(D_j|E_q)$, $\beta^-_{SD}(D_j|E_q)$ are defined and extremal (i.e. $= 1$), then put

$$\mu_{R^+_{SD}}(E_q, D_j) = \beta^+_{SD}(D_j|E_q) \tag{2.16}$$

$$\mu_{R^-_{SD}}(E_q, D_j) = \beta^-_{SD}(D_j|E_q) \tag{2.17}$$

Case 3:
Assume that $\beta^+_{SD}(D_j|E_q)$, $\beta^-_{SD}(D_j|E_q)$ are defined and nonextremal (i.e. $\neq 1$) and $\mu_{R^+_{SD}}(E_q, D_j)$, $\mu_{R^-_{SD}}(E_q, D_j)$ are not yet defined, E_q has some proper subconjunctions $E_q^{'}$ such that $\beta^+_{SD}(D_j|E_q^{'})$, $\beta^-_{SD}(D_j|E_q^{'})$ are defined and for all such $E_q^{'}$, $\mu_{R^+_{SD}}(E_q^{'}, D_j)$, $\mu_{R^-_{SD}}(E_q^{'}, D_j)$ have been defined. Collect positive and negative knowledge M^+ and M^- for D_j under proper subconjunctions $E_q^{'}$ of E_q. Define the total knowledge $M^{tot} = M^+ \ominus M^-$, where M^+, M^- are defined as follows:

$$M^+ = Max_{E_q^{'} \subset E_q}[\mu_{R^+_{SD}}(E_q^{'}, D_j)] \tag{2.18}$$

$$M^- = Max_{E_q^{'} \subset E_q}[\mu_{R^-_{SD}}(E_q^{'}, D_j)] \tag{2.19}$$

We consider the following cases:

a) **If** $M^{tot} = \beta^{tot}_{SD}(D_j|E_q)$ then put

$$\mu_{R^+_{SD}}(E_q, D_j) = \beta^{tot}_{SD}(D_j|E_q)$$

if $\beta^{tot}_{SD}(D_j|E_q) \geq 0$ or

$$\mu_{R^-_{SD}}(E_q, D_j) = \beta^{tot}_{SD}(D_j|E_q)$$

if $\beta^{tot}_{SD}(D_j|E_q) < 0$

b) **If** $M^{tot} < \beta^{tot}_{SD}(D_j|E_q)$ then put

$$\mu_{R^+_{SD}}(E_q, D_j) = M^- \oplus \beta^{tot}_{SD}(D_j|E_q) \tag{2.20}$$

Operation $\oplus$ is defined as in (2.8)

c) **If** $M^{tot} > \beta^{tot}_{SD}(D_j|E_q)$ then put

$$\mu_{R^-_{SD}}(E_q, D_j) = M^+ \ominus \beta^{tot}_{SD}(D_j|E_q) \tag{2.21}$$

Operation $\ominus$ is defined as in (2.9)
and we get (2.13) for each (D_j, E_q) in the domain β.

Proving case 1:
One proves by induction on the length of E_q that eventually $\mu_{R^+_{SD}}(S_i, D_j)$, $\mu_{R^-_{SD}}(S_i, D_j)$ are uniquely defined for each E_q such that $\beta^+_{SD}(D_j|E_q)$, $\beta^-_{SD}(D_j|E_q)$ are defined. We

have (by definition of MaxMin composition of CADIAG-2)

$$\mu_{R^{+}_{PD}}(P_q, D_j) = Max_{E'_q \subseteq E_q} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{+}_{SD}}(E'_q, D_j)]$$
$$= Max_{E'_q \subseteq E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)]$$

(because $\mu_{R^{tot}_{PS}}(P_q, E'_q) = 1$ from (2.0), if E'_q exists, then $\mu^{-}_{R_{PS}}(P_q, E'_q) = 0$)

$$= Max(Max_{E'_q \subset E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)], \mu_{R^{+}_{SD}}(E_q, D_j)) = Max(0, \mu_{R^{+}_{SD}}(E_q, D_j))$$
$$= \mu_{R^{+}_{SD}}(E_q, D_j) = \beta^{+}_{SD}(D_j|E_q)$$

because $Max_{E'_q \subset E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)] = 0$ (due to $\mu_{R^{+}_{SD}}(E'_q, D_j)$ is unknown, when $E'_q \subset E_q$)
In an analogous way, we get

$$\mu_{R^{-}_{PD}}(P_q, D_j) = \mu_{R^{-}_{SD}}(E_q, D_j) = \beta^{-}_{SD}(D_j|E_q)$$

and thus

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \mu_{R^{+}_{PD}}(P_q, D_j) \ominus \mu_{R^{-}_{PD}}(P_q, D_j) = \beta^{tot}_{SD}(D_j, E_q)$$

and the equation (2.13) holds.

Proving case 2:
Given $\beta^{+}_{SD}(D_j, E_q) = 1$ (or $\beta^{-}_{SD}(D_j, E_q) = 1$) we have

$$\mu_{R^{+}_{PD}}(P_q, D_j) = Max_{E'_q \subseteq E_q} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{+}_{SD}}(E'_q, D_j)]$$
$$= Max_{E'_q \subseteq E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)]$$

(because $\mu^{+}_{R_{PS}}(P_q, E'_q) = 1$ from (2.0), if E'_q exists, then $\mu^{-}_{R_{PS}}(P_q, E'_q) = 0$)

$$= Max[Max_{E'_q \subset E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)], \mu_{R^{+}_{SD}}(E_q, D_j)] = \mu_{R^{+}_{SD}}(E_q, D_j) = \beta^{+}_{SD}(D_j|E_q)$$

(because $\mu_{R^{+}_{SD}}(E_q, D_j) = 1$ by condition)
In an analogous way, we get

$$\mu_{R^{-}_{PD}}(P_q, D_j) = \mu_{R^{-}_{SD}}(E_q, D_j) = \beta^{-}_{SD}(D_j|E_q)$$

and thus

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \mu_{R^{+}_{PD}}(P_q, D_j) \ominus \mu_{R^{-}_{PD}}(P_q, D_j) = \beta^{tot}_{PD}(D_j, E_q)$$

and the equation (2.13) holds.

Proving case 3:
a) **When** $M^{tot} = \beta^{tot}_{SD}(D_j|E_q)$:

First, we consider the case $M^{tot} = \beta^{tot}_{SD}(D_j|E_q) \geq 0$

By definition of MaxMin composition of CADIAG-2, we have:

$$\mu_{R^+_{PD}}(P_q, D_j) = Max_{E'_q \subseteq E_q} Min[\mu^+_{R_{PS}}(P_q, E'_q); \mu_{R^+_{SD}}(E'_q, D_j)]$$
$$= Max_{E'_q \subseteq E_q}[\mu_{R^+_{SD}}(E'_q, D_j)]$$

(because $\mu^+_{R_{PS}}(P_q, E'_q) = 1$ from (2.0)),

$$= Max(Max_{E'_q \subset E_q}[\mu_{R^+_{SD}}(E'_q, D_j)], \mu_{R^+_{SD}}(E_q, D_j))$$

From definition (2.16), having

$$Max_{E'_q \subset E_q}(\mu_{R^c_{SD}}(E'_q, D_j)) = M^+$$

(because $\mu_{R^+_{SD}}(E'_q, D_j) = M^+$ for some $E'_q > 0$) and by condition, put

$\mu_{R^+_{SD}}(E_q, D_j) = \beta^{tot}_{SD}(D_j|E_q)$, we get

$$\mu_{R^+_{PD}}(P_q, D_j) = Max(M^+, \beta^{tot}_{SD}(D_j|E_q)) = M^+$$

because $M^- \geq 0$, $M^{tot} = M^+ \ominus M^- = \beta^{tot}_{SD}(D_j|E_q) \geq 0$, then $M^+ \geq \beta^{tot}_{SD}(D_j|E_q)$
In an analogous way, we get

$$\mu_{R^-_{PD}}(P_q, D_j) = Max(M^-, 0) = M^-$$

because $\mu_{R^-_{SD}}(E_q, D_j) = 0$
and thus

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \mu_{R^+_{PD}}(P_q, D_j) \ominus \mu_{R^-_{PD}}(P_q, D_j) = M^+ \ominus M^- = \beta^{tot}_{SD}(D_j|E_q)$$

and the equation (2.13) holds.

Second, for the case $M^{tot} = \beta^{tot}_{SD}(D_j|E_q) < 0$
The proof is quite similar.
We have

$$\mu_{R^+_{PD}}(P_q, D_j) = Max(M^+, 0) = M^+$$

and

$$\mu_{R^-_{PD}}(P_q, D_j) = Max(M^-, \beta^{tot}_{SD}(D_j|E_q)) = M^-$$

because $M^+ \geq 0$, $M^{tot} = M^+ \ominus M^- = \beta^{tot}_{SD}(D_j|E_q) < 0$, then $M^- > \beta^{tot}_{SD}(D_j|E_q)$, we get

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \mu_{R^+_{PD}}(P_q, D_j) \ominus \mu_{R^-_{PD}}(P_q, D_j) = M^+ \ominus M^- = \beta^{tot}_{SD}(D_j|E_q)$$

and the equation (2.13) holds.

b) **When** $M^{tot} = M^+ \ominus M^- < \beta^{tot}_{SD}(D_j|E_q)$:

We have

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \mu_{R^{+}_{PD}}(P_q, D_j) \ominus \mu_{R^{-}_{PD}}(P_q, D_j)$$

$$= Max_{E'_q \subseteq E_q} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{+}_{SD}}(E'_q, D_j)] \ominus$$

$$Max_{E'_q \subseteq E_q} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{-}_{SD}}(E'_q, D_j)]$$

$$= Max_{E'_q \subseteq E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)] \ominus Max_{E'_q \subseteq E_q}[\mu_{R^{-}_{SD}}(E'_q, D_j)]$$

$$= Max(Max_{E'_q \subset E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)]; \mu_{R^{+}_{SD}}(E_q, D_j)) \ominus$$

$$Max(Max_{E'_q \subset E_q}[\mu_{R^{-}_{SD}}(E'_q, D_j)]; \mu_{R^{-}_{SD}}(E_q, D_j))$$

Put

$$\mu_{R^{+}_{SD}}(E_q, D_j) = M^{-} \oplus \beta^{tot}_{SD}(D_j|E_q)$$

We have now $\mu_{R^{+}_{SD}}(E_q, D_j) > 0$, because $0 \leq M^{+} < M^{-} \oplus \beta^{tot}_{SD}(D_j|E_q)$ and $M^{-} \geq 0$. We get

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = max[M^{+}, M^{-} \oplus \beta^{tot}_{SD}(D_j, E_q)] \ominus max[M^{-}, 0]$$

and finally, we have

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = (M^{-} \oplus \beta^{tot}_{SD}(D_j, E_q)) \ominus M^{-} = \beta^{tot}_{SD}(D_j|E_q)$$

Thus the equation (2.13) holds.
c) **When** $M^{tot} > \beta^{tot}_{SD}(D_j|E_q)$:

In similar way, we have

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \mu_{R^{+}_{PD}}(P_q, D_j) \ominus \mu_{R^{-}_{PD}}(P_q, D_j)$$

$$= Max_{E'_q \subseteq E_q} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{+}_{SD}}(E'_q, D_j)] \ominus$$

$$Max_{E'_q \subseteq E_q} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{-}_{SD}}(E'_q, D_j)]$$

$$= Max_{E'_q \subseteq E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)] \ominus Max_{E'_q \subseteq E_q}[\mu_{R^{-}_{SD}}(E'_q, D_j)]$$

$$= Max(Max_{E'_q \subset E_q}[\mu_{R^{+}_{SD}}(E'_q, D_j)]; \mu_{R^{+}_{SD}}(E_q, D_j)) \ominus$$

$$Max(Max_{E'_q \subset E_q}[\mu_{R^{-}_{SD}}(E'_q, D_j)]; \mu_{R^{-}_{SD}}(E_q, D_j))$$

Put

$$\mu_{R^{-}_{SD}}(E_q, D_j) = M^{+} \ominus \beta^{tot}_{SD}(D_j|E_q)$$

We have now $\mu_{R^{-}_{SD}}(E_q, D_j) > 0$, because $0 \leq M^{-} < M^{+} \ominus \beta^{tot}_{SD}(D_j|E_q)$ and $M^{+} \geq 0$. We get

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = max[M^{+}, 0] \ominus max[M^{-}, M^{+} \ominus \beta^{tot}_{SD}(D_j|E_q)]$$

$$= M^{+} \ominus (M^{+} \ominus \beta^{tot}_{SD}(D_j|E_q)) = \beta^{tot}_{SD}(D_j|E_q)$$

that the equation (2.13) holds. This completes the proof of the theorem.

The following example shows that (2.21) may be undefined for usual subtraction $-$:

Let given a conditional weight system β:

$\beta^+_{SD}(D|S_1) = 0.3$ $\beta^-_{SD}(D|S_1) = 0,$
$\beta^+_{SD}(D|S_2) = 0.4$ $\beta^-_{SD}(D|S_2) = 0$
$\beta^+_{SD}(D|S_1 \wedge S_2) = 0$ $\beta^-_{SD}(D|S_1 \wedge S_2) = 0.7$

Applying Möbius transform according to case 3:
- From (2.10), we get:

$$\beta^{tot}_{SD}(D|S_1 \wedge S_2) = \beta^+_{SD}(D|S_1 \wedge S_2) - \beta^-_{SD}(D|S_1 \wedge S_2)$$

$$= 0 - 0.7 = -0.7$$

- Now we calculate M^{tot} from (2.18), (2.19), we get

$$M^{tot} = Max(0.3, 0.4) \ominus Max(0, 0) = 0.4 \ominus 0 = 0.4$$

We have $M^{tot} > \beta^{tot}_{SD}(D|S_1 \wedge S_2)$ then put

$$\mu_{R^-_{SD}}(S_1 \wedge S_2, D) = M^+ \ominus \beta^{tot}_{SD}(D_j|S_1 \wedge S_2)$$

$$= 0.4 \ominus -0.7 = 0.4 \oplus -(-0.7) = 0.4 \oplus 0.7$$

Apply operation $\oplus$ in (2.8), we get

$\mu_{R^-_{SD}}(S_1 \wedge S_2, D) = 0.8593$

Remark: Now if we use an usual subtraction $-$ for $\ominus$, we have

$$\mu_{R^-_{SD}}(S_1 \wedge S_2, D) = M^+ - \beta^{tot}_{SD}(D_j|S_1 \wedge S_2) = 0.4 - (-0.7) = 1.1 > 1$$

But from definition 5, $\mu_{R^-_{SD}}(S_1 \wedge S_2, D)$ must be in [0,1], that means *(2.21) is undefined for usual subtraction* $-$ in our example.
More than that the example shows that if $\mu_{R^{tot}_{PD}}(P_q, D_j)$ were defined in (2.7) using $-$ instead of $\ominus$ then we could not construct a rule base Θ such that $\mu_{R^{tot}_{PD}}(P_q, D_j) = \beta^{tot}_{SD}(D_j|E_q)$ for $E_q = S_1,\ S_2,\ S_1 \wedge S_2$
Now we would have to construct the following new rule base:

$$S_1 \longrightarrow D(0.3), S_1 \longrightarrow \neg D(0)$$

$$S_2 \longrightarrow D(0.4), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0), S_1 \wedge S_2 \longrightarrow \neg D(w)$$

such that

$$\mu_{R^+_{PD}}(P_q, D) - \mu_{R^-_{PD}}(P_q, D) = -0.7 = \mu_{R^{tot}_{PD}}(P_q, D) \tag{2.22}$$

But $\mu_{R^+_{PD}}(P_q, D) = 0.4$, $\mu_{R^-_{PD}}(P_q, D) = w$, which gives

$$0.4 - w = -0.7$$

w = 1.1, which > 1.

3 Some examples:

We discuss the following conditional weight systems β. We apply the above algorithm to compute new weights using MinMax composition of rules of CADIAG-2:
For every example, we apply Möbius transfrom to the given β of using MaxMin Composition of CADIAG-2 that we find new weights $\mu_{R^+_{SD}}(S_i, D_j)$ and $\mu_{R^-_{SD}}(S_i, D_j)$ such that

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \beta^{tot}_{SD}(D_j|E_q)$$

In all examples, we assume $\mu^+_{R_{PS}}(P_q, S_1) = \mu^+_{R_{PS}}(P_q, S_2) = 1$
We use PROSPECTOR group operation $\oplus$ and $\ominus$ defined in (2.8), (2.9)

1) **Example 1**:
$\beta^+_{SD}(D|S_1) = 0.7$ $\quad$ $\beta^-_{SD}(D|S_1) = 0$,
$\beta^+_{SD}(D|S_2) = 0.7$ $\quad$ $\beta^-_{SD}(D|S_2) = 0$
$\beta^+_{SD}(D|S_1 \wedge S_2) = 0.7$ $\quad$ $\beta^-_{SD}(D|S_1 \wedge S_2) = 0$

- Möbius transform for **example 1**:
a) **Calculating** M^{tot}**,** $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$:

$$M^+ = max_{E'_q \subset S_1 \wedge S_2}(\mu^+_{R_{PS}}(P_q, E'_q) \wedge \mu_{R^+_{SD}}(E'_q, D))$$

= max(0.7,0.7) = 0.7
In similar way, we get

$$M^- = max_{E'_q \subset S_1 \wedge S_2}(\mu_{R^+_{SD}}(E'_q, D)) = max(0, 0) = 0$$

Then $M^{tot} = 0.7 \ominus 0 = 0.7$
On the other hand,

$$\beta^{tot}_{SD}(D|S_1 \wedge S_2) = \beta^+_{SD}(D|S_1 \wedge S_2) - \beta^-_{SD}(D|S_1 \wedge S_2) = 0.7$$

b) **Compare** M^{tot} **with** $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$:
From results above, having $M^{tot} = \beta^{tot}_{SD}(D|S_1 \wedge S_2) = 0.7 > 0$, then put

$$\mu_{R^+_{SD}}(S_1 \wedge S_2, D) = \beta^{tot}_{SD}(D|S_1 \wedge S_2)$$

= 0.7
We receive the following **new rule base**:

$$S_1 \longrightarrow D(0.7), S_1 \longrightarrow \neg D(0)$$

$$S_2 \longrightarrow D(0.7), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0.7), S_1 \wedge S_2 \longrightarrow \neg D(0)$$

such that

$$\mu_{R^{tot}_{PD}}(P_q, D) = \beta^{tot}_{SD}(D|S_1 \wedge S_2) = 0.7 \qquad (e.1)$$

c) **Verifying (e.1)**:
From (2.5) we have

$$\mu_{R^{tot}_{PD}}(P_q, D) = \mu_{R^{+}_{PD}}(P_q, D) \ominus \mu_{R^{-}_{PD}}(P_q, D)$$

$$= Max_{E'_q \subseteq S_1 \wedge S_2} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{+}_{SD}}(E'_q, D)]$$

$$\ominus Max_{E'_q \subseteq S_1 \wedge S_2} Min[\mu^{+}_{R_{PS}}(P_q, E'_q); \mu_{R^{-}_{SD}}(E'_q, D)]$$

$$= max(0.7; 0.7; 0.7) \ominus max(0; 0; 0) = 0.7 \ominus 0 = 0.7$$

thus equation (e.1) holds.

2) **Example 2**:

$\beta^{+}_{SD}(D|S_1) = 0.7$ $\qquad$ $\beta^{-}_{SD}(D|S_1) = 0$
$\beta^{+}_{SD}(D|S_2) = 0.7$ $\qquad$ $\beta^{-}_{SD}(D|S_2) = 0$
$\beta^{+}_{SD}(D|S_1 \wedge S_2) = 0$ $\qquad$ $\beta^{-}_{SD}(D|S_1 \wedge S_2) = 0.7$

- Möbius transform for **example 2**:
a) **Calculating M^{tot}, $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$**:

$$M^{+} = max_{E'_q \subset S_1 \wedge S_2}(\mu^{+}_{R_{PS}}(P_q, E'_q) \wedge \mu_{R^{+}_{SD}}(E'_q, D))$$

= max(0.7,0.7) = 0.7
In similar way, we get

$$M^{-} = max_{E'_q \subset S_1 \wedge S_2}(\mu_{R^{+}_{SD}}(E'_q, D)) = max(0, 0) = 0$$

Then $M^{tot} = M^{+} \ominus M^{-} = 0.7 \ominus 0 = 0.7$
On the other hand

$$\beta^{tot}_{SD}(D|S_1 \wedge S_2) = \beta^{+}_{SD}(D|S_1 \wedge S_2) - \beta^{-}_{SD}(D, S_1 \wedge S_2) = 0 - 0.7 = -0.7$$

b) **Compare M^{tot} with $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$**:
From results above, having $M^{tot} > \beta^{tot}_{SD}(D, S_1 \wedge S_2)$ then put

$$\mu_{R^{-}_{SD}}(S_1 \wedge S_2, D) = M^{+} \ominus \beta^{tot}_{SD}(D|S_1 \wedge S_2)$$

$= 0.7 \ominus - 0.7 = 0.7 \oplus 0.7 = 0.9395$
We receive the following **new rule base**:

$$S_1 \longrightarrow D(0.7), S_1 \longrightarrow \neg D(0)$$

$$S_2 \longrightarrow D(0.7), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0), S_1 \wedge S_2 \longrightarrow \neg D(0.7 \oplus 0.7)$$

where $0.7 \oplus 0.7 = 0.9395$ (using (2.8))
such that

$$\mu_{R_{PD}^{tot}}(P_q, D) = \beta_{SD}^{tot}(D|S_1 \wedge S_2) = -0.7 \qquad (e.2)$$

c) **Verifying (e.2)**:
From (2.5) we have

$$\mu_{R_{PD}^{tot}}(P_q, D) = \mu_{R_{PD}^{+}}(P_q, D) \ominus \mu_{R_{PD}^{-}}(P_q, D)$$

$$= max(0.7; 0.7; 0) \ominus max(0; 0; 0.7 \oplus 0.7) = 0.7 \ominus (0.7 \oplus 0.7) = -0.7$$

Thus

$$\mu_{R_{PD}^{tot}}(P_q, D) = \beta_{SD}^{tot}(D|S_1 \wedge S_2) = -0.7$$

Thus the equation (e.2) holds.

3) **Example 3**:

$\beta_{SD}^{+}(D\|S_1) = 0.3$	$\beta_{SD}^{-}(D\|S_1) = 0$
$\beta_{SD}^{+}(D\|S_2) = 0.3$	$\beta_{SD}^{-}(D\|S_2) = 0$
$\beta_{SD}^{+}(D\|S_1 \wedge S_2) = 0.7$	$\beta_{SD}^{-}(D\|S_1 \wedge S_2) = 0$

- Möbius transform for **example 3**:
a) **Calculating** M^{tot}**,** $\beta_{SD}^{tot}(D|S_1 \wedge S_2)$:

$$M^{+} = max_{E_q' \subset S_1 \wedge S_2}(\mu_{R_{PS}}^{+}(P_q, E_q^{'}) \wedge \mu_{R_{SD}^{+}}(E_q^{'}, D))$$

= max(0.3,0.3) = 0.3
In similar way, we get

$$M^{-} = max_{E_q' \subset S_1 \wedge S_2}(\mu_{R_{SD}^{+}}(E_q^{'}, D)) = max(0, 0) = 0$$

Then $M^{tot} = M^{+} \ominus M^{-} = 0.3 \ominus 0 = 0.3$
On the other hand

$$\beta_{SD}^{tot}(D, S_1 \wedge S_2) = \beta_{SD}^{+}(D, S_1 \wedge S_2) - \beta_{SD}^{-}(D, S_1 \wedge S_2) = 0.7 - 0 = 0.7$$

b) **Compare** M^{tot} **with** $\beta_{SD}^{tot}(D|S_1 \wedge S_2)$:
From results above, having $M^{tot} < \beta_{SD}^{tot}(D, S_1 \wedge S_2)$ then put

$$\mu_{R_{SD}^{c}}(S_1 \wedge S_2, D) = M^{-} \oplus \beta_{SD}^{tot}(D|S_1 \wedge S_2)$$

$= 0 \oplus 0.7 = 0.7$
We receive the following **new rule base**:

$$S_1 \longrightarrow D(0.3), S_1 \longrightarrow \neg D(0)$$

$$S_2 \longrightarrow D(0.3), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0.7), S_1 \wedge S_2 \longrightarrow \neg D(0)$$

such that

$$\mu_{R_{PD}^{tot}}(P_q, D) = \beta_{SD}^{tot}(D|S_1 \wedge S_2) = 0.7 \qquad (e.3)$$

c) **Verifying (e.3)**:
From (2.5) we have

$$\mu_{R^{tot}_{PD}}(P_q, D) = \mu_{R^+_{PD}}(P_q, D) \ominus \mu_{R^-_{PD}}(P_q, D)$$

$$= max(0.3; 0.3; 0.7) \ominus max(0; 0; 0) = 0.7 \ominus 0 = 0.7$$

Thus

$$\mu_{R^{tot}_{PD}}(P_q, D) = \beta^{tot}_{SD}(D|S_1 \wedge S_2) = 0.7$$

and the equation (e.3) holds.

4) **Example 4**:

$\beta^+_{SD}(D\|S_1) = 0$	$\beta^-_{SD}(D\|S_1) = 0.3$
$\beta^+_{SD}(D\|S_2) = 0.3$	$\beta^-_{SD}(D\|S_2) = 0$
$\beta^+_{SD}(D\|S_1 \wedge S_2) = 0.7$	$\beta^-_{SD}(D\|S_1 \wedge S_2) = 0$

- Möbius transform for **example 4**:
a) **Calculating** M^{tot}**,** $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$:

$$M^+ = max_{E'_q \subset S_1 \wedge S_2}(\mu^+_{R_{PS}}(P_q, E'_q) \wedge \mu_{R^+_{SD}}(E'_q, D))$$

= max(0,0.3) = 0.3
In similar way, we get

$$M^- = max_{E'_q \subset S_1 \wedge S_2}(\mu_{R^c_{SD}}(E'_q, D)) = max(0.3, 0) = 0.3$$

Then $M^{tot} = M^+ \ominus M^- = 0.3 \ominus 0.3 = 0$
On the other hand

$$\beta^{tot}_{SD}(D|S_1 \wedge S_2) = \beta^+_{SD}(D|S_1 \wedge S_2) - \beta^-_{SD}(D|S_1 \wedge S_2) = 0.7 - 0 = 0.7$$

b) **Compare** M^{tot} **with** $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$:
From results above, having $M^{tot} < \beta^{tot}_{SD}(D, S_1 \wedge S_2)$ then put

$$\mu_{R^+_{SD}}(S_1 \wedge S_2, D) = M^- \oplus \beta^{tot}_{SD}(D|S_1 \wedge S_2)$$

$= 0.3 \oplus 0.7 = 0.8264$ (using (2.8))
We receive the following **new rule base**:

$$S_1 \longrightarrow D(0), S_1 \longrightarrow \neg D(0.3)$$

$$S_2 \longrightarrow D(0.3), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0.3 \oplus 0.7), S_1 \wedge S_2 \longrightarrow \neg D(0)$$

such that

$$\mu_{R^{tot}_{PD}}(P_q, D) = \beta^{tot}_{SD}(D|S_1 \wedge S_2) = 0.7 \qquad (e.4)$$

c) **Verifying (e.4)**:
From (2.5) we have

$$\mu_{R^{tot}_{PD}}(P_q, D) = \mu_{R^+_{PD}}(P_q, D) \ominus \mu_{R^-_{PD}}(P_q, D)$$

$$= max(0; 0.3; 0.3 \oplus 0.7) \ominus max(0.3; 0; 0) = (0.3 \oplus 0.7) \ominus 0.3) = 0.7$$

Thus

$$\mu_{R^{tot}_{PD}}(P_q, D) = \beta^{tot}_{SD}(D|S_1 \wedge S_2) = 0.7$$

and the equation (e.4) holds.

5) **Example 5**:

$\beta^{+}_{SD}(D|S_1) = 0$ $\beta^{-}_{SD}(D|S_1) = 0.3$
$\beta^{+}_{SD}(D|S_2) = 0.3$ $\beta^{-}_{SD}(D|S_2) = 0$
$\beta^{+}_{SD}(D|S_1 \wedge S_2) = 0$ $\beta^{-}_{SD}(D|S_1 \wedge S_2) = 0.7$

- Möbius transform for **example 5**:
a) **Calculating M^{tot}, $\beta^{tot}_{SD}(D|S_1 \wedge S_2)$**:

$$M^{+} = max_{E'_q \subset S_1 \wedge S_2}(\mu^{+}_{R_{PS}}(P_q, E'_q) \wedge \mu_{R^{+}_{SD}}(E'_q, D))$$

= max(0,0.3) = 0.3
In similar way, we get

$$M^{-} = max_{E'_q \subset S_1 \wedge S_2}(\mu_{R^{+}_{SD}}(E'_q, D)) = max(0.3, 0) = 0.3$$

Then $M^{tot} = M^{+} \ominus M^{-} = 0.3 \ominus 0.3 = 0$
On the other hand

$$\beta^{tot}_{SD}(D|S_1 \wedge S_2) = \beta^{+}_{SD}(D|S_1 \wedge S_2) - \beta^{-}_{SD}(D, S_1 \wedge S_2) = 0 - 0.7 = -0.7$$

b) **Compare M^{tot} with $\beta^{tot}_{SD}(D, S_1 \wedge S_2)$**:
From results above, having $M^{tot} > \beta^{tot}_{SD}(D, S_1 \wedge S_2)$ then put

$$\mu_{R^{-}_{SD}}(S_1 \wedge S_2, D)) = M^{+} \ominus \beta^{tot}_{SD}(D|S_1 \wedge S_2)$$

$= 0.3 \ominus - 0.7 = 0.3 \oplus 0.7 = 0.8264$ (using (2.8))
We receive the following **new rule base**:

$$S_1 \longrightarrow D(0), S_1 \longrightarrow \neg D(0.3)$$

$$S_2 \longrightarrow D(0.3), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0), S_1 \wedge S_2 \longrightarrow \neg D(0.3 \oplus 0.7)$$

where $0.3 \oplus 0.7 = 0.8264$

such that

$$\mu_{R^{tot}_{PD}}(P_q, D) = \beta^{tot}_{SD}(D|S_1 \wedge S_2) = -0.7 \qquad (e.5)$$

c) **Verifying (e.5)**:
From (2.5) we have

$$\mu_{R^{tot}_{PD}}(P_q, D) = \mu_{R^{+}_{PD}}(P_q, D) \ominus \mu_{R^{-}_{PD}}(P_q, D)$$

$$= max(0; 0.3; 0) \ominus max(0.3; 0; 0.3 \oplus 0.7) = 0.3 \ominus (0.3 \oplus 0.7) = -0.7$$

Thus

$$\mu_{R_{PD}^{tot}}(P_q, D) = \beta_{SD}^{tot}(D|S_1 \wedge S_2) = -0.7$$

and the equation (e.5) holds.

6) **Example 6**:

$\beta_{SD}^{+}(D|S_1) = 0.7$ $\beta_{SD}^{-}(D|S_1) = 0$

$\beta_{SD}^{+}(D|S_2) = 0.3$ $\beta_{SD}^{-}(D|S_2) = 0$

$\beta_{SD}^{+}(D|S_1 \wedge S_2) = 0.7$ $\beta_{SD}^{-}(D|S_1 \wedge S_2) = 0$

- Möbius transform for **example 6**:

a) **Calculating M^{tot}, $\beta_{SD}^{tot}(D|S_1 \wedge S_2)$**:

$$M^{+} = max_{E_q' \subset S_1 \wedge S_2}(\mu_{R_{PS}}^{+}(P_q, E_q') \wedge \mu_{R_{SD}^{+}}(E_q', D))$$

= max(0.7,0.3) = 0.7

In similar way, we get

$$M^{-} = max_{E_q' \subset S_1 \wedge S_2}(\mu_{R_{SD}^{+}}(E_q', D)) = max(0,0) = 0$$

Then $M^{tot} = M^{+} \ominus M^{-} = 0.7 \ominus 0 = 0.7$

On the other hand

$$\beta_{SD}^{tot}(D|S_1 \wedge S_2) = \beta_{SD}^{+}(D|S_1 \wedge S_2) - \beta_{SD}^{-}(D|S_1 \wedge S_2) = 0.7 - 0 = 0.7$$

b) **Compare M^{tot} with $\beta_{SD}^{tot}(D, S_1 \wedge S_2)$**:

From results above, having $M^{tot} = \beta_{SD}^{tot}(D, S_1 \wedge S_2) = 0.7 > 0$, then put

$$\mu_{R_{SD}^{+}}(S_1 \wedge S_2, D) = \beta_{SD}^{tot}(D, S_1 \wedge S_2) = 0.7$$

We receive the following **new rule base**:

$$S_1 \longrightarrow D(0.7), S_1 \longrightarrow \neg D(0)$$

$$S_2 \longrightarrow D(0.3), S_2 \longrightarrow \neg D(0)$$

$$S_1 \wedge S_2 \longrightarrow D(0.7), S_1 \wedge S_2 \longrightarrow \neg D(0)$$

such that

$$\mu_{R_{PD}^{tot}}(P_q, D) = \beta_{SD}^{tot}(D|S_1 \wedge S_2) = 0.7 \qquad (e.6)$$

c) **Verifying (e.6)**:

From (2.5) we have

$$\mu_{R_{PD}^{tot}}(P_q, D) = \mu_{R_{PD}^{+}}(P_q, D) \ominus \mu_{R_{PD}^{-}}(P_q, D)$$

$$= max(0.7; 0.3; 0.7) \ominus max(0; 0; 0) = 0.7 \ominus 0 = 0.7$$

Thus

$$\mu_{R_{PD}^{tot}}(P_q, D) = \beta_{SD}^{tot}(D|S_1 \wedge S_2) = 0.7$$

and the equation (e.6) holds.

4 Conclusion

In this study, we have described an algorithm using Möbius transform to compute new rule base for CADIAG-2. We have extended CADIAG-2 by including fuzzy negative knowledge. To apply Möbius transform for CADIAG-2 means to find new weights $\mu_{R^+_{SD}}(S_i, D_j)$ and $\mu_{R^-_{SD}}(S_i, D_j)$ of fuzzy rules such that for each patient P_q whose data $\mu^+_{R_{PS}}(P_q, S_i); \mu^-_{R_{PS}}(P_q, S_i)$ are three-valued (therefore E_q exists) such that

$$\mu_{R^{tot}_{PD}}(P_q, D_j) = \beta^{tot}_{SD}(D_j|E_q)$$

Thus this algorithm garantees that using generalized MaxMin inference of CADIAG-2 the inference machine will reproduce the expert's stated conditional beliefs as total degrees of confirmation and exclusion. To illustrate this algorithm, several examples are examined.

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