Slightly generalized regular space decompositions

Kolcun, Alexej
2014

Slightly generalized regular space decompositions

A. Kolcun

Institute of Geonics AS CR, Ostrava

Space decomposition methods represent an important part of numerical modelling process. In many applications it is suitable to use the simplest case of decomposition – regular rectangular grid: the whole raster graphic concept can be seen from this point of view. Different raster concept, based on regular hexagonal mesh, is analyzed e.g. in [18]. Generalized task – decomposition of the space to the set of the identical elements have been presented in various research areas since a long time, and it is connected with Hilbert's 18th problem [9], [10].

Within the numerical methods development, the space discretization became an important tool of the shape expressivity. The domain of interest is decomposed to simple polyhedral elements. In the simplest case of linear approximation, the triangles for 2D tasks and tetrahedra for 3D tasks are used. There is a wide range of generators used for the decomposition, e.g. [7], [6], which produces meshes with irregular structure of nodes coincidency. For good interpolation properties, the condition like Delaunay one [3], [5] is required in the case of the isotropical environment. (In 2D case, the Delaunay triangulation maximizes the minimum angle. Compared to any other triangulation of the points, the smallest angle in the Delaunay triangulation is at least as large as the smallest angle in any other. However, the Delaunay triangulation does not necessarily minimize the maximum angle.)

Due to the fact, that for geometric modelling (CAD) parametrical models based on the tool of NURBS curves, surfaces and volumes are used, discretization with regular structure of node coincidency is important and wide spread. Moreover, the common basis for both geometric and physically based modelling can be founded [11], [4].

Using current methods, creating 3D models is an extremely time-consuming, unreliable, and labour-intensive process. So, when the geometry information is obtained e.g. from computer tomograph or similar devices, i.e. in the form of pixel/voxel grid, it is reasonable to create the space decomposition in the same or similar way. The discretization error – aliasing – has a very local character in this case only [2]. Moreover, this error can be eliminated as mentioned below.

Finer mesh This way, however, leads to substantial increasing of memory demands.

Adjusting the geometry Some of the grid nodes are shifted according to prescribed geometry. It is proved [16] that even in the simplest case of adjusting (local displacement of the grid nodes only, which are the most close to the prescribed shape) resulting mesh holds the Delaunay property.

Pixel/voxel partitioning This approach gives a wide variability of the shape expression: from four types of possible tetrahedra we can create 72 different conform decompositions [12], [1], [17]. On the other hand, not all configurations of diagonals are admissible: in some cases only nonconform decompositions are possible, and in some cases no decompositions are possible. This fact can be considered as a generalization of the decomposition of Schönhardts polyhedron [19], [20]. This drawback can be eliminated. We can move the node in such way, that planar quadrilateral, which is nonconformly partitioned, become a tetrahedron. In this case voxel can be decomposed to 6-13 tetrahedra. [13]. So, in this case both regularity of geometry and nodes coincidency is spoilt.

Goldberg's tiling There are several methods how to decompose 3D space into the set of the same tetrahedra [8], [9], [21]. Comparison of these decompositions in [14] shows the benefit of Goldbergs tiling. Within this class of decompositions we can find such one, based on tetrahedron, close to the regular one (each face is isoscelles triangle with edges ratio $\sqrt(\frac{4}{3}) : 1 : 1$). Moreover, Cartesian indexation of nodes with three-indexes can be used and there are four different assembling schemes how we can compose the voxel form these six tetrahedra [15].

Acknowledgement:
This work has been supported by the grants FR-TI3/579 and GA13-18652S.

References

[1] Apel, T., Duvelmeyer, N.: *Transformation of Hexahedral FE-Mesh into Tetrahedral Meshes according to Quality Criteria*. Computing 71 (2003), pp 293–304.

[2] Arbenz, P., Flaig, C.: *On Smoothing in Voxel Based Finite Element Analysis of Trabecular Bone*. In: LSSC2007 proc. (Lirkov,I., Margenov,S., Wasniewski J. eds.), Lecture Notes in Computer Science 4818, Springer 2008, pp. 69–77.

[3] de Berg, M.: *Computational Geometry – Algorithms and Applications*. Springer-Verlag 2008.

[4] Cottrell, J.A., Hughes, T.J.R., Bazilevs, Y.: *Isogeometric Analysis: Toward Integration of CAD and FEA*. John Wiley & Sons 2009.

[5] Delaunay Triangulation, `http://en.wikipedia.org/wiki/Delaunay_triangulation`. Retrieved 2014-01-06.

[6] Edelsbrunner, H.: *Geometry and Topology for Mesh Generation*. Cambridge University Press 2001.

[7] Frey, P.J., George, P.L.: *Mesh generation: Application to Finite Elements*. Hermes Science 2000.

[8] Goldberg M.: *Three Infinite Families of Tetrahedral Space/Fillers*. Journal of Combinatorial Theory (A) 16, 1978, pp.348–354.

[9] Grunbaum, B., Shepard, G.C.: *Tilings with Congruent Tiles*. Bulletin of American Math. Soc. Vol. 3, No.3, November 1980. pp. 951–973.

[10] Hilberts problems, `http://en.wikipedia.org/wiki/Hilbert's_problems`. Retrieved 2014-01-06.

[11] Hughes, T.J.R., Cottrell, J.A., Bazilevs, Y.: *Isogeometric analysis: CAD, finite elements, NURBS,exact geometry and mesh refinement*. Comput. Methods Appl. Mech. Engrg. (Elsevier) 194(2005). pp.4135–4195.

[12] Kolcun, A.: *Conform Decompositions of Cube*. In: Proceedings of Spring School on Computer Graphics, Comeniu University Bratislava, 1994. pp. 185–191.

[13] Kolcun, A.: *Non-Conformity Problem in 3D Grid Decomposition*. Journal of WSCG, Vol. 10, No.1,(2002), pp. 249–254.

[14] Kolcun, A.: *(Semi) regular tetrahedral tilings.* In: WSCG 2013 Conference - Communication Papers Proceedings, ZČU Plzeň, pp. 145–150.

[15] Kolcun, A.: *Voxel representation in the context of Goldbergs tilings of 3D space.* In: Proceedings of 33-rd Conference on Geometry and Computer Graphics. VŠB-TU Ostrava 2013. pp. 149–154.

[16] Kahánek, P., Kolcun, A.: *Bresenham's regular mesh deformation and angle criteria.* In: Proc of the 25-th Conference on Geometry and Computer Graphics. VŠB-TU Ostrava 2005. pp. 95–102.

[17] Lubojacký, J.: *Basic algorithms of rasterization for generalized raster lattice* (in czech). Master Thesis, University of Ostrava, 2013.

[18] Middleton, M., Sivaswamy, J.: *Hexagonal Image processing.* Springer 2005.

[19] Rambau, J.: *On a Generalization of Schönhardts Polyhedron.* Combinatorial and Computational Geometry, Vol. 52(2005), pp. 501-516.

[20] Schönhardt's polyhedron, `http://en.wikipedia.org/wiki/Schonhardt_polyhedron`. Retrieved 2014-01-06.

[21] Senechal, M.: *Which tetrahedra Fill Space?* Mathematical Magazine, Vol. 54, No. 5, 1981, pp. 227-243.