Confidence Cycles and Liquidity Hoarding

Audzei, Volha
2016

WORKING PAPER SERIES 7

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2016

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WORKING PAPER SERIES

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7/2016

CNB WORKING PAPER SERIES

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Confidence Cycles and Liquidity Hoarding

Volha Audzei [*]

Abstract

Market confidence has proved to be an important factor during past crises. However, many existing general equilibrium models do not account for agents' expectations, market volatility, or overly pessimistic investor forecasts. In this paper, we incorporate a model of the interbank market into a DSGE model, with the interbank market rate and the volume of lending depending on market confidence and the perception of counterparty risk. In our model, a credit crunch occurs if the perception of counterparty risk increases. Our results suggest that changes in market confidence can generate credit crunches and contribute to the depth of recessions. We then conduct an exercise to mimic some central bank policies: targeted and untargeted liquidity provision, and reduction of the policy rate. Our results indicate that policy actions have a limited effect on the supply of credit if they fail to influence agents' expectations. Interestingly, a policy of a low policy rate worsens recessions due to its negative impact on banks' revenues. Liquidity provision stimulates credit slightly, but its efficiency is undermined by liquidity hoarding.

Abstrakt

Důvěra na trzích se během minulých ekonomických krizí ukázala být důležitým faktorem. Přesto mnoho existujících modelů všeobecné rovnováhy neuvažuje očekávání ekonomických subjektů, volatilitu trhů nebo příliš pesimistické výhledy investorů. V této práci je do modelu DSGE zakomponován model mezibankovního trhu, přičemž úroková sazba na mezibankovním trhu a objem půjček záleží na tržní důvěře a vnímání rizika spojeného s protistranou. K úvěrové krizi v modelu dojde při vnímání nárůstu rizika protistrany. Výsledky naznačují, že změny v důvěře na trzích mohou generovat úvěrové krize a přispívat k hloubce ekonomických recesí. V článku jsou také provedeny simulace napodobující některé měnové politiky centrálních bank: cílované a necílované poskytování likvidity a snížení měnověpolitických sazeb. Výsledky naznačují, že bez ovlivnění očekávání ekonomických subjektů mají tyto politiky na nabídku úvěrů pouze omezený dopad. Zajímavé je také zjištění, že nízké úrokové míry v tomto modelu zhoršují ekonomickou recesi skrze negativní dopad na bilance bank. Poskytování likvidity mírně stimuluje úvěry, ale efektivita tohoto kroku je podrývána hromaděním likvidity.

Nontechnical Summary

The expectations of financial agents affect the functioning of financial markets and can propagate shocks to the real economy or become a source of shocks themselves. These expectations are not necessarily perfect. Agents can have limited information or a limited ability to process it. Studies[1] have shown that the expectations of professional forecasters demonstrate inertia, and it takes time for them to learn when changes occur. Therefore, after crisis episodes, agents can have pessimistic forecasts. Imperfect information and/or overly pessimistic expectations influence the efficiency of policy actions aimed at mitigating the recession and can undermine their effect or lead to unintended consequences.

As regards the recent financial crisis with the subsequent recession and unconventional policy responses by leading central banks, there are papers that show how banks' liquidity constraints affect interbank market allocations.[2] Policies aimed at relaxing these constraints are found to be efficient in restoring the markets. At the time of the credit crunch in 2008–2009, however, banks demonstrated a reluctance to lend and hoarded some of the liquidity they obtained from central banks.

This paper contributes to the literature by addressing how imperfect information among financial agents – banks – influences the functioning of the interbank market and the supply of credit to the real economy. We start with a simple model where the supply of interbank market credit depends on banks' expectations about economic activity. Banks have heterogeneous expectations about risky asset returns and are endogenously divided into lenders and borrowers. The lenders assess counterparty risk as borrowers' ability to meet their obligations given their portfolio returns. After periods with low returns, lenders anticipate higher risk on the interbank market and demand a higher interbank rate. Given the banks' beliefs distribution, lower market beliefs result in less lending and can even lead to an interbank market crunch. As banks are creditors to the real sector, the functioning of the interbank market then affects the real economy, generating or amplifying business cycle fluctuations. Within this framework we study possible central bank policy actions for stimulating credit or restoring the interbank market. We show that when lending is impaired because of low market sentiment, policy actions have a rather limited effect.

We further proceed with a workhorse DSGE model where banks' return expectations depend on their backward-looking economic model and heterogeneous expert assessments. In this environment we simulate three types of crisis: a negative shock to returns, a negative shock with an additional shock to investor sentiment, and just a sentiment shock. After a large negative shock to returns, banks tend to underestimate future returns, reduce interbank lending, and increase hoarding, which results in lower credit to the economy and amplifies the recession. This effect is stronger when the crisis shock is accompanied by sentiment shocks. Also, sentiment shocks alone are able to generate a recession by reducing the supply of credit and interbank market lending.

Within this framework, we consider the following central bank policy responses: (i) liquidity provision, targeted and untargeted, (ii) a policy rate cut, and (iii) relaxation of the collateral constraint on the interbank market. Our findings suggest that liquidity provision can help restore credit to the real sector, but its effect is limited by banks' pessimism, with a significant share of the central bank's funds ending in reserves. Interestingly, reducing the policy rate results in a worse outcome than the scenario with no policy response. In our model, the policy rate is the reserves rate, with reserves

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[1] Examples being Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013).
[2] For example Gale and Yorulmazer (2013), Heider et al. (2009) and Allen et al. (2009).

being the only safe asset. The low return on reserves erodes banks' revenues, resulting in a smaller supply of credit and a subsequent fall in capital accumulation.

1. Introduction

The recent financial crisis was one of the deepest and longest in modern history. Having started in the financial sector, it then spread into the real economy, causing a recession the length of which has yet to be determined. Not surprisingly, it drew the attention of academics and policy makers to the interconnections between the financial and real sectors. A possible explanation of the origin and development of the crisis, or at least of its depth, lies in market imperfections and the limited rationality of economic agents. Figure 1 illustrates some change in European bankers' expectations

Figure 1: Net Tightening of Banks' Lending Standards
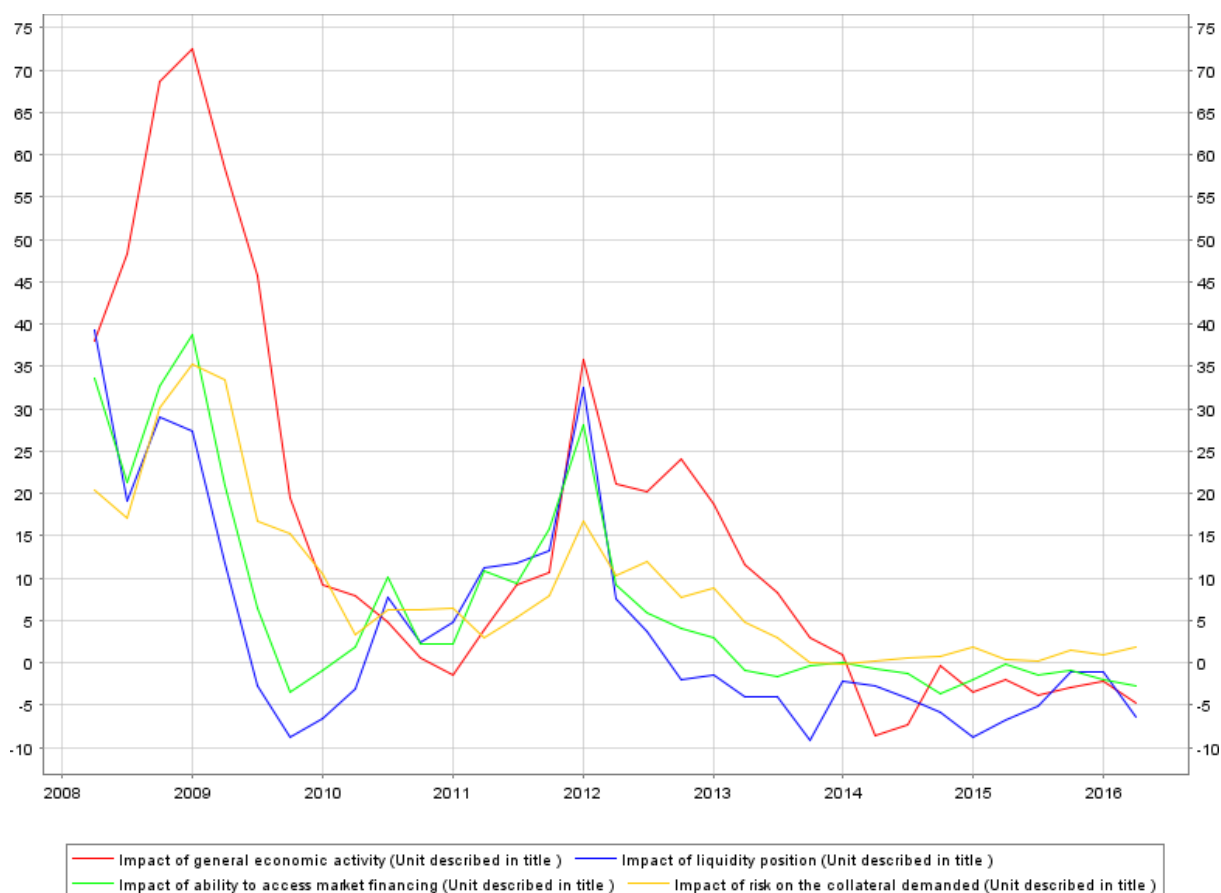
Source: ECB Bank Lending Survey

occurring at the start of the crisis.[3] The figure shows the tightening of banks' lending standards in response to different factors. With the start of the crisis in 2008, there is a huge spike in the percentage of banks who tightened their credit standards due to the impact of general economic activity. This can be interpreted as a rise in banks' concerns about the economy. Other factors also contributed significantly to the tightening of lending standards, with banks' liquidity position being the least important. One can see another, somewhat smaller, spike around 2012, when most of the factors had the same impact. This spike can be attributed to the recent euro crisis. As banks'

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[3] In the Bank Lending Survey conducted by the ECB, one of the questions was about how selected factors contributed to the lending standards of the bank: either to tightening or to easing. Net tightening is defined as the difference between the percentage of respondents who tightened their lending standards and those who eased them. Here, we show the lending standards applied to enterprises.

assessment of the economy becomes more optimistic and monetary policy actions mitigate their liquidity or collateral risk concerns, their lending standards become slightly eased (negative values on the graph) or factors become irrelevant for lending standards (values around zero).

The question could be if the banks were overly pessimistic or just rationally predicting the downturn? There is no obvious way to find hard proof of overly pessimistic or overly optimistic expectations. One possible proxy is the survey of professional forecasters. A number of papers studying the survey of professional forecasters, examples being Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013), have found sluggishness in forecasters' expectations. We interpret this finding as meaning that agents form their forecasts based on backward-looking data. It is not surprising, then, that after an episode of low returns or high risk, forecasters underestimate returns or overestimate risks in the next period. Therefore, it is not unrealistic to consider that after the crisis, when central banks started to implement unconventional measures, banks had overly pessimistic expectations.

This paper contributes to the literature by addressing banks' imperfect information about general economic activity and counterparty risk in a DSGE model. Unlike a number of papers which regard interbank market collapse as being due to a liquidity shortage, we focus on the role of counterparty risk. In our model, banks lend to the real economy depending on their heterogeneous return expectations. A decline in return expectations increases their evaluation of counterparty risk on the interbank market. When lenders expect a low return on a risky asset, they assign a high probability to the scenario of their borrowers not being able to honor the debt. These expectations can drive the interbank market rate to a level where no bank is willing to borrow. Without access to the interbank market, even the most optimistic banks reduce their lending to the real economy and pessimistic banks just hoard their funds, i.e., keep them in reserves.

Such an enriched banking sector is incorporated into a workhorse DSGE model, with only the moments of the beliefs distribution entering the equilibrium solution. With the number of expert surveys and market volatility indices at hand, our developed framework becomes a tractable version of a DSGE model for analyzing the role of expectational shocks and their propagation to the real economy. We also consider the question of the efficiency of the policy measures applied during the economic downturn. Our model allows us to account for the hoarding behavior by banks observed during the crisis, which is often missing from DSGE models analyzing unconventional central bank policy. Hoarding was observed in the form of banks being reluctant to lend while keeping funds in excessive reserves or investing in short-term assets.[4] We consider several types of central bank policy actions that resemble those taken during the crisis and the subsequent recession, including liquidity provision to all banks at a fixed rate and targeted liquidity provision to support lending to the real sector. We also consider the policies of reducing the rate on reserves and relaxing collateral constraints on the interbank market.[5].

Our findings suggest that investors' expectations and their uncertainty instigate large swings in the real economy, where manufacturers are dependent on credit. In our model, when banks are concerned about economic prospects the liquidity provision policy dampens the magnitude of the crisis but neither stops nor shortens it. Moreover, a significant share of funds received from the central bank is invested in safe assets instead of flowing into the real economy. This result is in

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[4] For evidence on hoarding see Gale and Yorulmazer (2013) and Heider et al. (2009) and references therein.

[5] The policy of relaxing collateral constraints actually involved widening the set of assets accepted as collateral by the central bank. In our model, it takes the form of banks being willing to lend up to a larger fraction of borrowers assets.

line with the banks' observed behavior. This also suggests that making policy evaluations without accounting for investors' sentiment and market volatility may overstate policy efficiency. Lowering policy rate makes hoarding less attractive, but reduces the banks' revenues, resulting in even worse outcomes than in the case of no central bank action.

This paper is related to several strands of literature. The first concentrates on the role of the financial sector and credit in the economy. Studies have incorporated the banking sector into general equilibrium models. Examples include Gertler and Karadi (2011), Curdia and Woodford (2011), Del Negro et al. (2011), and Gertler and Kiyotaki (2010). Having introduced the financial sector, these papers address central banks' crisis-mitigation policies. While the first two papers consider the effect of policies on the transfer of credit between households and financial intermediaries, the latter two analyze credit supply to entrepreneurs subject to a liquidity constraint of the Kiyotaki and Moore (2008) type. Most of these papers (aside from Curdia and Woodford, 2011) conclude that central bank policy unambiguously helped prevent a far more severe recession. Curdia and Woodford (2011) condition the relevance of the policy on its implementation and on agents' perceptions of the future path of interest rates. Our study also addresses the efficiency of central bank policy, but accounts for the role of investor sentiment. The closest to our paper is Gertler and Karadi (2011). We use their framework as a backbone, allowing banks to have imperfect expectations and to lend to each other. We also use a similar approach to simulate the crisis and consider policy efficiency.

There are papers on interbank market structure related to our model. They include Gale and Yorulmazer (2013), Heider et al. (2009), and Allen et al. (2009), who consider liquidity hoarding through the interbank market structure. In these models the reason for banks to hoard liquidity is anticipation of a liquidity shock. Our interbank market structure can allow for a liquidity shock, but we consider the role of counterparty risk in breaking the interbank market. Another related paper is by Bianchi and Bigio (2014). They focus on the liquidity management problem and assume no default risk on the interbank market and analyze the possible causes of crises and policy responses. Here, we focus more on counterparty risk on the interbank market and its implications for monetary policy. Bank heterogeneity in a DSGE model is introduced by Hilberg and Hollmayr (2011), who study liquidity provision and relaxation of collateral constraints. In Hilberg and Hollmayr (2011), bank heterogeneity is caused by exogenous separation into investment and commercial banks; only investment banks are allowed to borrow from the central bank. We consider a different interbank market structure consisting of a number of ex-ante identical banks who differ ex post depending on their subjective interpretation of public information.

There is body of literature suggesting that market expectations and uncertainty about the future can be important factors in generating economic fluctuations.[6] The importance of sentiment shocks is empirically supported by Fuhrer (2011) and Beaudry et al. (2011), Beber et al. (2013) and Bloom (2009). The informational structure of the model is motivated by a branch of empirical literature analyzing survey data on economic forecasts.[7] Theoretical support for persistent heterogeneity in agents' beliefs and its impact on the business cycle is provided by Acemoglu et al. (2006), Pfajfar

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[6] Among the seminal contributions to this branch of literature are Woodford (2001) and Mankiw and Reis (2007).
[7] Among the papers analyzing diversity and systematic mistakes in professional and central bank forecasts are Franses et al. (2011) and Fildes et al. (2009), who study the bias in expert adjustment of forecasts. Andrade and Le Bihan (2013) and Nason and Smith (2013) analyze professional forecasts and find qualitative support for the inattentiveness and noisy information theories. A test of the most acknowledged models of information rigidities on survey forecast data is conducted by Coibion and Gorodnichenko (2012). Siklos (2013) and Patton and Timmermann (2010) study the sources of heterogeneity in economic forecasts. Empirical and theoretical support for dispersion and bias in inflation forecasts is provided by Capistrán and Timmermann (2009). The usefulness of survey data for estimating the term structure model is examined by Kim and Orphanides (2005).

(2013), and Lorenzoni (2009). Experimental evidence on heterogeneous expectations can be found in Hommes (2011).

This paper proceeds as follows. We first analyze a simple model of the interbank market to illustrate the role of market expectations in causing a credit crunch and consider policy actions by a central bank. Next, the general equilibrium model is completed. Within a DSGE model, we show the implications of market mood swings for the propagation of crises and policy efficiency when there is feedback from household decisions and market prices.

2. A Simple Model of Credit and the Interbank Market

In this section we describe the main mechanism of the model in a simplified setting. Later in the paper, the described sector is incorporated into a DSGE model to compare our results with the literature and to consider the general equilibrium effects of policy actions.

There are two time periods and two types of investment opportunities for banks: a storage asset pays R^{res} and a risky asset R^k in the next period. Decisions are made in period 1 and payoffs are realized in period 2. At $t = 1$ banks attract deposits d from the household. We simplify the problem by assuming that deposits are distributed equally among all banks and set $d = 1$. Banks pay R to depositors in the next period. The time subscripts are dropped.

There is a continuum of banks normalized to 1 and indexed by i, each with different expectations about the risky asset return, $E^i \hat{R}^k$. In the general equilibrium context the risky asset is credit to the real sector, so in the simple model we sometimes refer to the risky asset position as credit. Banks can participate in the interbank market. If a bank chooses to borrow on the interbank market, we limit its borrowing to its share of liabilities $- \lambda_b d = \lambda_b$. The interbank market rate, R^{ib}, is determined endogenously by clearing the market. Clearly, portfolio decisions depend on the bank's expectations about the risky asset return, the safe asset return, the state of the interbank market (functioning or not), and the interbank market rate. We show below that, given the safe asset rate, the distribution of banks' beliefs defines their decisions and interbank market conditions. Then we consider possible policy actions and show how they affect liquidity hoarding and credit. In this framework we consider the state of the interbank market and the amount of credit given the moments of the beliefs distribution – the average market belief and its standard deviation.

The timing is straightforward: in the first period banks make decisions on investing in the risky asset, lending to the real economy, and lending or borrowing on the interbank market. At the moment they make these decisions, banks know the interbank rate and the safe rate (on reserves) and they have some estimate of the risky asset return. In the next period, the risky asset return is realized, the return on the safe asset is paid, and borrowers (if they can) repay the interbank market loans. A bank's balance sheet is illustrated in Table 1. Lending on the interbank market is risky: there is a probability that due to low portfolio returns borrowers will not repay their loans. We assume that not repaying part of a loan and not paying the full amount are equally costly for borrowers and that the cost is exclusion from the interbank market. That is, the lender only considers the probability that the borrower's return is smaller than his liabilities and disregards the set of possible partial loan repayments, which simplifies the math significantly. We also abstract from the agency problem here, assuming that banks will honor their debt unless their returns do not allow it.

Survey-based expectations and their volatility are studied by Clark and Davig (2011). What this literature agrees on is the heterogeneity of forecasts and the practice of using expert adjustments.

Table 1: A Bank's Balance Sheet

Assets	Liabilities
Manufacturers' claims	Deposits
Reserves	Interbank borrowing
Interbank lending	Net worth

Suppose for simplicity that beliefs are distributed uniformly among banks with mean m and variance σ^2.[8] We think of each banker as being a statistician making her best forecast conditional on the available information. Each bank's individual estimate, $E^i \hat{R}^k$, is then assumed to be distributed uniformly with the same variance σ^2. That is, each banker has her own prediction of the risky asset return, $E^i \hat{R}^k$, with variance σ^2, and these predictions are distributed uniformly among banks with mean $\bar{E} \hat{R}^k = m$ and the same variance σ^2. These assumptions are made for the sake of simplicity and more intuitive presentation of the results. Later in the paper we relax these simplifying assumptions and let banks have a model-consistent beliefs distribution.

Every bank is risk neutral and optimizes its next-period return by maximizing the following function:

$$\max_{\alpha^i, h^i, L^i} \alpha^i E^i \hat{R}^k + h^i * R^{res} + \left(1 - \alpha^i - h^i\right) p^i R^{ib} + (E^i \hat{R}^k - R^{ib}) * \Lambda^i \tag{1}$$

subject to a collateral constraint on the interbank market: $\Lambda^i = \lambda_b$ or 0.

Bank i chooses the portfolio shares α^i (the share of the risky asset) and h^i (the share of hoarded or reserve assets), and the rest of the assets $\left(1 - \alpha^i - h^i\right)$ are then lent on the interbank market. The return then consists of the expected return on the risky asset $\alpha^i E^i \hat{R}^k$, the return on the safe asset $h^i * R^{res}$, and the expected return on interbank lending $p^i \left(1 - \alpha^i - h^i\right) R^{ib}$. The lenders are uncertain whether the borrowers will be able to repay their debt, hence they assign a loan repayment probability p^i. Those banks which are willing to borrow on the interbank market and invest in the risky asset get the expected return on borrowed funds: $(E^i \hat{R}^k - R^{ib}) * \Lambda^i$,[9] where Λ^i is the amount borrowed on the interbank market.

Because every bank is risk neutral, the problem results in a corner solution. If the expected return on the risky asset is the highest among those on the available assets, everything is invested in the risky asset: $\alpha^i = 1$. Otherwise, $\alpha^i = 0$. If the expected return on the interbank market is higher than the expected return on the risky asset and the reserve rate, the bank becomes a lender on the interbank market: $\left(1 - \alpha^i - h^i\right) = 1$. Accordingly, those believing that reserves pay the most hoard everything they have: $h^i = 1$.

Now, some of the banks may believe that the risky asset return is higher than the interbank market rate: $E^i \hat{R}^k > R^{ib}$. In this case, a bank would like to borrow on the interbank market to invest in the risky asset. Because a banker is risk neutral, she would like to borrow as much as possible. At the same time, when a desired interbank loan is too large, the lender will expect a lower loan

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[8] The bounds of the uniform distribution a and b are then: $a = m - \sigma\sqrt{3}$ and $b = m + \sigma\sqrt{3}$. In this simplest model, a can be negative.

[9] Banks only borrow on the interbank market to invest in the risky asset. Consider the case where a banker borrows and invests in the safe asset. This would mean that $R^{res} > R^{ib}$. In this case, no one would lend on the interbank market. Therefore, the interbank market only functions when $R^{res} < R^{ib}$.

repayment probability. If banks' borrowing is not restricted, they will ask for an infinitely large loan, lenders will not grant it or will ask for an infinitely large interbank rate, and the interbank market will collapse. This happens because banks do not internalize the impact of their decisions on the interbank market. We therefore have to restrict borrowing on the interbank market. In the simplified model, we set it as a fraction of liabilities: λ_b.

Let us now consider the subjective loan repayment probability. The probability of a loan being repaid, p^i, is lender i's subjective probability that the borrower will repay the loan, in other words, that the borrower's return on the risky asset will be higher than her payments on the loan and other liabilities. Because of risk neutrality, all borrowers invest everything in the risky asset. By construction, all banks have an equal amount of deposits and also borrow the same amount on the interbank market: λ_b. That is, from the lender's perspective, each borrower has the same amount of assets and liabilities. And p^i is determined by the lender's belief about the risky asset return, $E^i \hat{R}^k$:

$$p^i = Prob\left\{(1+\lambda_b)E^i\hat{R}^k \geq R + \lambda_b R^{ib}\right\} \tag{2}$$

In (2) the borrower's expected return is $(1+\lambda_b)E^i\hat{R}^k$, where 1 is the borrower's own funds and λ_b is the share borrowed on the interbank market. The liabilities are then $R*1$ to households and $\lambda_b R^{ib}$ to the interbank market lender. If the return is higher than the liabilities, a bank pays interbank market debt. As we have assumed for this section that the bank's estimate of the risky asset return is uniformly distributed with variance σ^2, we can write p^i as a cumulative density function of uniform distribution:

$$p^i = 1 - U_{E^i\hat{R}^k,\sigma_v^2}\left(\frac{R+\lambda_b R^{ib}}{1+\lambda_b}\right) = \frac{1}{2} - \frac{\left(R+\lambda_b R^{ib}\right)}{2\sigma\sqrt{3}(1+\lambda_b)} + \frac{E^i\hat{R}^k}{2\sigma\sqrt{3}} \tag{3}$$

Equation (3) shows the connection between the individual probabilities and the interbank interest rate. Aggregating the interbank market, we show how these individual probabilities translate into the interbank market rate.

Every bank has different risky asset return expectations, which results in different portfolio decisions. Banks' choices are illustrated in Figure 2 (for the case where the interbank market functions). In Figure 2 banks are distributed according to their risky asset return expectations. Let us start with the most pessimistic bankers. Their estimates of the risky asset return are so low that: 1) they are lower than the rate on reserves: $E^i\hat{R}^k < R^{res}$, 2) the subjective probability of loan repayment on the interbank market is so low that the expected interbank market return is lower than the rate on reserves: $p^i R^{ib} < R^{res}$. These bankers hoard (invest in reserves). The less pessimistic bankers assign a higher loan repayment probability; for them $p^i R^{ib} \geq R^{res}$. At the same time, they do not expect the risky asset to pay more than the interbank market: $E^i\hat{R}^k < p^i R^{ib}$. We denote a banker who is indifferent between lending and hoarding as a marginal lender with beliefs $E^l\hat{R}^k$ such that $p^l R^{ib} = R^{res}$. The optimistic banks – those investing in the risky asset – believe that the risky asset pays more than lending on the interbank market $E^i\hat{R}^k > p^i R^{ib}$. The marginal investor is then a banker indifferent between investing and lending; we denote her beliefs as $E^m\hat{R}^k = p^m R^{ib}$. Among the optimists there are those who believe that the risky asset pays even more than the interbank market rate: $E^i\hat{R}^k > R^{ib}$. They borrow on the interbank market and invest everything in the risky asset. The marginal borrower is then defined simply as $E^b\hat{R}^k = R^{ib}$.

For the interbank market to clear, lending must be equal to borrowing. Lenders are risk neutral and lend everything they have, and all have the same funds (equal to 1), so the amount of lending is

Figure 2: Banks' Expectations and Investment Decisions
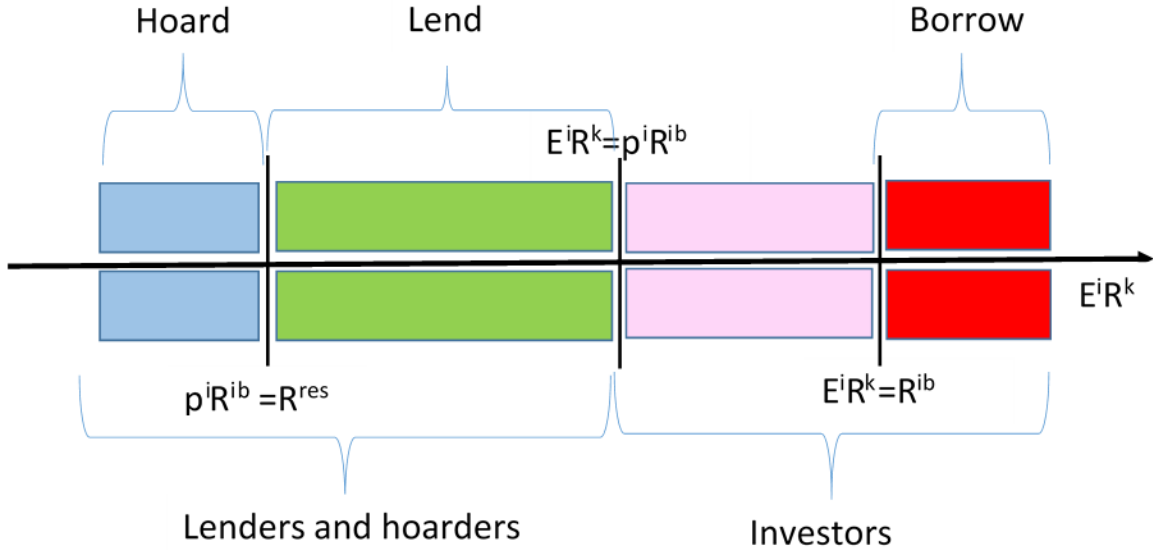
given by:

$$\underbrace{\frac{1}{}}_{\text{lenders' funds}} * \underbrace{\int_{E^l \hat{R}^k}^{E^m \hat{R}} f(x)d(x)}_{\text{share of lenders}} * \underbrace{\frac{1}{}}_{\text{number of banks}}$$

where $f(x)$ is a probability density function of uniform distribution. To calculate the share of lenders, we use the frequency of banks between the marginal lender and the marginal investor. Multiplying the share by the total number of banks and lenders' funds we get the total supply of interbank lending.

Similarly, the amount of borrowing on the interbank is:

$$\underbrace{1}_{borrower's\ funds} * \underbrace{\lambda_b}_{collateral\ constraint} * \underbrace{\int_{R^{ib}}^{\bar{R}} f(x)\,dx}_{share\ of\ borrowers} * \underbrace{1}_{number\ of\ banks}$$

where the upper bound on return expectations is given by $\bar{R}$. Multiplying the share by the total number of banks we get the number of borrowers. As every borrower faces the collateral constraint $\lambda_b * borrower's\ funds$, the demand for interbank funds is given by the number of borrowers times this collateral constraint.

Knowing the distribution of beliefs across banks, in the case of uniform distribution we can write the interbank market clearing condition as follows (a detailed derivation is given in Appendix A):

$$E^m \hat{R}^k - E^l \hat{R}^k = \lambda_b \left(\sigma \sqrt{3} + m - R^{ib} \right) = \lambda_b \left(\bar{R} - R^{ib} \right) \tag{4}$$

The interbank market rate, R^{ib}, clears the market, and m and σ are the mean and the standard deviation of banks' beliefs distribution. The supply of loans is given simply by the difference

between the belief of the marginal investor and that of the marginal lender: $E^m \hat{R}^k - E^l \hat{R}^k$. The demand for loans is the difference between the largest belief in the market, $\bar{R}$, and the belief of the marginal borrower. Note that the marginal borrower expects the risky asset return to be equal to the interbank market rate. When the distribution is uniform, the largest belief can be written as the sum of the mean and the standard deviation $\sigma\sqrt{3} + m$. Each individual probability is also a function of the standard deviation.

The solution to the model is then given by (4), (3), and the definitions of the marginal lender and investor:

$$
\begin{aligned}
p^l R^{ib} &= R^{res} \\
E^m \hat{R}^k &= p^m R^{ib}
\end{aligned}
$$

The marginal lender's belief determines the lower bound on the interbank interest rate. It must be such that there exists at least one banker whose expected interbank market return, $p^l R^{ib}$, is greater than or equal to the reserve rate, R^{res}. And at the same time, her belief about the risky asset return must be lower than the interbank market return:

$$
E^l \hat{R}^k \leq p^l R^{ib} \geq R^{res} \tag{5}
$$

Equation (5) also shows where the non-linearity in the model comes from. If the market beliefs are such that there is no banker expecting the interbank market return to be higher than the reserve rate or the risky asset return, there is no lending. The reason for this may be that market beliefs about the risky asset are very low. Then, a lender would expect interbank lending to be very risky (p^i, the probability of loan repayment, is very low) and demand a high interbank rate, as $R^{ib} \geq \frac{E^i \hat{R}^k}{p^i}$ and $R^{ib} \geq \frac{R^{res}}{p^i}$. At the same time, no banker would believe that the risky asset pays more than this high interbank rate, and none would be willing to borrow. The marginal borrower determines the upper bound on the interbank market rate. It should not exceed the largest belief in the economy: $R^{ib} \leq \bar{R}$. This upper bound, however, disappears in the full model with normal distribution of banks' beliefs.

Depending on the beliefs and their mean and dispersion, the interbank market can have three different states: a functioning market, or a market where no one lends, or a market where no one borrows.

Proposition 2.0.1. *Given the bounds of the beliefs distribution, for an interbank market to exist the interbank rate must satisfy*

$$
R^{res} < R^{ib} < \frac{1+\lambda_b}{\lambda_b} \sigma\sqrt{3}
$$

for $\sigma > \frac{\lambda_b R^{res}}{(1+\lambda_b)\sqrt{3}}$.

The mathematical proof is given in Appendix A. Proposition 2.0.1 states that for an interbank market to exist, beliefs must be sufficiently diverse. It also determines the upper bound on the interbank rate proportionally to banks' diversity. If the diversity of beliefs is small, the beliefs of borrowers and lenders do not differ much, borrowers' beliefs are not much more optimistic than those of lenders, and borrowers are not willing to pay a high interbank rate. In the opposite case, with large diversity, borrowers expect much higher returns from the risky asset than lenders do, and the interbank rate is higher.

Proposition 2.0.2. *Low market beliefs result in a lower interbank rate and lower lending.*

The mathematical proof is given in Appendix A. The proposition is illustrated in panel *a* of Figure 3. If the dispersion is fixed, a decrease in market beliefs about the risky asset return means a shift in the bounds of the beliefs distribution: the most pessimistic banker becomes even more pessimistic and the most optimistic banker becomes less optimistic. Borrowers (the red area) use interbank loans to invest in the risky asset. Intuitively, when borrowers expect a lower return on the risky asset they are willing to pay less for the interbank loan. With all bankers being less optimistic about the risky asset return, there is a larger share of those who do not invest themselves and expect a lower loan repayment probability: there is more hoarding (the blue area). Those bankers who are considering whether to lend on the interbank market or to invest in the risky asset evaluate these two options at a lower interbank rate. This makes interbank lending less attractive and the share of lenders (the green area) also shrinks.

As risky asset return expectations decline, the interbank market rate falls, but there is less lending. On the demand side, borrowers are willing to pay less for the interbank loan, as they expect to receive lower returns on loans to the real sector. One the supply side, lenders expect a lower loan repayment probability. They believe it is harder for borrowers to repay interbank market debt, as they believe that borrowers' investment in the risky asset pays little. The decline in the interest rate increases their p^i slightly, but not enough to compensate for the fall in beliefs. The lenders would ask for a higher interbank rate, but then no one would borrow at such a price. Therefore, some lenders leave the market and become hoarders. Only those with higher beliefs are still willing to lend at this reduced interbank market rate. However, the reduction in the interbank market rate is not enough to make all the borrowers stay on the market, and some of them – the less optimistic ones – stop borrowing. This results in a lower interbank rate and lower interbank market lending.

Let us now consider the possible scenarios for the functioning of the interbank market.

Case 1. No one lends.

With a functioning interbank market, a marginal lender compares the return on the safe asset and the expected interbank market return. However, if the expected interbank market return is lower than the expected risky asset return, a banker prefers to invest herself rather than lend on the market. From proposition 2.0.2 it follows that the interbank rate falls as market beliefs decrease. Suppose, then, that market beliefs are so low that borrowers are willing to pay an interest rate so small that no one is willing to lend. In terms of a marginal lender this would imply $p^l R^{ib} = R^{res} \leq E^l \hat{R}^k$: a banker who is indifferent between hoarding and interbank lending is actually better off investing in the risky asset instead.

Case 2. No one borrows.

It is also possible that the risky asset is so attractive to all the bankers that the lenders ask for a high interbank rate (to compensate them for not investing in the risky asset themselves). If the interbank rate is higher than the upper bound of the distribution $R^{ib} > \bar{R}$, no one borrows. This case, however, is absent from the normal distribution without distribution bounds. Consequently, even with a higher interbank rate there will be borrowers, but their share will be very small.

Case 3. A functioning interbank market. In this case, market beliefs are such that on the demand side $R^{ib} < \bar{R}$, and on the supply side $p^l R^{ib} = R^{res} > E^l \hat{R}^k$. That is, there exists a marginal lender and

Figure 3: Banks' Beliefs Distribution

(a)   *(b)*
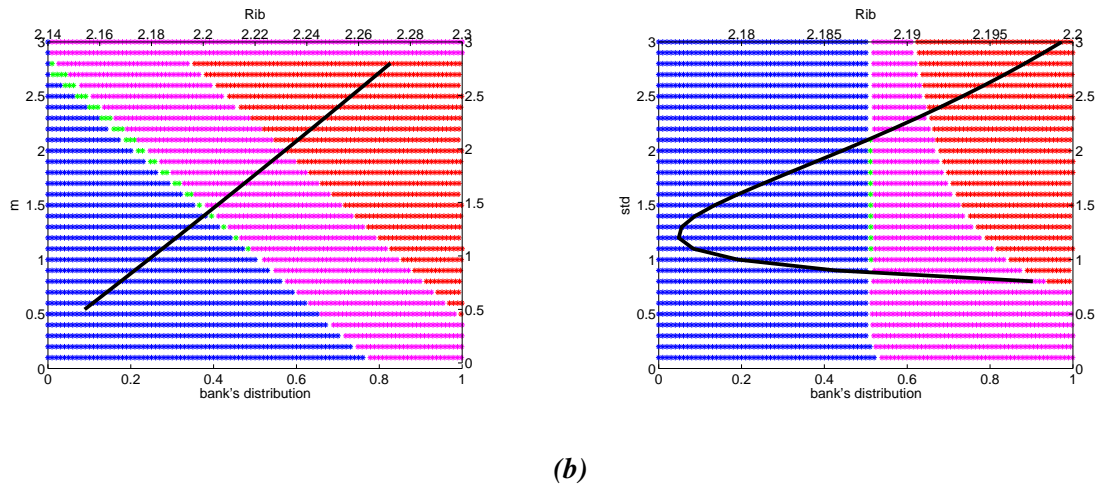
Note: Numerical simulations. Blue area – hoarders, green area – lenders, pink area – investors who do not borrow, red area – investors who borrow, solid line – interbank rate (upper horizontal axis)

a marginal borrower. All three cases are illustrated in Figure 3, where panel *a* shows the impact of mean market beliefs on banks' equilibrium allocations and panel *b* shows the impact of the beliefs dispersion and the standard deviation of banks' forecasts. On the horizontal axis are the shares of bankers: hoarders, lenders, investors that do not borrow, and investors that borrow. The black line is the interbank market rate, measured on the upper horizontal axis. When the interbank market rate is not defined, the interbank market collapses.

Corollary 2.0.1. *With very low beliefs diversity, there is no lending on the interbank market. With very high beliefs diversity, lending is possible but small.*

From proposition 2.0.1 there is a lower bound on beliefs diversity $\sigma > \frac{\lambda_b R^{res}}{(1+\lambda_b)\sqrt{3}}$. Because a lender compares the expected interbank market return with the return on reserves, the rate on reserves defines this upper bound. The role of the standard deviation is two-fold in the model, as shown in panel *b* of Figure 3. First, it measures the dispersion of beliefs among banks. With very low dispersion there is little difference in beliefs across bankers and there is little or no lending. Second, it reflects how each bank is uncertain about its own estimate of the future return. If a lender is almost certain about her low return expectation, she will assign a low loan repayment probability and ask for a high interbank market rate. At the same time, the borrowers are more convinced about their high return expectations and are willing to pay that high rate. Consequently, with a low standard deviation, there is either no lending, or very low lending at a high interbank rate. As the standard deviation increases, so does the uncertainty among the borrowers. They are willing to pay a lower interbank rate. For the same reason, the lenders are less certain about their pessimistic returns. They expect a higher loan repayment probability and agree to a lower interbank rate.

When the uncertainty and dispersion are very large, there is still lending, but its volume is negligible (see Figure 3). If the diversity of beliefs is large, the bounds of the distribution widen and there are some very optimistic borrowers. This pushes the interbank market rate up.

Policy effects
A functioning interbank market.

Figure 4: Banks' Beliefs Distribution and Interest Rate Decline
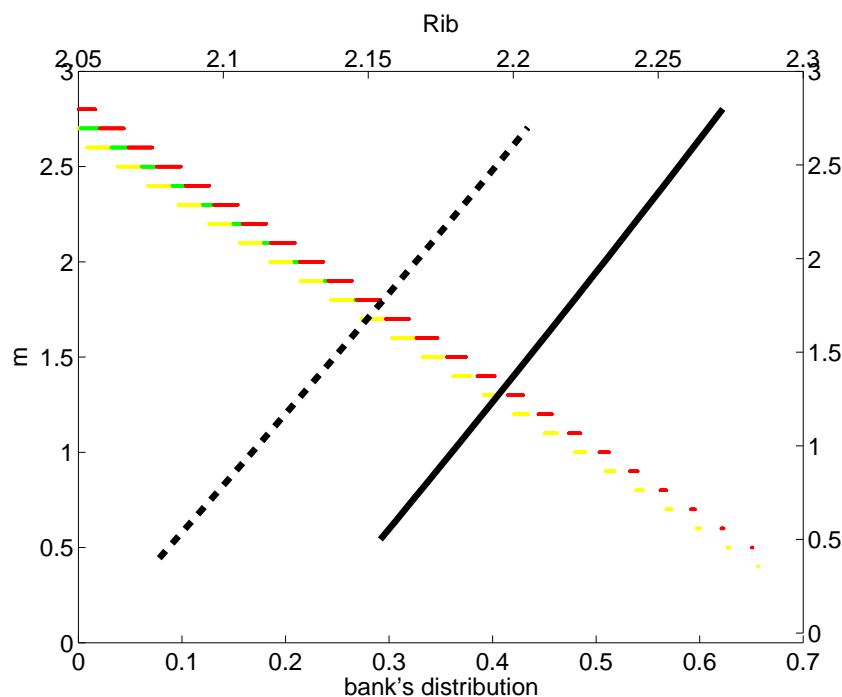
Note: Numerical simulations. Red area – old lenders, yellow area – new lenders, green area – their interception. Solid line – old interbank rate, dashed line – new interbank rate.

Let us now consider the impact of policy actions in the context of the simple model we have developed. First, suppose that the interbank market is functioning, but some superior agent, which we call the central bank, would like to increase interbank lending and/or stimulate credit to the real economy.

Proposition 2.0.3. *A low policy rate increases lending and lowers the interbank market rate, and increases the supply of credit to the real economy.*

The mathematical proof is given in the appendix. First, consider a change in the policy rate, which is the rate on the safe asset – the reserve rate, R^{res}. Suppose there is a decline in the reserve rate. This makes the safe asset less attractive, implying that some of the hoarders would like to become lenders: $E^l \hat{R}^k$ falls. The larger supply of lending results in a lower interbank market rate. From the point of view of the marginal investor, the risky asset becomes more attractive, as the interbank market return falls. A less optimistic bank becomes a marginal investor, as $E^m \hat{R}^k$ falls too. But what about the set of lenders $E^m \hat{R}^k - E^l \hat{R}^k$ as both of the bounds fall? It turns out that there is a slight increase in the share of lenders, as the first-order effect of the policy rate decrease dominates the decline of the interest rate: $E^l \hat{R}^k$ falls more than $E^m \hat{R}^k$. Figure 4 shows the shift in the interbank rate and the distribution of lenders. The set of new lenders that used to hoard liquidity is marked in yellow. Those who lend under the new and old policy rate are marked with green, and those who used to lend under the old policy rate but now choose to invest are marked in red. It is immediately clear from Figure 4 that the increase in the set of lenders is very small. On the other hand, as lenders are now shifted to the left of the beliefs distribution, they crowd out hoarders and make more room for investors. Recall that all funds that are not kept in reserves are transferred to the real sector, either directly or through interbank market lending. Thus, even when lending increases by only a small amount, there is an increase in credit to the real sector.

So, in the simple model framework, a reduction in the safe interest rate can increase the interbank volume and reduce the interbank rate. More importantly, it increases the supply of credit to the real economy. However, in the general equilibrium context discussed below, a reduction in the policy rate results in a worse outcome than no policy action. This is because of a decline in banks' net worth.

Proposition 2.0.4. *Relaxing the collateral constraint increases lending and the interbank market rate, but lowers credit to the real economy.*

The mathematical proof is given in the appendix. The collateral constraint in our model is more a mathematical restriction and is not meant to represent the real-life interbank market or the central bank's policy. However, we find it instructive to analyze what impact relaxing the constraint could have on the interbank market. When borrowers are less restricted, one would expect there to be more interbank market lending and more credit supply to the real economy. In our model, allowing borrowers to borrow more does indeed increase lending and the interbank market rate. With relaxed restrictions and a higher interbank rate, lenders expect a lower loan repayment probability, with the higher interbank rate partially compensating for it. However, the most pessimistic lenders leave the market. That is, the marginal lender must now have higher return expectations, and all bankers that have lower beliefs (and there are now more of them, as the marginal lender shifts to the right) hoard. That is, hoarding increases.

The higher interbank rate attracts some of the investors, who now prefer to lend than to invest themselves. Because of them, the subset of lenders expands. Also, the high interbank rate makes it unattractive for some of the borrowers. That is, there are fewer borrowers, but each of them borrows more. There are also fewer investors, but those of them who do borrow attract more interbank market funds. In our model, credit to the real sector depends on how many bankers do not hoard funds. Even though lenders do not invest themselves, they lend all their funds to the borrowers, who invest them. As a result, credit to the real sector falls when there are more hoarders.

Liquidity provision. In this simple framework without liquidity concerns, the provision of liquidity to banks, whether targeted or untargeted, does not affect the functioning of the interbank market. All the bankers would allocate all their available funds according to the decision rules discussed above. The provision of funds to optimists increases credit to the real economy, given that optimists exist. If the liquidity provision is untargeted and the funds are distributed equally among the banks, the pessimistic banks hoard it, as their main concern is counterparty risk and a low risky asset return. In this regard, targeting only optimistic banks can increase credit. Again, in the general equilibrium context, the feedback from prices and banks' balance sheets reverses the predictions: the untargeted policy results in better general outcomes than the targeted one.

To sum up, in the light of our model, if a central bank wants to increase lending on a market where banks are concerned about counterparty risks, a policy that does not address those concerns can do very little. Moreover, it could even have opposite-than-desired effects.

Interbank market collapse. Let us now consider the case where the interbank market collapses due to low market expectations about the risky asset return.

Suppose that expectations are such that no lenders are willing to lend. Obviously, providing them with additional funds will not revive interbank lending, but, if provided to optimists, such funds could increase credit to the real economy. The size of this effect is conditional on the share of investors: the more investors there are in the economy, the more efficient the policy will be. On

the other hand, if the funds are provided to pessimistic banks, the policy increases hoarding and has only a small impact on credit to the real sector.

Proposition 2.0.5. *The effect of a policy rate reduction is limited by the mean market belief.*

A formal proof is provided in the appendix. The only tool that might have a potential effect is **a reduction in the policy rate**. Given the average beliefs and the initial policy rate, it is possible to restore some fraction of the interbank market. In Figure 4 there is a small range of mean market beliefs where the interbank market collapses at the old interest rate, but there is still some lending at the new interest rate. However, this policy has a very limited or zero effect if market beliefs are very low, which also means a very low interbank rate. In this case, even with a low reserve rate, hoarding is still more attractive than interbank lending. Formally, when no one lends because of a low equilibrium interbank rate, $E^l R^k > p^l R^{ib} = R^{res}$. Lower reserve rates would imply that a banker with lower beliefs would now be willing to lend: p^l falls, so $E^l R^k$ decreases, although only if the resulting interbank rate is high enough. This only happens when the mean market belief is not too low (see proposition 2.0.2). Otherwise, even the lower $E^l R^k$ is still smaller than the expected interbank market return.

Proposition 2.0.6. *Relaxing the collateral constraint does not restore the functioning of the interbank market or credit to the real economy*

Proof. Suppose that no one lends in the interbank market. This means that the marginal lender is better off investing herself than lending: $E^l R^k > p^l R^{ib} = R^{res}$. To restore lending, the policy should bring about $E^l R^k \leq p^l R^{ib} = R^{res}$. Proposition 2.0.4 showed that the marginal lender's belief increases with increasing collateral constraint λ_b. In this case, an increase in λ_b means an increase in $E^l R^k$, that is, lending is not restored. Intuitively, as was discussed in proposition 2.0.4, when borrowers are more leveraged lenders expect a lower loan repayment probability. In the notation of our model, this means shifting the marginal lender's belief, $E^l R^k$, to the right, even further away from $p^l R^{ib}$. □

To sum up, if banks are concerned about a low risky asset return and expect a low loan repayment probability, policy actions have a very limited effect. Liquidity provision policies enhance credit through optimistic bankers only, with the rest of the funds ending up in reserves. A low interest rate policy restores the market only if market beliefs are not very low, and stimulates credit to the real economy among banks expecting the risky asset to pay more than the storage asset.

3. Closing the General Equilibrium Model

In this section we drop our simplifying assumptions about banks' beliefs. In particular, banks now form expectations based on past data on risky asset returns and private signals about future returns. A bank's belief is then the result of the Kalman filter and follows a normal distribution. We then input the banking sector developed above into a linearized DSGE model as in Gertler and Karadi (2011). In their model, agents have perfect expectations about future risky asset returns. We modify it so that risky asset returns are harder to predict. Besides publicly observable risky returns, our banks have private signals about future returns. These signals are heterogeneous, though correlated among banks, and are subject to mood swings, that is, mood shocks, which we model as a decline in banks' average private expectations about asset returns. Another difference is that, in Gertler and Karadi (2011), banks frictionlessly transfer their liabilities to credit to the real sector. In our model, we allow the banks to keep (hoard) liquidity if they choose to. Thus, it is possible to address the

question of whether the liquidity provided by the central bank is transmitted to the real economy, or ends up in bank reserves. Last but not least, heterogeneous expectations give rise to an interbank market. In our model, the interbank market serves as a propagation mechanism, increasing or decreasing the credit supply as interbank market conditions change.

We start with a description of the main building blocks of the model: the financial sector with heterogeneous beliefs and the interbank market. Then we proceed to complete the general equilibrium model and consider crisis and policy effects when there is feedback from the rest of the economy. The rest of the sectors are standard as in Smets and Wouters (2007) and Gertler and Karadi (2011), so we outline them only briefly. For a more rigorous discussion the reader is referred to these papers.

3.1 The Financial Sector

There is a continuum of banks normalized to one. Every period, a fraction $(1 - \theta)$ of banks exit the sector and join the households. At the same time, the same number of household members become bankers and receive starting capital from the households. This starting capital equals a share of total banking sector assets. Banks receive deposits from the households, paying a gross real rate R_t. We model deposits as distributed equally among the banks regardless of their portfolio holdings. Banks allocate their funds between a safe asset paying the same gross real rate R_t, a risky asset with an uncertain gross real return, R_{t+1}^k, and interbank market lending with a gross real return R_t^{ib}. Banks are aware of the risk that some borrowers may not repay their debt. The debt repayment probability is reflected in the interbank market rate.[10] In order not to track the distribution of each banker's wealth, we treat the bankers as members of one family, where each member maximizes his own return. At the beginning of a new period, before making investment decisions, they all average their net worth.

The risky asset in the model is credit to the real sector. Banks buy the claims of non-financial firms, S_t, at price Q_t and return R_{t+1}^k. The non-financial firms are intermediate goods manufacturers that need funding to buy capital. They transfer the return on the capital as a payment on S_t. The uncertainty comes from a so-called capital quality shock, ξ_{t+1}, as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). As opposed to physical depreciation, δ_t, it is intended to capture unexplained fluctuations in the value of capital: it influences not only the capital stock $\delta_t \xi_t K_{t-1}$, but also its value $Q_t \xi_t K_{t-1}$. The value of undepreciated capital is then defined as the difference between the value of new capital and the value of depreciated capital.

$$(Q_{t+1} - \delta_{t+1}) \xi_{t+1} K_t \tag{6}$$

The return on capital consists of the value of the marginal product of capital, $\alpha \frac{P_{m,t+1} Y_{t+1}}{\xi_{t+1} K_t}$, plus the value of new capital, Q_{t+1}, minus depreciated capital, δ_{t+1}. The quality shock, ξ_{t+1}, then influences expectations of the return on capital:

$$R_{t+1}^k = \frac{\left(\alpha \frac{P_{m,t+1} Y_{t+1}}{\xi_{t+1} K_t} + Q_{t+1} - \delta_{t+1} \right) \xi_{t+1}}{Q_t} \tag{7}$$

Equations (6) and (7) are identical to those in Gertler and Karadi (2011). Now, though, we adjust the process for the quality shock. It is observable by all the sectors, but the composition of the

---

[10] Note the timing of the interest. Although it is paid in period $t + 1$, the rate on the safe asset and the interbank market rate are set in period t. For the rest of the model description we use the same convention to refer to the timing of the variables when they are decided upon.

shock is unobservable. With this process we intend to capture developments in capital value which are not predictable by the market. We assume that capital quality is subject to two types of shocks – persistent and transitory. The combination of these two shocks creates uncertainty in predicting future values of capital quality.

$$\xi_t = \rho_\xi \xi_{t-1} + \mu_t + \varepsilon_{\xi,t} \tag{8}$$

μ_t is a persistent shock

$$\mu_t = \rho_\mu \mu_{t-1} + v_t \tag{9}$$

where ρ_μ and ρ_ξ are persistence parameters, v_t and $\varepsilon_{\xi,t}$ are transitory Gaussian shocks, serially uncorrelated with zero contemporaneous correlation and variances σ_v^2 and σ_ε^2. Neither the intermediate goods producers nor the banks observe either μ_t or $\varepsilon_{\xi,t}$. Next we explain how the banks set their expectations about ξ_{t+1}.

3.1.1 Expectations Formation

Banks do not observe whether the change in ξ in (8) is due to a transitory or a persistent shock. They have access to past data on returns and they use it to form a homogeneous economic forecast. There are, however, private signals about the value of μ_t. The motivation for the use of private signals can come from different aspects of forecasting practice. First, they can provide qualitative information, such as news about the future, which is used as an additional source of information on the current state variable. Pieces of news are interpreted differently by each bank's experts, although their interpretation may be correlated.[11] Second, banks may have access to private information not visible to other banks. The inclusion of expert forecast adjustments is motivated by an extensive literature that provides evidence of the widespread use of expert factors in forecasting practice.[12] Expert adjustments are widely used in both the private and public sectors and in central bank forecasts, and are often found to improve forecast quality.[13] It is beyond the scope of this paper to analyze experts' motivation and behavior, so we consider these adjustments as just noisy signals about the current value of the permanent shock μ_t. The incorporation of heterogeneous expert opinions results in heterogeneous bank expectations.

We model expert adjustment as an additional signal about the value of μ_t:

$$\theta_t^i = \rho_\theta \theta_{t-1}^i + \eta_t^i \tag{10}$$

where η_t^i is the noise in the opinion of bank i's expert, with η_t^i being correlated draws from $N(\mu_t, \sigma_\eta)$.

---

[11] For example, there may be disagreement about the size of the effect, but agreement about whether the effect is positive or negative.

[12] For an example of this literature and a survey see Franses et al. (2011) or Fildes et al. (2009).

[13] They are used to take into account "special events" (e.g. a new wave of quantitative easing by the ECB) or upcoming changes that cannot be reflected in the model-based forecast. However, expert adjustments tend to be both biased and inefficient. Moreover, there are behavioral aspects of expert adjustments: experts tend to be overly optimistic, or may see non-existent patterns in the noise, or overestimate their own knowledge of the subject. There may also be incentives problems, where experts adjust their forecasts only to show supervisors that they are "doing their job." That is, expert adjustments may introduce bias into the forecast, despite the fact that they are often found to be helpful in increasing forecast accuracy.

Figure 5: Formation of the Bank's Forecast
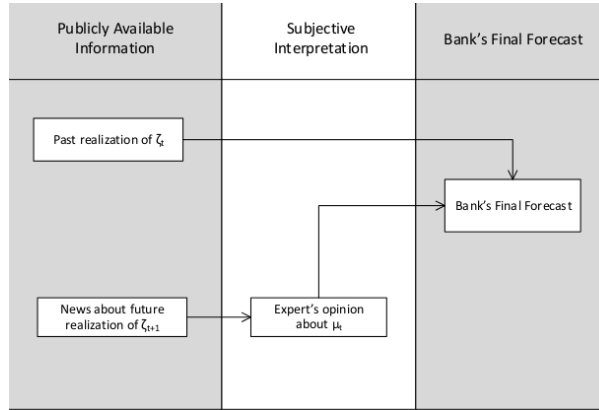
The noise in expert opinions is correlated with correlation coefficient ρ^c.[14] This correlation can be interpreted in two ways. First, experts tend to react to similar news in a similar fashion – being overly optimistic or overly pessimistic. Second, even though formally they do not share their forecasts with each other, we retain the possibility of convergence of their opinions or coordination on an additional public signal. That is, when the correlation coefficient is one, experts' opinions are fully converged and are the same. Conversely, when ρ^c is zero, they are fully diverged. We assume that the correlation coefficient lies between zero and unity. Appendix B shows that such a correlation among expert errors shifts the average of the draws away from the distribution mean, so that the error in expert opinions is not averaged away.

A question can be raised here: why do the experts' opinions differ in the first place if they observe the same pieces of news? Though we do not model how the experts form their opinions implicitly, the intuition for the difference among them can be found, for example, in Acemoglu et al. (2006), where experts who are uncertain about the **interpretation** of a signal have heterogeneous beliefs that do not necessarily converge asymptotically. In our simple case, the uncertainty could be about the informativeness of news or about the efficiency of a particular action (such as technological or administrative changes on a manufacturer's site). Belief formation is illustrated in Figure 5. Banks have two sources of information – the past and current realizations of ξ_t and the expert opinion about μ_t. Banks use the Kalman filter to combine the two signals, with the weights of the signals in the final forecasts depending on their relative variance. A description of the Kalman filter setup is given in Appendix C.

3.1.2 The Interbank Market and Banks' Problem

The banks' problem is very similar to the one in the simplified model. At time t, banks choose their portfolio allocation: invest a share in the risky asset, α_t^i, leave a share in the reserves (or hoard), h_t, lend on the interbank market, $\left(1 - \alpha_t^i - h_t^i\right)$, or borrow on the interbank market, Λ_t^i. The banks are risk neutral and when choosing the portfolio they maximize their expected return. The expected return of bank i consists of the expected return on the risky asset, the return on the safe asset, the expected return on interbank market lending if the bank is a lender, and the expected net return on funds borrowed on the interbank market if the bank is a borrower:

$$\max_{\alpha_t^i, h_t^i, L_t^i} E_t^i \hat{R}_{t+1}^k \alpha_t^i + h_t^i * R_t^{res} + p_t^i \left(1 - \alpha_t^i - h_t^i\right) R_t^{ib} + (E_t^i \hat{R}_{t+1}^k - R_t^{ib}) * \Lambda_t^i \tag{11}$$

---

[14] ρ^c is the Pearson correlation coefficient for each pair of experts.

subject to

$$\Lambda_t^i = 0 \text{ or } \lambda_b$$

where R_t^{ib} is the gross real interbank market rate to be paid at $t+1$, and p_t^i is the subjective probability that the loan will be repaid. $E^i \hat{R}_{t+1}^k$ is bank i's subjective expectation about the risky asset return, and R_t^{res} is the gross real safe asset return. If a bank is a borrower on the interbank market, borrowing is restricted to a fraction λ_b of its net worth. Because we assume that net worth is averaged up at the beginning of the period, all borrowers borrow the same amount $- \lambda_b$. For a lender $\Lambda_t^i = 0$.

Because banks are risk neutral, their problem has a corner solution. With a functional interbank market, a bank will invest as much as possible in the risky asset, $\alpha_t^i = 1$, if $\hat{E}_t^i R_{t+1}^k \geq p_t^i R_t^{ib}$, and will not invest at all, $\alpha_t^i = 0$, if $\hat{E}_t^i R_{t+1}^k < p_t^i R_t^{ib}$. A bank hoards its funds, $h_t^i = 1$, if the expected return on the interbank market is lower than the reserve rate $p_t^i R_t^{ib} < R_t^{res}$, and lends on the market $\left(1 - \alpha_t^i - h_t^i\right) = 1$ if $p_t^i R_t^{ib} > R_t^{res} \ \hat{E}_t^i R_{t+1}^k < p_t^i R_t^{ib}$. Those which are more optimistic about the risky asset return borrow on the interbank market $\hat{E}_t^i R_{t+1}^k > R_t^{ib}$. The interbank market rate, R_t^{ib}, is determined by the market clearing condition.

The main modification from the simple case is a different beliefs distribution. The distribution affects the subjective loan repayment probability and the share of bankers hoarding, etc. Recall that each bank's subjective probability of borrowers being able to meet their obligations is:

$$p_t^i = 1 - F_{E_t^i \hat{R}_{t+1}^k, \sigma_R^2} \left(\frac{\left(R_t d_t + \lambda_b R_t^{ib}\right)}{(1 + \lambda_b)} \right) \tag{12}$$

where $(1 + \lambda_b) E_t^i \hat{R}_{t+i}^k$ is the bank's expected risky asset return on its own funds plus those borrowed on the interbank market, λ_b. For a bank to be able to honor its interbank market loan (assuming that debt to the household has priority) the return should be higher than payments to the household, R_t, times the amount of deposits per bank, d_t, and the interbank loan repayment, $\lambda_b R_t^{ib}$, because each bank's belief is distributed normally with variance σ_R^2. Now, p_t^i is the cumulative density function of the normal distribution.

However, to consider what fraction of banks actually invest in credit to the real sector or borrow in the interbank market, we need the distribution across banks. The distribution is Gaussian with some mean, m, and the same variance σ_R^2. The variance of the forecasts is the same for all the banks because they use the same observable and we assume the same variance in their expert adjustments. Both the mean and the variance enter the rest of the model as state variables. Appendix B shows how the correlation of banks' expert adjustment affects the mean and variance of the forecast distribution across banks. When simulating the general equilibrium model, we use only the mean and the variance of the forecast across banks and evaluate the cumulative distribution function for the beliefs of the marginal bankers – the investor, the lender, and the borrower. To sum up, we have a continuum of banks with beliefs distributed across banks $E_t^i \hat{R}_{t+i}^k \sim N(m, \sigma_R)$. Thus, the share of banks investing in the real economy is simply the share of banks with beliefs equal to or higher than the marginal investor's, whose belief is $E_t^m \hat{R}_{t+1}^k$:

$$s_t^{inv} = \int_{E_t^m \hat{R}_{t+1}^k}^{\infty} f(x) \, dx = 1 - F_{m, \sigma_R^2} \left(E_t^m \hat{R}_{t+1}^k \right) \tag{13}$$

$$E_t^m \hat{R}_{t+1}^k = p_t^i R_t^{ib} \tag{14}$$

where $F_{m,\sigma_R^2}\left(E_t^m \hat{R}_{t+1}^k\right)$ is normal cdf with mean m and variance σ_R^2. With a functioning interbank market, the marginal investor is indifferent between lending to the real sector and lending on the interbank market.

The share of banks borrowing on the interbank market can be defined as the probability that their belief is higher than the interbank interest rate. The belief of a marginal borrower then equals the interbank market rate $E_t^b \hat{R}_{t+1}^k = R_t^{ib}$. Again, we express the share of borrowers through the normal cdf.

$$s_t^b = \int_{R_t^{ib}}^{\infty} f(x)\,dx = 1 - F_{m,\sigma_v^2}\left(R_t^{ib}\right) \tag{15}$$

The share of banks lending is then defined as the probability that the belief is higher than the belief of a marginal lender, $E_t^l \hat{R}_{t+1}^k$, who is indifferent between hoarding and lending. His belief is defined through his loan repayment probability:

$$p_t^l R_t^{ib} = R_t^{res}$$

and the share of lenders is the probability that beliefs lie between the marginal investor's and the marginal lender's:

$$
\begin{aligned}
s_t^l &= \int_{E_t^l \hat{R}_{t+1}^k}^{E_t^m \hat{R}_{t+1}^k} f(x)\,dx = \\
&= F_{m,\sigma_R^2}\left(E_t^m \hat{R}_{t+1}^k\right) - F_{m,\sigma_R^2}\left(E_t^l \hat{R}_{t+1}^k\right)
\end{aligned}
$$

The share of those keeping money in reserves (hoarding) is then defined as those neither investing nor lending $\left(1 - s_t^b - s_t^l\right)$. Multiplying these shares by the total funds of the banking family, we get the respective amounts of credit, borrowing, lending, and hoarding.

3.1.3 Interbank Market Clearing

For the interbank market to clear, demand should be equal to supply. Each borrower demands $\Lambda_t^i = \lambda_b$, and each lender supplies 1 if she finds the interbank market rate more attractive than alternative investments (hoarding or risky asset investment). So, market clearing is:

$$s_t^l = \lambda_b s_t^b$$

Supply on the interbank market is the share of lenders, as they supply all their available funds. Demand is then the share of borrowers multiplied by the funds demanded – λ_t^b. Plugging in the expressions for the shares, one can re-write the market clearing condition as:

$$\int_{E_t^l \hat{R}_{t+1}^k}^{E_t^m \hat{R}_{t+1}^k} f(x)\,dx = \lambda_b \int_{R_t^{ib}}^{+\infty} f(x)\,dx \tag{16}$$

where $f(x)$ is a normal density function. Alternatively:

$$F_{m,\sigma_R^2}\left(E_t^m \hat{R}_{t+1}^k\right) - F_{m,\sigma_R^2}\left(E_t^l \hat{R}_{t+1}^k\right) = \lambda_b \left(1 - F_{m,\sigma_R^2}\left(R_t^{ib}\right)\right)$$

The banks' mean beliefs and their variance enter (16) as the moments for the cumulative density function. Then the variance enters the definition of the marginal investor and the marginal lender: $E_t^m \hat{R}_{t+1}^k = p_t^m R_t^{ib}$ and $E_t^l \hat{R}_{t+1}^k$ such that $p_t^l R_t^{ib} = R_t^{res}$. Combining these two definitions and (16) one gets a solution for the interbank market rate and the corresponding amount of lending.

3.1.4 Bank's Net Worth and the Financial Accumulator

Similarly to Gertler and Karadi (2011) we define the bank's net worth as N_t^i, and B_t^i are the deposits from households. In our model, however, banks can invest in three types of assets: risky, safe (hoarding), and interbank loans. So, the bank's balance sheet in our model is given by:

$$Q_t S_t^i + Res_t^i + Lend_t^i = N_t^i + B_t^i \tag{17}$$

where $\frac{Q_t S_t}{N_t^i + B_t^i} = \alpha_t^i$, $\frac{Res_t^i}{N_t^i + B_t^i} = h_t^i$, and $\frac{Lend_t^i}{N_t^i + B_t^i} = \left(1 - \alpha_t^i - h_t^i\right)$ for the lender or $\frac{Lend_t^i}{N_t^i + B_t^i} = \lambda_b$ for the borrower in (11). Given each asset return, the evolution of a bank's net worth over time can be formulated for a lending bank as

$$N_{t+1}^i = R_{t+1}^k Q_t S_t + R_t^{ib} Lend_t^i + R_t^{res} Res_t^i - R_t B_t^i \tag{18}$$

From (17) $B_t^i = N_t^i - \left(Q_t S_t^i + Res_t^i + Lend_t^i\right)$, that is:

$$N_{t+1}^i = \left(R_{t+1}^k - R_t\right) Q_t S_t + \left(R_t^{ib} - R_t\right) Lend_t^i + (R_t^{res} - R_t) Res_t^i + R_t N_t^i \tag{19}$$

For a borrower on the interbank market, the net worth dynamics are modified:

$$N_{t+1}^i = \left(R_{t+1}^k - R_t\right) Q_t S_t^i + \left(R_{t+1}^k - R_t^{ib}\right) Lend_t^i + (R_t^{res} - R_t) Res_t^i + R_t N_t^i \tag{20}$$

Aggregating the net worth over all the banks, we define the evolution of the sector's net worth as:

$$N_{t+1} = \sum_i N_{t+1}^i = \left(R_{t+1}^k - R_t\right) \left(1 - s_t^h\right) (Q_t S_t + Res_t) +$$

$$+ (R_t^{res} - R_t) s_t^h (Q_t S_t + Res_t) + R_t N_t \tag{21}$$

where s_t^h is the share of banks that hoard. To get the result, we used the interbank market clearing condition that the amount of borrowing equals the amount of lending and, on aggregate, transfers from borrowers to lenders sum to zero. We also use the result that for a bank, the share of risky assets, α_t^i, is either 1 or 0. That is, the total amount of risky assets in the economy is

$$Q_t S_t = \left(1 - s_t^h\right) (N_t + B_t) \tag{}$$

Equation (21) defines total banking sector net worth. As we treat all bankers as members of a family where everyone shares the net worth and receives the same amount of deposits, it is also reasonable to address the agency problem between households and banks on the family level. As the agency problem is a slight modification of that in Gertler and Karadi (2011), we put the solution in Appendix D and here present the resulting constraint. Note that our agency problem differs in three respects. First, in our model banks have the possibility of putting their funds in reserves. Second, banks are heterogeneous, with a share of them investing in risky assets. And last but not

least, some banks participate in the interbank market, transferring some funds from pessimistic to optimistic banks.

The resulting constraint limits the amount of assets the banking family can acquire to be proportional to the family's net wealth:

$$(Q_t S_t + Res_t) = \frac{\eta_t}{\lambda - \nu_t \left(1 - s_t^h\right)} N_t = \varphi_t N_t \tag{22}$$

where φ_t is the banking sector leverage ratio, and ν_t and η_t are described in Appendix D.

To finalize the law of motion for banks' net worth, recall that in each period a fraction $(1 - \theta)$ of the bankers exit and take a $(1 - \theta)$ share of the banking family's assets. At the same time, households transfer a fraction $\frac{\omega}{1-\theta}$ of the exit value to the new bankers. That is, the law of motion of banks' net wealth is given by:

$$N_{t+1} = \theta \left\{ \left[\left(1 - s_t^h\right) \left(R_{t+1}^k - R_t\right) + s_t^h \left(R_t^{res} - R_t\right) \right] \varphi_t + R_t \right\} N_t + \omega \left(Q_t S_{t-1} + Res_{t-1}\right) \tag{23}$$

3.1.5 Credit Support Policies

We consider several credit support policies. Under the first two, the central bank funds asset purchases through intermediaries. The untargeted liquidity provision is modeled as the funding of a share ψ_t of banks' asset purchases:

$$Q_t S_t + Res_t = \varphi_t N_t + \psi_t \left(Q_t S_t + Res_t\right)$$

As a result:

$$Q_t S_t + Res_t = \frac{\varphi_t}{1 - \psi_t} N_t$$

A bank pays R_t for central bank support. One can think of this support as being financed through selling government debt to households paying R_t, so that it does not appear in the government budget constraint. There are, however, operational costs of conducting the policy, $\tau \psi_t \left(Q_t S_t + Res_t\right)$. As in Gertler and Karadi (2011) the central bank selects ψ_t as a proportion of the rise in the risk premium. When there are disturbances in the economy, the risk premium rises above the steady-state level.

$$\psi_t = \kappa \left(R_{t+1}^k - R_t - \overline{(Rk - R)} \right) \tag{24}$$

where κ is a reaction parameter.

Similarly, for targeted credit support, the central bank funds a fraction of the asset purchases, but limits the set of assets to risky claims on firms. Let ψ_t^{tar} denote the fraction of risky assets funded by the central bank. Then

$$Q_t S_t + Res_t = \varphi_t N_t + \psi_t^{tar} Q_t S_t$$

Using $Q_t S_t = \left(1 - s_t^h\right) \left(Q_t S_t + Res_t\right)$ we get:

$$Q_t S_t + Res_t = \frac{\varphi_t}{1 - \psi_t^{tar} \left(1 - s_t^h\right)} N_t$$

We assume this policy is as costly as unlimited liquidity provision, with the operational costs being $\tau \psi_t^{tar} Q_t S_t$. The central bank's decision rule is the same:

$$\psi_t^{tar} = \kappa \left(R_{t+1}^k - R_t - (\overline{Rk - R}) \right)$$

We further consider relaxing the collateral constraint on the interbank market and lowering the real gross return on the safe asset, R_t^{res}, both of which policies involve no operational costs. Relaxing the collateral constraint takes the form of increasing the fraction of borrowers' liabilities up to which borrowing is restricted – λ_b. An increase in this fraction, denoted as ∇_t^λ, and a reduction in R_t^{res}, ∇_t^R, follow the same decision rule as the two previous policies considered:

$$\nabla_t^i = \kappa^i \left(R_{t+1}^k - R_t - (\overline{Rk - R}) \right)$$

where i stands either for λ or for R^{res}. We allow for a different feedback parameter κ^i in the rules.

The Household There is a representative risk-averse household in the economy which has utility from consumption and disutility from labor. The household solves the following problem subject to a budget constraint:

$$\max_{C_t, L_t, D_t} E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i-1}^{1+\phi} \right] \tag{25}$$

$$\text{s.t. } C_t + B_t = W_t L_t + R_{t-1} B_{t-1} + \Pi_t + T_t \tag{26}$$

where C, L, B, and T stand for consumption, labor supply, deposits in banks, and tax, respectively. W and R are the real wage and the real gross return on bank deposits. Π_t is net transfers from financial and non-financial firms to the household. β, ϕ, $\chi > 0$, and $\beta < 1$.

Bank deposits are guaranteed by the government, which, in the case of bank insolvency, pays the deposits and interest to the household.

Denoting the marginal utility of consumption as ρ_t, we can write the first-order conditions as:

$$[C_t] \ \rho_t = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1} \tag{27}$$

$$[L_t] \ \rho_t W_t - \chi L_t^\phi = 0 \tag{28}$$

$$\Omega_{t,t+1} \equiv \frac{\rho_{t+1}}{\rho_t} \tag{29}$$

$$[B_t] \ E_t \beta \Omega_{t,t+1} R_{t+1} = 1 \tag{30}$$

where $\Omega_{t,t+1}$ is a stochastic discount factor. The first-order conditions state that the marginal disutility of labor is equal to the marginal utility of consumption and that the nominal return on bank deposits should, at the margin, compensate the consumer for postponing consumption to the next period.

Intermediate Goods Producers The sector is perfectly competitive. Producers combine labor and capital using the Cobb-Douglas production function:

$$Y_t = A_t \left(U_t \xi_t K_{t-1} \right)^\alpha L_t^{1-\alpha} \tag{31}$$

where K_{t-1} stands for capital, L_t stands for labor, and A_t is total factor productivity. U_t is the utilization rate of capital. That is, shock ξ_t influences effective capital.

Investment in capital should be made one period in advance. In other words, to produce in period $t+1$ the investment should be made in period t. To invest in the next period's capital, K_t, intermediate goods producers issue claims S_t at price Q_t^S. The value of the capital they can buy at price Q_t^K is then:

$$Q_t^K K_t = Q_t^S S_t \tag{32}$$

In the next period, intermediate goods producers sell the depreciated capital to capital producers at the market price Q_{t+1}^K. Because of the perfect competition among intermediate goods producers, the price of capital equals the price of producers' claims: $Q_t^K = Q_t^S \equiv Q_t$. The amount of depreciated capital is equal to $\delta_t(U_t)\xi_t K_{t-1}$, where δ_t is the physical depreciation rate and ξ_t reflects the capital quality shock discussed above. At $t+1$, the firm pays a gross return $R_{k,t+1}$ to the bankers per each unit of investment. As firms are identical, investment in capital pays the same return to all banks.

In each period, an intermediate goods producer chooses labor demand and demand for capital to maximize its current and next-period profits. Profit consists of the revenues from production and the resale value of the depreciated capital net of payments on claims S_t and labor costs. The price of a unit of the intermediate good is $P_{m,t}$, the cost of replacing used capital is unity, and the cost of buying new capital is Q_t. The producer then chooses the utilization rate and labor demand as:

$$
\begin{aligned}
(1-\alpha)\frac{P_{m,t}Y_t}{L_t} &= W_t \\
(\alpha)\frac{P_{m,t}Y_t}{U_t} &= \delta'(U_t)\xi_t K_{t-1}
\end{aligned}
$$

As firms make zero profit, they distribute the return on capital to holders of their claims:

$$R_t^k = \frac{\left(\alpha\frac{P m_t Y_t}{K_{t-1}} + Q_t - \delta(U_t)\right)\xi_t}{Q_{t-1}} \tag{33}$$

The first-order conditions determine the expected return on the firm's claim as the expected value of the marginal product of capital plus the expected resale value of the capital divided by the price of the claim. The wage is then determined by the marginal product of labor. The price of the intermediate good equals the marginal costs.

Capital-Producing Firms Capital goods producers are competitive firms. They buy depreciated capital from intermediate goods producers and renovate it at the unit costs and sell it at the unit price. They also produce new capital and sell it at price Q. There are no adjustment costs for renovating worn-out capital, but there are flow adjustment costs when producing new capital. Capital producers are risk neutral and maximize the following utility:

$$\max_{In_t} E_t \sum_{k=t}^{\infty} \beta^{T-k}\Omega_{t,k}\left((Q_k-1)I_{nk} - f\left(\frac{In_k+I_{ss}}{In_{k-1}+I_{ss}}\right)(In_k+Iss)\right) \tag{34}$$

where In is net investment, defined as $In_t \equiv I_t - \delta(U_t)\xi_t K_{t-1}$, where $\delta(U_t)\xi_t K_{t-1}$ is the quantity of renovated capital. Iss is steady-state investment and Q_t is the price of capital. Function f is

an investment adjustment cost function satisfying the following properties $f(1) = f'(1) = 0$ and $f''(1) > 0$.

The first-order conditions for investment give the price of capital, Q_t:

$$[I_t] : Q_t = 1 + f(\cdot) + \frac{In_k + I_{ss}}{In_{k-1} + I_{ss}} f'(\cdot) - E_t \beta \Omega_{t,t+1} \left(\frac{In_k + I_{ss}}{In_{k-1} + I_{ss}} \right)^2 f'(\cdot) \tag{35}$$

Final Goods Producers (retailers) Retailers combine output from intermediate goods producers using the production function:

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{36}$$

where Y_{ft} is composite goods output from retailer f and ε is the elasticity of substitution. We follow the Calvo-pricing convention and each period allow only a fraction γ of firms to optimize their prices. The rest of the firms are allowed to index their prices to inflation. Firms are monopolistic competitors and maximize their profit:

$$\max_{P_t^*} \sum_{i=0}^{\infty} \gamma^i \beta^i \Omega_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p} - P_{m,t+i} \right] Y_{ft+i} \tag{37}$$

subject to demand from households:

$$Y_{ft} = \left(\frac{P_{ft}^*}{P_t} \right)^{-\varepsilon} Y_t \tag{38}$$

where P_t^* is the optimal price set in period t, γ is the fraction of firms which cannot reset their prices but only index to inflation, and $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the one-period inflation rate.

The problem results in the first-order condition:

$$\sum_{i=0}^{\infty} \gamma^i \beta^i \Omega_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p} - \mu P_{m,t+i} \right] Y_{ft+i} = 0 \tag{39}$$

where $\mu \equiv \frac{1}{1-\frac{1}{\varepsilon}}$ is a monopolistic mark-up.

The resulting equation for the price dynamics takes the form:

$$P_t = \left[\int_0^1 P_{ft}^{\frac{1}{1-\varepsilon}} df \right]^{1-\varepsilon} \tag{40}$$

$$P_t = \left[(1-\gamma)(P_t^*)^{1-\varepsilon} + \gamma \{ (1 + \pi_{t+k-1})^{\gamma_p} P_{t-1} \}^{1-e} \right]^{\frac{1}{1-\varepsilon}} \tag{41}$$

The Government and the Central Bank The government collects lump-sum taxes from households, T_t, and accepts reserves (the safe asset), Res_t. It also bears some costs of conducting policy, Po_t. We assume that only liquidity provision (targeted or not) is operationally costly for the central bank, and that lowering the interest rate or relaxing the collateral constraint does not imply any operational costs. The government's budget constraint is satisfied when the following holds:

$$G_t + Po_t = T_t + Res_t - R_t Res_{t-1} \tag{42}$$

That is, the government budget constraint equalizes government spending and payment on due government claims with taxes and new purchases of government claims. As in Gertler and Karadi (2011), we assume that steady-state government expenditures are exogenously fixed at level G. We also set expenditure on policy as a fraction τ of the assets funded via central bank programs: $\tau \psi_t (Q_t K_t + Res_t)$ for unlimited liquidity provision and $\tau \psi_t^{tar} Q_t K_t$ for targeted credit support.

The resources in the economy are then distributed between consumption, investment, and government expenditure on policy:

$$Y_t = C_t + I_t + f\left(\frac{In_t + I_{ss}}{In_{t-1} + I_{ss}}\right)(In_t + Iss) + G_t + Po_t \tag{43}$$

The central bank conducts monetary policy according to the simple rule:

$$i_t = (1 - \rho_i)\left(i + \kappa_\pi \pi_t + \kappa_y(\log Y_t - \log Y_t^*)\right) + \rho_i i_{t-1} + \varepsilon_t \tag{44}$$

where Y^* is flexible output, ε_t is an exogenous monetary policy shock, and i is the steady-state nominal rate. ρ_i is a smoothing parameter lying between zero and one. The real and nominal interest rates are linked via the Fisher equation:

$$1 + i_t = R_t E_t (1 + \pi_{t+1})$$

4. Calibration and Simulations

To compare our results with the literature, where possible we follow the calibration choices of Gertler and Karadi (2011); we list their parameter choices in Table 2. There are, however, some parameters specific to our model: σ_R^2, λ_b, ω, and $\bar{E}\hat{\xi}$. We set average expectations of the capital quality shock, $\bar{E}\hat{\xi}$, to be equal to the steady-state value of $\bar{\xi} = 1$.

Table 2: Calibrated Parameters from Gertler and Karadi (2011)

β	0.99	household's discount rate
h	0.815	habit parameter
χ	3.409	relative utility weight of labor
ϕ	0.276	inverse Frisch elasticity of labor supply
λ	0.381	fraction of capital to be diverted
θ	0.972	survival rate of bankers
α	0.33	capital share
U	1	steady-state capital utilization rate
$\delta(U)$	0.025	steady-state depreciation rate
ζ	7.2	elasticity of marginal depreciation with respect to utilization rate
η_i	1.728	inverse elasticity of net investment to price of capital
ε	4.167	elasticity of substitution
γ	0.779	probability of keeping prices fixed
γ_p	0.241	measure of price indexation
κ_π	1.5	inflation coefficient of Taylor rule
κ_y	0.125	output gap coefficient of Taylor rule
ρ_i	0.8	smoothing parameter of Taylor rule
$\frac{G}{Y}$	0.2	steady-state proportion of government expenditure
τ	0.001	cost of government policy
κ	10	reaction parameter for government policy

The variance of banks' forecasts and the dispersion between them, σ_R^2, the collateral constraint, λ_b, and the transfer to new entering bankers, ω, are meant to be suggestive. We set their values to match the following pre-crisis data: the interbank market rate, the share of interbank loans in banks' portfolios, and the share of loans in banks' portfolios. In our model, banks exchange loans for one period on the interbank market, with the period being a quarter. Therefore, the 3-month Euribor is a natural choice for the empirical counterpart for the model interbank rate. The share of interbank loans resembles the share of lenders in our model and is calculated as the ratio of euro area banks' loans to monetary and financial institutions to total assets. Similarly, the share of loans is the ratio of total loans to the total assets of European banks. Total loans include loans to enterprises and to other banks. In our model, what is not lent either to banks or to firms is hoarded. That is, the share of loans is useful for calculating the share of hoarded assets as (1 - share of loans).[15]

We choose the parameters to roughly match an interbank rate of 1.31%, a share of hoarded assets of 40%, and a share of assets lent on the interbank market of 20%, and choose σ_R^2, λ_b, and ω to be 0.1, 0.24, and 0.0059, respectively. The parameters for Kalman filter updating, σ_v^2, σ_η^2, σ_e^2, and $\sigma_{\varepsilon\eta}$, are set to match the steady-state variance of the bank's forecast, σ_R^2. Recall that in our model, ξ is subject to two shocks: a persistent one and transitory one: $\xi_t = \rho_\xi \xi_{t-1} + \mu_t + \varepsilon_t$, so the variance of ξ, $\sigma_\xi^2 = \left(\sigma_v^2 + \sigma_\varepsilon^2\right)/(1 - \rho_\xi^2)$. At the same time, σ_R^2 is a function of σ_ξ^2, as shown by (7). That is, the steady-state value of σ_R^2 defines the sum of the variances of the persistent and transitory shocks, which are set at 0.001 and 0.03, respectively. We choose the variance of the expert opinion shock, σ_η^2, so that the share of expert adjustments in the final forecast lies within the bounds defined

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[15] In our model, reserves represent safe assets, but in reality there are a number of assets that can be considered "safe." Consequently, we cannot use the amount of reserves as an empirical counterpart for hoarding.

by the literature on forecasting.[16] We set σ_η^2 to be equal to 0.5, and the resulting share of expert adjustments is then 0.37. The persistence of the capital quality shock is $\rho_\xi = 0.66$, as in Gertler and Karadi (2011). We set the persistence of all other shocks to be the same, at 0.66 for both the persistent shock and the expert opinions shock.[17] The policy reaction parameter for the reserve rate is set to match the decline in the real policy rate during 2008–2009 relative to the pre-crisis 10-year average: the resulting deviation from the steady state during the crisis is a fall of 0.23 percentage points. The collateral constraint in our model does not have an intuitive empirical counterpart. The value is set for illustrative purposes and alternative values are discussed. The parameters are listed in Table 3.

Table 3: Calibrated Parameters Specific to our Model

ω	0.0059	proportional transfer to entering bankers
λ_b	0.24	collateral constraint on interbank market
σ_R^2	0.1	variance of return expectations
σ_v^2	0.001	variance of persistent shock to capital quality
σ_η^2	0.5	variance of expert opinion shock
σ_e^2	0.03	variance of capital quality transitory shock
$\sigma_{\varepsilon\eta}$	0.1	covariance of errors in econometric and expert forecasts
ρ_θ	0.66	persistence of expert opinion shock
ρ_ξ	0.66	persistence of capital quality shock
ρ_μ	0.66	persistence of persistent shock to capital quality
ρ_c	0.62	correlation of experts' opinion
κ^λ	2.5	policy reaction for collateral constraint
κ^R	0.1	policy reaction for reserve rate

With the parameters described above, we then proceed with an analysis of the linearized model and its performance relative to Gertler and Karadi (2011). The model is simulated using Dynare 4.

4.1 Defining a Crisis

We consider a crisis to be a transitory shock to capital quality, ξ_t. To make the dynamics of our model comparable to the literature, we consider a crisis to be a 5% decline, as in Gertler and Karadi (2011). They set this value to match a 10% decline in the effective capital stock over a two-year period. We consider several types of crises: a drop in ξ_t of 5%, a drop in ξ_t of 5% combined with banks believing that this was a permanent shock,[18] and an unchanged ξ_t with banks believing in a 5% drop in the persistent component of ξ_t. In other words, we consider a crisis without an expectational shock, a crisis with an expectational shock, and a pure expectational shock, respectively. In the simulations without an expectational shock, banks observe a change in ξ_t, but they do not have perfect information on how persistent this change is. They use past and current observations via the Kalman filter to predict ξ_{t+1}. In the simulations with an expectational shock, in addition to a change

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[16] For example, Fildes et al. (2009) analyze a data set containing 70,000 business organizations and their forecasts. They find the mean expert adjustment for monthly forecasts to vary between 18% and 46% depending on the type of business.

[17] In previous research – Audzei (2012) – we calculated the persistence of the expert opinion shock using the SPF GDP forecast. The resulting value was very close to the current calibration – 0.61.

[18] This is modeled as a shock to banks' average belief about the drop in the persistent shock. Recall that in the model this is the average belief that matters for the simulations.

Figure 6: Role of Expectations
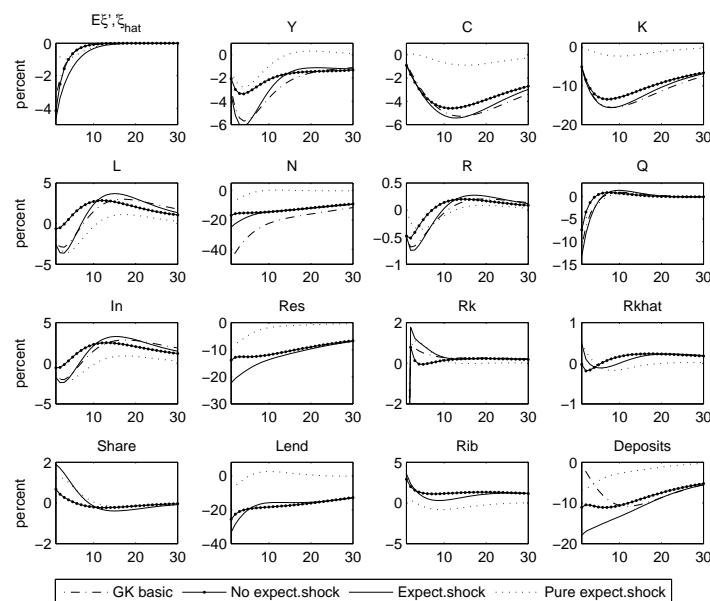
Note: Y – output, C – consumption, K – capital, L – labor supply, N – banks' net worth, R – deposit rate, Q
– price of capital, In – net investment, Res – safe asset, Rk – return on capital, Rkhat – expected
return on capital, Share – share of investors, Lend – interbank lending, Rib – interbank interest rate,
Deposits – deposits.

in ξ_t, banks get a "pessimistic shock": experts start to believe that a persistent shock has occurred.
These expert opinions are combined with past observations again via the Kalman filter.

4.2 The Role of Expectations

In our economy, expectations determine credit to the real sector. They also affect the functioning of
the interbank market: the numbers of borrowers and lenders and the equilibrium interbank market
rate. As was shown in the simple model case, when market expectations become too low, this results
in an interbank market crunch, with no lending occurring. In the general equilibrium model, we
consider model responses linearized around the steady state with a functioning interbank market.
A decrease in market expectations then results in lower credit supply and lower lending between
banks.

Figure 6 shows the dynamics of the model in response to the crisis scenarios with no central bank
policy. In these scenarios, the policy rate, R_t^{res}, is set to be equal to the deposit rate, so that banks
earn nothing on safe asset. Figure shows the responses of our model with the three types of crisis
considered and Gertler and Karadi (2011) model where applicable.[19] Model by Gertler and Karadi
(2011) serves here as a benchmark with perfect information,[20] the absence of an interbank market,
and only one asset for banks to invest in. In period 1 there is a 5% temporary shock to ξ_t. Because
ξ_t is itself a persistent process, it stays below the steady-state value for about ten periods. The
first subplot shows the expectations about ξ_{t+1}. In a model with perfect expectations, this will be

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[19] Obviously, such variables as interbank market rate or safe asset holdings do not exist in their paper.
[20] Our model with imperfect information would be identical to Gertler and Karadi (2011) when the risky asset
return is higher than the reserve rate: without heterogeneity there is no interbank market, and risk-neutral banks
invest everything in the risky asset.

$\overline{E}_t \xi_{t+1} = \rho_\xi * \xi_t$, resulting in a decline of 3.3% in the first period. It coincides with the decline in the ξ_{t+1} in the Gertler and Karadi (2011) model. However, in our model it is not observable whether this was a transitory or a persistent shock. Recall that agents combine two observables to form their forecast: historical values of ξ and expert opinions about the value of the persistent shock to ξ_t. Even when the experts consider the persistent shock to be zero, analysis of historical observations leaves a possibility for it to exist. Consequently, even in a model where expert opinions are not disturbed, the future values of ξ_t are underestimated. This underestimation is even larger and more persistent when experts believe there has been a 5% decline with a persistent shock to ξ. When, on the contrary, there has been no change in ξ_t, but the experts have decided that there has been a decline in the persistent component, the fall in expectations is much smaller, but the persistence reflects the persistence of the expert opinions. In this case, the historical analysis does not find any change in ξ_t, but the expert opinion is still given some weight in the final forecast, because the weights are influenced by past performance. As a result, the final bank forecast is a decline in ξ_{t+1}.

When ξ_t is hit by a shock, there is an immediate decline in the contemporaneous return on banks' investment – R_t^k, as in (33). This lowers banks' returns and has a negative effect on their net wealth, N_t. In the scenarios with expectational shocks, low expectations contribute to lower demand for capital claims. With a lower net wealth of banks, current net investment falls and the price of capital goes down. Recall that Q_t in (35) is a function of the change in net investment, and a fall in net investment results in a fall in Q_t. Because capital and, accordingly, net investment fall, there is less demand for capital and capital producers sell it at a lower price, Q_t. Q_t also reflects the resale value of capital. Consequently, a fall in Q_t contributes to a further decline in banks' wealth. A fall in N_t leads to a decline in both safe and risky asset holdings. With smaller net wealth, banks are able to attract less deposits from households, as their deposits are limited to a fraction of their net wealth through the agency problem. Hence, net investment falls even more, as banks simply have less funds for it.

With investment falling, the return on it rises. The return on the banks' investment is the sum of the marginal product of capital and the resale value of capital. Although the resale value is low in a crisis, the marginal product is large. That leads to a higher expected return on the risky asset, *Rkhat* (albeit smaller than the actual future return, R_{t+1}^k) and a larger share of investors. Banks would like to invest more with such returns, but their investment is already reduced by the fall in net wealth.

Note that in the simple model, a fall in expectations results in lower lending and a lower interbank market rate. All the crisis scenarios result in a fall in lending, *Lend$_t$*. The scenario with lower beliefs – with expectational shocks – leads to the largest fall in lending. However, the reaction of the interbank rate, R_t^{ib}, is different in general equilibrium because of the feedback from banks' investment to the risky asset and the risky asset return. The share of lenders shrinks because more banks would like to invest themselves. This puts upward pressure on the interbank rate, which therefore rises. The rise is the highest in the model with the expectational shock and the lowest in the model with the pure expectational shock.

Comparing the responses to a crisis with and without the expectational shock, note that the difference in net worth, N_t, is explained solely by the asset price, Q_t, as the shock to the actual ξ_t is the same. That is, net worth falls the most in the model with expectational shocks. Net worth affects banks' ability to attract deposits and influences the deposit rate, R_t. Smaller deposits result in a lower interest rate on them, making household savings less attractive. In the model, there are no frictions on the labor side, so it is labor that adjusts, with the fall in consumption, C_t, being almost the same under all scenarios. Output, Y_t, falls in response to the fall in capital and labor, the largest drop being in the scenario with the expectational shock.

When only an expectational shock hits the economy and there is no actual drop in ξ_t (a pure expectational shock), banks underestimate ξ_t for some period of time. This generates a decline in net investment, a decrease in the price of capital, and a fall in the current return on capital, followed by a decline in net wealth. Capital falls initially by 0.1%. The decline in net wealth accelerates the fall in capital in the following periods, with the maximum decline being 1.244%. That is, a persistent pessimistic shock can generate a small recession, as investment falls, leading to a decline in in output and consumption. If the crisis is interpreted as a combination of a shock to ξ and a shock to agents' expectations, the resulting responses look like the sum of the pure expectational shock and no expectational shock scenarios.

Thus, the main difference from the simple model is the presence of feedback from expectations to asset prices and net worth and from investment to the risky asset and the return on it. These differences will influence the model response to the policy actions described in the following section.

Now compare Gertler and Karadi (2011) and our model with no expectational shock. In our model, banks do not invest all their funds in the risky asset, but leave some share in the safe one. Consequently, only a proportion of banks' net wealth is affected by the fall in the risky asset return. Our law of motion of net wealth (23) is similar to the dynamics of N_t in Gertler and Karadi (2011), but the term $\left(R_t^k - R\right)$ is also multiplied by the share of banks investing in the risky asset. As a result, our model demonstrates almost half as large a fall in $I_{n,t}$ and N_t compared to Gertler and Karadi (2011). The fall in capital is comparable in the two models, though smaller in ours. The small difference is explained by counteracting forces: lower expectations about the capital return in our model and a higher net wealth of banks. The higher Q_t in our model also contributes to a higher initial R_t^k, reflecting the resale value of capital. The higher return on capital also contributes to a higher net wealth of banks, N_t. With a smaller fall in capital and higher labor, there is a smaller fall in output and a slightly smaller fall in output in our model. Thus, without the expectational shock, our model with uncertainty and diversified bank portfolios results in a comparable, but slightly smaller, decline in the capital stock (roughly 9%). Our banks, though, experience a smaller drop in net wealth, and there is a smaller decrease in output. In the following periods, with a larger stock of capital there is smaller return on capital and the risk premium $\left(R_{t+1}^k - R\right)$ is lower in our model.

Imperfect information results in a lower predicted risky asset return relative to the perfect information in Gertler and Karadi (2011). As banks expect a smaller stream of future wealth, they have a lower "continuation value" in their agency problem and the resulting private leverage ratio ϕ in (22) is lower. Even without expectational shocks, therefore, the fall in deposits is more pronounced in our model, regardless of the higher net wealth of banks. To limit the decrease in consumption, households supply more labor and this has some mitigating effect on the fall in output. The model with the expectational shock delivers a crisis of a similar magnitude to Gertler and Karadi (2011). However, net worth still falls much less and deposits decrease much more in our model.

To summarize, the expectational shock alone can generate some need for a policy response by the central bank. Combined with the occurrence of an actual crisis, this leads to a more severe recession and a larger policy response. That is, investor sentiment can be an important factor for policy design and evaluation. Without the expectational shock, our model predicts a milder recession than Gertler and Karadi (2011), as banks in our model have an opportunity to diversify their assets and are thus less impacted by the crisis. With the expectational shock, our model has similar predictions to the baseline regarding the dynamics of output, capital, labor, and consumption.

Figure 7: Policy Effects
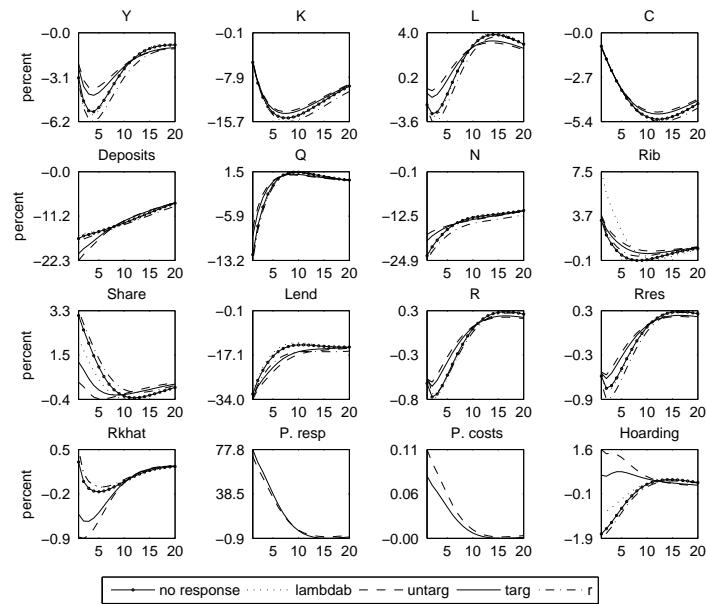
Note: Y – output, C – consumption, K – capital, L – labor supply, N – banks' net worth, R – deposit rate, Q
   – price of capital, Rres – policy rate, Rkhat – expected return on capital, Share – share of investors,
   Lend – interbank lending, Rib – interbank interest rate, Deposits – deposits, P. response – policy
   response, P. costs – policy costs.

4.3 Policy Results

For the policy analysis, we consider a "crisis" shock defined as a decline in capital quality, ξ_t, in combination with a wave of investor pessimism. The results are presented in Figure 7. The policy exercises without the expectational shock are presented in Appendix E and have the same qualitative results.

First, consider the two types of liquidity provision: targeted (*targ*) and untargeted (*untarg*). The two policies have a very similar effect on output, consumption, and capital, and mitigate the crisis relative to the simulation with no policy response (*no response*). The policy response is the total amount of funds supplied to banks – $\psi(QK + Res)$ and $\psi^{tar}(QK)$ for untargeted and targeted policies, respectively. *Hoarding* is the share of storage assets in banks' total assets. The main difference between the two policies is in the share of hoarded assets, *Hoarding*, which is almost twice as large in the case of untargeted policy. These predictions are in line with the simple model results: liquidity provision helps restore credit to the real economy, but also increases reserve holdings. However, in a general equilibrium setup, it also has some impact on the interbank market, because it changes the return on capital. Liquidity provision stimulates investment, reducing the marginal product of capital. At the same time, banks' expectations of capital value ξ_{t+1} are pessimistic, as they are affected by the crisis shock. This makes the expected risky asset return, *Rkhat*, lower than the actual realization under all policies. The larger the impact of a policy on investment, the smaller the return on investment. The policies partially crowd out the interbank market and deposits, as banks use more central bank funds. Also, a decrease in banks' return expectations relative to the no-policy scenario reduces the amount of interbank lending, *Lend*, and increases the interbank rate, *Rib*. With targeted credit support, banks expect a share of their risky asset purchases to be financed by the central bank. For those with high expectations about the risky asset return, this means less need to

borrow from households and on the interbank market. Note that this is only true for banks with high return expectations. In the case of untargeted liquidity provision, where both assets are financed by the central bank, all banks have less need for household funding. Banks' deposits therefore fall more with untargeted support. The most important difference between the two policies is in their efficacy in restoring capital accumulation. An insight obtained from the simple model is that with targeted support, there is less hoarding. In a general equilibrium context, targeted support also results in a lower asset price, Q: as the central bank finances more risky asset purchases under targeted support, there is less private investment. The lower asset price reduces banks' net worth, making banks poor and reducing investment even more. Thus, targeted support results in lower hoarding but also in lower capital and output.

Interest rate policy is the least efficient policy in our simulations. It is modeled as a decline in the reserve rate, $Rres$, below the deposit rate R, meaning that banks are making negative returns on their reserves. In line with the simple model results, such policy increases the share of investors, *share*, as the safe asset becomes less attractive, and lowers the share of hoarded assets in banks' portfolios. However, it reduces banks' net wealth, leading to a large drop in investment. This drop in banks' net wealth leads to even worse outcomes than in the case of no policy action.

Relaxing the collateral constraint on the interbank market by raising λ_b allows borrowers to borrow a larger fraction of their net wealth. The larger demand for interbank credit drives up the interbank market rate (Rib), reducing the number of banks willing to borrow. Thus, there are fewer borrowers on the market, but they borrow more. As a result, interbank market lending (*lend*) increases. The high interbank market rate makes interbank lending more attractive relative to risky asset invest- ment, so some potential investors become lenders on the interbank market and the share of those investing in the risky asset (*share*) falls. As a result, despite the larger volume on the interbank market, credit supply to the real economy is almost unchanged, as are safe asset positions.[21]

It is also instructive to compare the policy results from our model with no expectational shock with those from Gertler and Karadi (2011). This allows us to show how hoarding and imperfect information influence policy efficiency. As the policy exercise in Gertler and Karadi (2011) in- volves untargeted liquidity provision, we limit our comparison to this policy only. As was shown in Figure 6, with no expectational shock, the impact of crisis shocks in our model is smaller. For comparison, therefore, we simulate our model with a larger shock to ζ, [22] so that the initial fall in capital, K, is similar in both models with no policy. The policy response is an endogenous reaction to an increase in the risk premium. For this exercise, in our model we substitute the policy reaction, ψ, with an autoregressive process to match the policy response in the baseline model. Figure 8 shows the policy responses: the difference in the responses with and without policy in panel a, and the impulse responses in the baseline and in our model with and without policy in panel b. Even with a larger shock, there is smaller fall in banks' net wealth, as banks have a diversified portfolio. Imperfect information results in a more dramatic fall in deposits and an increase in labor supply relative to the baseline model (see Figure 7). As expected, a policy response of initially similar size has a delayed effect in our model, as banks are less optimistic about future returns and store some part of central bank funds in the safe asset. Also, with a higher price of capital, Q, the cost of the policy (modeled as $\tau\psi QK$) is larger in our model when the policy response is the same.

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[21] In alternative simulations we considered different response parameters for relaxing the collateral constraint: from 0.4 to 2.5. The difference in the output and capital responses is negligible. Also, with stronger easing, there is less investment.

[22] The shock in our model is 1.35 times larger.

Figure 8: Policy Effects vs Baseline
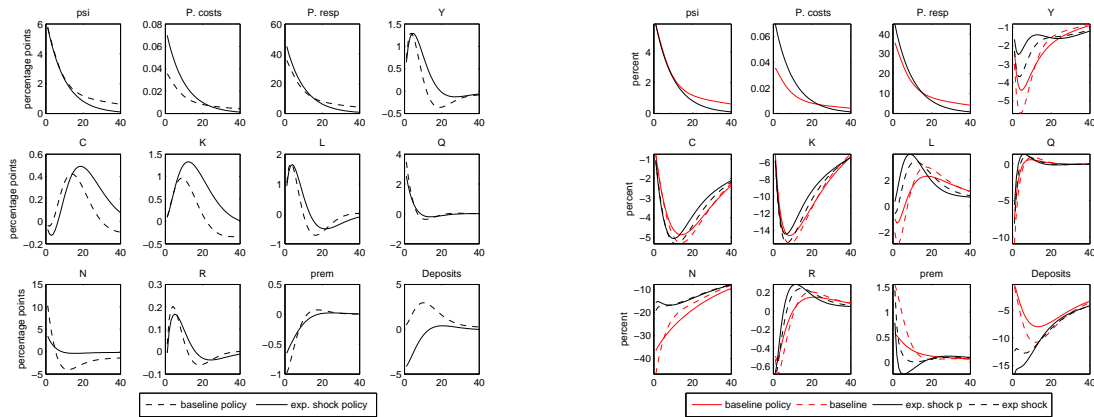
(a) differences   *(b) impulse responses*

Note: Y – output, C – consumption, K – capital, L – labor supply, N – banks' net worth, R – deposit rate, Q – price of capital, Deposits – deposits, psi – policy reaction, P. costs – policy costs, P. response – policy response, prem – risk premium.

To conclude, liquidity provision policies help mitigate the simulated crisis. However, relative to the baseline model of Gertler and Karadi (2011) our model with imperfect information and a storage asset displays low efficiency of liquidity provision, with higher costs and delayed responses. The policy of targeted liquidity provision is slightly less effective in our model than that of untargeted liquidity provision, as it disturbs the price of capital. Low policy rates have a negative impact on banks' net worth, leading to worse outcomes than in the case of no policy.

5. Conclusion

In this paper we address the role of imperfect market expectations in interbank lending and in amplifying economic fluctuations. In particular, we show that besides a liquidity shortage, assessment of counterparty risk can be one of the factors contributing to a credit crunch. In a simple finite-horizon model, we consider an expectations-driven credit crunch and show that policy effects in this case are very limited.

We then develop a workhorse DSGE model and consider linearized responses around the steady state. To study market expectations, we incorporate a heterogeneous banking sector with a continuum of risky asset return expectations. The heterogeneity of expectations gives rise to an interbank market where lenders take into account the possibility of a borrower failing to repay the loan. Imperfect information among the bankers results in higher assessment of counterparty risk after crisis episodes, as bankers are not sure how persistent the negative shock is. Several types of crisis are considered: with and without pessimism shocks, and purely driven by pessimism.

We consider several types of central bank policy response, including unlimited liquidity provision, targeted credit support, and varying the interest rate on reserves. Our model predicts that the efficiency of the policy depends on market confidence. Market pessimism dampens the positive effects of policies, making banks hoard central bank funds in reserves instead of transferring them through

the bank lending channel. Low policy rates in our model devastate banks' balance sheets and result in a worse recession than in the case of no policy response.

References

ACEMOGLU, D., V. CHERNOZHUKOV, AND M. YILDIZ (2006): "Learning and Disagreement in an Uncertain World." NBER Working Papers 12648, National Bureau of Economic Research, Inc.

ALLEN, F., E. CARLETTI, AND D. GALE (2009): "Interbank market liquidity and central bank intervention." *Journal of Monetary Economics,* 56(5):639–652.

ANDRADE, P. AND H. LE BIHAN (2013): "Inattentive professional forecasters." *Journal of Monetary Economics,* 60(8):967–982.

AUDZEI, V. (2012): "Efficiency of central bank policy during the crisis: role of expectations in reinforcing hoarding behavior." Working Paper Series 477, CERGE-EI.

BEAUDRY, P., D. NAM, AND J. WANG (2011): "Do Mood Swings Drive Business Cycles and Is It Rational?" NBER Working Paper Series 17651, National Bureau of Economic Research, Inc.

BEBER, A., M. W. BRANDT, AND M. LUISI (2013): "Distilling the Macroeconomic News Flow." NBER Working Papers 19650, National Bureau of Economic Research, Inc.

BIANCHI, J. AND S. BIGIO (2014): "Banks, Liquidity Management and Monetary Policy." NBER Working Papers 20490, National Bureau of Economic Research, Inc.

BLOOM, N. (2009): "The Impact of Uncertainty shocks." *Econometrica,* 77(3):623–685.

CAPISTRÁN, C. AND A. TIMMERMANN (2009): "Disagreement and Biases in Inflation Expectations." *Journal of Money, Credit and Banking,* 41(2-3):365–396.

CLARK, T. E. AND T. DAVIG (2011): "Decomposing the declining volatility of long-term inflation expectations." *Journal of Economic Dynamics and Control,* 35(7):981–999.

COIBION, O. AND Y. GORODNICHENKO (2015): "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts." *American Economic Review,* 105(8):2644–78.

COIBION, O. AND Y. GORODNICHENKO (2012): "What Can Survey Forecasts Tell Us about Information Rigidities?" *Journal of Political Economy,* 120(1):116 – 159.

CURDIA, V. AND M. WOODFORD (2011): "The Central-Bank Balance Sheet as an Instrument of Monetary Policy." *Journal of Monetary Economics,* 58(1):54–79.

DEL NEGRO, M., G. EGGERTSSON, A. FERRERO, AND N. KIYOTAKI (2011): "The Great Escape? A Quantitative Evaluation of the Fed's Non-standard Monetary Policy." Staff Reports 520, Federal Reserve Bank of New York.

FILDES, R., P. GOODWIN, M. LAWRENCE, AND K. NIKOLOPOULOS (2009): "Effective forecasting and judgmental adjustments: an empirical evaluation and strategies for improvement in supply-chain planning." *International Journal of Forecasting,* 25(1):3–23.

FRANSES, P. H., H. C. KRANENDONK, AND D. LANSER (2011): "One model and various experts: Evaluating Dutch macroeconomic forecasts." *International Journal of Forecasting,* 27(2):482–495.

FUHRER, J. (2011): "The Role of Expectations in U.S. Inflation Dynamics." Working Paper Series 11-11, Federal Reserve Bank of Boston.

GALE, D. AND T. YORULMAZER (2013): "Liquidity hoarding." *Theoretical Economics,* 8(2): 291–324.

GERTLER, M. AND P. KARADI (2011): "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics,* 58(1):17–34.

GERTLER, M. AND N. KIYOTAKI (2010): *Financial Intermediation and Credit Policy in Business Cycle Analysis,* volume 3 of *Handbook of Monetary Economics,* chapter 11, pages 547–599., 1 edition.

HEIDER, F., M. HOEROVA, AND C. HOLTHAUSEN (2009): "Liquidity Hoarding and Interbank Market Spreads the Role of Counterparty Risk." Working Paper Series 1126, European Central Bank.

HILBERG, B. AND J. HOLLMAYR (2011): "Asset prices, collateral and unconventional monetary policy in a DSGE modeL." Working Paper Series 1373, European Central Bank.

HOMMES, C. (2011): "The heterogeneous expectations hypothesis: Some evidence from the lab." *Journal of Economic Dynamics and Control,* 35(1):1–24.

KIM, D. H. AND A. ORPHANIDES (2005): "Term structure estimation with survey data on interest rate forecasts." Finance and Economics Discussion Series 2005-48, Board of Governors of the Federal Reserve System (U.S.)

KIYOTAKI, N. AND J. MOORE (2008): "Liquidity, Business Cycles and Monetary Policy." mimeo, Princeton University and LSE.

LORENZONI, G. (2009): "A Theory of Demand Shocks." *American Economic Review,* 99(5): 2050–2084.

MANKIW, N. G. AND R. REIS (2007): "Sticky Information in General Equilibrium." *Journal of the European Economic Association,* 5(2-3):603–613.

NASON, J. M. AND G. W. SMITH (2013): "Reverse Kalman filtering U.S. inflation with sticky professional forecasts." Working Papers 13-34, Federal Reserve Bank of Philadelphia.

PATTON, A. J. AND A. TIMMERMANN (2010): "Why do forecasters disagree? Lessons from the term structure of cross-sectional dispersion." *Journal of Monetary Economics,* 57(7): 803–820.

PFAJFAR, D. (2013): "Formation of rationally heterogeneous expectations." *Journal of Economic Dynamics and Control,* 37(8):1434–1452.

SIKLOS, P. L. (2013): "Sources of disagreement in inflation forecasts: An international empirical investigation." *Journal of International Economics,* 90(1):218–231.

SMETS, F. AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review,* 97(3):586–606.

WOODFORD, M. (2001): "Imperfect Common Knowledge and the Effects of Monetary Policy." NBER Working Papers 8673, National Bureau of Economic Research, Inc.

Appendix A: Derivations for the Simple Model

Interbank market clearing.

Proof. The market clearing condition for the interbank market with a uniform beliefs distribution is:

$$F_{m,\sigma_v^2}\left(E^m \hat{R}^k\right) - F_{m,\sigma_v^2}\left(E^l \hat{R}^k\right) = \lambda_b \left(1 - F_{m,\sigma_v^2}\left(R^{ib}\right)\right)$$

The cumulative distribution function for the continuous uniform distribution is $\frac{x-a}{b-a}$. Then the market clearing condition is rewritten as:

$$\frac{E^m \hat{R}^k - a}{b-a} - \frac{E^l \hat{R}^k - a}{b-a} = \lambda_b \left(1 - \frac{R^{ib} - a}{b-a}\right)$$

$$\Rightarrow E^m \hat{R}^k - a - E^l \hat{R}^k + a = \lambda_b \left(b - a - R^{ib} + a\right)$$

$$\Rightarrow E^m \hat{R}^k - E^l \hat{R}^k = \lambda_b \left(b - R^{ib}\right)$$

where b is the upper bound on the beliefs distribution, denoted as $\bar{R}$ in the text. $\square$

Proposition 2.0.1. *Given the bounds of beliefs for the interbank market to exist, the interbank rate must satisfy*

$$R^{res} < R^{ib} < \frac{1+\lambda_b}{\lambda_b} \sigma \sqrt{3}$$

for $\frac{\lambda_b R^{res}}{(1+\lambda_b)\sqrt{3}} < \sigma < \frac{\bar{R}}{2}$

Proof. For the interbank market to exist, the marginal lender's belief from $p^l R^{ib} = R^{res}$ should be smaller than $p^l R^{ib}$. Otherwise, the marginal lender invests herself and the set of lenders vanishes. Using (3) the marginal lender's belief can be rewritten as:

$$\left(\frac{1}{2} - \frac{\left(R + \lambda_b R^{ib}\right)}{(1+\lambda_b)2\sigma\sqrt{3}} + \frac{E^l \hat{R}^k}{2\sigma\sqrt{3}}\right) R^{ib} = R^{res}$$

or

$$\left(\frac{1}{2} - \frac{\lambda_b R^{ib}}{(1+\lambda_b)2\sigma\sqrt{3}}\right) R^{ib} = R^{res} - \left(\frac{E^l \hat{R}^k}{2\sigma\sqrt{3}} - \frac{R}{(1+\lambda_b)2\sigma\sqrt{3}}\right) R^{ib}$$

Because $R^{ib} > 0$, it follows that either 1) $\left(\frac{1}{2} - \frac{\lambda_b R^{ib}}{(1+\lambda_b)2\sigma\sqrt{3}}\right) > 0$ and $R^{res} - \left(\frac{E^l \hat{R}^k}{2\sigma\sqrt{3}} - \frac{R}{(1+\lambda_b)2\sigma\sqrt{3}}\right) R^{ib} > 0$ or 2) $\left(\frac{1}{2} - \frac{\left(\lambda_b R^{ib}\right)}{(1+\lambda_b)2\sigma\sqrt{3}}\right) < 0$ and $R^{res} - \left(\frac{E^l \hat{R}^k}{2\sigma\sqrt{3}} - \frac{R}{(1+\lambda_b)2\sigma\sqrt{3}}\right) R^{ib} < 0$. However, in this case $\frac{1}{2} - \frac{\left(R + \lambda_b R^{ib}\right)}{(1+\lambda_b)2\sigma\sqrt{3}} < 0$, which violates $p^i > 0$ for small $E^i R^k$

For 1)

$$R^{ib} \quad < \quad \frac{(1+\lambda_b)\sigma\sqrt{3}}{\lambda_b} \tag{A1}$$

and

$$R^{ib} \quad < \quad \frac{(1+\lambda_b)2\sigma\sqrt{3}R^{res}}{(1+\lambda_b)E^l\hat{R}^k - R} \tag{A2}$$

Suppose that $E^l\hat{R}^{k23} = R^{res} = R$, then the second inequality is

$$R^{ib} < \frac{(1+\lambda_b)2\sigma\sqrt{3}}{\lambda_b} \tag{A3}$$

If $E^l\hat{R}^k < R^{res}$, the upper bound in (A3) increases, leaving (A1) as the most restrictive. The lower bound for R^{ib} is the rate on the residuals. That is, the interbank market rate must satisfy:

$$R^{res} < R^{ib} < \frac{(1+\lambda_b)\sigma\sqrt{3}}{\lambda_b}$$

Finally, for the interbank market to exist, $R^{res} < \frac{(1+\lambda_b)\sigma\sqrt{3}}{\lambda_b}$. This gives the condition for σ and λ_b :

$$\frac{\lambda_b R^{res}}{\sqrt{3}(1+\lambda_b)} < \sigma$$

Also, with $\frac{(1+\lambda_b)}{\lambda_b} > 2$, we can write $R^{ib} < 2\sigma\sqrt{3}$ $\square$

Proposition 2.0.2. Low market beliefs result in a lower interbank rate and lower lending.

Proof. In the simple model, lending is given by $E^m R^k - E^l R^k$. Deriving with respect to the average market belief, m:

$$\frac{\partial\left(E^m R^k - E^l R^k\right)}{\partial m} \quad = \quad \frac{\partial E^m R^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m} - \frac{\partial E^l R^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m} =$$

$$= \quad \frac{\partial R^{ib}}{\partial m}\left(\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}\right) = \frac{\left(\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}\right)}{\left(1 + \frac{1}{\lambda}\left(\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}\right)\right)}$$

where the last equality is derived from the interbank market clearing condition

$$R^{ib} = m + \sqrt{3}\sigma - \frac{1}{\lambda}\left(E^m R^k - E^l R^k\right)$$

with the derivative with respect to the average market belief being

$$\frac{\partial R^{ib}}{\partial m} = 1 - \frac{1}{\lambda}\left(\frac{\partial E^m R^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m} - \frac{\partial E^l R^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m}\right)$$

---

[23] R^{res} is the upper bound on $E^l R^k$ and is set to be equal to R in the steady state.

or

$$\frac{\partial R^{ib}}{\partial m} = \frac{1}{\left(1 + \frac{1}{\lambda}\left(\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}\right)\right)}$$

With the marginal lender and the marginal investor defined, respectively, as $E^m R^k = R^{ib} p^m$ and $R^{res} = R^{ib} p^l$, and $p^i = \frac{1}{2} - \frac{(R + \lambda_b R^{ib})}{2\sigma\sqrt{3}(1+\lambda_b)} + \frac{E^i \hat{R}^k}{2\sigma\sqrt{3}}$, we get

$$\frac{\partial p^l}{\partial R^{ib}} + \frac{\partial E^l R^k}{\partial R^{ib}} = -\frac{R^{res}}{\left(R^{ib}\right)^2} \text{ and}$$

$$\frac{\partial E^l R^k}{\partial R^{ib}} = \frac{\lambda_b}{(1+\lambda_b)} - \frac{2\sigma\sqrt{3}R^{res}}{\left(R^{ib}\right)^2}$$

and

$$\frac{\partial E^m R^k}{\partial R^{ib}} = \frac{\partial p^m}{\partial R^{ib}} R^{ib} + \frac{\partial E^m R^k}{\partial R^{ib}} R^{ib} + p^m \text{ and}$$

$$\frac{\partial E^m R^k}{\partial R^{ib}} = \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}}$$

$\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}$ then can be rewritten as

$$\frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{(1+\lambda_b)} + \frac{2\sigma\sqrt{3}R^{res}}{\left(R^{ib}\right)^2} > 0$$

To prove that this expression is positive, we use the result from proposition 2.0.1 that $R^{ib} < \frac{(1+\lambda_b)\sigma\sqrt{3}}{\lambda_b}$ and $\lambda < 1$, so that $\frac{1+\lambda}{\lambda} > 2$, $R^{ib} < 2\sigma\sqrt{3}$. Then the above expression can be negative only with $p^m < \frac{\lambda}{1+\lambda} + \frac{R^{res}(R^{ib} - 2\sigma\sqrt{3})}{\left(R^{ib}\right)^2}$ and $\lambda > \frac{R^{res}}{R^{ib} - R^{res}}$. With $\frac{\lambda}{1+\lambda}$ increasing in λ, $\frac{\lambda}{1+\lambda} > \frac{R^{res}}{R^{ib}}$, so $p^m < \frac{R^{res}}{R^{ib}} - \frac{R^{res}(2\sigma\sqrt{3} - R^{ib})}{\left(R^{ib}\right)^2}$. Multiplying both sides by R^{ib} we get $R^{ib}p^m < R^{res} - \frac{R^{res}(2\sigma\sqrt{3} - R^{ib})}{\left(R^{ib}\right)}$, meaning that $E^m R^i = R^{ib}p^m < R^{res}$, which contradicts $E^m R^i > R^{res}$. $\quad\square$

Proposition 2.0.3 *A low policy rate increases lending and lowers the interbank market rate.*

Proof.

$$\frac{\partial R^{ib}}{\partial R^{res}} = -\frac{1}{\lambda}\left(\frac{\partial E^m R^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}}\right)$$

with

$$\frac{\partial E^m R^k}{\partial R^{ib}} = \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}}$$

and

$$E^l R^k = \frac{R}{1+\lambda_b} + \frac{\lambda_b R^{ib}}{1+\lambda_b} - \sqrt{3}\sigma + \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}}$$

$$\frac{\partial E^l R^k}{\partial R^{res}} = \frac{\lambda_b}{1+\lambda_b}\frac{\partial R^{ib}}{\partial R^{res}} + \frac{2\sqrt{3}\sigma}{R^{ib}} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\frac{\partial R^{ib}}{\partial R^{res}}$$

Then

$$\frac{\partial R^{ib}}{\partial R^{res}} = \frac{\frac{2\sqrt{3}\sigma}{R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}$$

With $\lambda > 0$, the size of the derivative is determined by the denominator. $\lambda_b > \frac{\lambda_b}{1+\lambda_b}$. Consider if $\frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2} > 0$ with $\frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2} = \frac{2\sqrt{3}\sigma p^l}{R^{ib}}$:

$$\frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} + \frac{2\sqrt{3}\sigma p^l}{R^{ib}} =$$

$$= \frac{(1+\lambda_b)2\sigma\sqrt{3}\left(R^m - R^{res}\right) + (1+\lambda_b)\left(2\sqrt{3}\sigma\right)^2 p^l - \lambda_b R^{ib}}{(1+\lambda_b)\left(2\sigma\sqrt{3} - R^{ib}\right)}$$

The first term is positive for $E^m R^k > R^{res}$ and the second term is positive with $2\sqrt{3}\sigma > R^{ib}$ and $\left(2\sqrt{3}\sigma\right)^2 p^l > R^{ib}p^l = R^{res}$

$$\frac{(1+\lambda_b)\left(2\sqrt{3}\sigma\right)^2 p^l}{\lambda_b} = \frac{(1+\lambda_b)2\sqrt{3}\sigma * kR^{res}}{\lambda_b} > R^{ib}$$

That is, $\frac{\partial R^{ib}}{\partial R^{res}} > 0$.

Consider $\frac{\partial E^m R^k}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}}$.

$$\frac{\partial E^m R^k}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}} = \frac{\partial E^m R^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}} =$$

$$= \frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\left(\frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\right)}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}} - 1\right) < 0$$

□

Proposition 2.0.4 *Relaxing the collateral constraint increases lending and the interbank market rate.*

Proof. The proof is based on deriving the interbank market rate:

$$\frac{\partial R^{ib}}{\partial \lambda} = \frac{1}{\lambda^2}\left(\frac{\partial E^m R^k}{\partial \lambda} - \frac{\partial E^l R^k}{\partial \lambda}\right)$$

$$\frac{\partial E^m R^k}{\partial \lambda} = \frac{\frac{R-R^{ib}}{(1+\lambda_b)^2}R^{ib} + \left(\sigma\sqrt{3} - \frac{(R+2\lambda_b R^{ib})}{(1+\lambda_b)} + E^m \hat{R}^k\right)\frac{\partial R^{ib}}{\partial \lambda}}{2\sigma\sqrt{3} - R^{ib}}$$

$$\frac{\partial E^l R^k}{\partial \lambda} = \frac{R^{ib} - R}{(1+\lambda_b)^2} + \left(\frac{\lambda_b}{1+\lambda_b} - \frac{2\sqrt{3}\sigma p^l}{R^{ib}}\right)\frac{\partial R^{ib}}{\partial \lambda}$$

$$\frac{\partial R^{ib}}{\partial \lambda} = \frac{\frac{R-R^{ib}}{(1+\lambda_b)^2}\left(\frac{R^{ib}}{(2\sigma\sqrt{3}-R^{ib})} + 1\right)}{\frac{\lambda}{(2\sigma\sqrt{3}-R^{ib})}\left(\lambda\left(2\sigma\sqrt{3} - R^{ib}\right) - \frac{(E^m\hat{R}^k - E^l\hat{R}^k)}{\lambda}\frac{2\sigma\sqrt{3}}{R^{ib}}\right) + \frac{2\sigma\sqrt{3}}{(2\sigma\sqrt{3}-R^{ib})R^{ib}}\left(\frac{\lambda R^{ib} - p^l(2\sigma\sqrt{3})^2(1+\lambda_b)}{(1+\lambda_b)}\right)} > 0$$

$\left(R - R^{ib}\right) < 0$ and $2\sigma\sqrt{3} - R^{ib} > 0$, that is, the nominator is negative. The denominator consists of two elements: $\frac{2\sigma\sqrt{3}}{(2\sigma\sqrt{3}-R^{ib})R^{ib}}\left(\frac{\lambda R^{ib} - p^l(2\sigma\sqrt{3})^2(1+\lambda_b)}{(1+\lambda_b)}\right) < 0$ (as was shown in the proof of proposition 2.0.3, $\lambda R^{ib} - p^l\left(2\sigma\sqrt{3}\right)^2(1+\lambda_b) < 0$). To see that the first term is also negative, recall that from interbank market clearing $\frac{(E^m\hat{R}^k - E^l\hat{R}^k)}{\lambda} = \left(\bar{R} - R^{ib}\right) > \left(2\sigma\sqrt{3} - R^{ib}\right) > \lambda\left(2\sigma\sqrt{3} - R^{ib}\right) > \lambda\left(2\sigma\sqrt{3} - R^{ib}\right)\frac{R^{ib}}{2\sigma\sqrt{3}}$

$$\frac{\partial E^m R^k}{\partial \lambda} - \frac{\partial E^l R^k}{\partial \lambda} = \frac{R-R^{ib}}{(1+\lambda_b)^2}\left(\frac{2\sigma\sqrt{3}}{2\sigma\sqrt{3} - R^{ib}}\right)$$

$$+ \left(\frac{\sigma\sqrt{3} - \frac{(R+2\lambda_b R^{ib})}{(1+\lambda_b)} + E^m\hat{R}^k}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma p^l}{R^{ib}}\right)\frac{\partial R^{ib}}{\partial \lambda} =$$

$$\frac{R-R^{ib}}{(1+\lambda_b)^2}\left(\frac{2\sigma\sqrt{3}}{2\sigma\sqrt{3} - R^{ib}}\right)\left(\frac{\lambda^2}{\lambda^2 - \frac{\left(\sigma\sqrt{3} - \frac{(R+2\lambda_b R^{ib})}{(1+\lambda_b)} + E^m\hat{R}^k\right)}{2\sigma\sqrt{3} - R^{ib}} + \left(\frac{\lambda_b}{1+\lambda_b} - \frac{2\sqrt{3}\sigma p^l}{R^{ib}}\right)}\right) > 0$$

Also note that $\frac{\partial E^l R^k}{\partial \lambda}$ consists of two elements, both of them positive. That is, when λ is raised, the marginal lender must have higher return expectations, and those with lower expectations hoard. $\square$

Proposition 2.0.5 *The effect of a policy rate reduction is limited by the mean market belief.*

Proof. Suppose that $E^l R^k > R^{res}$. Then, for the policy rate reduction to restore lending, the change should be such that $E^l R^k < R^{res}$

$$\frac{\partial E^l R^k}{\partial R^{res}} = \frac{\lambda_b}{1+\lambda_b}\frac{\partial R^{ib}}{\partial R^{res}} + \frac{2\sqrt{3}\sigma}{R^{ib}} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\frac{\partial R^{ib}}{\partial R^{res}} =$$

$$= \frac{\frac{2\sqrt{3}\sigma}{R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}\left(\frac{\lambda_b}{1+\lambda_b} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\right) + \frac{2\sqrt{3}\sigma}{R^{ib}}$$

$$= \frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}\right) =$$

The nominator is smaller than the denominator:

$$\frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}\right) > 1$$

$$2\sqrt{3}\sigma\left(\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}\right)$$

$$> R^{ib}\left(\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\right)$$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}p^m - \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}} - \frac{\lambda_b}{(1+\lambda_b)}R^{ib} > -\frac{R^{ib}\lambda_b}{1+\lambda_b}$$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}p^m - \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}} > 0$$

with $\frac{R^{res}}{R^{ib}} = p^l$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}\left(p^m - p^l\right) > 0$$

This is true, as $2\sqrt{3}\sigma - R^{ib} > 0$ (the result from propositions 2.0.1 and 2.0.2) and $p^m - p^l > 0$. That is, the derivative $\frac{\partial E^l R^k}{\partial R^{res}} > 1$.

Now consider how the difference $E^l R^k - R^{res}$ changes with respect to R^{res} :

$$\frac{\partial E^l R^k}{\partial R^{res}} - 1 > 0$$

That is, the function is increasing in R^{res} and is increasing faster than R^{res}. A downward shift in reserves reduces both the right and left-hand sides of the inequality $E^l R^k > R^{res}$, with $E^l R^k$ declining

faster than R^{res}. Thus, if the difference between the marginal lender's belief and the policy rate is small, it is possible to reverse this inequality and restore lending: there will be some banker who would be better off lending on the interbank market at the low policy rate than investing herself or hoarding. However, with a large difference between $E^l R^k$ and R^{res}, which happens with very low market expectations (see proposition 2.0.2), it is not possible to restore lending with a positive policy rate $\qquad\square$

Appendix B: Correlation of Experts' Opinions, the Mean Market Belief, and Its Variance

Expert opinions are defined in the text as

$$\theta_t = \rho_\theta \theta_{t-1} + \eta_t^i$$

where η_t^i is the noise in the opinion of bank i's expert, with $\eta_t^h \sim N(\mu_t, \sigma_\eta)$. We assume that the noise in experts' opinions is correlated. That is, when one expert overestimates/underestimates the value of a persistent shock, others tend to do the same. Technically, we model correlated draws in the following way. First, there are N^{24} independent draws from $N(\mu_t, \sigma_\eta)$. Then, each of the independent draws is rescaled:

$$\bar\eta_t^i = \rho^c \eta_t^1 + \sqrt{1 - (\rho^c)^2}\,\eta_t^i,\; h \neq 1 \tag{B1}$$

where η_t^i is one of the independent draws and ρ^c is the correlation coefficient

$$\rho^c = \frac{Cov(\bar\eta_t^i, \bar\eta_t^j)}{\sqrt{Var\left(\eta_t^i\right) Var\left(\eta_t^j\right)}}, i \neq j$$

where $Var\left(\eta_t^i\right) = Var\left(\eta_t^j\right) = Var(\bar\eta_t) = \sigma_\eta^2$. The last equality comes with the observation that $Var\left(\bar\eta_t^h\right) = (\rho^c)^2 Var\left(\eta_t^1\right) + \left(1 - (\rho^c)^2\right) Var\left(\eta_t^h\right)$. With η_t^h and η_t^1 being drawn from the same distribution, $Var\left(\bar\eta_t^h\right) = \left((\rho^c)^2 + 1 - (\rho^c)^2\right) Var\left(\eta_t^h\right) = Var\left(\eta_t^h\right)$.

Using (B1), we thus obtain a sequence of random variables, correlated with each other with correlation coefficient ρ^c. Because in equilibrium only the average shock to market beliefs matters, we now proceed to derive its properties. The expected average belief shock can be defined as:

$$\frac{1}{N}E\left(\eta_t^1 + \sum_{h=2}^N \bar\eta_t^h\right) = \frac{1}{N}E\left(\eta_t^1 + (N-1)\eta_t^1\rho^c + \sqrt{1-(\rho^c)^2}\sum_{h=2}^N \eta_t^h\right) \tag{B2}$$

Note that η_t^1 and $\eta_t^{h,h \neq 1}$ are independent and drawn from the same distribution. This means that the expectation of their sum equals the sum of their expectations, which are unconditional expectations μ_t. The expected average belief shock is then:

$$\mu_t \frac{1}{N}\left(1 + (N-1)\left(\rho^c + \sqrt{1-(\rho^c)^2}\right)\right)$$

---

[24] In the text we assume the existence of a continuum of H banks, normalized to 1. Here, for computational purposes, we use N as the number of banks and set it equal to a "large number": $N = 100$.

Note that with $\rho^c = 1$ in the case of perfect correlation and with $\rho^c = 0$ in the case of no correlation, the expected average of the correlated draws corresponds to the unconditional mean. Also, unless μ_t is zero, the average belief shock is not equal to the distributional mean.

The variance of the average belief shock is then:

$$\sigma_\eta^2 \left(1 + (N-1)^2 \left((\rho^c)^2 + 1 - (\rho^c)^2 \right) \right) = \sigma_\eta^2 \frac{(2 + N^2 - 2N)}{N^2} \tag{B3}$$

Appendix C: The Bank's Filtering Problem

The state-space representation of the filtering problem is given by the following equations.

The state equation is:

$$(\mu_t) = (\ \rho_\mu\ ) \times (\mu_{t-1}) + (v_t) \tag{C1}$$

where q is the variance of the i.i.d. Gaussian shock v_t.

The measurement vector consists of two types of signals: data on ξ_t and the expert opinion, $\tilde{\xi}_t^{ex}$. The measurement equation is:

$$\begin{pmatrix} \xi_t \\ \tilde{\xi}_t^{ex} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \times \mu_t + \begin{pmatrix} \rho_\xi \\ \rho_\theta \end{pmatrix} \times \xi_{t-1} + \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix},$$

where ε_t is Gaussian and η_t is uniformly distributed. The measurement equation can be rewritten as:

$$\xi_t = C\mu_t + D\xi_{t-1} + \varpi_t$$

where

$$C = \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ D = \begin{pmatrix} \rho_\zeta \\ \rho_\theta \end{pmatrix},\ \tilde{\xi}_t = \begin{pmatrix} \xi_t \\ \tilde{\xi}_t^{ex} \end{pmatrix}$$

and $\varpi_t = (v_t,\ \eta_t)'$ is a vector of measurement errors with the variance-covariance matrix:

$$R = \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta}^2 \\ \sigma_{\varepsilon\eta}^2 & \sigma_\eta^2 \end{pmatrix}$$

where $\sigma_{\varepsilon\eta}^2$ is the covariance of errors in econometric and expert forecasts.

Appendix D: The Agency Problem

Recall that our agency problem differs from that of Gertler and Karadi (2011) in several respects. First, in our model banks have the possibility to put their funds in reserves. Second, banks are heterogeneous, with a share of them investing in a risky asset. Last but not least, some banks participate in the interbank market, transferring some funds from pessimistic to optimistic banks.

The banking family maximizes the terminal wealth of each member, discounted by the stochastic discount factor $\beta^j \Omega_{t,t+j}$ arising from the household problem. The value

$$V_t = \max E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^{j+1} \Omega_{t,t+1+j}(N_{t+1+j}) =$$

$$= \max E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^{j+1} \Omega_{t,t+1+j} \left\{ \left(R_{t+1+j}^k - R_{t+j} \right) Q_{t+j} S_{t+j} + \left(R_{t+j}^{res} - R_{t+j} \right) Res_t + R_{t+j} N_{t+j} \right\}$$

(D1)

Equation (D1) resembles the terminal wealth equation in Gertler and Karadi (2011), the only difference being that we are applying it on the average level. Note that the banks' family budget constraint is

$$Q_t S_t + Res_t = N_t + B_t$$

Also, only those banks with the lowest return expectations hoard funds in reserves (others either invest themselves or lend funds to be invested by others): $Res_t = s_t^h (N_t + B_t) = s_t^h (Q_t S_t + Res_t)$ with s_t^h being the share of hoarders. And $Q_t S_t = \left(1 - s_t^h\right)(Q_t S_t + Res_t)$. The terminal wealth is:

$$E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta \Omega_{t,t+1+j} \{ \left(R_{t+1+j}^k - R_{t+j} \right) \left(1 - s_{t+j}^h\right)(Q_t S_{t+j} + Res_{t+j}) +$$

$$+ \left(R_{t+j}^{res} - R_{t+j} \right) s_{t+j}^h (Q_t S_{t+j} + Res_{t+j}) + R_{t+j} N_{t+j} \} =$$

$$= E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^{j+1} \Omega_{t,t+1+j} \{ \left(\left(1 - s_{t+j}^h\right) R_{t+1+j}^k + s_{t+j}^h R_{t+j}^{res} - R_{t+j} \right)(Q_t S_{t+j} + Res_{t+j}) +$$

$$+ R_{t+j} N_{t+j} \}$$

We then have to restrict banks from borrowing from the household. Otherwise, for a non-negative $\beta^j \Omega_{t,t+j} \left(R_{t+1+j}^k - R_{t+j} \right)$ a bank would like to borrow indefinitely from the household. To avoid this, a moral hazard problem is introduced. At the beginning of the period, a banker can choose to divert a fraction λ of its assets. The depositors can recover the remaining fraction $(1-\lambda)$ of the banks' assets. For a depositor willing to participate, the banks must meet the incentives constraint:

$$V_t \geq \lambda (Q_t S_t + Res_t)$$

where V_t is the wealth the banker would lose by diverting, and $\lambda (Q_t S_t + Res_t)$ is the gain from diverting. That is, the continuation value should be larger than the gain from deviating. We rewrite (D1) as

$$V_t = v_t (Q_t S_t + Res_t) + \eta_t N_t$$

where

$$v_t = E_t \left\{ (1-\theta)\beta \Omega_{t,t+1} \left(\left(1 - s_t^h\right) R_{t+1}^k + s_t^h R_t^{res} - R_t \right) + \beta \Omega_{t,t+1} \theta \chi_{t,t+1} v_{t+1} \right\}$$

$$\eta_t = E_t \left\{ (1-\theta) + \beta \Omega_{t,t+1} \theta z_{t,t+1} \eta_{t+1} \right\}$$

$$\chi_{t,t+1} = \frac{Q_{t+1} S_{t+1} + Res_{t+1}}{Q_t S_t + Res_t}$$

$$z_{t,t+1} = \frac{N_{t+1}}{N_t}$$

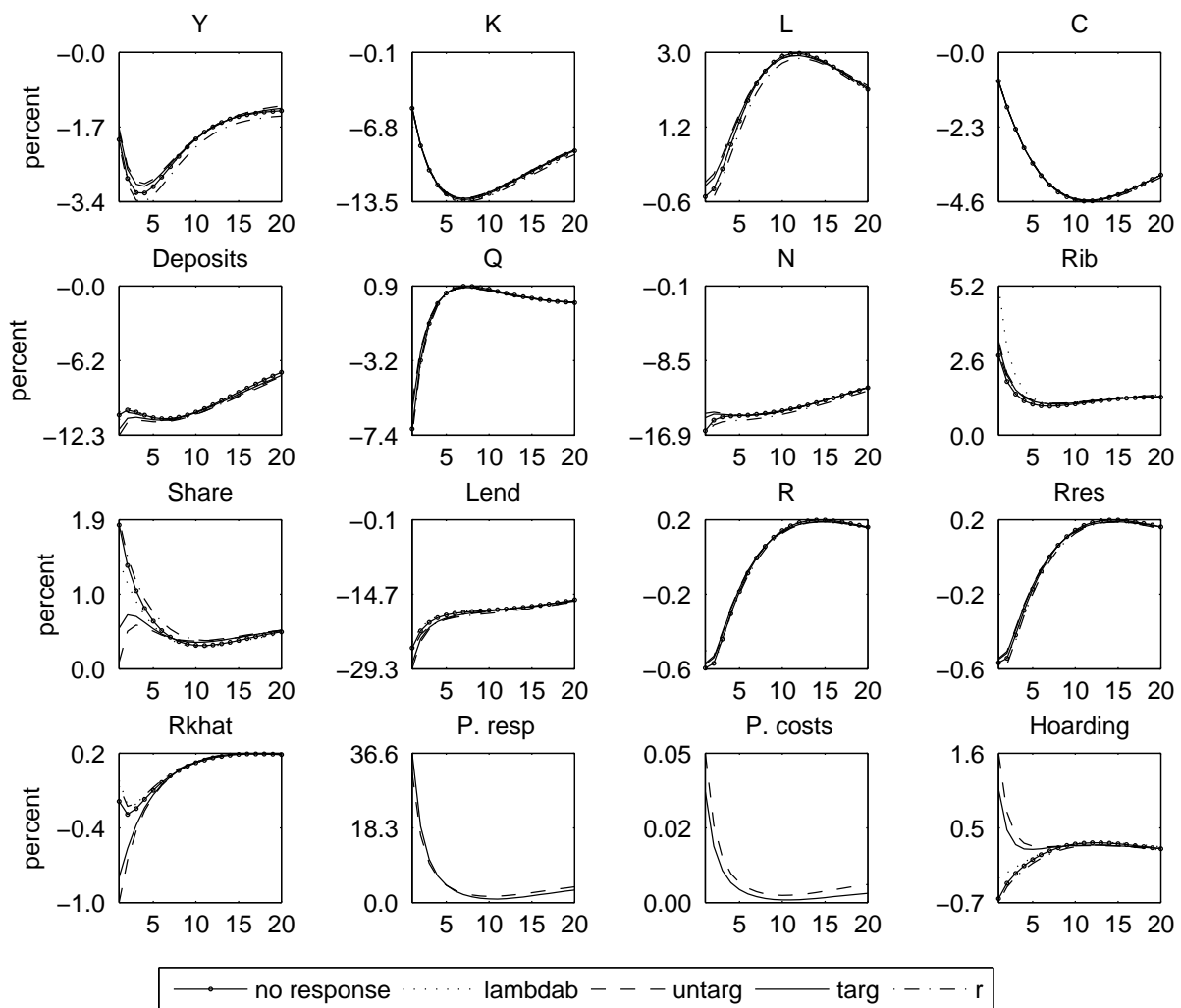and finally we have the expression for the financial accelerator:

$$Q_t S_t + Res_t = \frac{\eta_t}{\lambda - v_t} N_t = \varphi_t N_t$$

where φ_t is the leverage ratio, limiting the amount of assets an intermediary can acquire as a proportion of net wealth.

To determine the leverage ratio, the household needs to form expectations about the future risky asset return. We assume that the household has a belief equal to the mean market belief.

Appendix E: Policy Effects without an Expectational Shock

Figure E1: Policy Effects

Note: Y – output, C – consumption, K – capital, L – labor supply, N – banks' net worth, R – deposit rate, Q – price of capital, Rres – policy rate, Rkhat – expected return on capital, Share – share of investors, Lend – interbank lending, Rib – interbank interest rate, Deposits – deposits, P. response – policy response, P. costs – policy costs.

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