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Malík, Josef
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J. Malík

Institute of Geonics, Academy of Sciences of the Czech Republic,
Studetská 1768,708 00 Ostrava–Poruba

1 Introduction

The collapse of the original Tacoma suspension bridge has been studied in many papers. On 7 November 1940 around 10 a.m. the torsional oscillations appeared on the deck of the original Tacoma bridge after the loosening of one midspan cable band, which resulted in the lateral asymmetry of the construction. It seems that the loosening of the midspan cable band had a significant impact on the behavior of the bridge and in the end it resulted in the collapse.

The model of the central span , depicted in Fig. 1,and the cable system studied in this paper is described by two functions corresponding with vertical and torsional motions of the central span and was formulated in [1]. The cable stays are modeled as a continuum. The model is based on the equilibrium state given by the gravitational forces acting on the whole construction. The two functions mentioned above describe the deflection from the equilibrium state. We analyze the action of lateral wind on the center span. These forces are relatively small comparing to the gravitational forces. The formulation describes the mutual reaction of the center span and the cable system as well as the reaction of the diagonal ties on the midspan cable bands. Three different types of evolution variational problems are formulated and analyzed.

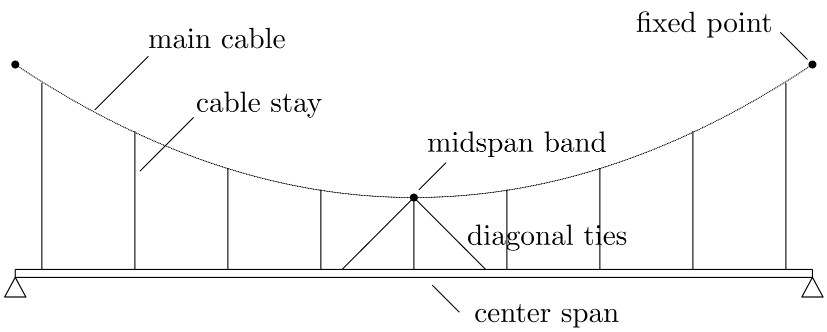

Fig. 1. Specification of center span

The equations formulated here describe the deflections from the equilibrium state due to the forces induced by lateral wind. The analysis of the derived equations reveals that the action of lateral wind can cause torsional oscillations if just one midspan cable band loosens.

2 Formulation of problems and main results

The analysis is based on the variational equations derived in [1]. Let us remind the parameters of the deck and the cable system.

- The width of the deck is denoted $2D$.

- The length of the central span is L.
- The sag of the main cables is L_1.
- The mass of the deck per unit length along the span is M_D.
- The mass of the main cable per unit length is M_C.
- The modulus of elasticity of the deck is E_D.
- The moment of inertia of the deck cross section with respect to the horizontal line through its centroid is I_D.
- The polar mass moment of inertia of the deck is I_P.
- The shear modulus of the deck is G_D.
- The torsional constant of the deck is J_D.
- The gravitational acceleration is g.

The formulation of the linearized model is based on the Hamilton principle. The starting point is the equilibrium under gravitational forces. Then we look for a new equilibrium, which is a stationary point of the functional defined below. The deflection of the center span from the original equilibrium is described by functions $u(x,t), \theta(x,t)$, where $u(x,t)$ corresponds to vertical deformations and $\theta(x,t)$ corresponds to torsional deformations of the center span The formulation of the linearized models is based on the following hypotheses formulated in [1]. Let us define the bilinear form

$$a_c(u,v) = \int_{-\frac{L}{2}}^{\frac{L}{2}} H \left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right) \frac{\mathrm{d}u}{\mathrm{d}x}\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x.$$

Then the potential energy of the main cables can be expressed in the form

$$a_c(u,u) + D^2 a_c(\theta,\theta)\,.$$

Let us define another two bilinear forms

$$a_{ver}(u,v) = \int_{-\frac{L}{2}}^{\frac{L}{2}} E_D I_D \frac{\mathrm{d}^2 u}{\mathrm{d}x^2}\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}\mathrm{d}x\,, \quad a_{tor}(\theta,\varphi) = \int_{-\frac{L}{2}}^{\frac{L}{2}} G_D J_D \frac{\mathrm{d}\theta}{\mathrm{d}x}\frac{\mathrm{d}\varphi}{\mathrm{d}x}\mathrm{d}x\,,$$

which are connected with the bending and torsional deformation energy of the deck. To simplify our equations for the dynamic problems, we define the bilinear forms

$$m_{ver}(u,v) = \int_{-\frac{L}{2}}^{\frac{L}{2}} M_{ver} uv\mathrm{d}x, \quad m_{tor}(\theta,\varphi) = \int_{-\frac{L}{2}}^{\frac{L}{2}} M_{tor}\theta\varphi\mathrm{d}x,$$

where M_{ver}, M_{tor} are functions on $(-L/2, L/2)$ defined by

$$\begin{aligned} M_{ver}(x) =& 2M_C \left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)^{\frac{1}{2}} + M_D\,, \\ M_{tor}(x) =& 2D^2 M_C \left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)^{\frac{1}{2}} + I_P\,. \end{aligned}$$

The equations for the dynamic problems will be derived from the Hamilton principle. The variational equation reads

$$m_{ver}(\ddot{u}, v) + m_{tor}(\ddot{\theta}, \varphi) + 2a_c(u, v) + 2D^2 a_c(\theta, \varphi) + a_{ver}(u, v) + a_{tor}(\theta, \varphi) = \int_{-\frac{L}{2}}^{\frac{L}{2}} F_{ver} v \, \mathrm{d}x + \int_{-\frac{L}{2}}^{\frac{L}{2}} F_{tor} \, \varphi \, \mathrm{d}x$$

and holds for all sufficiently smooth functions $v(x), \varphi(x)$ defined on $(-L/2, L/2)$. In our models we assume that the main span is hinged in its end points, so the functions u, θ satisfy the boundary conditions

$$u\,(-L/2,\, t) = u\,(L/2,\, t) = \theta\,(-L/2,\, t) = \theta\,(L/2,\, t) = 0\,.$$

So far we have not consider the fact that the main cables are inextensible and fixed at the end points and fastened at the midspan cable bands. Let us suppose that the deck deforms and the deformation transfers on the main cables via the inextensible suspenders. To simplify our considerations, we define three linear forms

$$h(u) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x\,, \quad h_r(u) = \int_{-\frac{L}{2}}^{0} \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x\,, \quad h_l(u) = \int_{0}^{\frac{L}{2}} \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x\,.$$

If both main cables are fixed in their end points, then u and θ satisfy the relations

$$h(u) = h(\theta) = 0\,.$$

In the case both main cables are fixed at the midspan cable bands as well, the following relations

$$h_r(u) = h_r(\theta) = h_l(u) = h_l(\theta) = 0$$

hold. In the end let us study the case, where both main cables are fixed at the end points and only one main cable is fixed at the midspan cable band, then the relations

$$h_r(u - D\theta) = h_l(u - D\theta) = h(u + D\theta) = 0$$

Now we are going to formulate three dynamic problems connected with the way how the main cables are fixed. The first dynamic problem describes oscillations of the center span if the main cables are fixed at the end points. The functions $u(t), \theta(t)$ are a solution to $\mathcal{D}_1$ if these functions satisfy the relations

$$h(u(t)) = h(\theta(t)) = 0$$

for all t, and the variational equation. The variational equation holds for all v, φ which satisfy the relations

$$h(v) = h(\varphi) = 0.$$

The initial conditions are compatible with $\mathcal{D}_1$, which is defined in [1].

The functions $u(t), \theta(t)$ are a solution to the dynamic problem $\mathcal{D}_2$ if these functions satisfy the relations

$$h_r(u(t)) = h_r(\theta(t)) = h_l(u(t)) = h_l(\theta(t)) = 0$$

for all t, the boundary conditions , and the variational equation. The variational equation holds for all v, φ which satisfy the relations

$$h_r(v) = h_r(\varphi) = h_l(v) = h_l(\varphi) = 0$$

and the boundary conditions. The initial conditions are compatible with $\mathcal{D}_2$, which is defined in [1].

The functions $u(t)$, $\theta(t)$ are a solution to the third dynamic problem $\mathcal{D}_3$ if these functions satisfy the relations

$$h_r(u(t) - D\theta(t)) = h_l(u(t) - D\theta(t)) = h(u(t) + D\theta(t)) = 0\,,$$

for all t, the boundary conditions, and the variational equation. The variational equation holds for all v, φ which satisfy the relations

$$h_r(v - D\varphi) = h_l(v - D\varphi) = h(v + D\varphi) = 0$$

and the boundary conditions. The initial conditions are compatible with $\mathcal{D}_3$, which is defined in [1].

The existence and continuous dependence on data is proved in [2].

3 Conclusion

The original Tacoma bridge exhibited relatively small vertical oscillations from the time that it was opened. The bridge was stable with respect to torsional oscillations until one midspan cable band loosened. This led to torsional oscillations which lasted for approximately one hour and then the deck broke. The new evolution variational equations were derived. These equations describe the behavior of the center span and main cables in the three different situations, where the both main cables have the fastened midspan cable bands, only one cable has the fastened midspan cable band, and the main cables have no fastened midspan cable bands. The analysis revealed that the behavior of the center span depends on the direction of lateral wind and vertical and torsional oscillations of the center span are connected if just one midspan cable band loosens.

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