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Technical report No. 604

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Abstract

It is proved that two previously known sufficient conditions for regularity of interval matrices are equivalent in the sense that they cover the same class of interval matrices.

Keywords

Interval matrix, regularity

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Let $A_c, \Delta \in R^{n \times n}, \Delta \geq 0$. The interval matrix

$$A^I = \{A; A_c - \Delta \leq A \leq A_c + \Delta\}$$

is called regular if each $A \in A^I$ is nonsingular, and is said to be singular otherwise. It has been proved recently [5] that the problem of checking regularity of interval matrices is NP-hard. Therefore, in practical computations we must resort to verifiable sufficient regularity conditions (which are not necessary). The most commonly used sufficient condition is due to Beeck [1]: if

$$\rho(|A_c^{-1}|\Delta) < 1 \tag{0.2}$$

holds, then A^I is regular (here, ρ denotes the spectral radius and the absolute value of a matrix $A = (a_{ij})$ is defined by $|A| = (|a_{ij}|)$). To avoid the use of the exact inverse A_c^{-1}, Rump [9] proposed a modified condition employing instead
an arbitrary matrix R which is specified only by some condition: if

$$\rho(G_R) < 1 \tag{0.3}$$

holds for some matrix R, where

$$G_R = |I - RA_c| + |R|\Delta \tag{0.4}$$

(I is the unit matrix), then the interval matrix A^I is regular. It follows obviously from (0.3) that (0.1) is a special case of (0.2) for $R = A_c^{-1}$. Therefore, the condition (0.2) seems to be more general than (0.1). In this note we show that in fact it is not so, since both (0.1) and (0.2) are equivalent in the sense that they cover the same class of interval matrices prescribed by the condition (0.1) (the so-called strongly regular interval matrices [3]). This equivalence will be a consequence of the following result which, moreover, shows that we can never do better than with $R = A_c^{-1}$:

Theorem 1 *Let (0.2) hold for some R. Then A_c is nonsingular and we have*

$$\rho(|A_c^{-1}|\Delta) \leq \rho(G_R) . \tag{0.5}$$

Proof. First, since

$$I - RA_c \leq |I - RA_c| \leq G_R,$$

we have

$$\rho(I - RA_c) \leq \rho(|I - RA_c|) \leq \rho(G_R) < 1, \tag{0.6}$$

hence the matrix

$$RA_c = I - (I - RA_c) \tag{0.7}$$

is nonsingular, which gives that A_c is nonsingular. Now, assume to the contrary that

$$\rho(|A_c^{-1}|\Delta) > \rho(G_R) \tag{0.8}$$

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holds. Then we can choose an α satisfying

$$\rho(G_R) < \alpha < \min\{1, \rho(|A_c^{-1}|\Delta)\}. \qquad (0.9)$$

Since $\rho(G_R) < \alpha$, in view of Corollary 3.2.3 in [3] there exists a vector $x \in R^n$, $x > 0$ satisfying

$$G_R x < \alpha x,$$

hence also

$$\alpha |I - RA_c| x + |R|\Delta x < \alpha x$$

(since $\alpha < 1$), which implies

$$|R|\Delta x < \alpha(I - |I - RA_c|)x. \qquad (0.10)$$

Because of (0.5), the matrix $I - |I - RA_c|$ is nonnegative invertible. Hence, premultiplying the inequality (0.9) by its inverse, we obtain

$$(I - |I - RA_c|)^{-1} |R|\Delta x < \alpha x. \qquad (0.11)$$

On the other hand, from (0.6) we have

$$A_c^{-1} = \sum_{j=0}^{\infty} (I - RA_c)^j R,$$

hence

$$|A_c^{-1}| \leq \sum_{j=0}^{\infty} |I - RA_c|^j |R| = (I - |I - RA_c|)^{-1} |R|.$$

Then, premultiplying this inequality by the nonnegative vector Δx and using (0.10), we finally obtain

$$|A_c^{-1}|\Delta x < \alpha x$$

where $x > 0$, hence, again using Corollary 3.2.3 in [3], we conclude that

$$\rho(|A_c^{-1}|\Delta) < \alpha$$

holds, which contradicts (0.8). Hence the assumption (0.7) leads to a contradiction. Therefore (0.4) holds, which completes the proof. ∎

Remark. A related result using a norm instead of the spectral radius can be found in Neumaier [4, Theorem 6], see also Krawczyk [2] and Scheu [10].

The main result of this paper is now obtained as a simple consequence of Theorem 1:

Theorem 2 *For a square interval matrix A^I, the following two conditions are equivalent:*

(i) A_c is nonsingular and (0.1) holds,

(ii) there exists an R such that (0.2) holds (where G_R is given by (0.3)).

If any of them is satisfied, then A^I is regular.

Proof. If (i) holds, then it suffices to set $R := A_c^{-1}$ to have (ii) satisfied. The converse implication follows directly from Theorem 1, and the last assertion is simply a restatement of the sufficient conditions by Beeck [1] and Rump [9]. ∎

In this way we have proved that the conditions (0.1) and (0.2) cover the same class of interval matrices. Obviously, the condition (0.2) is more appropriate for practical computations since it allows the use of the computer inverse R of A_c instead of the exact inverse A_c^{-1}. The conditions are not necessary: an example of a 3×3 regular interval matrix with $\rho(|A_c^{-1}|\Delta) > 1.7$ is given in [8, p. 71]. Theorem 1 implies that if $\rho(|A_c^{-1}|\Delta) \geq 1$ holds, then there does not exist a matrix R with $\rho(G_R) < 1$; see [6] and also [7], where a counterexample is given.

Finally we note that to achieve the inequality $\rho(G_R) < 1$ provided $\rho(|A_c^{-1}|\Delta) < 1$ holds, R must be chosen as a sufficiently close approximation to A_c^{-1}.

Bibliography

[1] H. Beeck, Zur Problematik der Huellenbestimmung von Intervallgleichungs-systemen, in: *Interval Mathematics* (K. Nickel, ed.), Lecture Notes in Comput. Sci. 29, Springer-Verlag, Berlin 1975, pp. 150-159

[2] R. Krawczyk, Interval Extensions and Interval Iterations, *Computing* 24(1980), 119 - 129

[3] A. Neumaier, *Interval Methods for Systems of Equations*, Cambridge University Press, Cambridge 1990

[4] A. Neumaier, New Techniques for the Analysis of Linear Interval Equations, *Linear Algebra and Its Applications* 58(1984), 273 - 325

[5] S. Poljak and J. Rohn, Checking Robust Nonsingularity Is NP-Hard, *Mathematics of Control, Signals, and Systems* 6(1993), 1-9

[6] G. Rex, Zur Loesungseinschliessung linearer Gleichungssysteme, *Wissenschaftliche Zeitschrift TH–Leipzig* 15(1991), 441 - 447

[7] G. Rex, Unique Existence of a Solution of a System of Equations, *KMU–NTZ– Bericht*, Nr. 13, University of Leipzig, 1988

[8] J. Rohn, Systems of Linear Interval Equations, *Linear Algebra and Its Applications* 126(1989), 39 - 78

[9] S. M. Rump, Solving Algebraic Problems with High Accuracy, in: *A New Approach to Scientific Computation* (U. Kulisch and W. Miranker, eds.), Academic Press, New York 1983, pp. 51 - 120

[10] G. Scheu, Ueber eine Wahl des Parameters beim Parallelenverfahren, *Computing* 20(1978), 17 -26