The principle of anti-superposition in QM and the local solution of the Bell's inequality problem

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The principle of anti-superposition in QM and the local solution of the Bell's inequality problem

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Abstract. In this paper we identify the superposition principle as a main source of problems in QM (measurement, collapse, non-locality etc.). Here the superposition principle for individual systems is substituted by the anti-superposition principle: no non-trivial superposition of states is a possible individual state (for ensembles the superposition principle is true).

The modified QM is based on the anti-superposition principle and on the new type of probability theory (Extended Probability Theory [1]), which allows the reversible Markov processes as models for QM.

In the modified QM the measurement is a process inside of QM and the concept of an observation of the measuring system is defined. The outcome value is an attribute of the ensemble of measured systems. The collapse of the state is substituted by the Selection process. We show that the derivation of Bell's inequalities is then impossible and thus QM remains a local theory.

Our main results are: the locality of the modified QM, the local explanation of EPR correlations, the non-existence of the wave-particle duality, the solution of the measurement problem. We show that QM can be understood as a new type of the statistical mechanics of many-particle systems.

1 Introduction

There are (at least) two un-solved problems (in fact, contradictions) in QM.

(i) universality of QM

(ii) non-locality of QM.

1.1 The problem of the universality of QM.

The key principle of QM is the **principle of superposition**: let ϕ and ψ be two possible states of the system and let α, β are complex numbers. Then

$$\alpha\phi + \beta\psi,$$

is also the possible state of this system.

Let us consider the two possible states of an electron, one is localized in Tokyo and the second in Paris. Then the state

$$\psi^{electron} = \alpha\psi^{electron}_{Tokyo} + \beta\psi^{electron}_{Paris}, \quad \alpha, \beta \neq 0, \ \alpha, \beta \in \mathbb{C} \tag{1A1}$$

is a possible state of an electron (at least in the standard QM).

Let us consider the cat (which is alive) and the two of its possible states; ψ^{cat}_{Tokyo}, ψ^{cat}_{Paris}. It is evident, that (for $\alpha, \beta \neq 0$)

$$\psi^{cat} = \alpha\psi^{cat}_{Tokyo} + \beta\psi^{cat}_{Paris} \tag{1A2}$$

is **not** the possible state of the cat[1].

The principle of the universality of QM asserts that QM holds equally for the microworld as for the macroworld. This universality of QM implies that (1A1) is wrong, since it contradicts to (1A2).

[1]The diameter of the support of the cat is smaller than 1 meter. Thus the support of the cat cannot intersect both Tokyo and Paris. It is then clear that (1A2) cannot be the state of the cat.

The basis of our approach is the idea that not only the principle of superposition is not true, but the idea that the **opposite is true**.

The principle of anti-superposition.

Let ϕ and ψ are two distinct states of the individual system S and let $\alpha, \beta \neq 0$ are two complex numbers. Then

$$\alpha\phi + \beta\psi$$

is not a possible state of S. □

By this way the modified QM arises, which is based on the following

(i) for states of the individual system the principle of the anti-superposition holds

(ii) for ensembles the principle of superposition holds.

By an ensemble we mean the set of systems prepared in the same way.

The principle of superposition for ensembles implies that all calculations in the standard QM can be transported to the modified QM. It is clear that each assertion of QM can be reformulated for ensembles. In fact, this reformulation is natural since all QM assertions have probabilistic character (QM predicts only probabilities of results).

Such reformulated assertion will be true also in the modified QM..

1.2 The problem of the non-locality of QM

Bell's inequalities imply that

$$\text{QM+locality} \Rightarrow \text{contradiction.}$$

The standard conclusion of this implication is the non-locality of QM. (The proof is given by the contradiction, so that it is non-constructive, i.e. no description of the non-local mechanism is proposed.)

The problem stays in the fact that the theory of elementary particles is based on

(i) The special theory of relativity

(ii) QM

We clearly have

$$\text{Special relativity} \Rightarrow \text{locality}$$

$$\text{QM+locality} \Rightarrow \text{contradiction}$$

so that

$$\text{Special relativity+QM} \Rightarrow \text{contradiction}\ ^{2}.$$

The true problem is the local explanation of EPR correlations.

Alice and Bob measure the value of the spin (in the z-direction which is chosen arbitrarily by them) for the EPR pair and the result is always opposite. The problem is to explain this.

(i) Let us assume that the Alice's value is random. Then this random value cannot be locally (causally) transported to Bob.

(ii) There is another possibility, that the Alice's and Bob's result are pre-determined. But then there are Bell's inequalities as a consequence of this pre-determination. Bell's inequalities contradicts to QM.

(iii) As a consequence we obtain the impossibility to locally explain the EPR correlations. If we have no local explanation of EPR correlations, we must reject the locality of QM!

Now we shall show (very roughly) that in the modified QM the local explanation of EPR correlations is possible. The procedure is roughly the following (details are given bellow):

[2] It is often mentioned the problem of the incompatibility between General Relativity and QM. But the contradiction between Special Relativity and QM is effective as well and these theories are used in the same region, i.e. in the micro-world.

(i) The individual experiment can be considered as an element of an ensemble $\mathbb{E}$, which can be obtained as many-times repetitions of the given experiment (so-called virtual ensemble consider bellow). The virtual ensemble $\mathbb{E}$ can be the true ensemble, if the experiment is repeated.

(ii) Let us assume that Alice has obtain the value $spin = +1$. (The value $+1$ is an example, the case $spin = -1$ is similar.)

This situation means that the individual system S will be the element of the sub-ensemble

$$\mathbb{E}' = \{S \in \mathbb{E} \mid \text{Alice has measured } spin = +1 \text{ on } S_1\}$$

(iii) Now, one can show that for each $S \in \mathbb{E}'$ the Bob's measurement gives the values $spin = -1$

There is a question.

How the information that Alice has obtained $spin = +1$ is locally transferred to Bob? The answer will be given bellow.

1.3 The organization of the paper

In the second part we shall study the probabilistic model for the real QM based on the new type of the probability theory (EPT=Extended Probability Theory) introduced in [1].

The real QM is much simpler than the true complex QM. In the complex QM there is one more problem: the inner complex structure which is absent in the real QM.

The real QM is quite close to EPT and this makes the study of the real QM quit clear.

Sections 2.1.-2.10. contain the necessary concepts and results. Sect. 2.11. contains the main result: the local explanation of EPR correlations.

In the third part the methods and results obtained for the real QM are generalized to the case of complex QM. Results are similar, the local explanation of the EPR correlations is analogical to the real QM case (Sect. 3.4).

In the forth part we summarized the principles and results of the modified QM and we stress the main result – the locality of QM based on the anti-superposition principle.

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2 The model for the real Quantum Mechanics

We shall start with the study of the real QM, which is simpler then the true QM and is closer to the extended probability theory.

2.1 States of individual systems and their observability

We start with the Principle of the anti-superpostion. It states that the state of the individual system cannot be a superposition of other states of this individual system. Then it can be expected that

$$D = D_S = \{s_1, \dots, s_n\}. \tag{2A1}$$

No s_i is the superposition of other states of D.

In principle D could be countable infinite but we shall assume that D is finite. S is a simple system if its state space D_S is finite.

Simple systems correspond to systems in the standard QM which Hilbert space of states is finite dimensional.

In each probability theory it is possible to observe the individual system S and to find the state $s \in D$ in which the system actually occures. This observation has no influence on the state of S.

2.2 The probability distributions and ensembles

In each probability theory it is possible to assume that the system S can be found in different states $s \in D$ with some probabilities.

It is clear that the classical (Kolmogorov) probability theory cannot describe QM (see [1]). The main reason is the time reversibility of the time evolution in QM. In fact, in the standard probability theory only trivial reversible evolution is possible.

This implies that a certain non-standard probability theory must be used in the description of QM. This non-standard probability theory must contain the possibility of the time-reversible evolution.

Such a probability theory was proposed and developed in [1] under the name of Extended Probability Theory (EPT). In EPT there is an extended space of events containing certain non-classical events. These non-classical events arise as indistinguishable unions of standard events (see [1] for the detailed description of EPT). Here we shall use only basic consequences of EPT obtained in [1]. The main concepts in EPT are the incompatibility of events and the concept of a context.

In the classical probability theory the probability distribution is function

$$p : D \to [0, \infty) \tag{2B1}$$

satisfying

$$\sum_{x \in D} p(x) = 1. \tag{2B2}$$

In EPT the probability distribution must determine the standard probability distribution in each context. It is shown in [1] that then the extended probability distribution is a function

$$p : D \times D \to \mathbb{R}. \tag{2B3}$$

where the following conditions are satisfied

$$p(x, y) = p(y, x), \quad \forall x, y \in D \quad \text{(symmetry)} \tag{2B4a}$$

$$\sum_{x,y \in D} p(x, y) f(x) f(y) \geq 0, \quad \forall f : D \to \mathbb{R} \quad \text{(positivity)} \tag{2B4b}$$

$$\sum_{x \in D} p(x, x) = 1 \quad \text{(normalization)}. \tag{2B4c}$$

The spectral decomposition theorem says that there exists vectors

$$u_i \in \mathcal{H}_D = \{u : D \to \mathbb{R}\}, \quad i = 1, \ldots, n$$

and

$$\lambda_1, \dots, \lambda_n \geq 0$$

such that

$$p = \sum_{i=0}^{n} \lambda_i u_i \otimes u_i \tag{2B5}$$

where

$$(u \otimes v)(x, y) = u(x)v(y). \tag{2B6}$$

Definition 2B1. The (extended) probability distribution $p : D \times D \to \mathbb{R}$ is non-dissipative if there exists a vector $u \in \mathcal{H}_D$ such that

$$p = u \otimes u \tag{2B7}$$

(see [1]).

The vector u is called the state vector of p or a generating vector of p.

The spectral decomposition formula (2B5) then says that each extended probability distribution can be written as a convex combination of non-dissipative probability distributions.

The extended probability distribution p from (2B3) then defines the probability distribution in all possible experiments (see [1]).

We shall understood p as a state of an ensemble resulting from a given preparation process.

2.3 The time evolution

In the probability theory, there is usually given the law of the time evolution of the probability distribution.

In EPT, the time evolution need not be reversible, but in QM the reversibility is required. There is a question, which reversible time evolution in EPT is possible.

It can be shown that the "reasonable" reversible time evolution in EPT is given by the group of orthogonal transformations (see [2]).

We shall use the following definition of the time evolution in the real QM.

Definition 2C1.

(i) Each possible time transformation of the probability distribution in the real QM is generated by an orthogonal transformation

$$O : \mathcal{H}_D \to \mathcal{H}_D \tag{2C1}$$

by the formula

$$Tp = OpO^{\top} \tag{2C2}$$

i.e.

$$T(p)(x,y) = \sum_{u,v \in D} O(x,u)O(y,v)p(u,v)$$

where $\mathcal{H}_D = \{f : D \to \mathbb{R}\}$ is the real Hilbert space with norm $\|f\| = (\sum f(s)^2)^{1/2}$.

(ii) The time evolution of the system in the real QM is given by the one-parameter group of orthogonal transformations

$$G = \{O_t\}_{t \in \mathbb{R}}$$

where the evolution group $\{T_t\}_{t \in \mathbb{R}}$ is given by

$$T_t p = O_t p O_t^{\top}, \quad t \in \mathbb{R}.$$

The group $\{T_t\}_{t \in \mathbb{R}}$ describes the reversible time evolution of the extended probability distribution p. □

2.4 Ensembles

It is the general property of any probabilistic system that the probability distribution is not an attribute (a property) of an individual system, but it is an attribute of an ensemble.

The ensemble of systems is generally defined as a set of systems prepared in the same way

$$\mathbb{E} = \{S_1, \ldots, S_N\}. \tag{2D1}$$

It is assumed that N is sufficiently large (matematically $N \to \infty$). We assume that the spaces of individual states are same

$$D_{S_1} = \ldots D_{S_n} = D_{\mathbb{E}}. \tag{2D2}$$

We shall assume also that systems $S_1, \ldots, S_N$ are statistically independent. This means that what happens with the system S_i has no influence on the system S_j, $j \neq i$.

We shall use the basic theorem of the standard probability theory, that there exists the stabilisation of relative frequences in the ensemble $\mathbb{E}$ (if $N \to \infty$).

This means that to each ensemble $\mathbb{E}$ there esists an extended probability distribution

$$p : D_{\mathbb{E}} \times D_{\mathbb{E}} \to \mathbb{R}. \tag{2D3}$$

The proof of this statement needs more details from the Extended probability theory, so we shall use this stabilization as an assumption. We shall assume that to each ensemble $\mathbb{E}$ prepared in a certain way there exists an extended probability distribution $p : D_{\mathbb{E}} \times D_{\mathbb{E}} \to \mathbb{R}$ which describes the state of an ensemble $\mathbb{E}$.

We shall say that two ensembles $\mathbb{E}_1, \mathbb{E}_2$ are similar if they have the same spaces of individual states

$$D_{\mathbb{E}_1} = D_{\mathbb{E}_2}.$$

We shall say that two similar ensembles $\mathbb{E}_1, \mathbb{E}_2$ are equivalent if they have the same extended probability distribution (i.e. they are in the same state). This means that two equivalent ensembles behave in the same way.

Definition 2D1.

(i) We shall denote by $\mathcal{E}(D;p)$ the class of equivalent ensembles having the same space D of individual states and being in the same state p. The time evolution of ensembles from $\mathcal{E}(D;p)$ are the same

$$T_t : \mathbb{E} \in \mathcal{E}(D;p) \mapsto \mathbb{E}' \in \mathcal{E}(D;p') \tag{2D4}$$

where

$$p' = O_t p O_t^{\top}$$

The time evolution maps ensembles from $\mathcal{E}(D;p)$ onto ensembles from $\mathcal{E}(D;p')$.

(ii) We say that p is the state of an ensemble $\mathbb{E}$ if $\mathbb{E} \in \mathcal{E}(D;p)$. □

There is a useful inequality which is a consequence of the positivity (2B4b) (see [1])

$$|p(x,y)|^2 \leq p(x,x) \cdot p(y,y), \quad x,y \in D. \tag{2D5}$$

and also

$$p(x,x) \geq 0, \quad x \in D. \tag{2D6}$$

As a consequence we obtain the implication

$$p(x,x) = 0 \Rightarrow p(x,y) = p(y,x) = 0, \quad \forall x,y \in D. \tag{2D7}$$

There are basic concepts connected with the extended probability distribution p.

Definition 2D2.

(i) $\operatorname{spt}^1 p = \{x \in D \mid p(x,x) > 0\}$

(ii) $\operatorname{spt} p = \{(x,y) \in D \times D \mid p(x,y) \neq 0\}$

(iii) the state p is deterministic if there exists $x_0 \in D$ such that $\operatorname{spt} p = (x_0, x_0)$

(iv) the state p is semi-deterministic, if

$$\operatorname{spt}^1 p \neq D. \qquad \square$$

There are simple consequences of these definitions

$$\operatorname{spt} p \subset \operatorname{spt}^1 p \times \operatorname{spt}^1 p. \tag{2D8}$$

$$\text{If } p = f \otimes f, \quad f \in \mathcal{H}_D \text{ i.e. } p \text{ is non-dissipative} \tag{2D9}$$

then

$$\operatorname{spt}^1 p = \operatorname{spt} f, \quad \operatorname{spt} p = \operatorname{spt} f \times \operatorname{spt} f.$$

Let $\mathbb{E}$ is an ensemble in the state p, i.e.

$$\mathbb{E} = \{S_1, \dots, S_N\}, \quad \mathbb{E} \in \mathcal{E}(\Sigma; p).$$

We shall describe the relation between states of individual systems $S_1, \dots, S_N$ and the state of an ensemble $\mathbb{E}$.

We have a function ISt (=the individual state function)

$$\operatorname{ISt} : S \in \mathbb{E} \mapsto \operatorname{ISt}(S) \in D$$

and a function CSt (=collective state function)

$$\operatorname{CSt} : \mathbb{E} \mapsto \operatorname{CSt}(\mathbb{E}) = p = \text{ the state of ensemble } \mathbb{E}.$$

The individual state $\operatorname{ISt}(S)$ can be observed without influencing the system.

The probability postulate.

In the sequence

$$\mathrm{ISt}\,(S_1), \ldots, \mathrm{ISt}\,(S_N)$$

the state $s \in D$ occures with the relative frequency $p(s,s)$, i.e. the probability distribution of individual states in $\mathbb{E}$ is given by the classical probability distribution

$$p^{cl}(s) = p(s,s), \quad s \in D. \tag{2D10}$$

We shall assume the rule stating that events with zero probability never happen. As a consequence we obtain

$$\mathrm{ISt}\,(S) \in \mathrm{spt}^{\,1} p, \quad \forall S \in \mathbb{E} \tag{2D11}$$

Let $\delta_{s_0} : D \to \mathbb{R}, \quad s_0 \in D$ is the function

$$\delta_{s_0} : s \in D \mapsto \delta_{s_0 s} \in \{1, 0\} \tag{2D12}$$

where $\delta_{s_0 s} = 1$ for $s = s_0$, $\delta_{s_0 s} = 0$ otherwise.

In the special situation, where the ensemble $\mathcal{E}$ is in the deterministic state we have the simple but very important consequence.

The principle of individualization.

If the state p of an ensemble $\mathbb{E}$ is deterministic, $p = \delta_{s_0} \otimes \delta_{s_0}$ for some $s_0 \in D$, then

$$\mathrm{ISt}\,(S) = s_0, \quad \forall S \in \mathbb{E}$$

(this is the consequence of (2D11) since $\mathrm{spt}^{\,1}(\delta_{s_0} \otimes \delta_{s_0} = \{s_0\})$

What is the relation between the individual state of $S \in \mathbb{E}$ and the state of an ensemble $\mathbb{E}$? We have already considered the condition (2D11) as a necessary condition. But is also (2D11) also a sufficient condition?

In general, in each probability theory there is an assumption, that each event can be independently repeated. This means repeated on another places or in another times. As a consequence we obtain that each individual system can be considered as an element of certain ensemble. The probability distribution (non-deterministic) cannot be an attribute of the individual system , but it can be an attribute of an ensemble. There is a question of possible states of this ensemble. The answer is given by the Principle of Virtual Ensemble. It says that the condition (2D11) is also sufficient.

The principle of the virtual ensemble.

Let S_0 is a system in the individual state

$$\mathrm{ISt}\,(S_0) = s_0 \in D$$

and let $p : D \times D \to \mathbb{R}$ be an extended probability distribution on D.

If

$$s_0 \in \mathrm{spt}\,^1 p$$

then there exists an ensemble $\mathbb{E} \in \mathcal{E}(D; p)$ such that

$$S_0 \in \mathbb{E}.$$

There is a question how to construct an ensemble with the given probability distribution p.

The procedure is the following:

(i) We assume that we have a classical probability distribution

$$\lambda : D \to [0,1], \quad \sum_{s \in D} \lambda(s) = 1.$$

In the standard way it is possible to construct the ensemble $\mathbb{E}^{(in)}$ with the classical distribution λ, where the relative frequency of the state $s \in D$ is equal to $\lambda(s)$. This state is described by the extended probability distribution

$$p^{(in)} = \sum_{s \in D} \lambda(s) \delta_s \otimes \delta_s \tag{2D13}$$

(i.e. $p^{(in)}(s,s) = \lambda(s), \quad p^{(in)}(s,t) = 0, \quad \forall s \neq t$).

(ii) Let $\{f_i\}_{i=1}^n$ is an orthogonal bases in $\mathcal{H}_D$. We shall define the orthogonal transformation O by

$$O : \delta_{s_i} \mapsto f_i, \quad \forall i = 1, \dots, n. \tag{2D14}$$

This orthogonal transformation defines the new ensemble $\mathbb{E}' \in \mathcal{E}(D; p')$ where

$$p' = O p^{(in)} O^\top = \sum_{i=1}^{n} \lambda(s_i) f_i \otimes f_i \tag{2D15}$$

(iii) In this way we obtain the ensemble in the state (2D15). But any possible state of an ensemble can be written in the form (2D15).

2.5 Superpositions and ensembles

For individual systems the principle of anti-superposition holds:

no non-trivial linear combination of individual states is a possible individual state. For ensembles, the superposition principle holds in the full extend.

The so-called principle of superposition is not internally completely consistent (the principle is not in-correct, but its formulation is not usually correct).

The standard formulation is the following.

Proposition 2E1. Let

$$p = f \otimes f, \quad q = g \otimes g$$

are two non-dissipative states of an ensemble $\mathbb{E}$. Let $\alpha,\ \beta \in \mathbb{R}$ are two real numbers

Then the state

$$r = N^{-1} h \otimes h, \quad h = \alpha f + \beta g$$

is also a possible state of this ensemble where N is a normalization factor.

Proof.

$$\sum r(x,y)\eta(x)\eta(y) = \sum h(x)h(y)\eta(x)\eta(y) = |\sum h(x)\eta(x)|^2 \geq 0 \qquad \square$$

The statement of the superposition principle is not correct, since f and g are not uniquelly determined by p and q. We have

$$p \leftrightarrow \{\varepsilon f \mid \varepsilon = \pm 1\} \tag{2E1}$$

As a superposition we have

$$\{(\alpha f + \varepsilon \beta g) \otimes (\alpha f + \varepsilon \beta g) \mid \varepsilon = \pm 1\} =$$

$$= \{\alpha^2 f \otimes f + \beta^2 g \otimes g + \varepsilon \alpha \beta [f \otimes g + g \otimes f] \mid \varepsilon = \pm 1\} \qquad \square$$

The comparison of individual states with collective states is following:

(I1) The space of possible states of an individual system D is finite and any superpositions are forbidden.

(E1) The space of possible states of an ensemble is the set $\boldsymbol{\sigma}_D$ given by

$$\{p : D \times D \to \mathbb{R} \mid p \text{ is symm.}, \quad p \geq 0, \quad \operatorname{tr} p = 1\} \tag{2E2}$$

which is un-countable and satisfies the principle of superposition. Moreover, this state space is closed with respect to convex combinations.

(I2) The time evolution of the state of the individual system is random and there does not exist an operator of time evolution of the individual system.

(E2) The time evolution of the state of the ensemble is continuous, deterministic and time-reversible given by the group of transformations $\{\mathcal{T}_t\}_{t \in \mathbb{R}}$ where

$$\mathcal{T}_t : \boldsymbol{\sigma}_D \to \boldsymbol{\sigma}_D, \quad \mathcal{T}_t(p) = O_t p O_t^{\top}$$

and $\{O_t\}_{t \in \mathbb{R}}$ is the one-parametric group of orthogonal transformations in the real Hilbert space $\mathcal{H}_D$. If the state $\mathcal{T}_{t_0}(p)$ is deterministic then, in general, the state $\mathcal{T}_t(p), \quad t \neq t_0$ is not deterministic

(I3) Observing the individual system S we find the individual state $\operatorname{ISt}(S) \in D$.

(E3) If we observe individual states $\mathrm{ISt}\,(S)$ of system $S \in \mathbb{E}$ we can obtain (calculating relative frequences of $S \in \mathbb{E}$) the probability distribution

$$p^{diag}(s) = p(s,s), \quad s \in D.$$

For the calculating of the non-diagonal elements $p(e,e'), \quad e' \neq e$, we must consider the ensemble $\mathbb{E}$ in other contexts (see [1]). The exact procedure will be described in another paper [4].

In general, the information on the individual system S can be obtained from the fact that $S \in \mathbb{E}$, i.e. from the participation to certain ensemble. This will be analyzed below.

2.6 The selection process and the corresponding probability

The process of the selection is a necessary part of any probability theory. It is based on the possibility to create a subensemble $\mathbb{E}'$ from the given ensemble $\mathbb{E}$. This possibility of choice depends on the set of attributes of an individual system $S \in \mathbb{E}$. But the individual system has only one proper attribute - the individual state $\mathrm{ISt}\,(S) \in D$ of S. There is a unique possibility - to choose the set $A \subset D$, $A \neq 0$ and to set

$$\mathbb{E}' = \mathbb{S}_A(\mathbb{E}) = \{S \in \mathbb{E} \mid \mathrm{ISt}\,(S) \in A\}. \tag{2F1}$$

Example. (The Brownian particle.)

Let us consider the situation, where the possible positions of the particle forms the finite set $D \subset \mathbb{R}^3$ (the discrete approximation, for example). The state of an ensemble will be given by the probability distribution

$$p : D \to [0,1], \quad \sum p(s) = 1.$$

Let us consider the set $A \subset D, \quad A \neq 0$. If $\mathbb{E} \in \mathcal{E}(D;p)$ and $\sum_{s \in A} p(s) > 0$, then

$$\mathbb{E}' = \{S \in \mathbb{E} | \ \mathrm{ISt}\,(S) \in A\} = \mathbb{S}_A(\mathbb{E})\} \tag{2F2}$$

will satisfy

$$\mathbb{E}' \in \mathcal{E}(D;p')$$

where

$$p' = p \cdot \mathcal{X}(A;\cdot) \;\cdot\; P_A^{-1} \tag{2F3}$$

$P_A = \sum_{s \in A} p(s)$ = "probability that $s \in A$" ; here $\mathcal{X}(A';\cdot)$ is the characteristic function of A. Thus the change of an ensemble

$$\mathbb{S}_A : \mathbb{E} \mapsto \mathbb{E}'$$

creates the change of the probability distribution

$$\mathbb{C}_A : p \mapsto p' \tag{2F4}$$

described above (assuming $P_A > 0$). □

It is completely natural that the change $\mathbb{S}_A$ of an ensemble generates the change $\mathbb{C}_A$ of the probability distribution. Thus the so-called "probabilistic collapse" $\mathbb{C}_A$ is a derived effect (the natural consequence) of the original change $\mathbb{S}_A : \mathbb{E} \mapsto \mathbb{E}'$ which describe the selection process.

The transformation $\mathbb{S}_A : \mathbb{E} \mapsto \mathbb{E}'$ is the "instanteous, non-local and global" change, since it is the change of an ensemble, i.e. the change of the definition of an ensemble. The change of the definition is a mental process. The corresponding (derived) change $\mathbb{C}_A$ of the probability distribution is a consequence of $\mathbb{S}_A$. $\mathbb{C}_A$ cannot be considered as a change of the state of a given ensemble - it is the ensemble, which is primarily changed. □

The situation in EPT is analogous.

For $A \subset D$, $A \neq 0$ we define

$$\mathbb{S}_A : \mathbb{E} \mapsto \mathbb{E}' = \{S \in \mathbb{E} | \ \mathrm{ISt}\,(S) \in A\} = \mathbb{S}_A(\mathbb{E}) \tag{2F5}$$

Proposition 2F1. Let $\mathbb{E} \in \mathcal{E}(D;p)$, and $\sum_{s \in A} p(s,s) > 0$. Then

$$\mathbb{S}_A(\mathbb{E}) = \mathbb{E}' \in \mathcal{E}(D;p')$$

where $p' = \mathbb{C}_A(p)$ and

$$p'(s,\overline{s}) = p(s,\overline{s}) \cdot \mathcal{X}(A;s) \cdot \mathcal{X}(A;\overline{s}) \cdot P_A^{-1}$$

$$P_A = \sum_{s \in A} p(s,s) = \text{"probability that } s \in A\text{"}. \tag{2F6}$$

Proof. This assertion is intuitively clear, but it needs a careful analysis. We shall postpone this analysis to the next paper. This needs the consideration of so-called semi-classical contexts (see [1] for more details). □

We can define the selection process

$$\mathbb{S}_A : \mathbb{E} \mapsto \mathbb{E}', \quad \mathbb{E} \in (D; p)$$

where

$$\mathbb{E}' \in \mathcal{E}(D; p')$$

and the corresponding change

$$p \mapsto p' = \mathbb{C}_A(p)$$

of the probability distribution.

These changes ($\mathbb{S}_A$ and $\mathbb{C}_A$) are instantaneous and non-local but they are generated by the mental change - the redefinition of an original ensemble.

The selection process can also happen in the situation, where we observe something. We observe the individual system S.

(i) At the beginning we have the situation that

$$S \in \mathbb{E} \in \mathcal{E}(D; p)$$

(ii) By the observation (appropriately prepared) we find that

$$\mathrm{ISt}\,(S) \in A$$

for some $A \subset D$ satisfying

$$A \neq 0, \quad \sum_{s \in A} p(s,s) > 0.$$

(iii) We can conclude that

$$S \in \mathbb{E}' = \mathbb{S}_A(\mathbb{E}) \in \mathcal{E}(D; p'), \quad p' = \mathbb{C}_A(p)$$

(iv) the individual state of S was not changed, only its participation to some ensembles has changed:

$$S \in \mathbb{E} \mapsto S \in \mathbb{E}'$$

It is useful to compare the selection process in EPT to the collapse in the standard QM:

(i) If the set D of possible individual states of a system is finite, then the set of possible selections $\mathbb{S}_A$ is finite, since each possible selection is determined by the set $A \subset D$.

(ii) In the standard QM the set of possible collapses is parametrized by subspaces $M \subset \mathcal{H}$, where $\mathcal{H}$ is a Hilbert space of the system. So the set of all possible collapses is un-countable, even if the space $\mathcal{H}$ is finite dimensional.

(iii) In the standard QM the collapse is understood as an instantaneous change of the state of an individual system. This is clearly very problematic, since this change must be non-local if the state of the system (e.g. the support of the wave function) is non-local.

(iv) In the modified QM the state of an individual system is unchanged during the selection process, since the selection process consists in the change of the ensemble.

2.7 The composition of systems and probabilities

Let us consider the composition of two systems

$$S = S_1 \oplus S_2.$$

Let spaces of individual states are

$$D_1 = D_{S_1}, \quad D_2 = D_{S_2}$$

then the composed system S has the space of individual states

$$D = D_S = D_1 \times D_2 = \{(s_1, s_2) \mid s_1 \in D_1, \ s_2 \in D_2\}. \tag{2G1}$$

Let us assume that we have two ensembles of these systems $\mathbb{E}_1$, $\mathbb{E}_2$

$$\mathbb{E}_1 = \{S_1^1, \dots, S_N^1\},\ \mathbb{E}_2 = \{S_1^2, \dots, S_M^2\} \tag{2G2}$$

and

$$\mathbb{E}_1 \in \mathcal{E}(D_1; p_1),\ \mathbb{E}_2 \in \mathcal{E}(D_2; p_2) \tag{2G3}$$

Definition 2G1.

(i) The union of ensembles $\mathbb{E}_1$ and $\mathbb{E}_2$ is defined by

$$\mathbb{E}_1 \oplus \mathbb{E}_2 = \{S_1 \oplus S_2 \mid S_1 \in \mathbb{E}_1,\ S_2 \in \mathbb{E}_2\}. \tag{2G4}$$

(ii) The tensor product $p = p_1 \otimes p_2$ is defined as probability distribution $p : D \times D \to \mathbb{R}$

$$p((s_1, s_2), (\overline{s}_1, \overline{s}_2)) = p_1(s_1, \overline{s}_1) \cdot p_2(s_2, \overline{s}_2) \tag{2G5}$$

where $s_1,\ \overline{s}_1 \in D_1,\ s_2,\ \overline{s}_2 \in D_2$.

(iii) The probability distribution $p : D \times D \to \mathbb{R}$ is separable if there exist probability distributions $p_1 : D_1 \times D_1 \to \mathbb{R}$, $p_2 : D_2 \times D_2 \to \mathbb{R}$ such that $p = p_1 \otimes p_2$. If this is not the case, the distribution p is called entangled.

(iv) Ensembles $\mathbb{E}_1$ and $\mathbb{E}_2$ are called (statistically) independent if the probability distribution of their union is separable.

(v) The marginal probabilities are defined by

$$\tilde{p}_1(s_1, \overline{s}_1) = \sum_{s_2 \in D_2} p((s_1, s_2), (\overline{s}_1, s_2),\ s_1,\ \overline{s}_1 \in D_1$$

and analogously $\tilde{p}_2(s_2, \overline{s}_2),\ s_2,\ \overline{s}_2 \in D_2$. Clearly, if $p = p_1 \otimes p_2$, then $\tilde{p_1} = p_1$, $\tilde{p_2} = p_2$. □

Proposition 2G1.

(i) If two independent ensembles $\mathbb{E}_1$ and $\mathbb{E}_2$ in states p_1 and p_2, resp. are unified, then their union $\mathbb{E}_1 \oplus \mathbb{E}_2$ will be in the state $p_1 \otimes p_2$. In this situation we have also

$$S_1 \in \mathbb{E}_1,\ S_2 \in \mathbb{E}_2 \Rightarrow S_1 \oplus S_2 \in \mathbb{E}_1 \oplus \mathbb{E}_2.$$

(ii) If $\mathbb{E} \in \mathcal{E}(D_1 \times D_2; p)$ and the probability distribution p is separable, $p = p_1 \otimes p_2$, then $\mathbb{E}$ can be considered as an union of two independent ensembles in states p_1 and p_2 resp.. Thus the composed ensemble of systems $\mathbb{E}$ can be decomposed into $\mathbb{E}_1 \in \mathcal{E}(D_1; p_1)$ and $\mathbb{E}_2 \in \mathcal{E}(D_2; p_2)$.

(iii) The marginal probability distributions are true states.

(iv) If $\operatorname{spt}{}^1 p \subset A = \{s_0\} \times D_2$ for some $s_0 \in D_1$, then p is separable and $p = p_1 \times p_2$ where $p_1(s_1, \overline{s}_1) = \delta_{s_0,s_1}\delta_{s_0,\overline{s}_1}$ is deterministic and

$$p_2(s_2, \overline{s}_2) = p((s_0, s_2), (s_0, \overline{s}_2)),\ \ s_2, \overline{s}_2 \in D_2$$

Proof.

(i) This needs detailed consideration of contexts. The diagonal part

$$(p_1 \otimes p_2)((s_1, s_2), (s_1, s_2)) = p_1(s_1, s_1) \cdot p_2(s_2, s_2)$$

is trivially true. But the non diagonal elements

$$(p_1 \otimes p_2)((s_1, s_2), (\overline{s}_1, \overline{s}_2)) = p_1(s_1, \overline{s}_1) p_2(s_2, \overline{s}_2)$$

must be considered in different contexts and we shall consider these arguments in the next paper.

(ii) It follows from definitions.

(iii) The symmetry of $\tilde{p_1}$ follows from the symmetry of p. To obtain the positivity of $\tilde{p_1}$ it is sufficient to apply positivity of p to $f(s_1, s_2) = f_1(s_1)\delta_{s_2 r}$ then sum the result over r. The trace properly is obtained immediately.

(iv) is simple. □

2.8 The measurement

In the modified QM the measurement is the process within the theory, it is not the external process like in the standard QM. This internal process of the measurement is exactly defined but it has a certain property, which we call the non-reality which contrasts the standard QM. The basic point which solves this problem is the anti-superposition principle.

We shall assume that

$$D_M = \{m_0, \ldots, m_{n-1}\},\ \ D_S = \{s_0, \ldots, s_{n-1}\}$$

$$\{f_0, \ldots, f_{n-1}\} \text{ is a given orthogonal bases of}$$

$$\mathcal{H}_S = \{f \mid f : D_S \to \mathbb{R}\}.$$

We shall consider the schematic measurement process, which consist of three phases

(i) We start with an ensemble $\mathbb{E}_M$ of measuring systems, the ensemble of $\mathbb{E}_S$ of measured systems and in this first step the union ensemble

$$\mathbb{E} = \mathbb{E}_M \oplus \mathbb{E}_S \text{ is created.}$$

It is supposed that the initial state of the ensemble $\mathbb{E}_S$ is $\Psi \in \mathcal{H}_S$ and it is supposed that the initial state of the measuring ensemble $\mathbb{E}_M$ is the deterministic state δ_{m_0}. Thus the state of an ensemble $\mathbb{E}_M \oplus \mathbb{E}_S$ is

$$\delta_{m_0} \otimes \Psi$$

(it is considered as a vector generator of the non-dissipative state $(\delta_{m_0} \otimes \Psi) \otimes (\delta_{m_0} \otimes \Psi)$).

(ii) The composed ensemble $\mathbb{E}_M \oplus \mathbb{E}_S$ in the state $\delta_{m_0} \otimes \Psi$ is transformed by the orthogonal transformation (this is the so-called measuring transformation) defined by

$$O : \delta_{m_i} \otimes f_j \mapsto \delta_{m_{i\oplus j}} \otimes f_j, \quad i, j = 0, \ldots, n-1 \tag{2H2}$$

where

$$\begin{aligned} i \oplus j &= i + j \quad \text{if } i + j < n \\ i \oplus j &= i + j - n \quad \text{if } i + j \geq n. \end{aligned} \tag{2H3}$$

These conditions define the unique transformation O, since $\{\delta_{m_i} \otimes f_j\}_{i,j=0}^{n-1}$ is the orthogonal base of the space $\mathcal{H}_D$, $D = D_M \times D_S$. This transformation gives

$$O : \delta_{m_0} \otimes f_j \mapsto \delta_{m_j} \otimes f_j, \quad j = 0, \dots, n-1 \tag{2H4}$$

and exactly this is the concept of the entangling transformation. The initial state of $\mathbb{E}_M \oplus \mathbb{E}_S$ was separable while the new state is not separable, in general.

(iii) It is then found in which individual state the measuring system M occures. Let this state is the deterministic state

$$\delta_{m_k}, \quad \text{for some } k \in \{0, \dots, n-1\}.$$

As a consequence it is found that the state of $\mathbb{E}_M \oplus \mathbb{E}_S$ is

$$\delta_{m_k} \otimes f_k.$$

Since k is a concrete value, we can deduce the state of the ensemble of measured system S is f_k. The state $\delta_{m_k} \otimes f_k$ must be considered as the generating vector of the state

$$(\delta_{m_k} \otimes f_k)^{\otimes 2} = (\delta_{m_k} \otimes f_k) \otimes (\delta_{m_k} \otimes f_k) \tag{2H5}$$

of the ensemble $\mathbb{E}_M \oplus \mathbb{E}_S$, $k \in \{0, \dots, n-1\}$.

Now we shall evaluate the operator

$$O : \mathcal{H}_D \to \mathcal{H}_D, \quad D = D_M \times D_S$$

in its natural bases $\{\delta_{m_i} \otimes \delta_{s_j}\}_{i,j=0}^{n-1}$.

Here

$$\delta_{m_i} : m \in D_M \mapsto \delta_{m_i m}, \quad i = 0, \dots, n-1. \tag{2H6}$$

The vectors $v, v' \in \mathcal{H}_D$ can written as

$$v = \sum v_{ij} \delta_{m_i} \oplus \delta_{s_j}, \quad v' = \sum v'_{kl} \delta_{m_k} \oplus \delta_{s_l} \tag{2H7}$$

Then the transformation O can be written as

$$O : v \in \mathcal{H}_D \mapsto v' \in \mathcal{H}_D$$

$$v'_{kl} = \sum_{ij} O_{kl,ij} v_{ij}, \quad k, l = 0, \ldots, n-1. \tag{2H8}$$

The orthogonal base $\{f_r\}_{r=0}^{n-1}$ in $\mathcal{H}_S$ can be defined by the relation

$$f_r = \sum_{t=0}^{n-1} f_{rt} \delta_{s_t}, \quad r = 0, \ldots, n-1. \tag{2H9}$$

Proposition 2H1. Then we have

$$O_{kl,ij} = f_{k\ominus i,l} \cdot f_{k\ominus i,j}, \quad k, l, i, j = 0, \ldots, n-1. \tag{2H10}$$

where

$$\begin{aligned} k\ominus j &= k - i \quad \text{if } i \leq k \\ k\ominus j &= k - i + n \quad \text{if } i > k. \end{aligned} \tag{2H11}$$

Proof. This relation is simply proved by the inspection that the map defined by (2H10) verifies the conditions (2H2). □

For the special case we have

$$O : \delta_{m_o} \otimes f_\beta \mapsto \delta_{m_\beta} \otimes f_\beta, \quad \beta = 0, \ldots, n-1.$$

and this is the non-Neumann's schema of the measurement, where the initial state of the measurement device is δ_{m_0}.

We shall consider the non-trivial measurement where

$$\langle f_i, \delta_{s_j} \rangle \neq 0, \quad \forall i, j = 0, \ldots, n-1.$$

This is the idea of a general measurement.

What will be the result of a measurement? The result will not be deterministic. Let $\Psi = \delta_{s_0}$, for example , the initial state is $(\delta_{m_0} \otimes \delta_{s_l})^{\otimes 2}$. The resulting state is

$$O(\delta_{m_0} \otimes \delta_{s_l})^{\otimes 2} = O(\delta_{m_0} \otimes \delta_{s_l}) \otimes O(\delta_{m_0} \otimes \delta_{s_l}).$$

The result of the measurement (in the bases $\{f_0, \ldots, f_{n-1}\}$) will not be unique - in the sense that, under the repeating the experiment, the measurement will give the different results. This non-determination of the results implies the necessity of considering the ensembles.

Let us consider the situation when the state of the measured system is δ_{s_0}. Then we have the ensemble

$$\mathbb{E} = \mathbb{E}_M \oplus \mathbb{E}_S : \{M_i + S_j \mid i, j = 1, \dots, N\}$$

in the deterministic state $(\delta_{m_0} \otimes \delta_{s_0})^{\otimes 2}$, i.e.

$$\mathbb{E} \in \mathcal{E}\left(D; (\delta_{m_0} \otimes \delta_{s_0})^{\otimes 2}\right).$$

By the application of the measuring transformation we obtain

$$\mathbb{E}' \in \mathcal{E}(D; p')$$

where

$$p'(ij, \bar{i}\,\bar{j}) = \sum_{k,r} f_{k0} f_{r0} \delta_{ki} f_{kj} \delta_{r\bar{i}} f_{r\bar{j}}$$

since

$$O : \delta_{m_0} \otimes \delta_{s_0} \mapsto \sum_k f_{k0} \cdot \delta_{m_k} \otimes f_k$$

and then

$$O \otimes O : (\delta_{m_0} \otimes \delta_{S_0})^{\otimes 2} \mapsto \sum f_{k0} f_{r0} (\delta_{m_k} \otimes f_k) \otimes (\delta_{m_r} \otimes f_r).$$

In the observation of the measuring instrument we shall obtain the probability distribution (the so-called trace-rule)

$$\tilde{p}(i, \bar{i}) = \sum_j p'(ij, \bar{i}\,j) = \sum_k f_{k0} \cdot f_{k0} \cdot \delta_{ki} \delta_{k\bar{i}} = \delta_{i\bar{i}} f_{i0}^2$$

and this corresponds to the standard QM.

The paradoxical situation will happen if the initial state will be

$$\delta_{m_0} \otimes f_\alpha, \quad \alpha \in \{0, \dots, n-1\}.$$

This is the non-deterministic state and this cannot be attributed to the individual system (the anti-superposition principle).

Let us consider the ensemble of composed systems in the state

$$(\delta_{m_0} \otimes f_\alpha)^{\otimes 2}$$

after the application of the transformation O we obtain the ensemble in the state

$$(\delta_{m_\alpha} \otimes f_\alpha)^{\otimes 2}$$

The measuring system (alone) will be in the deterministic state $\delta_{m_\alpha}^{\otimes 2}$, i.e. each system M will be in the individual state m_α.

In this ensemble we have the certainty (100% probability) that

$$\mathrm{ISt}\,(M) = m_\alpha.$$

But the individual state of S is not determined. The measured system S cannot be in the individual state f_α (the principle of anti-superposition). Thus each individual measured system S will be in some (random) individual state. It can be said that the individual system S is an element of an ensemble, which is in the state f_α.

This is the negation of the so-called (EPR) principle of reality: the 100% certainty of the state of $M \Rightarrow$ the state of S.

In other words:

the 100% certainty of the measured result implies the "state of reality" of the measured system.

This "reality principle" is not true in the modified QM. In the extended meaning there exists an element of reality: this is the ensemble of measured systems that are in the state f_α (which is not the deterministic state). But the original meaning of (EPR) principle of reality means the state of the individual measured system.

Our modified QM excludes the reality principle: individual state of $M \not\Rightarrow$ individual state of S.

There is no paradox. The individual state of the measuring system does not imply the individual state of the measured system. I.e. the individuality of $M \not\Rightarrow$ individuality of S.

2.9 The problem of EPR correlations and Bell's inequalities

EPR correlations were introduced in 1935 in the paper of Einstein, Podolsky and Rosen. Then they were reformulated as correlation of "spins" and in this form they are the basic problem of QM.

We shall assume here the locality of QM.

Let us consider the composed system $S_1 \oplus S_2$ of two "*spins*" in the entangled state

$$\psi_0 = \frac{1}{\sqrt{2}}(|+\rangle^1 \otimes |-\rangle^2 - |-\rangle^1 \otimes |+\rangle^2) \tag{2J1}$$

The systems S_1 and S_2 are transposed into distant regions of the space. Then the value of spin of S_1 is measured in the direction of the z axis. After the small time interval the value of the spin of S_2 is measured along the axis z (these two measurements are supposed to be in the space-like relation, i.e. casually independent).

Then in all known events these measurements give opposite values of spins. Exactly this 100% anti-correlation between results of two distant measurements creates the EPR-correlation problem.

(i) Let us assume that the value of the spin of S_1 is the random result of the "un-controlled" interaction of S_1 with the measurement apparatus M_1. The result of this interaction cannot be transferred into measurement of S_2, so that the complete 100% anti-correlation between result of these two measurements is impossible. The assumption (i) must be rejected. (Generally the hypothesis that the un-controlable distribution in the interaction between S_1 and M_1 is the basic argument for non-deterministic nature of QM in the original Copenhagen interpretation of QM.)

(ii) It is possible to assume certain pre-determination of values of spins (these values are pre-determinated already by the original construction of the entangled pair ψ_0). This is the hypotheses of the pre-determination (this is more or less equivalent to the hypotheses of the hidden parameters, but the pre-determination is conceptually simpler). The pre-determination explains EPR correlations: there are in each case only two possibilities (+,-) or (-,+). But the predetermination hypotheses means that for any choice of the axis z and any case of the experiment spins are either (+,-) or (-,+).

But exactly this pre-determination of values of the spin (for all possible choices of the axes z - in fact three particular choices are sufficient.) is the source of Bell's inequalities (1964). Bell's inequalities contradict QM, so that the assumption (ii) of the pre-determination must be rejected.

(iii) Considering the impossibility of (i) and (ii) there is known no local mechanism explaining the EPR correlations. There is a unique way-out: to reject to locality of QM.

(iv) The general conclusion from the study of Bell's inequalities are

  (a) QM is non-local

  (b) it is not known any concrete non-local mechanism explaining the EPR correlations.

2.10 EPR problem in the modified real QM

We shall consider two systems S_1, S_2 with the spaces of their individual states

$$D_{S_i} = \{s_0^{(i)}, s_1^{(i)}\}, \quad i = 1, 2. \tag{2K1}$$

For the composed system $S_1 \oplus S_2$ we have

$$D_{S_1 \oplus S_2} = D_{S_1} \times D_{S_2}, \quad |D_{S_1 \oplus S_2}| = 2^2 = 4. \tag{2K2}$$

We shall suppose that we have given an ensemble $\mathbb{E}_0$ of such systems in the state

$$\begin{aligned} p_0 &= v_0 \otimes v_0 \\ v_0 &= \tfrac{1}{\sqrt{2}}\big(\delta_{s_0^{(1)}} \otimes \delta_{s_1^{(2)}} - \delta_{s_1^{(1)}} \otimes \delta_{s_0^{(2)}}\big) \end{aligned} \tag{2K3}$$

This is the well-known EPR state which is non-deterministic and not separable (it is entangled). As a non-deterministic state, p_0 can be the state of an ensemble, but it cannot be the state of an individual system.

Most of states considered here will be non-dissipative states which can be generated by state vectors. We shall describe them by their state vectors.

We shall consider also two measured devices M_1 interacting with S_1, M_2 with S_2. Their domains will be

$$D_{M_i} = \{m_0^{(i)}, m_1^{(i)}\}, \quad i = 1, 2. \tag{2K4}$$

Then we shall consider the composed system

$$M_1 \oplus S_1 \oplus S_2 \oplus M_2 \tag{2K5}$$

in the state $p = v \otimes v$,

$$v = \tfrac{1}{\sqrt{2}} \delta_{m_0^{(1)}} \otimes \big(\delta_{s_0^{(1)}} \otimes \delta_{s_1^{(2)}} - \delta_{s_1^{(1)}} \otimes \delta_{s_0^{(2)}} \big) \otimes \delta_{m_0^{(2)}}$$

This state p is separable with respect to subsystems M_1, $S_1 \oplus S_2$, M_2. We can write this state as

$$\begin{aligned} v &= \sum v_{ij\alpha\beta} \delta_{m_i}^{(1)} \otimes \delta_{s_j}^{(1)} \otimes \delta_{s_\alpha}^{(2)} \otimes \delta_{m_\beta}^{(2)} \\ v_{ij\alpha\beta} &= \delta_{i0} \cdot \varepsilon_{j\alpha} \cdot \delta_{\beta 0} \cdot \tfrac{1}{\sqrt{2}} \end{aligned} \tag{2K6}$$

where $\varepsilon_{01} = -\varepsilon_{10} = 1$, $\varepsilon_{00} = \varepsilon_{11} = 0$ is the totally antisymmetric tensor.

Now, we shall apply the measuring transformation $O^{(1)}$ onto the subsystem $M_1 \oplus S_1$. This transformation is determined by the orthogonal base

$$f^{(\theta)} = \{f_0^{(\theta)}, f_1^{(\theta)}\}, \quad f_r^{(\theta)} = \sum_{t=0}^{1} f_{rt}^{(\theta)} \delta_{s_t^{(1)}}, \; r = 0, 1, \tag{2K7}$$

where $f_{00}^{(\theta)} = f_{11}^{(\theta)} = \cos\theta$, $f_{10}^{(\theta)} = -f_{01}^{(\theta)} = \sin\theta$.

After the application of $O^{(1)}$ to the state v from (2K6) we obtain

$$v' = \sum v'_{kl\alpha\beta} \delta_{m_k^{(1)}} \otimes \delta_{s_l^{(1)}} \otimes \delta_{s_\alpha^{(2)}} \otimes \delta_{m_\beta^{(2)}} \tag{2K8}$$

where (using (2H10), (2H13) and the factor δ_{i0})

$$\begin{aligned} v'_{kl\alpha\beta} &= \sum O_{kl,ij}^{(1)} v_{ij\alpha\beta} \\ &= \sum f_{k\ominus i,l}^{(\theta)} f_{k\ominus i,j}^{(\theta)} \delta_{i0} \varepsilon_{j\alpha} \delta_{\beta 0} \cdot \tfrac{1}{\sqrt{2}} \\ &= \sum f_{kl}^{(\theta)} f_{kj}^{(\theta)} \varepsilon_{j\alpha} \delta_{\beta 0} \cdot \tfrac{1}{\sqrt{2}}. \end{aligned} \tag{2K9}$$

Then we apply the measurement transformation $O^{(2)}$ to the subsystem $S_2 \oplus M_2$. Here $O^{(2)}$ is determined by the base

$$f^{(\sigma)} = \{f_0^{(\sigma)}, f_1^{(\sigma)}\},\ \ f_r^{(\sigma)} = \sum f_{rt}^{(\sigma)} \delta_{s_t^{(2)}},\ \ r = 0, 1, \tag{2K10}$$

where $f_{00}^{(\sigma)} f_{11}^{(\sigma)} = \cos\sigma,\ f_{10}^{(\sigma)} = -f_{01}^{(\sigma)} = \sin\sigma.$

As a result we obtain

$$v'' = \sum v''_{kl\gamma\delta} \delta_{m_k^{(1)}} \otimes \delta_{S_l^{(1)}} \otimes \delta_{S_\gamma^{(2)}} \otimes \delta_{m_\delta^{(2)}}, \tag{2K11}$$

$$\begin{aligned} v''_{kl\gamma\delta} &= \sum O^{(2)}_{\gamma\delta,\alpha\beta} v'_{kl\alpha\beta} \\ &= \sum f^{(\sigma)}_{\delta\ominus\beta,\gamma} f^{(\sigma)}_{\delta\ominus\beta,\alpha} v'_{kl\alpha\beta} \\ &= \sum f^{(\sigma)}_{\delta\gamma} \cdot f^{(\sigma)}_{\delta\alpha} \cdot f^{(\theta)}_{kl} \cdot f^{(\theta)}_{kj} \cdot \varepsilon_{j\alpha} \cdot \tfrac{1}{\sqrt{2}} \end{aligned} \tag{2K12}$$

In the case of the EPR correlations we have $\theta = \sigma$ and then

$$v''_{kl\gamma\delta} = \sum f^{(\theta)}_{\delta\gamma} f^{(\theta)}_{kl} \cdot \left(\sum f^{(\theta)}_{kj} \cdot f^{(\theta)}_{\delta\alpha} \varepsilon_{j\alpha}\right) \cdot \tfrac{1}{\sqrt{2}} \tag{2K13}$$

Since the tensor ε is invariant with respect to the orthogonal transformations, we obtain

$$v''_{kl\gamma\delta} = f^{(\theta)}_{\gamma\delta} f^{(\theta)}_{kl} \cdot \varepsilon_{k\delta} \cdot \tfrac{1}{\sqrt{2}} \tag{2K14}$$

2.11 The local explanation of the EPR correlations in the modified real QM

At the end of the preceding section we have obtained formulas (2K11) and (2K14) which describe the state of the system $S = M_1 \oplus S_1 \oplus S_2 \oplus M_2$ after Alice and Bob made their measurement (at the same angle $\theta = \sigma$).

In the resulting formula (2K14) there is a coefficient $\varepsilon_{k\delta}$ which make sure that

$$v''_{1l\gamma 1} = v''_{0l\gamma 0} = 0,\ \ \forall l, \gamma.$$

This means (the principle of probability) that P(Alice finds spin=1 and Bob finds spin=1) = $\sum_{l,\gamma} (v''_{1l\gamma 1})^2 = 0$.

The same holds for both spins=0.

As a consequence we obtain 100% anti-correlation between Alice's and Bob's measurements.

But this explanation is a little formal. The problem lies in the question, how to explain locally the transmition of the (random) value of spin observed by Alice to Bob! This transmition must work in each individual case, since the anti-correlation is absolute (=100%).

The detailed causal description of the EPR correlations is the following.

(i) The ensemble
$$\mathbb{E}^0 = \{S_1^r \oplus S_2^r \mid r = 1, \dots, N\} \tag{2L1}$$
of EPR pairs is in the entangled state generated by the vector
$$v^0 = \tfrac{1}{\sqrt{2}} \textstyle\sum_{j,\alpha}^1 \varepsilon_{j\alpha} \delta_{s_j^{(1)}} \otimes \delta_{s_\alpha^{(2)}}, \tag{2L2}$$
where $\varepsilon_{01} = -\varepsilon_{10}$, $\varepsilon_{00} = \varepsilon_{11} = 0$ is the totally antisymmetric tensor. The state of $\mathbb{E}^0$ is $v^0 \otimes v^0$.

(ii) Then the subsystems $S_1^1, \dots, S_1^N$ are transferred to the region $\mathcal{R}_{Alice}$ and subsystems $S_2^1, \dots, S_2^N$ are transferred into the region $\mathcal{R}_{Bob}$.

It is supposed that regions $\mathcal{R}_{Alice}$ and $\mathcal{R}_{Bob}$ have no causal connection, i.e. these space-time regions are in the space-like position.

(iii) Alice connects system $S_1^1, \dots, S_1^N$ to the measure net systems $M_1^1, \dots, M_1^N$ which are all in the individual state $\delta_{m_0^{(1)}}$. The result will be the ensemble
$$\mathbb{E}^1 = \{M_1^r \oplus S_1^r \oplus S_2^r \mid r = 1, \dots, N\} \tag{2L3}$$
in the state (given by the vector)
$$v^1 = \tfrac{1}{\sqrt{2}} \textstyle\sum \varepsilon_{j\alpha} \delta_{m_0^{()1}} \otimes \delta_{s_j^{(1)}} \otimes \delta_{s_\alpha^{(2)}}. \tag{2L4}$$

(iv) Alice will make the measurement transformation $O^{1(\theta)}$ on the subsystem $M_1 \oplus S_1$. The transformation $O^{1(\theta)}$ will be specified by the bases $f^{(\theta)} = \{f_0^{(\theta)}, f_1^{(\theta)}\}$ where θ is the angle of the position of the measuring apparatus M_1. Using formulas from the preceding section the ensemble

$\mathbb{E}^1$ changes its state and it transforms into the ensemble $\mathbb{E}^2$ in the state (given by its generating vector)

$$v^2 = \frac{1}{\sqrt{2}} \sum \varepsilon_{j\alpha} f^{\theta}_{kl} f^{(\theta)}_{kj} \delta_{m^{(1)}_k} \otimes \delta_{s^{(1)}_l} \otimes \delta_{s^{(2)}_\alpha} \tag{2L5}$$

where $f^{(\theta)}_{00} = f^{(\theta)}_{11} = \cos\theta$, $f^{(\theta)}_{10} = -f^{(\theta)}_{01} = \sin\theta$. It is clear that this transformation is localized in the region $\mathcal{R}_{Alice}$.

(v) The ensemble $\mathbb{E}^2$ can be now (after realizing $O^{1(\theta)}$) divided into two subensenbles using the selection process. The selection process depends on the individual state of the subsystem M_1. There are two possibilities: $\mathrm{ISt}\,(M^r_1) = m^{(1)}_0$, $\mathrm{ISt}\,(M^r_1) = m^{(1)}_1$. So that we obtain two subensembles ($k_0 = 0, 1$)

$$\mathbb{E}^{3(k_0)} = \{M^r_1 \oplus S^r_1 \oplus S^r_2 \in \mathbb{E}^2 \mid \mathrm{ISt}\,(M^r_1) = m^{(1)}_{k_0}\}. \tag{2L6}$$

These subsystems are created by the selection process with ($k_0 = 0, 1$)

$$A^{(k_0)} = \{m^{(1)}_{k_0}\} \times D_{S_1} \times D_{S_2}. \tag{2L7}$$

The state of the ensemble $\mathbb{E}^{3(k_0)}$ is given by the rules for the selection process and the result is the following

$$v^{3(k_0)} = \sum \varepsilon_{j\alpha} f^{(\theta)}_{kl} f^{(\theta)}_{kj} \delta_{kk_0} \delta_{m^{(1)}_k} \oplus \delta_{s^{(1)}_l} \oplus \delta_{s^{(2)}_\alpha} \tag{2L8}$$

(the factor $\frac{1}{\sqrt{2}}$ is missing by the normalization).

The state $v^{3(k_0)}$ can be rewritten as

$$v^{3(k_0)} = \delta_{m^{(1)}_{k_0}} \otimes \left(\sum f^{(\theta)}_{k_0 l} \delta_{s^{(1)}_l}\right) \otimes \left(\sum f^{(\theta)}_{k_0 j} \varepsilon_{j\alpha} \delta_{s^{(2)}_\alpha}\right) \tag{2L9}$$

This shows that the state of $v^{3(k_0)}$ is completely separable.

The creation of states $v^{3(k_0)}$, $k_0 = 0, 1$ is possible only after the application of the measurement application $O^{1(\theta)}$, i.e. only after the specification of the angle θ

As a result we obtain that $\mathbb{E}^3 = \mathbb{E}^{3(0)} \cup \mathbb{E}^{3(1)}$. It is an important fact, that the possible action of Alice of looking on the individual state of M_1 is irrelevant. Alice can only record what it is. Alice cannot (by observing M_1) change the actual reality. There is no such possibility as a standard collapse of the state in QM.

(vi) Bob now associates its system S_2 with measuring system M_2. Bob is going to do the interaction transformation $O^{2(\theta)}$ on the system $S_2 \oplus M_2$.

We could consider the complete system $M_1 \oplus S_1 \oplus S_2 \oplus M_2$, but it is not necessary since the state of $M_1 \oplus S_1 \oplus S_2$ is separable. It is sufficient to consider only the state of the subsystem S_2. The ensemble

$$\mathbb{E}^4 = \{S_2 \mid M_1 \oplus S_1 \oplus S_2 \in \mathbb{E}^{3(k_0)}\} \tag{2L10}$$

is in the state

$$v^4 = \sum f^{(\theta)}_{k_0 j} \varepsilon_{j\alpha} \delta_{s_{\alpha^{(2)}}}. \tag{2L11}$$

After the association the system S_2 with the measuring system M_2 in the individual state $\delta_{m_0^{(2)}}$ we obtain the ensemble

$$\mathbb{E}^5 = \{S_2 \oplus M_2 \mid S_2 \in \mathbb{E}^4\} \tag{2L12}$$

in the state

$$v^5 = \sum f^{(\theta)}_{k_0 j} \varepsilon_{j\alpha} \delta^{(2)}_{s_\alpha} \otimes \delta_{m_0^{(2)}}. \tag{2L13}$$

(vii) Bob applies the measuring transformation $O^{2(\theta)}$ onto the system $S_2 \oplus M_2$.

By the operation the ensemble $\mathbb{E}^5$ changes its state and we shall denote it as $\mathbb{E}^6$.

The state of $\mathbb{E}^6$ will be

$$v^6 = \sum f^{(\theta)}_{k_0 j} \varepsilon_{j\alpha} f^{(\theta)}_{\varrho\sigma} f^{(\theta)}_{\varrho\alpha} \delta_{s_\alpha^{(2)}} \otimes \delta_{m_\varrho^{(2)}}. \tag{2L14}$$

Using the relation $\sum f^{(\theta)}_{k_0 j} f^{(\theta)}_{\varrho\alpha} \varepsilon_{j\alpha} = \varepsilon_{k_0 \varrho}$ we obtain

$$v^6 = \sum \varepsilon_{k_0 \varrho} f^{(\theta)}_{\varrho\sigma} \delta_{s_\sigma^{(2)}} \otimes \delta_{m_\varrho^{(2)}}. \tag{2L15}$$

Now we can use the formula

$$\varepsilon_{k_0 \varrho} = (-1)^{k_0} \delta_{\widehat{k}_0 \varrho}$$

where $\widehat{k}_0 = 1 - k_0$. We obtain

$$v^6 = (-1)^{k_0} \sum \delta_{\widehat{k}_0 \varrho} f^{(\theta)}_{\varrho\sigma} \delta_{s_\sigma^{(2)}} \otimes \delta_{m_p^{(2)}} = (-1)^{k_0} \left(\sum f_{\widehat{k}_0 \sigma} \delta_{s_\sigma^{(2)}} \right) \otimes \delta_{m_{\widehat{k}_0}^{(2)}}. \tag{2L16}$$

This implies that the subsystem M_2 is in the deterministic state $\delta_{m^{(2)}_{\widetilde{k}_0}}$ and, as a consequence of the principle of individualization each subsystem M_2 is in the individual state $m^{(2)}_{\widetilde{k}_0}$. This can be applied on the actual state of M_2 which is $m^{(2)}_{\widetilde{k}_0}$ and this individual state is anti-correlated with the state $m^{(1)}_{k_0}$ of M_1.

By this argument, there is a complete anti-correlation between states of M_1 and M_2.

For subsystems S_1 and S_2, there is no anti-correlation between their individual states. But their collective states are anti-correlated, as can be seen from the fact that the state $\sum f^{(\theta)}_{k_0 j}\varepsilon_{j\alpha}\delta_{s^{(2)}_{\alpha}}$ is orthogonal to the state $\sum f^{(1)}_{k_0 l}$ in the formula (2L9).

So we have the complete individual anti-correlation between states of subsystems M_1 and M_2, while for S_1 and S_2 are anti-correlated only as ensembles and this anti-correlation exists only after the transformation $O^{1(\theta)}$ is applied.

Now we are able to make the summary of this procedure.

(i) Alice makes the interaction $O^{1(\theta)}$ on $M_1 \oplus S_1$ and the ensemble is transformed into $\mathbb{E}^2$ in the state v^2.

(ii) The ensemble $\mathbb{E}^2$ can be divided into two parts

$$\mathbb{E}^2 = \mathbb{E}^{3(0)} \cup \mathbb{E}^{3(1)}$$

and the actual system is contained in one of these subensembles.

The Alice's findings that the actual system lies in, say, $\mathbb{E}^{3(0)}$ (i.e. $k_0 = 0$) has no consequences on the physical processes (there is no collapse postulate in the modified QM). This decomposition can be done only after the transformation $O^{1(\theta)}$ has been applied.

(iii) Resulting steps are

(a) the state $v^{3(0)}$ is separable

(b) the state of S_2 is v^4 and $S_2 \in \mathbb{E}^4$

(c) Bob associates S_2 with $\mathbb{E}^6$ in the state v^6

(d) the state v^6 is separable and the subsystem M_2 is in the deterministic state $\delta_{m_1^{(2)}}$.

(iv) Bob uses the principle of the individualization and the individual state of M_2 is $m_1^{(2)}$ - the anti correlated state to the state $m_0^{(1)}$ of M_1.

3 The modified complex QM

Our approach to the complex QM will be based primarily on the principle of anti-superposition. We shall use, of course, the section 2. as a motivation.

3.1 States and ensembles

Let us consider the standard quantum system S described by the finite dimensional complex space $\mathcal{H}$, $\dim \mathcal{H} = n$. The state space is the set of density operators in $\mathcal{H}$

$$\boldsymbol{\sigma}_{\mathcal{H}} = \{\sigma : \mathcal{H} \times \mathcal{H} \to \mathbb{C} \mid \sigma \text{ is Hermitean, positive and } \operatorname{tr} \sigma = 1\}.$$

The state σ is non-dissipative, iff there exists a vector $u \in \mathcal{H}$, such that $\sigma = u \otimes u^*$.

In both, standard and modified QM states σ describe (in general) the states of ensembles.

In the standard QM, non-dissipative state (called often the "pure" state) can be attributed to the individual systems (the superposition principle).

In the modified QM, the anti-superposition principle implies that individual states must be linearly independent, so that maximal number of individual states will be $n = \dim \mathcal{H}$. We shall assume that this maximum will be achieved, so that the set of individual states in the modified QM we be a (linear) base of $\mathcal{H}$

$$D_S = \{s_1, \ldots, s_n\}, \; s_1, \ldots, s_n \in \mathcal{H}.$$

Each non-dissipative state generated by $u \in \mathcal{H}$ can be written as

$$u = \sum u_i s_i, \; u_i \in \mathbb{C}.$$

The corresponding state $\sigma = u \otimes u^*$ will be $\sigma(s_i, s_j) = u_i u_j^*$. For the probability we obtain $(A \subset D_S)$

$$\operatorname{Prob}[\operatorname{ISt}(S) \in A] = \sum_{s \in A} \sigma(s, s) = \sum_{s_i \in A} |u_i|^2$$

If we require the standard relation between probability and the norm in $\mathcal{H}$, we arrive at $\|s_i\| = 1$ (since $s_i = \sum \delta_{ij} s_j$) and that $\|u\|^2 = \sum |u_i|^2$. As a conclusion we obtain that D must be an orthonormal base in $\mathcal{H}$.

Using the base D we can identify $\mathcal{H}$ with $\mathbb{C}^n$ by

$$\mathcal{H} \approx \mathcal{H}_D^{\mathbb{C}} = \{u : D \to \mathbb{C} \mid \|u\|^2 = \sum_{s \in D} |u(s)|^2\}$$

The formula for $\|u\|^2$ implies that

$$(u, v)_{\mathcal{H}} = \sum_{s \in D} u(s) v(s)^*.$$

For $s_i \in D$ the corresponding vector $\delta_{s_i} \in \mathcal{H}_D^{\mathbb{C}}$ will be

$$\delta_{s_i}(s) = \delta_{s_i s}, \ \ s \in D.$$

The corresponding state will be $s_i \mapsto \delta_{s_i} \otimes \delta_{s_i}^*$. The non-dissipative state $u_0 \otimes u_0^*$ can be generated by vectors $(\varepsilon u_0) \otimes (\varepsilon u_0)^*$, where $\varepsilon \in \mathbb{C}$, $|\varepsilon| = 1$.

States of ensembles may be identified with maps $q : D \times D \to \mathbb{C}$ satisfying

$$q(s, s') = q(s', s)^*$$

$$\sum_{s, s' \in D} q(s, s') f(s) f(s')^* \geq 0, \ \ \forall f \in \mathcal{H}_D^{\mathbb{C}}$$

$$\sum_s q(s, s) = 1.$$

By the spectral theorem for each q there exists an orthonormal base $u_1, \dots, u_n \in \mathcal{H}_D^{\mathbb{C}}$ and real numbers $\lambda_1, \dots, \lambda_n \geq 0$ such that

$$q = \sum \lambda_i u_i \otimes u_i^*.$$

Properties of q:

(i) $|q(s, s')|^2 \leq q(s, s) \cdot q(s', s')$, $q(s, s) \geq 0$

(ii) $q(s, s) = 0 \Rightarrow q(s, s') = q(s', s) = 0$

(iii) we have

$$\operatorname{spt}^1 q = \{s \in D \mid q(s, s) > 0\}$$

$$\operatorname{spt} q \subset \operatorname{spt}^1 q \times \operatorname{spt}^1 q$$

(iv) q is deterministic $\Leftrightarrow q = \delta_{s_0} \times \delta_{s_0}$ for some $s_0 \in D$ i.e. only $\delta_{s_i} \otimes \delta_{s_0},\ i = 1, \ldots, u$ are deterministic.

We shall assume that each system (in a given time) is in certain individual state

$$S \mapsto \mathrm{ISt}\,(S) \in D.$$

The ensemble is the set (N large)

$$\mathbb{E} = \{S_1, \ldots, S_N\}$$

of systems prepared in the same way. We shall assume that the set of individual states are the same for all systems

$$D_{S_1} = \cdots = D_{S_N} = D_{\mathbb{E}}.$$

We shall assume that with each ensemble there is associated its (collective) state $q : D \times D \to \mathbb{C}$. The set of all ensembles with given D and q will be denoted $\mathcal{E}(D; q)$.

The assertion that the system S is in the (non-deterministic) state q must be understood as

$$\exists \mathbb{E} \text{ such that } S \in \mathbb{E} \in \mathcal{E}(D; q).$$

Principle of individualization:

Let the state of $\mathbb{E}$ is $q = \delta_{s_0} \otimes \delta_{s_0}$ for some $s_0 \in D$ then $S \in \mathbb{E} \Rightarrow \mathrm{ISt}\,(S) = s_0$.

Principle of virtual ensemble:

Let $\mathrm{ISt}\,(S) = s_0 \in D,\ q : D \times D \to \mathbb{C}$ such that $s_0 \in \mathrm{spt}^{\,1} q$. Then there exists an ensemble $\mathbb{E} \in \mathcal{E}(D; q)$ such that $S \in \mathbb{E}$.

Probability postulate:

Let $\mathbb{E} = \{S_1, \ldots, S_N\} \in \mathcal{E}(D; q)$. Then $s \in D$ occures in the sequence

$$\mathrm{ISt}\,(S_1), \ldots, \mathrm{ISt}\,(S_N)$$

with the relative frequence $q(s, s)$.

Superposition principle (for ensembles): Let $q = u \otimes u^*$ and $q' = u' \otimes u'^*$ are two possible states of an ensemble $\mathbb{E}$. Let α, β are complex numbers.

Then $N^{-1}(\alpha u + \beta v) \otimes (\alpha u + \beta v)^*$ is also a possible state of $\mathbb{E}$ where N is a normalization constant.

3.2 Evolution, Selection and composition in the modified QM

The (reversible) time evolution is in the modified (as in the standard) QM generated by the group of unitary operators

$$\{\mathcal{U}_t\}_{t \in \mathbb{R}},\ \mathcal{U}_t : \mathcal{H}_D^{\mathbb{C}} \to \mathcal{H}_D^{\mathbb{C}}.$$

Then states are transformed by

$$\mathcal{T}_t(q) = \mathcal{U}_t q \mathcal{U}_t^+,\ t \in \mathbb{R}$$

where $\mathcal{U}_t^+$ denotes the Hermitte conjungation, i.e.

$$\mathcal{T}_t(q))_{em} = \sum \mathcal{U}_{tek} \mathcal{U}_{tmj}^* q_{kj}.$$

The transformation of ensembles satisfies

$$\mathbb{E} \in \mathbb{E}(D, q) \Rightarrow \mathbb{E}' \in \varepsilon(D; q')$$

$$q' = \mathcal{T}_t(q) = \mathcal{U}_t q \mathcal{U}_t^+$$

Let $q = u \otimes u^*$, then

$$q' = \mathcal{U}_t q \mathcal{U}_t^+ = \mathcal{U}_t (u \otimes u^*) \mathcal{U}_t^+ = (\mathcal{U}_t u) \otimes (\mathcal{U}_t u)^*.$$

The selection process is identical to the selection process described in sect. 2.6. For each $A \subset D,\ A \neq \varnothing$ there exists a map

$$\mathbb{E} \mapsto \mathbb{E}' = \mathbb{S}_A(\mathbb{E}) = \{S \in \mathbb{E} \mid \mathrm{ISt}\,(S) \in A\}.$$

If $\mathbb{E} \in \mathbb{E}(D; q)$, then $\mathbb{E}' \in \mathbb{E}(D; q')$ where

$$q'(s, s') = q(s, s') \cdot \mathcal{X}(A; s) \cdot \mathcal{X}(A; s') \cdot P_A^{-1},\ P_A = \sum_{s \in A} q(s, s)$$

assuming that $P_A > 0$.

The description of the procedure of the composition is only formally different from sect.2.7. If $S = S_1 \oplus S_2$, then $D_S = D_{S_1} \times D_{S_2}$ and the state $q = q_1 \otimes q_2$ is defined by

$$q\left((s_1, s_2), (\overline{s}_1, \overline{s}_2)\right) = q_1(s_1, \overline{s}_1) \cdot q_2(s_2, \overline{s}_2).$$

The state q on D_S is separable if there exists state q_1 on D_{S_1} and q_2 on D_{S_2} such that $q = q_1 \otimes q_2$. Separability means the statistical independence of S_1 and S_2.

Marginal states of q are defined by

$$\tilde{q}_1(s_1, \overline{s}_1) = \sum_{s \in D_{S_2}} q\left((s_1, s), (\overline{s}_1, s)\right)$$

$$\tilde{q}_2(s_2, \overline{s}_2) = \sum_{s \in D_{S_1}} q\left((s, s_2), (s, \overline{s}_2)\right).$$

If $q = q_1 \otimes q_2$, then $\tilde{q}_1 = q_1$, $\tilde{q}_2 = q_2$.

3.3 Measurement and the EPR state

The description of the measurement process is analogous to the sect. 2.8. of course, there are necessary changes related to the complex structure of $\mathcal{H}_D^{\mathbb{C}}$ and $\mathcal{U}_t$.

(i) We shall assume domains of the measurement apparatus M and the measured system S are

$$D_= \{m_0, \dots, m_{n-1}\}, \; D_s = \{s_0, \dots, s_{n-1}.$$

We shall assume that the measurement process is specified by the orthogonal base $\{g_0, \dots, g_{n-1}\}$ in the space $\mathcal{H}_S^{\mathbb{C}}$.

We shall assume that the ensemble $\mathbb{E}_S$ of measured systems is in the non-dissipative state generated by $\Psi \in \mathcal{H}_S^{\mathbb{C}}$ and that the ensemble of measurement apparatuses $\mathbb{E}_M$ is in the deterministic state δ_{m_0}.

(ii) The state of $\mathbb{E}_M \oplus \mathbb{E}_S$ is transformed by the measuring transformation

$$\mathcal{U} : \mathcal{H}_D^{\mathbb{C}} \to \mathcal{H}_D^{\mathbb{C}}, \; D = D_M \times D_S \tag{3C1}$$

satisfying

$$\mathcal{U} : \delta_{m_i} \otimes g_j \mapsto \delta_{m_{i\oplus j}} \otimes g_j, \; i, j = 0, \dots, n-1 \tag{3C2}$$

where $i \oplus j$ is defined in (2H3) and in the special case $i = 0$ we have

$$\mathcal{U} : \delta_{m_0} \otimes g_j \mapsto \delta_{m_j} \otimes g_j. \tag{3C3}$$

The description of $\mathcal{U}$ from (3C1) may be described explicitely. Let

$$\mathcal{U} : v = \sum v_{ij} \delta_{m_i} \otimes \delta_{s_j} \mapsto v' = \sum v'_{kl} \delta_{m_k} \otimes \delta_{s_l}. \tag{3C4}$$

Then we have

$$v'_{kl} = \sum \mathcal{U}_{kl,ij} v_{ij}. \tag{3C5}$$

If g_r are written as

$$g_r = \sum_{t=0}^{n-1} g_{rt} \delta_{s_t} \tag{3C6}$$

then we have the basic formula

$$\mathcal{U}_{kl,ij} = g_{k\ominus i,l} g^*_{k\ominus i,j}, \; k, l, i, j = 0, \dots, n-1 \tag{2C7}$$

where $k \ominus i$ is defined in (2H11). The proof is given by the examination that (2C7)$\Rightarrow$ (2C2).

Now we shall give the description of the EPR pairs. The ensemble of EPR pairs $S_1 \oplus S_2$ is in the state $q_0 = v_0 \otimes v_0^*$, $v_0 = \frac{1}{\sqrt{2}}(\delta_{s_0^{(1)}} \otimes \delta_{s_1^{(2)}} - \delta_{s_1^{(1)}} \otimes \delta_{s_0^{(2)}})$.

Then measurement apparatuses are introduced and the ensemble of systems $M_1 \oplus S_1 \oplus S_2 \oplus M_2$ is in the state $q = v \otimes v^*$ where

$$v = \delta_{m_0^{(1)}} \otimes v_0 \otimes \delta_{m_0^{(2)}} = \sum v_{ij\alpha\beta} \delta_{m_i^{(1)}} \otimes \delta_{s_j^{(1)}} \otimes \delta_{s_{\alpha^{(2)}}} \otimes \delta_{m_\beta^{(}2)}, \tag{3C8}$$

$$v_{ij\alpha\beta} = \frac{1}{\sqrt{2}} \delta_{i0} \varepsilon_{j\alpha} \delta_{\beta 0}. \tag{3C9}$$

The measuring transformation $\mathcal{U}^{(1)}$ (applied by Alice) operates only on $M_1 \oplus S_1$ and it is specified by the basic $g^{(\theta,\phi)} = \{g_0, g_1\}$ in $\mathcal{H}^{\mathbb{C}}_{S_1}$ where

$$g_r = \sum_{t=0}^{1} g_{rt} \delta_{s_t^{(1)}}, \; r = 0, 1, \tag{3C10}$$

$$g_{00} = g_{11} = \cos\theta,\ \ g_{10} = \sin\theta e^{-i\phi},\ \ g_{01} = -\sin\theta e^{i\phi} \tag{3C11}$$

By the application of $\mathcal{U}^{(1)}$ onto the vector v we obtain a vector

$$v' = \sum v'_{kl\alpha\beta}\delta_{m_k^{()1}} \otimes \delta_{s_l^{(1)}} \otimes \delta_{s_\alpha^{(2)}} \otimes \delta_{s_\beta^{(2)}}$$

where

$$v'_{kl\alpha\beta} = \sum \mathcal{U}^{(1)}_{kl,ij} v_{ij\alpha\beta} = \frac{1}{\sqrt{2}} \sum g_{k\ominus i,l} g^*_{k\ominus i,j} \delta_{i0} \varepsilon_{j\alpha} \delta_{\beta 0} = \frac{1}{\sqrt{2}} \sum g_{kl} g^*_{kj} \varepsilon_{j\alpha} \delta_{\beta 0} \tag{3C12}$$

Then Bob applies his measuring transformation $\mathcal{U}^{(2)}$ (with angles θ', ϕ' - corresponding g is denoted as g') onto $S_2 \oplus M_2$ and obtains

$$v'' = \sum v''_{kl\gamma\sigma}\delta_{m_k^{(1)}} \otimes \delta_{s_l^{(1)}} \delta_{s_\gamma^{(2)}} \otimes \delta_{m_\sigma^{(2)}} \tag{3C13}$$

where

$$\begin{aligned} v''_{kl\gamma\sigma} &= \sum \mathcal{U}^{(2)}_{\gamma\sigma,\alpha\beta} v'_{kl\alpha\beta} = \\ &= \sum g'_{\sigma\ominus\beta,\gamma} {g'}^*_{\sigma\ominus\beta,\alpha} v'_{kl\alpha\beta} = \\ &= \frac{1}{\sqrt{2}} \sum g'_{\sigma\ominus\beta,\gamma} {g'}^*_{\sigma\ominus\beta,\alpha} g_{kl} g^*_{kj} \varepsilon_{j\alpha} \delta_{\beta 0} = \\ &= \frac{1}{\sqrt{2}} \sum g'_{\sigma\gamma} {g'}^*_{\sigma\alpha} g_{kl} g^*_{kj} \varepsilon_{j\alpha}. \end{aligned} \tag{3C14}$$

In the situation where $(\theta', \phi') = (\theta, \phi)$, i.e. Alice and Bob apply the same measurement we obtain

$$v''_{kl\gamma\sigma} = \frac{1}{\sqrt{2}} \sum g_{\sigma\gamma} g^*_{\sigma\alpha} g_{kl} g^*_{kj} \varepsilon_{j\alpha}. \tag{3C15}$$

The totally antisymmetric tensor $\varepsilon_{j\alpha}$ is invariant for unitary transformations, i.e.

$$\sum g^*_{\sigma\alpha} g^*_{kj} \varepsilon_{j\alpha} = \varepsilon_{k\sigma} = \sum g_{kj} g_{\sigma\alpha} \varepsilon_{j\alpha}, \tag{3C16}$$

$$\sum \varepsilon_{k\sigma} g_{kl} g_{\sigma\gamma} = \varepsilon_{l\gamma}. \tag{3C17}$$

Using this we obtain

$$v''_{kl\gamma\sigma} = \frac{1}{\sqrt{2}} g_{\sigma\gamma} g_{kl} \varepsilon_{k\sigma}. \tag{3C18}$$

3.4 The local nature of the EPR correlations in the modified complex QM

For the EPR pair we obtain (using (2C18))

$$v''_{1l\gamma 1} = v''_{0l\gamma 0} = 0, \ \forall l, \gamma \tag{3D1}$$

and then

$$\sum_{l,\gamma} |v''_{1l\gamma 1}|^2 = \sum_{l,\gamma} |v''_{0l\gamma 0}|^2 = 0 \tag{3D2}$$

This implies the 100% anti-correlation between Alice's and Bob's measurements.

The local explanation given in Sect. 2.11. (steps (i)-(vii)) is true also in the actual situation - with the appropriate change in formulas.

The key point is clear form the formula (2L8). Assume that Alice has observed that M_1 is in the individual state $m_0^{(1)}$ (the case of $m_1^{(1)}$ is analogical). Then S_1 is in the (collective) state

$$\sum_l g_{0l} \delta_{s_l^{(1)}} \tag{3D3}$$

and then S_2 is in the (collective) anti-correlated state

$$\sum_{j,\alpha} g_{0j} \varepsilon_{j\alpha} \delta_{s_\alpha^{(2)}}. \tag{3D4}$$

In the standard QM these states are attributed to the **individual** systems S_1 and S_2.

But then it is possible to deduce Bell's inequalities. This is the consequence of the individual anti-correlation between S_1 and S_2.

In the modified QM the anti-correlation between S_1 and S_2 still exists, but only on the level of ensembles. States (3D3) and (3D4) cannot be attributed to the individual system S_1 and S_2. As a consequence, the Bell's inequalities cannot be derived! All this is the direct consequence of the anti-superposition principle.

In the modified QM, the anti-correlation between S_1 and S_2. on the level of ensembles is still sufficient to obtain the **individual** anti-correlation between M_1 and M_2. This was shown clearly in Sect. 2.11. Schematically we proceeded as follows:

The individual state $\delta_{m_0^{(1)}}$ of M_1

→ the collective state (3D3) of S_1

→ the collective state (3D4) of S_2

→ the individual state $\delta_{m_1^{(2)}}$ of M_2.

The last step is crucial: the Bob's interaction $\mathcal{U}^{(2)}$ transforms the ensemble of M_2's into the deterministic state $\delta_{m_1^{(2)}}$ and this implies (the principle of individualization!) that the individual state of M_2 will be $m_1^{(2)}$.

Thus the information on $\delta_{m_0^{(1)}}$ is transported to Bob trough the anti-correlated ensembles of S_1's and S_2's.

But one has to consider the fact, that these ensembles (of S_1's and S_2's) may be only virtual (i.e. possible), but at a given time, only singular systems M_1, S_1, S_2, M_2 may exist. This shows that the principle of the existence of virtual ensembles is fundamental. But this assumption on the existence of virtual ensembles is fundamental for any probability theory.

Thus the necessary information is transferred from Alice to Bob through the virtual ensembles of S_1's and S_2's.

The superposition principle for individual systems implies that the anti-correlation between S_1 and S_2 is individual and as consequence one obtains: Bell's inequality, non-locality of QM and the contradiction in QM.

The possible escape from this consequences is rather fine:

(i) the anti-superposition principle for individual systems

(ii) the assumption on the existence of virtual ensembles.

From the point of view of the Probability theory, the existence of virtual ensembles is completely necessary, since the (non-deterministic) probability distribution (as a central object in the theory) can be associated only with ensembles.

4 Conclusions

4.1 The relation of the modified QM to the statistical interpretation of QM.

The statistical interpretation considers only ensembles and no individual systems ([3]).

In the modified QM both individual systems (and their states) and ensembles (and their states) are considered.

In the modified QM we have used ensembles only in this extend which is necessary in any probability theory.

The basic objects in the probability theory, like the probability distribution, are associated with ensembles, but cannot be associated with individual systems (assuming that the probability distribution is not deterministic).

Moreover, in the modified QM individual systems and their states play the basic role.

Our basic principle – the principle of anti-superposition considers only states of individual systems, but not ensembles.

Thus there is only a small (and rather trivial) intersection between our theory and the statistical interpretation of QM.

4.2 The summary of principles of the modified QM

Principles of the modified QM (in both real and complex form) can be summarized as follows

(A) the anti-superposition principle is the basis of all our approach to QM – it considers possible states of an individual system. It says: no non-trivial superpositions are possible.

(B) Individual states versus probability distributions and individual systems versus ensembles.

The fine interplay between individual systems (and their states) and ensembles (and their states) forms the firm basic of each possible probability theory. The full study of this relationship needs the detail considerations of possible contexts (see [1]) and this will be postponed to the next paper ([4])

(C) The (reversible) time evolution is generated by the one-parameter group of unitary transformations.

(D) Instead the concept of a measurement, there is a concept of an observation. Principially, for each individual system it can be observed in which individual state this system actually occures. In the practical situations, only for some individual systems this observation can be realized. These systems are then used as measurement apparatuses.

(E) There is no concept of the collapse in the modified QM. Instead of this, there is the concept of the Selection process: the selection of a subensemble from the given ensemble by using a given criterion. This criterion tests the individual states $\mathrm{ISt}\,(S) \in A$. This condition is verified with some probability and this is exactly the probability of an event $[\mathrm{ISt}\,(S) \in A]$. The Probability postulate (Sect. 2.4., p.13) is fundamental in the modified QM.

(F) The principle of the virtual ensemble and the principle of individualization (Sect. 2.4., pp. 14, 13 resp.).

The principle of the virtual ensemble is necessary in each probability theory. It says that relevant events are only those which can be (independently) repeated. (These repetitions than create this "virtual" ensemble.) The probability can be attributed only to repeatable events.

The Principle of individualization is able to create a relation between the state of an individual system and the state of an ensemble. This link between individual systems and ensembles is both evident and fundamental.

4.3 The differences between the modified QM and the standard QM.

(i) The wave-particle duality.

In the standard QM there is a duality between the coordinate representation an the momentum representation of quantum states. This is often expressed as: the quantum particle is as particle as it is a wave. This means also the equivalence between particle and wave description of the phenomena.

In the modified QM there is no duality between the coordinate and momentum representations. Usually the coordination representation corresponds to individual states of the system, while the momentum representation corresponds to the states of an ensemble.

In the modified QM the "particle properties" can be attributed to the individual systems, while "wave properties" can be attributed only to ensembles. This means that individual systems have no "wave properties" – **only ensemble can have wave properties**. There is no contradiction between particle and wave properties of quantum systems in the modified QM since they are attributed to different objects.

Wave properties can be attributed only to ensembles.

(ii) The problem of the measurement (e.g. the Schroedinger cat problem). This is the basic un-solved problem in the standard QM.

There is no such problem in the modified QM. In the modified QM there exists the "measuring" transformation (Sect. 2.8.,3.3.), but this is the standard (orthogonal, resp. unitary) transformation.

In the modified QM the measurement process is substituted by the observation process. This is the process in which it is found in which individual state the given individual measuring apparatus actually occures. This observation process does not change the state of the observed individual measuring apparatus and thus it creates no problems in the modified QM.

(iii) The quantum state and its collapse during the measurement process.

The collapse process of the individual state in the standard QM is substituted in the modified QM by the Selection process applied to the ensemble of systems.

In the modified QM the quantum state is the synonym for the (extended) probability distribution associated with an ensemble. The observation of the individual state of the measuring apparatus leads to the change of the original ensemble which is reduced to the sub-ensemble created by the use of the information obtained by the observation. This sub-ensemble has a new probability distribution described in the Sections 2.6.,2.8.,3.2. and 3.3.

Thus there is no collapse of the state in the modified QM, there is only the transition to the sub-ensemble created by the new information obtained in the process of the observation.

(iv) The locality.

It is proved (using Bell's inequalities) that the standard QM is non-local. This proof cannot be applied to the modified QM since Bell's inequalities cannot be proved in the modified QM.

Using this we can state the conjecture that the modified QM is the local theory. This means the following: modified QM+locality $\not\Rightarrow$ contradiction.

This is not the proof of the locality of the modified QM, but this locality is quite probable.

The comparison of the standard QM and the modified QM can be summarized in the following table.

Problem	**standard QM**	**modified QM**
Superpositions for individual systems	yes	no
Superpositions for ensembles	yes	yes
Measurement problem	yes	no
Locality	no	yes
Collapse	yes	no (selection)
Local explanation of EPR correlations	no	yes
Cloning of the individual system	no	yes
Schroedinger's cat paradox	yes	no
"pure" quantum state	attributed to individual system	probability distribution of an ensemble

Remarks:

The cloning of the individual system is possible in the modified QM. In fact, this is the standard measuring transformation with the trivial orthogonal bases for which (in the notation of (2H4))

$$\mathcal{U} : \delta_{m_0} \otimes \delta_{s_j} \mapsto \delta_{m_j} \otimes \delta_{s_j} .$$

4.4 The main consequences.

(i) The modified QM is the theory of reversible Markov processes in the Extended Probability theory complemented with certain inner complex structure.

In other words:

The modified QM is the statistical mechanics of many-particle systems based on the new probability theory (EPT) and statistics. The quantum state means the (extended) probability distribution.

(ii) **By the use of the modified QM it is possible to save the locality of QM and of all physics in general.**

References.

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[3] L.E. Ballentine, The statistical interpretation of Quantum Mechanics, Rev. Mod. Physics, vol. 42, num.4 (1970)

[4] J. Souček, The quantum theory of ensembles and contexts (in preparation)