New Axiomatization of Quantum Mechanics

Souček, Jiří
2017

New Axiomatization of Quantum Mechanics

Jiří Souček

Charles University in Prague, Faculty of Arts
U Kříže 8, Prague 5, 158 00, Czech Republic
jiri.soucek@ff.cuni.cz

Abstract

In this paper we describe a new (complete) axiomatization of quantum mechanics (QM) in which we add axioms describing the concept of an observation. We show that new axioms are clear and evident and based on common sense. Our approach is based on the idea of the observation of properties of an individual measuring system. We distinguish two concepts of a state: the properties of an individual measuring system and the state of an ensemble of systems. Then we **prove** that the ontic model of QM is inconsistent. This is our main result. This implies the necessity to consider "non-realistic" models for QM in which it is not true that each wave-function describes a possible state of an individual system. Our axiomatization is based on the postulate of the **existence of definite outputs** of an experiment. Thus the **evident reality** of the existence of definite outputs implies that our axiomatization is true and verifiable.

1. Introduction.

Our starting point is the fact that certain form of observability **must** be present in any reasonable theory. Without possibility to observe something, the theory cannot be tested and cannot be related to the reality. Since the standard axiomatization of quantum mechanics (QM) does not contain the concept of an observation, it cannot be considered as a complete theory.

The main idea in this paper is to introduce into the axiomatization of QM the concept of an observation.

Our axiomatization contains the postulate of **existence of definite outputs** of an experiment. Thus the **evident reality** of the existence of definite outputs implies that our axiomatization must be considered as evident.

The main message of this paper is following: one must to **choose** between two alternatives – to choose the ontic model of QM (i.e. the standard model of QM) or to choose the existence of definite outputs. This is a fundamental choice. But the existence of definite outputs is an **evident fact.**

We consider the minimal requirement on observability in the following form: it is possible to observe whether the given property of the individual measuring system (at a given moment of time) occurs.

In each probability theory, there is a duality between two concepts: the state of an ensemble and the state of an individual system. For example let us consider the dice. The space of individual states is $\Omega = \{1,2,3,4,5,6\}$ while the space of possible states of ensembles is the set of all probability distributions on Ω. Similarly for Brownian particle the state of an individual particle is the point in the physical space while the state of an ensemble is the probability distribution on the physical space.

For QM the situation is analogical: there are properties of an individual measuring system M and the possible states $\psi \in \mathbf{H}_M$ of the ensemble of such systems.

In this paper we discuss the measurement problem only schematically. This discussion can be found with many details in [4].

Our axiomatization of QM is based on this simple idea: **what is important is to observe properties of the individual measuring apparatus.** This concept is not contained in the standard QM and this creates its **incompleteness**.

The final output of the standard QM is the prediction of the probabilities of possible outputs. But the question is what is a probability and how it can be found. Probability cannot be directly observed. It needs to make repeated experiments and then calculate the relative frequency. But, at first, it is necessary to observe the output of a given individual case (only then the statistics can be applied). There is a question what is the individual output and how it can be observed. And exactly this topic is missing in the standard QM. The standard QM predicts probabilities and nothing else. There is an empty space in QM concerning individual outputs and their possible observation. And this creates the incompleteness of QM.

The main consequences of this new axiomatization of QM are:

(i) the inconsistence of the ontic model of QM;
(ii) the hybrid-epistemic model of QM (see [4]) as the only consistent model for QM.

In Sect. 2 we present in details the new axiomatic formulation of QM (Axioms S1 – S6, O1 – O4). In Sect. 3 we prove our basic theorems. In Sect. 4 we describe the possible models of QM and their interrelations. In Sect. 5 we discuss the principle of superposition, we solve the "Schrodinger cat paradox" and present the abstract measurement schema. In Sect. 6 we discuss our results. In Sect. 7 we summarize our results. In this paper we continue our study from [3], [4], [7], [9], [10].

2. A new complete axiomatization of quantum mechanics.

At first we state standard axioms which are common in all axiomatizations of QM.

Axiom S1.
To each system S there corresponds (for the simplicity) a finite dimensional *complex Hilbert space* of states $\mathbf{H}_S = \mathbf{H}(S)$.

At the beginning it is necessary to define clearly the state space. Let us consider the system S and its Hilbert space $\mathbf{H}_S$. The *state space of pure states* is defined as a set of rays by

$$\mathbf{P}_S = \{ [\psi] \mid \psi \in \mathbf{H}_S , \| \psi \| = 1 \}, \quad \text{where } [\psi] = \{ a\psi \mid a \in \mathbf{C} , |a| = 1 \}, \| \psi \| = 1 .$$

We shall suppose that all state spaces used below will have dimension greater or equal to two.

It is assumed that the state $[\psi] \in \mathbf{P}_S$, $\| \psi \| = 1$ is the state of an ensemble of systems prepared in certain way.
We shall consider always the state $[\psi] \in \mathbf{P}_S$ as a state of certain ensemble $\mathbf{E}$ of systems.

Axiom S2.
Let M and S be two systems, then there exists a *composite system* $T = M \oplus S$ for which we have $\mathbf{H}_T = \mathbf{H}_M \otimes \mathbf{H}_S$.

Axiom S3.
For each system S there exists a *unitary group* $\{ U_t \mid t \in \mathbf{R} \}$ in the space $\mathbf{H}_S$ such that the state vector $\psi(t)$ of S evolves along the standard rule $\psi(t) = U_t\, \psi(0)$. This unitary group is generated by the Hamiltonian of the system S.

Axiom S4.

There is a one-to-one correspondence between *observables* and self-adjoint operators on $\mathbf{H}_S$. The set of all self-adjoint operators on $\mathbf{H}_S$ is denoted by $\mathbf{A}(\mathbf{H}_S)$.

Each operator $A \in \mathbf{A}(\mathbf{H}_S)$ has a spectral decomposition

$$A = \Sigma_{s=1}^{k} \ a_s \ P_s , \ a_s \text{ are distinct real eigenvalues, } P_s \text{ are orthogonal projectors.}$$

The spectrum sp (A) is the set of eigenvalues $\{ a_1 , \dots , a_k \}$.

Axiom S5.
The *measurement* is a map which to each observable A and each state $[\psi] \in \mathbf{P}_S$ assigns a probability distribution $\{p_1, \dots ,p_k\}$ on the spectrum sp (A). This probability is given by *Born's law*

$$p_i = \mathrm{tr}\, P_i (\psi \otimes \psi^*) = (\psi \mid P_i \psi) = \| P_i \psi \|^2 , \ \text{ for } \ i = 1, \dots ,k .$$

Axiom S6.
If, in the measurement, the value a_i is observed, then the state $[\psi] \in \mathbf{P}_S$ of the observed system is changed to the *up-dated state* $[P_i \psi / \| P_i \psi \|]$.

We add new axioms describing the concept of an observation.

Definition.

(i) A *property* is a function v which to each system assigns a value from the set $\{1, 0\}$. The assignment $v(S) = 1$ means that the system S has the property v.

(ii) A finite set of properties $C = \{v_1, v_2, \dots ,v_n\}$, $n \geq 2$ is called a *classification* if each two properties are exclusive, i.e. if $v_i(S) = 1$ then $v_j(S) = 0$ for each $j \neq i$. Also the empty set $C = 0$ is considered as a classification.

(iii) A system S is *C-classifiable* if there exists a property $v \in C$ satisfying $v(S) = 1$. The set of all C-classifiable systems is *the base of C* defined by $\mathbf{B}(C) = \{ S \mid \text{there exists } v \in C, v(S) = 1 \}$

Axiom O1.

(i) For each system S there is defined a classification C(S) such that
   a. either the system S is C-classifiable (i.e. S has exactly one of properties from C)
   b. or $C(S) = 0$.

(ii) For each observable system $|C(S)| \geq 2$ holds

(iii) For each individual observable system S it is *possible to observe* (in a given moment of time) the value $v(S) \in \{1, 0\}$ for each property $v \in C(S)$.

(iv) Let us consider an *ensemble* $\mathbf{E} = \{S_1, S_2, \dots , S_N\}$, $(N \rightarrow \infty)$, of systems prepared in a same way. Then $C(S_1) = \dots = C(S_N)$. We shall also assume that $\mathbf{H}(S_1) = \dots = \mathbf{H}(S_N)$.

(v) We shall assume that for each system S and for each $v \in C(S)$ there exists $\varepsilon > 0$ such that for each N there exists an ensemble $\mathbf{E} = \{S_1, S_2, \dots , S_N\}$ in the state $[\psi] \in \mathbf{P}_S$ such that $S \in \mathbf{E}$ and $|\{R \in \mathbf{E} \mid v(R) = v\}| > \varepsilon N$. (This means that the property p will have the positive probability for some $[\psi] \in \mathbf{P}_S$.) Here $|\,.\,|$ denotes the number of elements.

Definition.
Let us consider the ensemble $\mathbf{E} = \{S_1, S_2, \dots, S_N\}$, N large, and let the state of $\mathbf{E}$ be $[\psi] \in \mathbf{P}_S$. In this way we obtain (by making an observation) for each $v \in C$ the *sequence of values*

$$v(S_1), v(S_2), \dots, v(S_N) \in \{1, 0\}.$$

We can define a *new ensembles* $\mathbf{E}^{(v)}$, $v \in C(S)$ in the following way:

$$\mathbf{E}^{(v)} = \{ S \in \mathbf{E} \mid v(S) = 1 \},$$

Then we define *relative frequency*, i.e. the *probability* by

$$p (v \mid \mathbf{E}) = \lim_{N\to\infty} N^{-1} \mid \mathbf{E}^{(v)} \mid, \quad v \in C(S)^{1}.$$

It means that to every ensemble **E** there corresponds a *probability distribution* $p(\,.\mid \mathbf{E})$ on the set C(S).

We shall assume that the resulting probability distribution on C(S) depends only on the state $[\psi]$ of an ensemble **E**, i.e.

Axiom O2.
For each $[\psi]$ and an ensemble **E** in the state $[\psi]$ the probability distribution $p(\,.\mid \mathbf{E})$ depends only on $[\psi]$, i.e.
$p(\,.\mid \mathbf{E}) = p(\,.\mid [\psi])$, for each ensemble **E** in the state $[\psi]$.

Definition.
(i) Let $v \in C(S)$. The state $[\psi]$ is a *v-homogeneous state* if $p(v \mid [\psi]) = 1$. The state $[\psi]$ is *homogeneous* if $[\psi]$ is v-homogeneous for some $v \in C(S)$. The set of all homogeneous states is denoted hom (S).
(ii) For each $v \in C(S)$ we define the set

$$L^{(v)} = \{ \lambda\psi \in \mathbf{H}_S \mid [\psi] \text{ is an v-homogeneous state, } \lambda \text{ is a complex number}\}.$$

We shall call any such $L^{(v)}$ a *homogeneous subspace* of $\mathbf{H}_S$. The set of all homogeneous subspaces will be denoted by Hom (S).

Proposition 1.
For each $v \in C(S)$ there exists at least one v-homogeneous state $[\psi]$.

Proof.

[1] The stabilization of the relative frequency is assumed.

We shall use the Axiom O1(v). Let us consider the sub-ensemble $\mathbf{E}^{(v)} = \{ S \in \mathbf{E} \mid v(S) = v \}$ defined above. From Axiom O1(iv) we obtain $|\mathbf{E}^{(v)}| > \varepsilon N$. We obtain that the state of $\mathbf{E}^{(v)}$ is v-homogeneous and that the number of its elements is sufficiently large. (The end of proof.)

Axiom O3.
We shall assume that $p(v \mid [\psi])$ depends quadratically on ψ (this is generally true in QM – see the Born's rule).

Proposition 2.
Let $v \in C(S)$ and let us assume that there exist a Hermitian operator K^v , such that

$$p(v \mid [\psi]) = (\psi \mid K^v \psi) \text{ , for each } \psi \in \mathbf{H}_S , \|\psi\| = 1.$$

Denote

$$N^v = \{ \psi \in \mathbf{H}_S \mid p(v \mid [\psi]) = 0 \} .$$

Then N^v is a linear subspace of $\mathbf{H}_S$.

Proof.

(i) We show that the operator K^v is positive. For $\psi \in \mathbf{H}_S$, $\|\psi\| = 1$ the positivity of $(\psi \mid K^v \psi)$ follows from the definition since $p(v \mid [\psi]) \geq 0$. For the vector $a\psi$, $a \in \mathbf{C}$, we have $(a\psi \mid K^v a\psi) = |a|^2 (\psi \mid K^v \psi) \geq 0$.

(ii) Let the spectral decomposition of K^v be $K^v = \Sigma_{s=1}^k \lambda_s \phi_s \otimes \phi_s^*$ where $\lambda_s > 0$ for each $s = 1, \dots, k \leq n$. We obtain

$$(\psi| K^v \psi) = \Sigma_s \lambda_s (\psi| \phi_s \otimes \phi_s^* \psi) = \Sigma_s \lambda_s (\psi \mid \phi_s) (\phi_s \mid \psi) = \Sigma_s \lambda_s |(\phi_s \mid \psi)|^2.$$

Then $N^v = \{ \psi \mid (\phi_s \mid \psi) = 0 \text{ for each } s = 1, \dots, k \}$ since $\lambda_s > 0$. (End of proof.)

Proposition 3.
The set $L^{(v)}$ is a linear subspace of $\mathbf{H}_S$ for each $v \in C(S)$.

Proof.
We have $L^{(v)} = \cap \{ N^w \mid w \neq v \}$.

Definition.

(i) Let *map* $P^{(v)}$ be an orthogonal projection from $\mathbf{H}_S$ onto $L^{(v)}$, $v \in C(S)$.

(ii) The state $[\psi]$ is *completely homogeneous* if and only if ψ is an element of a one-dimensional homogeneous subspace of $\mathbf{H}_s$ (see [1]).

Clearly, $\psi \in L^{(v)}$ if and only if $P^{(v)}(\psi) = \psi$.

We have a 1-1 map

$$v \in C(S) \rightarrow L^{(v)} \in \mathrm{Hom}(S).$$

Axiom O4. (The existence of observable systems – an evident requirement)

For each $n \geq 2$ there exists at least one observable system satisfying $| C(S) | = n$.

3. Basic theorems.

Axioms we have formulated above have important consequences.

Theorem 1. (Born's rule for observations)
For each state $[\psi] \in \mathbf{P}_S$ and for each property $v \in C(S)$, the Born's rule holds

$$p (v \mid [\psi]) = \| P^{(v)} (\psi) \|^2.$$

Proof.
Let us consider a property $v \in C(S)$. Let us consider the ensemble

$$\mathbf{E} = \{S_1, S_2, \ \dots, S_N\} \text{ and let the state of } \mathbf{E} \text{ be } [\psi] \in \mathbf{P}_S$$

We can consider also the sub-ensemble $\mathbf{E}^{(v)} = \{S \in \mathbf{E} \mid v(S) = 1\}$.
We shall consider the observable $P^{(v)}$ which is the projector onto $L^{(v)}$. This operator has an eigenvalue 1 in the subspace $L^{(v)}$ and an eigenvalue 0 in the subspace $L^{(v)\perp}$. Thus we have $P^{(v)} = 1 \,.\, P^{(v)} + 0 \,.\, P^{(v)\perp}$. Using Axiom S5 we obtain probabilities

$$p_1 = \| P^{(v)} \psi \|^2 , \quad p_0 = \| P^{(v)\perp} \psi \|^2 = 1 - \| P^{(v)} \psi \|^2$$

Thus the probability to obtain value 1 is $p_1 = \| P^{(v)} \psi \|^2$.

Observing properties of members of the ensemble $\{S_1, S_2, \dots, S_N\}$ we obtain values $\{v(S_1), \dots, v(S_N)\}$. By the definition of the probability as a relative frequency we obtain

$$p_1 = \| P^{(v)} \psi \|^2 = \lim_{N\to\infty} N^{-1} \mid \mathbf{E}^{(v)} \mid$$

By the definition of the observable probability we have

$$p (v \mid \mathbf{E}) = \lim_{N\to\infty} N^{-1} \mid \mathbf{E}^{(v)} \mid = \| P^{(v)} \psi \|^2 . \text{ (The end of proof.)}$$

Theorem 2.
If $v, w \in C(S)$ and $v \neq w$ then homogeneous subspaces $L^{(v)}$ and $L^{(w)}$ are orthogonal.

Proof.
Let us consider an ensemble $\mathbf{E} = \{S_1, \dots, S_N\}$ in the state $[\psi]$, $\psi \in L^{(w)}$. From this we obtain that for values $\{w(S_1), \dots, w(S_N)\}$ we have

$$\lim_{N\to\infty} N^{-1} \mid E^{(w)} \mid = 1$$

Since properties v and w are exclusive (i.e. they cannot be verified simultaneously) we have

$$\lim_{N\to\infty} N^{-1} \mid E^{(v)} \mid = 0 .$$

and then using Theorem 1 we obtain

$$\| P^{(v)} \psi \|^2 = p (v \mid [\psi]) = \lim_{N\to\infty} N^{-1} \mid E^{(v)} \mid = 0 .$$

From this it simply follows that ψ is orthogonal to the subspace $L^{(v)}$. (The end of proof.)

Theorem 3.
Assume that $C(S) \neq 0$. Then the set Hom (S) of all homogeneous subspaces is the complete orthogonal decomposition of the Hilbert space $\mathbf{H}_S$.

Proof.
We have already shown the orthogonality of homogeneous subspaces. Let us assume that Hom(S) is not a complete orthogonal decomposition of $\mathbf{H}_S$. Then there exists a state $[\psi] \in \mathbf{P}_S$ which is orthogonal to all elements of Hom (S), i.e. to all homogeneous subspaces. Using Born's rule we obtain

$$p (v \mid [\psi]) = \| P^{(v)} (\psi) \|^2 = 0, \text{ for each } v \in C(S).$$

This means that the function $p (. \mid [\psi])$ is trivially equal to 0 and this is impossible since this function is a probability distribution. Thus Hom (S) must be a complete orthogonal decomposition of $\mathbf{H}_S$.

4. The hybrid-epistemic model of QM

At first we introduce the basic classification of systems.

Definition.

(i) A system S is an *epistemic* system if S is not observable, i.e. it has no property, C(S)=0

(ii) A system S is *hybrid-epistemic* system if $\mid C(S) \mid \geq 2$ and Hom (S) is an orthogonal decomposition of $\mathbf{H}_S$.

(iii) A system S is a *hybrid* system if S is hybrid-epistemic system and each homogeneous subspace $L^{(v)}$ is one-dimensional. (Each hybrid system is a hybrid-epistemic system.)

(iv) A system S is an *ontic* system if $\mid C(S) \mid \geq 2$ and if each one-dimensional subspace of $\mathbf{H}_S$ is a homogeneous subspace.

A typical hybrid–epistemic system S is the composite of the hybrid system M and the epistemic system E, $S = M \oplus E$, $\mathbf{H}_S = \mathbf{H}_M \otimes \mathbf{H}_E$.

Theorem 4.
In the complete axiomatization of QM

(i) There **do not exist** ontic systems, i.e. any ontic model is inconsistent and impossible.
(ii) Only hybrid-epistemic or epistemic systems can exist
(iii) Some hybrid-epistemic systems must exist
(iv) The situation where only hybrid systems exist is possible (this model is monistic – see [11])
(v) Situation where Hom (S) is an incomplete orthogonal decomposition is impossible
(vi) Situation where only epistemic systems exist is impossible.
(vii) Each observable system must be either hybrid or hybrid-epistemic

Proof.

(i) It follows from Theorem 3.
(ii) If $C(S) \neq 0$ then we can apply Theorem 3. If $C(S) = 0$ then S is epistemic.
(iii) See Axiom O4.
(iv) All axioms are satisfied in this situation. This is exactly the starting point of our argumentation in [3].
(v) See Theorem 3.
(vi) See Axiom O4.[2]
(vii) Evident.

Definition.
The *hybrid-epistemic model* of QM is a theory satisfying axioms S1 – S6 and O1 – O4 where moreover some epistemic systems exist (this is a dualistic model following the taxonomy of [11]).

The non-existence of the ontic systems has important consequences.

(i) The von Neumann's postulate stating that each ensemble in the pure state is completely homogeneous means that each one-dimensional subspace of $\mathbf{H}_S$ is the homogeneous subspace and this is impossible.
(ii) The "standard" model in the Dirac – von Neumann manner is the ontic model characterized by the property that each wave function is a possible state of an individual system (i.e. a state of a homogeneous ensemble). Such model is not possible in the complete axiomatization of QM.
(iii) The statement "an (individual) system S is in a state ψ" is meaningful only if the state ψ is an element of a one-dimensional homogeneous subspace – otherwise this statement is meaningless. In general, an individual system is characterized by properties but not by the state – state is, in general, an attribute of an ensemble.

[2] Thus the Einstein's idea that the wave function describes the state of an ensemble was almost correct. On the other hand Bohr's idea of the ontic model (each wave function describes a possible state of an individual system) was completely incorrect.

5. The principle of superposition, the "Schrodinger's cat paradox" and the measurement problem

It is clear that the principle of superposition holds for states of ensembles, since states are vectors in the Hilbert space $\mathbf{H}_S$.

But for homogeneous states the principle of superposition does not hold.

Theorem 5.
In the hybrid-epistemic model the non-trivial superposition of two homogeneous states is a homogeneous state if and only if these homogeneous states are elements of the same homogeneous subspace.

Proof.
Let us consider the hybrid-epistemic model. If two homogeneous states belong to two distinct homogeneous subspaces, then their non-trivial superposition will not be the homogeneous state.

Theorem 6. ("Schrodinger's cat paradox")
The *"cat state"*

$$(1/2)^{1/2} (\psi_{alive} + \psi_{dead})$$

is not a homogeneous state, so that it can be interpreted only as a state of an ensemble.

Proof.
In this situation we have to consider two properties: v^{alive} and v^{dead} and the corresponding classification C (cat) = { v^{alive}, v^{dead} }. There are also the corresponding homogeneous subspaces

$$L^{(alive)} = \{ \lambda\psi \in \mathbf{H}_{cat} \mid p (v^{alive} \mid [\psi]) = 1, \lambda \in \mathbf{C} \} ,$$
$$L^{(dead)} = \{ \lambda\psi \in \mathbf{H}_{cat} \mid p (v^{dead} \mid [\psi]) = 1, \lambda \in \mathbf{C} \} ,$$

and corresponding projectors $P^{(alive)}$, $P^{(dead)}$. We have the orthogonal decomposition $\mathbf{H}_{cat} = L^{(alive)} + L^{(dead)}$.
Since $\psi_{alive} \in L^{(alive)}$ and $\psi_{dead} \in L^{(dead)}$, then the "cat state" is a nontrivial combination of two states from different homogeneous subspaces, and by Theorem 5 we obtain that this state is not homogeneous. It cannot be associated with any property of an individual system.
This state can be interpreted only as a state of an ensemble. Clearly, in this ensemble some cats are alive and some are dead. There is no paradox. (The end of proof.)

Now we shall describe the general schema of the *measurement process*.

Let us assume that the *measuring* system M is observable and that its decomposition into homogeneous subspaces is numbered by $\mathbf{H}_M = L^{(1)} + \ldots + L^{(n)}$, where these homogeneous subspaces correspond to properties $v_1, \ldots, v_n$.
We shall assume that there is given a *measured* system S and its observable $A = \Sigma_{i=1}^{n} a_i P_i$ (here a_i are distinct). The corresponding decomposition is given by $\mathbf{H}_S = K_1 + \ldots + K_n$ where $K_i = P_i (\mathbf{H}_S)$.

The composed system $T = M \oplus S$ will be based on the Hilbert space $\mathbf{H}_T = \mathbf{H}_M \otimes \mathbf{H}_S$.

Definition.
The *measuring transformation* is a unitary operator in the space $\mathbf{H}_T$ satisfying the following condition

$$U (L^{(1)} \otimes K_i) = L^{(i)} \otimes K_i , \quad \text{for } i = 1, \ldots , n .$$

The *measurement process* consists in the following steps:

(i) We prepare the ensemble of measured systems $\mathbf{E} = \{S_1, S_2, \ldots , S_N\}$ in the state $\psi \in \mathbf{H}_S$. The state ψ can be decomposed with respect to the decomposition $\mathbf{H}_S = K_1 + \ldots + K_n$ into $\psi = \alpha_1\psi_1 + \ldots + \alpha_n\psi_n$, where $\psi_i = P_i \psi / \|P_i \psi\| \in K_i$, $i = 1, \ldots, n$.

(ii) We prepare the ensemble of measuring systems $\mathbf{F} = \{M_1, M_2, \ldots , M_N\}$ in the state $\phi \in L^{(1)}$.

(iii) The ensemble $\mathbf{G} = \{M_1 \oplus S_1 , \ldots , M_N \oplus S_N\}$ of composed systems will be in the state $\phi \otimes \psi$.

(iv) To the state $\phi \otimes \psi$ of the ensemble $\mathbf{G}$ we apply the measuring transformation U and obtain an ensemble $\mathbf{G'}$ in the state $U (\phi \otimes \psi)$. We obtain

$$U(\phi \otimes \psi) = \alpha_1 U(\phi \otimes \psi_1) + \ldots + \alpha_n U(\phi \otimes \psi_n) , \quad \text{where } U(\phi \otimes \psi_i) \in L^{(i)} \otimes K_i .$$

(v) Let v be a property. This property can be directly extended on composed systems by the formula $v (M_i \oplus S_i) = v (M_i)$. Then we can define the sub-ensemble

$$\mathbf{G'}^{(v)} = \{ M_i \oplus S_i \mid v (M_i \oplus S_i) = v (M_i) = 1 , i = 1, \ldots, N \} .$$

If $v = v_j$, then the state of $\mathbf{G'}^{(v)}$ is $U(\phi \otimes \psi_j) \in L^{(j)} \otimes K_j$ and the relative frequency of $\mathbf{G'}^{(v)}$ in the ensemble $\mathbf{G'}$ is $N^{-1} | \mathbf{G'}^{(v)} | = p (v_j \mid U(\phi \otimes \psi)) = | \alpha_j |^2$.

(vi) If the individual system $M_i \oplus S_i$ is observed and it is found that $v_j(M_i) = 1$, then we can obtain that S_i is an element of the ensemble in the state $\psi_j = P_j \psi / \|P_j \psi\| \in K_j$.

6. The discussion

There are two contradicting positions:

- The ontic model of QM (it is equivalent to the Dirac – von Neumann's standard QM).
- The existence of definite outcomes of a given experiment.

The standard argumentation follows these steps:

The ontic model → the measurement → the impossibility of definite outcomes (i.e. the measurement problem [5]).

Our model is exactly opposite and follows other steps:

The postulate of the existence of definite outcomes (Axiom O1(iii)) → the observation of the properties of the individual measuring system → the inconsistence of the ontic model.

In a short form:
The standard von Neumann's model *implies* the impossibility of the definite outcomes.
The postulate on definite outcomes *implies* the inconsistence of the ontic model of QM.

Thus there is a necessary choice between two exclusive alternatives:

- The acceptance of the ontic model of QM.
- The existence of definite outcomes of an experiment.

The right choice is simple and evident: **the existence of definite outcomes is an evident objective fact** while the existence of an ontic model of QM is a **pure speculation** without any objective support (it exists either as a tradition or as a prejudice).

Axioms O1 – O4 describing the observation process are evident.

The main point of our axiomatization is the Axiom O1 (iii). It implies the existence of definite outcomes in the postulate of the existence of functions $v(S) \in \{1, 0\}$, $v \in C(S)$.

We obtain our general conclusion: **the existence of definite outcomes of an experiment implies the inconsistence of the ontic model of QM and the necessity to use the hybrid-epistemic model of QM.**

Remark.
In fact, the fact of the inconsistence of the ontic model (obtained in this paper) has already two predecessors :

(i) Bell's theorem asserts that
Locality *implies* Bell's inequality (Bell's inequality contradicts QM).
Wrong conclusion is the quantum nonlocality while the right conclusion is the inconsistence of the ontic model (the ontic model is used in the proof of Bell's inequality under the name of "realism")

(ii) Maudlin's proof of the measurement problem [5]: the impossibility of definite outputs.
The wrong conclusion is the existence of the so-called measurement problem while the right conclusion is the inconsistence of the ontic model since the ontic model is used in the Maudlin's proof.

7. Conclusions

The main input of our study is the axiomatization of the concept of observability in QM and the relevant consequences. Our main starting point is the assumption of the existence of definite outputs of an experiment.

We take the standard axioms S1 – S6 of QM and we add axioms O1 – O4 describing the concept of an observation. We have shown that the ontic model does not satisfies the complete set of axioms while the hybrid-epistemic model satisfies these axioms.

We have shown that

(i) The introduction the concept of an observation into QM is necessary – without such a concept QM is incomplete.
(ii) New axioms are natural
(iii) The hybrid-epistemic models satisfies all axioms
(iv) Standard QM is in-complete and un-defined in this respect since the ontic model, the hybrid model and the hybrid-epistemic model satisfy the standard axioms.

There are also positive consequences of our axiomatization:

- The solution to the "Schrodinger's cat paradox" is obtained and it is quite simple and natural.
- The rational and plausible solution to the problem of the superposition of macroscopic states is obtained (see the "Schrodinger's cat paradox").
- In [4] there is a complete description of the internal measurement process inside QM and this implies the solution to the measurement problem.
- There is no problem of collapse since now the "collapse" is considered as an update of an ensemble and of its state.

We think that our new axioms are so general so that they could be acceptable for most of experts in the foundations of QM.

We think that the von Neumann's assumption that the ensembles in the pure state are homogeneous (i.e. that any pure state can be a state of an individual system) is **provably wrong** and that it was the source of the most foundational problems of QM and that exactly this assumption has to be changed.

Our considerations enters exactly into the heart of the Bohr – Einstein debate. Bohr preferred the ontic model and Einstein preferred the epistemic model (Einstein was the first to consider the idea that the wave function does not describe the state of an individual system but the state of an ensemble, see also [6[). Both standpoints are unacceptable: the ontic model is inconsistent while the epistemic model has no possibility of any observation. But the Einsten's proposal is quite close to the hybrid-epistemic model.

References

[1] J. von Neumann, Mathematical Foundations of Quantum Mechanics (Princeton University Press, 1955), translated by R. T. Beyer from the German Mathematische Grundlagen der Quantenmechanik (Springer, 1932).

[2] Bell, J. S. Against 'measurement' , Physics World, 33-40, August, 1990.

[3] J. Soucek, The Principle of Anti-Superposition in QM and the Local Solution of the Bell's Inequality Problem, http://vixra.org/abs/1502.0088

[4] J .Soucek , The hybrid-epistemic model of quantum mechanics and the possible solution to the measurement problem, http://vixra.org/pdf/1702.0044v2.pdf

[5] Tim Maudlin. Three Measurement Problems, Topoi 14,7-15, 1995

[6] L. E. Ballentine, The statistical interpretation of quantum mechanics, Reviews of Modern Physics 42,358-381, 1970

[7] J. Soucek, The Restoration of Locality: the Axiomatic Formulation of the Modified Quantum Mechanics, http://viXra.org/abs/1503.0109

[8] Matthew S. Leifer, Is the Quantum State Real? An Extended Review of ψ-ontology Theorems, arXiv:1409.1570v2 [quant-ph]

[9] J. Soucek, Quantum Mechanics Without the Measurement Axiom, http://vixra.org/abs/1603.0076

[10] J. Soucek, The Local Explanation of EPR Correlations in the Modified Quantum Mechanics, http://vixra.org/abs/1507.0161

[11] N, Gisin, Collapse. What else?, arXiv:1701.08300v1