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Seidler, Jakub; Jakubík, Petr
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Jakub Seidler
Petr Jakubík

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The Merton Approach to Estimating Loss Given Default:
Application to the Czech Republic

Jakub Seidler and Petr Jakubík*

Abstract

This paper focuses on a key credit risk parameter – Loss Given Default (LGD). We illustrate how the LGD can be estimated with the help of an adjusted Mertonian structural approach. We present a derivation of the formula for expected LGD and show its sensitivity analysis with respect to other company structural parameters. Finally, we estimate the five-year expected LGDs for companies listed on Prague Stock Exchange and find that the average LGD for the analyzed sample is around 20–50%.

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* Jakub Seidler, Czech National Bank, and IES, Charles University, Prague, *E-mail: jakub.seidler@cnb.cz.*

 Petr Jakubík, Czech National Bank, and IES, Charles University, Prague, *E-mail: petr.jakubik@cnb.cz.*

Nontechnical Summary

In this paper we try to estimate the credit risk parameter Loss Given Default (LGD) for selected companies listed on the Prague Stock Exchange (PSE). The importance of estimating LGD stems from the fact that a lender's expected loss is the product of the probability of default (PD), credit exposure at default (EAD), and LGD. However, LGD has received considerable attention only in recent years as Basel II identified it as one of the key credit risk parameters and allowed financial institutions to apply their own estimates of LGD in the computation of regulatory capital. Thus, accurate estimation of LGD has become an important problem in current credit risk management.

This paper does not estimate LGD based on the historical LGD values of defaulted companies. Instead, we try to employ information in the stock market and estimate potential LGD in the case of default for companies which are currently listed on the stock exchange. We employ Merton's structural approach, which models default as the situation where the value of a company's assets is lower than the value of its debt at the time of maturity. Nonetheless, this approach is based on a number of simplifying assumptions. There are no taxes, the company's debt structure is represented by a single zero-coupon bond, and default can occur only on maturity of the debt, which we arbitrarily set at five years for all the companies analyzed.

The 15 most liquid non-financial companies listed on the PSE were analyzed in the time period 1999–2008. We estimated the expected LGDs at the five-year horizon, which were in the range of 20–50% on average. Because of the model's simplifications, there is uncertainty about the precise values of the estimated LGD. However, it can serve as a credit risk indicator capturing the evolution of a company's riskiness over time. Furthermore, the presented results are the first estimates of expected LGD based on market information for companies listed on the PSE and could therefore serve as a stepping stone for further improving such estimates.

1. Introduction

Credit risk techniques have undergone significant development in recent decades. This has led to the development of new methods for the estimation of the potential bankruptcy of borrowing entities and parameters specifying possible losses. These parameters include Loss Given Default (LGD), expressing the percentage of an exposure which will not be recovered after a counterparty defaults.[1] While the estimation of the probability of default (PD) has received considerable attention over the past 20 years, LGD has gained greater acceptance only in recent years as the New Basel Accord identified it as one of the key risk parameters.

LGD modeling is still quite a new and open problem in credit risk management and its estimation is not straightforward, because it depends on many driving factors, such as the seniority of the claim, the quality of collateral, and the state of the economy. Moreover, the insufficient database of experienced LGDs makes it more difficult to develop accurate LGD estimates based on historical data. Hence, the extraction of LGD for credit-sensitive securities based on market-observable information is an important issue in the current credit risk area and may produce further improvements in present credit risk management.

This paper therefore discusses this key risk parameter for single corporate exposures and deals with the possibility of LGD extractions from market information. This type of LGD is referred to as *implied market LGD*. We use so-called structural models, which are based on the initial Merton framework, and present the derivation of a closed-form formula for LGD and its sensitivity analysis with respect to other company structural parameters. Furthermore, we empirically implement this contingent claim approach for a set of companies in the Czech Republic. As a result, we estimate five-year expected LGDs for the 15 most liquid companies listed on the Prague Stock Exchange in the period 1999–2008.

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[1] Before Basel II formalized the use of LGD, this concept was also called *Severity* (see Stephanou and Mendoza, 2005).

2. Basic Characteristics of LGD

LGD is usually defined as the loss rate experienced by a lender on a credit exposure if the counterparty defaults.[2] Thus, despite default the lender still recovers 1 – LGD percent of the exposure. One minus LGD is therefore called the recovery rate (RR). In principle, LGD also comprises other costs related to default of the debtor, and the correct formula should rather be LGD = 1 – RR + Costs. Nevertheless, costs are relevant only in a specific type of LGD and are not usually so high as to influence losses markedly in comparison with the recovery rate. Therefore, we use the recovery rate as the complement of LGD in the following text and take these two parameters as being conceptually the same.

Usually three basic types of LGD for defaulted facilities are used. Market LGD employs the price of a bond after default as a proxy for the recovered amount. However, the post-default price is available only for the fraction of the debt that is traded and for which an after-default market exists – very often it is available only for corporate bonds issued by large companies.[3] Market LGD is therefore highly limited for defaulted bank loans, which are traditionally not traded. For them one must turn to another approach.

Workout LGD considers all relevant facts that may influence the final economic value of the recovered part of the exposure arising in the long-running workout process. However, bankruptcy claims are often settled not in cash, but with securities (equity, options, warrants, etc.) with no secondary market, which means that their value will be unclear for years. Another problem is that the appropriate discount rate (which should reflect the risk of holding the defaulted asset) is not known. Computation of workout LGD therefore depends on an unknown and variable discount rate which is difficult to estimate for a particular situation.[4]

The last method of measuring of LGD is the concept of Implied Market LGD, which is estimated ex ante from market prices of non-defaulted loans, bonds, or credit default instruments by structural or reduced-form models. The idea is that prices of risky instruments reflect the market's expectation of the loss and may be broken down into PD and LGD. Implied market LGD estimation does not rely on historical data and can be used especially for low-default facilities.

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[2] In principle we should denote the loss given default rate as LGDR and use LGD for the absolute amount of the loss. However, LGD is used to indicate the loss rate by many practitioners, including Basel II, while the absolute loss is indicated as LGD.EAD, where EAD is the exposure at default (see BCBS, 1988).

[3] What is more, outside the USA the market for defaulted bonds either is non-existent or does not have the required depth and liquidity.

[4] Sometimes a discount rate based on historical values is used. What discount factor should be used is dealt with in detail in, for example, Maclachlan (2005).

Recovery rates are ultimately determined by the value of the assets that can be seized in the case of default. Because many asset types differ between industries,[5] it is intuitive to assume that the debtor's industry characteristics can influence LGD. Although the type of industry seems like a straightforward determinant of RR, the literature does not give wholly unified answers (see Altman and Kishore, 1996, Grossman et al., 2001, or Acharya et al., 2003). Those studies have broken down the LGD of corporate bonds by industry and have found evidence that some industries, such as public utilities and chemicals, do evidently better than the others. Nonetheless, they have also shown that the standard deviation of RR per industry and within a given industry is still very large (see Table 1).

Table 1: Average recoveries per industry

Altman and Kishore			Acharya et al.			Moody's	
1971–1996			1982–1999			1982–2003	
Industry Description	Mean (%)	Std. Dev. (%)	Industry Description	Mean (%)	Std. Dev. (%)	Industry Description	Mean (%)
Public Utilities	70.5	19.5	Utilities	74	18.8	Utility-Gas	51.5
Chemicals*	62.7	27.1	Energy, Resources*	60	31.0	Oil and Oil Service	44.5
Machinery*	48.7	20.1	Financial Institutions	59	44.3	Hospitality	42.5
Services*	46.2	25.0	Healthcare, Chemicals	56	40.8	Utility-Electric	41.4
Food*	45.3	21.7	Building Products	54	42.1	Media and Broadc.*	38.2
Wholesale and retail	44.0	22.1	Telecommunications	53	38.1	Finance and Banking	36.3
Divers. manufacturing	42.3	25.0	Aerospace, Auto*	52	38.1	Industrial	35.4
Casino, hotel*	40.2	25.7	Leisure Time, Media	52	37.2	Retail	34.4
Building material*	38.8	22.9	High Technology*	47	32.4	Automotive	33.4
Transportation*	38.4	27.9	Consumer, Service	47	35.6	Healthcare	32.7
Communication*	37.1	20.8	Transportation	39	36.1	Consumer Goods	32.5
Financial institutions	35.7	25.7	Insurance and Real Es.	37	35.4	Construction	31.9

* Industry description is reduced

Source: Altman and Kishore (1996), Acharya et al. (2003), Moody's (2004)

An opposite view of industry influence is presented by Gupton et al. (2000) and Araten et al. (2004). These studies found no evidence of different LGDs across industries. They state that the use of recovery averages broken down by industry does not capture the industry variability in recovery rates across time. Some sectors may enjoy periods of high recoveries, but can fall below average recoveries at other times. This means that industry recovery distributions change over time and therefore cannot be expected to hold in the future.

These unambiguous results of different studies might be due to cyclicality of LGD in relation to the economic environment. Each industry can be at a different stage of the economic cycle. The cycle can influence LGD more than the industry-type itself because

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[5] For example, firms in some sectors have a large amount of assets that can be easily sold on the market in case of default, while other sectors can be more labor-intensive, for example.

LGD is not stable in time and there is underlying cyclical variability depending on the macroeconomic conditions. Acharya et al. (2003) showed that when the industry is in distress, the mean LGD is 10–20% higher on average than otherwise.

Behind the cyclical variation is the fact that as the economy enters into recession, default rates increase. Recoveries from collateral will depend on the possibility of selling the relevant assets. We can generally suppose that a greater supply of collateral assets will lead to lower prices of those assets, of course depending on the market size and structure observed for the particular asset. Moreover, the demand for these assets declines because non-defaulted companies are not able to invest the same amount of money in a recession as during an expansion. The result is that the macroeconomic situation can significantly influence the recovery rate. This has been demonstrated by several authors (see Araten et al., 2004, or Altman et al., 2005).

 Also, when a firm goes into bankruptcy[6] and there is no other option than liquidation, the capital structure of the firm and the absolute priority rule (APR)[7] are important determinants of the recovery rate. This means that the rate of recovery of a defaulted bond depends on where the claims are in the firm's capital structure. Empirical evidence on recovery rates is usually based on defaulted bonds because the LGD data are simply available. The results of several empirical studies have confirmed that RR increases with the seniority and security of the defaulted bond and decreases with the degree of subordination. The results also tend to be rather similar in terms of average recovery rates – for bank loans (70–84%) and for bonds: senior secured (53–66%), senior unsecured (48–50%), senior subordinated (34–38%), and subordinated (26–33%). All studies also reported a high standard deviation characterizing the recovery rate across all bond debt classes, regularly exceeding 20% (see Altman and Kishore, 1996, Castle and Keisman, 1999, and Keenan et al., 2000).

As said earlier, LGD is influenced by many factors, such as the facility's seniority and the presence of collateral, the borrower's industry characteristics, and more general factors such as the macroeconomic conditions. However, previous research gives ambiguous results concerning some LGD properties. The relatively rare occurrence of default events for some facilities can cause the research to be based on relatively small empirical samples. It is clear that further research is needed, and hopefully with the adoption of the Basel II accord, which sets rules for LGD data gathering and estimation, this research will be based on better data samples offering more exact outcomes. However, a major difficulty of such information is its complete dependence on historical data. LGD predictions based on past

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[6] Bankruptcy takes the form of either reorganization or liquidation.

[7] Eberhart and Weiss (1998) confirm that the APR is routinely violated because of speed of resolution. Creditors agree to violate the APR to resolve bankruptcies faster.

LGD data are not thus necessarily consistent with the evolution of fundamentals across time and can result in inaccurate estimates that cannot capture the real trend in the economy.

3. LGD Modeling

In this part we focus on analytical tools enabling forward-looking estimates of LGD to be obtained from market-observable information. We employ asset pricing models, which aim at determining the equilibrium arbitrage-free price of risky assets. Each risky asset should offer an expected return corresponding to its degree of risk; therefore, all risky parameters must be evaluated by the market in order to get the equilibrium price. This assumption that prices include all information is then used by credit risk pricing models, which use market information (e.g. share or bond prices) to measure credit risk and try to extract the key risk parameters such as PD or LGD from the prices. Those models are forward-looking, estimating the risk parameters which are expected by the market in the future and not those that occurred in the past. From the nature of this method such estimate of LGD is called *implied market LGD*.

These credit risk pricing models can be further classified as *structural* and *reduced-form* models. The category of *structural-form* models is based on the framework developed by Merton in 1974 using the theory of option pricing presented by Black and Scholes (1973). The intuition behind this model is quite straightforward: a company defaults when the value of its assets is lower than that of its liabilities when the debt matures. For that reason, the default process is driven by the value of the company's assets and the risk of default is explicitly related to asset variability.[8]

In contrast, *reduced-form* models generally assume that default is possible and is driven by some exogenous random variable. The result is that default and recovery are modeled independently of the firm's structural features, which lacks the clear economic intuition behind the default event. The basic input parameters for extracting LGD in the reduced-form approach are the prices of risky corporate bonds. However, companies in the Czech Republic are still using traditional bank loans more than bond issuance as a source of finance (see Dvořáková, 2003). As a result, the domestic corporate debt market is rather illiquid and incomplete and can hence barely reflect market expectations about the default and recovery risk of particular companies or their securities. The result is that reduced-form models which employ prices of corporate bonds are currently hardly applicable for LGD estimation in the Czech Republic.

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[8] The term structural comes from the fact that these models focus on structural characteristics of the company, such as asset volatility or leverage, which determine the relevant credit risk elements. Default and RR are a function of those variables.

The stock market provides an alternative source of information, assuming that share prices incorporate all available information, including the future prospects of the company and its creditworthiness.[9] Structural models for extracting a company's default risk typically use observed stock prices, stock volatility, and specifics about the company's capital structure. Even if the number of listed companies in the Czech Republic is also limited,[10] for some of them it seems to be sufficiently liquid to apply structural models and estimate the required credit risk parameters. As a result, we will use Merton's structural approach to derive a formula for implied market LGD for particular companies.

The seminal structural Merton (1974) model relies on many hypotheses, most of which derive from the Black–Scholes option-pricing theory. Some of them became sources of criticism and were later relaxed.[11] The original framework in which the process of valuing a firm's assets is embedded requires many assumptions for the application of standard corporate credit risk pricing. There are no transaction costs, taxes, or short-selling restrictions. The term structure of the risk-free interest rate is flat and known with certainty. The price of a riskless bond paying $1 at time T is hence $B_0(T) = \exp[-rT]$, where r is the instantaneous riskless interest rate. The total value of firm V is financed by equity E and one zero-coupon non-callable debt contract D, maturing at time T with face value F. It also holds that $V_t = D_t + E_t$. With the no-taxes assumption this implies that the value of the firm and the values of assets are identical and do not depend on the capital structure itself (the Modigliani–Miller theorem).

The dynamics of the firm's value through time can be described by a stochastic differential equation called geometric Brownian motion:

$$dV_t = \mu_V V_t dt + \sigma_V V_t dW_t^V$$

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[9] This is true only if the efficiency hypothesis holds, which has been doubted by some studies (see, for example, Sloan, 1996). There is also a question whether the volatility of stock prices is caused solely by the incorporation of new information about future stock returns, or if it is caused largely by trading itself (see French, 1980, or French and Roll, 1986).

[10] More about the stock market efficiency of the PSE can be found in, for example, Filacek et al. (1998) and Hajek (2007).

[11] Alternative approaches have been developed in an attempt to remove one or more of the drawbacks of the seminal model. Black and Cox (1976) introduced the possibility of a more complex capital structure of the company's liabilities, Geske (1977) introduced interest-paying debt, and Vasicek (1984) established a distinction between short and long-term debt. All these authors also enhanced the model by treating default as an event that can occur any time before debt maturity. More recent improvements, such as in the papers by Longstaff and Schwartz (1995) and Hull and White (1995), reject the constant risk-free interest rate and consider the interest rate as a stochastic variable instead. For a detailed account of later structural models, see, for example, Altman et al. (2005) and the references therein.

where μ_V is asset drift (i.e., the instantaneous expected rate of return on the firm's value V per unit time), σ_V is the standard deviation of its return, and dW_t^V is a standard Gauss–Wiener process.

Based on these assumptions, credit risk concerns the possibility that the stochastically evolving value of the company on the maturity day T will be less than the repayment value of the loan F. The debt holders receive at T either the value F *(if $V_T > F$)* or the entire value of the firm and the owners of the firm receive nothing *(if $V_T < F$)*. The risk of default is therefore explicitly linked to the volatility in the firm's asset value. Merton's contingent claim analysis shows how this risk should be priced. Merton derived a fundamental differential equation which determines the value of the debt at any time t as a function of the value of the firm. We use Merton's famous conclusion that the value of equity is identical to the formula for pricing "*...a European call option on a non-dividend-paying common stock where firm value corresponds to stock price and F corresponds to the exercise price*" (Merton 1974, p. 10). This is given as

$$E(V,0) = \max\left[0; V - F\right] \tag{1}$$

Indeed, at maturity time T, the equity holders will exercise the option and pay the debt holders the face value of liabilities if $V_T \geq F$, otherwise they let this option expire. By applying the Black–Scholes option pricing formula it is straightforward to get the solution for equation (1) as

$$E(V,\tau) = V\Phi\left(d_1\right) - Fe^{-r\tau}\Phi\left(d_2\right) \tag{2}$$

where $d_1 = \dfrac{\ln\dfrac{V}{F} + \left(r + \dfrac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$, $d_2 = d_1 - \sigma_V\sqrt{\tau} = \dfrac{\ln\dfrac{V}{F} + \left(r - \dfrac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$,

and $\Phi(.)$ is the cumulative standard normal distribution. And since $V = D(V,\tau) + E(V,\tau)$, where $\tau = T - t$ is the length of time until maturity, we can express the value of the debt at time τ as

$$D(V,\tau) = V\Phi\left(-d_1\right) + Fe^{-r\tau}\Phi\left(d_2\right).$$

Now we can look at how credit risk parameters such as PD and RR can be extracted. Default occurs when the firm's value drops below some *default barrier* (DB), which in the seminal Merton model is represented by the face value of the debt F at its maturity. The probability of default is therefore simply expressed as

$$PD = \Pr(V_T \leq F). \tag{3}$$

To obtain this probability, more information about the probability distribution of V has to be known. However, we can use the assumption that the value of the firm V is log-normally distributed, which according to Crouhy et al. (2000) is quite a robust hypothesis confirmed by actual data, and we can obtain the probability distribution of $\ln V_T$,[12] which is

$$\ln V_T \sim \Phi \left[\ln V_0 + \left(\mu_V - 0,5\sigma_V^2 \right) T, \sigma_V^2 T \right]. \tag{4}$$

From the properties of the natural logarithm, one can obtain the probability (3) expressed as

$$PD = \Pr(\ln V_T \leq \ln F)$$

Combining this equation with eq. (4) we can get

$$PD = \Phi \left(-\frac{\ln \dfrac{V_0}{F} + \left(\mu_V - \dfrac{1}{2}\sigma_V^2 \right) T}{\sigma_V \sqrt{T}} \right) = \Phi(-d_2^*) \tag{5}$$

which is the PD of the company at the time of maturity T expected at time $t = 0, (\tau = T)$, when the value of the firm V_0 is known with certainty.[13] $\Phi(d_2)$ is the probability that the European call option will be exercised by equity holders and the company will not default. The term $1 - \Phi(d_2) = \Phi(-d_2)$ then characterizes the default probability. However, while $\Phi(-d_2^*)$ in eq. (5) gives the real-world (physical) probability of default, $\Phi(-d_2)$ represents the default probability in the risk-neutral world. This is caused by using the riskless interest rate r instead of the expected rate of return μ_V in the formula for d_2. In the real world, investors demand more than the risk-free rate of return and therefore $d_2^* > d_2$, which implies $\Phi(-d_2^*) < \Phi(-d_2)$ and the fact that the risk-neutral PD overstates its physical measure. Similarly, one has to distinguish between the physical and risk-neutral RR.[14]

The recovery rate, assuming no liquidation costs after default, will be given by the ratio of the firm's value at T to the debt F, (VT / F). More formally expressed as

$$RR = E\left(\frac{V_T}{F} \Big| V_T < F \right) = \frac{1}{F} E\left(V_T \big| V_T < F \right) \tag{6}$$

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[12] Itô's Lemma can again be used to get the dynamics for $d\ln V_t$, and from that the parameters of the normal distribution for $\ln V_t$ can be determined.

[13] From (5) it can be seen that PD is a function of the distance between the current V_0 and the face value of the debt F, adjusted for the expected growth of asset μ_V relative to its volatility σ_V^2. d_2^* is thus called the distance to default (DD) and the higher it is, the lower is PD.

[14] As, for example, Delianedis and Geske (2003) state, the risk-neutral default probabilities can serve as an upper bound to the physical default probabilities. For recoveries the reverse relation holds – the risk-neutral expected recovery rate is less than its physical (real-world) counterpart (see Madan et al., 2006, p. 5).

as was already mentioned, V is the log-normal variable. Therefore, to get an explicit formula for RR we can use the method presented in Liu et al. (1997), which derives the conditional mean for a log-normal distributed variable, which is exactly the case of equation (6) (see Resti and Sironi, 2007).

Let's suppose that variable Y is log-normal and $\ln Y$ is normally distributed with mean μ and variance σ^2. Then variable $Z = (\ln Y - \mu)/\sigma$ has a standard normal distribution. The conditional mean of Y, given $Y < c$, can then be expressed as follows:

$$E(Y|Y < c) = E\left(\exp[\sigma Z + \mu] \big| \exp[\sigma Z + \mu] < c\right)$$

$$= E\left(\exp[\sigma Z + \mu] \big| Z < (\ln c - \mu)/\sigma\right). \tag{7}$$

To simplify this expression, let's define $g = (\ln c - \mu)/\sigma$ and $h = \Phi(g)$, where $\Phi(.)$ is the normal c.d.f. With these notations, equation (7) becomes

$$E(Y|Y < c) = h^{-1} \int_{-\infty}^{g} \exp[\sigma Z + \mu](2\pi)^{-1/2} \exp[-z^2/2] dz$$

$$= \exp[\mu + \sigma^2/2] h^{-1} \int_{-\infty}^{g} (2\pi)^{-1/2} \exp[-(z-\sigma)^2/2] dz$$

$$= \exp[\mu + \sigma^2/2] \frac{\Phi\big((\ln c - \mu)/\sigma - \sigma\big)}{\Phi\big((\ln c - \mu)/\sigma\big)}.$$

Considering the parameters of the normal distribution of $\ln V$ stated in eq. (4), we can express the mean of V_T, conditional on $V_T < F$, as

$$E(V_T|V_T < F) = \exp[\mu_v^* + \sigma_V^{*2}/2] \frac{\Phi\big((\ln F - \mu_v^*)/\sigma_V^* - \sigma_V^*\big)}{\Phi\big((\ln F - \mu_v^*)/\sigma_V^*\big)}$$

where $\mu_v^* = \ln V_0 + \left(\mu_V - 0,5\sigma_V^2\right)T$ and $\sigma_V^{*2} = \sigma_V^2 T$. After substituting and rearranging we get

$$E(V_T|V_T < F) = \exp[\ln V_0 + \mu_V T] \frac{\Phi\left(-\dfrac{\ln(V_0/F) + \left(\mu_V + 0,5\sigma_V^2\right)T}{\sigma_V \sqrt{T}}\right)}{\Phi\left(-\dfrac{\ln(V_0/F) + \left(\mu_V - 0,5\sigma_V^2\right)T}{\sigma_V \sqrt{T}}\right)}$$

$$= V_0 \exp[\mu_V T] \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)}.$$

Using the term in equation (6) we get the final expression for the expected recovery rate at time $t = 0$ in the form

$$RR = \frac{1}{F} E\left(V_T \,\middle|\, V_T < F\right) = \frac{V_0}{F} \exp[\mu_V T] \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} \tag{8}$$

which is the physical recovery rate, and the risk-neutral RR would be obtained by replacing μ_V with r. The RR function is homogeneous of degree zero in V_0 and F, which means that a proportional change in those variables does not influence its value (ceteris paribus). Moreover, RR, like PD, is dependent on the uncertain development of the firm's value and therefore is not constant through time but stochastic.

Using the expression presented for PD and RR, sensitivity analyses can be made with respect to other company structural parameters. Consider a firm with given $F = 80$, $V_0 = 100$, $\sigma^2 = 30\%$, $\mu = 10\%$, and $T = 1$. The variables will be shocked to see how PD and RR change.

Figure 1: Sensitivity analysis for PD and RR (LGD) – part 1
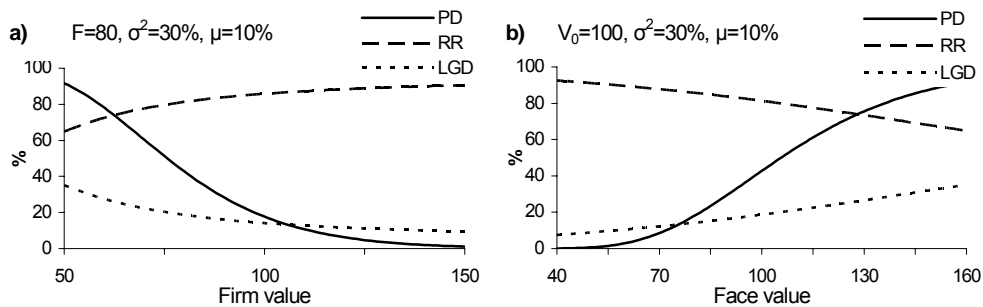
Source: computed from eq. (5) and (8)

The figure presents the results for RR and PD for the physical measure. It shows that the higher is the firm's value at the time of prediction of the risk parameters, the lower is the expected LGD and lower is PD (part a); the link is the reverse for the value of debt F (part b). An increase in the firm's leverage brings about higher both PD and LGD. An increase in asset volatility (leaving leverage unchanged) has a similar impact, causing higher uncertainty of the future value of the firm at maturity T and therefore a fall in RR.

Figure 2: Sensitivity analysis for PD and RR (LGD) – part 2
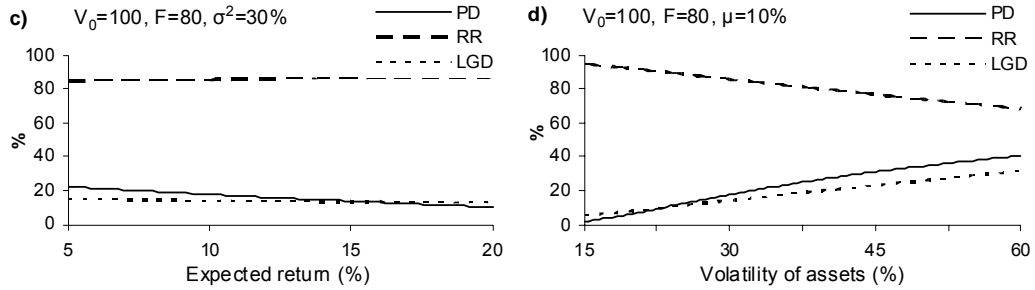
Source: computed from eq. (5) and (8)

In summary, Merton's approach evidently generates a negative correlation between PD and RR because both variables depend on the same structural characteristics of the firm. RR is significantly determined by the value of the firm's assets at the maturity time *T*.

However, the original Merton model does not include any payouts to security holders. Since the interest payouts occur over the life of the debt and are considerably lower than the principal amount, they represent lower default risk. However, disregarding the dividend stream, as Hillegeist et al. (2004) state, could introduce significant errors into the estimation of the current market value of the firm and its volatility and thereby influence the resulting LGD estimate.[15] Therefore, it is necessary to modify the seminal Merton approach and incorporate the payout of dividends into the model.

If we define the dividend rate δ as the ratio of the sum of the prior year's common and preferred dividends to the market value of the firm's assets, then the equation for the equity value reflecting the dividend stream paid by the firm accruing to equity holders would change as proposed by Hillegeist et al. (2004) into

$$E(V,T) = V \exp[-\delta T]\Phi(d_1) - Fe^{-rT}\Phi(d_2) + (1 - \exp[-\delta T])V \qquad (9)$$

where the additional $\exp[-\delta T]$ in the first term accounts for the reduction in asset value due to dividends distributed before maturity *T*. The last expression $(1 - \exp[-\delta T])V$ does not appear in the traditional equation for the call option on a dividend-paying stock since dividends do not accrue to option holders. Equation (9) is derived under the risk-neutral measure, therefore the risk-free rate is taken to be the expected rate of return on the firm's

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[15] We are more concerned about dividend payouts, since they lower the value of the company by transferring it to the shareholders, which implies a lower recovered amount for the debt holders if default occurs.

value. This rate, however, is lowered by the dividend rate and hence the terms d_1 and d_2 have to be modified to

$$d_1 = \frac{\ln(V_0/F) + \left(r - \delta + 0,5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}, \ d_2 = d_1 - \sigma_V\sqrt{T}$$

where all parameters are as defined above.

4. Implementation of the Model

The empirical use of any structural model is based on variables which are not directly observable. Similarly, in our case, the market value of assets V and also asset volatility σ_V must be estimated in order to compute the expected LGD.[16] A procedure for estimating these variables was first proposed by Jones et al. (1984) for publicly listed companies, exploiting the prices of their shares. Their approach is based on simultaneously solving two equations which match the value of equity E and its volatility σ_E with two unknown variables V and σ_V. Equity data is generally used since actual daily prices are observable and equity is the firm's most liquid security. Jones et al. (1984) used relation (2) as the first equation. However, this equation does not consider dividend payouts and we will thus use a modified equation (9). The second equation linking the observable and unknown values is in the form

$$\sigma_E E = \sigma_V \exp[-\delta T]V\Phi(d_1) \tag{10}$$

and its derivation uses Itô's lemma and the expression for equity delta (see Hillegeist et al., 2004). This system of two equations has to be solved to arrive at the unobservable market value of the firm's assets and its volatility. Due to the non-linearity of those equations it is necessary to solve the system iteratively.[17]

The accuracy of the expected LGD estimate is therefore dependent on the estimates of the parameters in equation (8). Although some of them, such as the face value[18] or maturity of the debt, are observable, some assumptions must be made about them to be able to implement Merton's simplifying approach. For example, the model requires us to reduce the firm's capital structure into a single liability. Since a large share of the firm's debt is not traded very often, we have to use book values as a proxy. As a result, the book value of total

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[16] The market value of the firm is the sum of the market value of its equity and its debt. However, the market value of the debt is not usually available since companies are not financed entirely by traded debt.

[17] To solve two non-linear equations of the form $F(x,y)=0$ and $G(x,y)=0$ we need to minimize the function $[F(x,y)]^2 + [G(x,y)]^2$ (see Kulkarni et al., 2005).

[18] This holds only if the debt is traded.

liabilities reported in firms' balance sheets is used as the notional face value of the zero coupon bond. This approach is often used because equity holders earn the residual value of the firm once all debt is paid off (see, for example, Helwege et al., 2004, or Hillegeist et al., 2004).[19]

To determine the maturity time of the zero coupon bond representing all the firm's liabilities, we could compute the weighted maturity of the individual claims' maturities.[20] However, our intention is to provide LGD comparable across the sample of the companies analyzed, which would hardly be practicable in case of different maturities (see the sensitivity analysis section). Therefore, we will assume a five-year debt maturity for all companies, which should take into consideration both short-term and long-term debt maturity.[21]

From our previous discussion it is obvious that the estimates of V and σ_V are highly dependent through the system of two equations on the value of equity and its volatility. While the market value of equity E is simply obtained as the closing price of shares at the end of the fiscal year multiplied by the outstanding number of stocks, the equity volatility value depends on the estimation method chosen. For that reason, it is desirable to use different types of estimation techniques for comparison.

 The standard approaches for estimating σ_E are based on the historical data of stock prices or on exploiting bond prices to obtain the so-called implied volatility. The implied volatility of a bond is obtained when one chooses the asset volatility such that the price generated by our model fits the bond's actual market value.[22] Nevertheless, since this volatility estimate incorporates all possible errors of the model used, and also considering our discussion about the illiquid and insufficient bond market, we will use only the historical approach based on stock returns.

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[19] Moody's KMV model specifies the notional default point as the book value of short-term liabilities plus half of the value of long-term liabilities (see Crosbie and Bohn, 2003). They put a greater weight on short-term obligations because debts due in the near term are more likely to cause a default. However, this approach is probably more convenient in the first-passage time models than in seminal Merton, where the default may occur only at debt maturity.

[20] Another method widely used among academics is to group the short-term and long-term obligations and find out the maturity by weighting the maturities of those two groups. For example, Dalianedis and Geske (2001) made an assumption of 1-year maturity for short-term debt and 10-year maturity for long-term debt. The weights would be the book values of claims.

[21] By setting the longer time horizon we should also avoid inaccuracies due to the fact that we use a poor diffusion process without possible jumps for the firm's asset value dynamics.

[22] Similarly, one could get the option-implied volatility for companies with options written on their stock by using the standard Black–Scholes formula for pricing options (see Hull, 2002).

Let P_i denote the closing price of the stock on day i. Then the continuously compounded one-day return r_i is defined as $r_i = lnP_i - lnP_{i-1}$ and the unbiased estimate of the one-day volatility using the m observations of r_i is

$$\sigma_E = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(r_i - \overline{r}\right)^2}$$

where $\overline{r}$ denotes the mean of the r_i's (see Hull, 2003). The appropriate observation interval depends on the time horizon which we are dealing with. Since we set the maturity time to five years, we should also use the long-term volatility for our predictions. For that reason we used a volatility of five trading years.[23] In addition, to take into account possible changes in volatility in the shorter term, we also estimate the last 250 trading days' volatility, similarly to, for example, Kulkarni et al. (2005).

An improvement over these traditional volatility estimation methods, which give equal weights to each observation, is estimation using the exponentially weighted moving average (EWMA), where more recent observations carry higher weights. This method, capturing the volatility dynamics better, is recommended in RiskMetrics[TM] (1996). For a given set of m observations, the exponentially weighted volatility can be computed as

$$\sigma_E = \sqrt{(1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}\left(r_i - \overline{r}\right)^2} \;,\; 0 < \lambda < 1$$

where λ is referred to as the *decay factor*, which determines the relative weights for particular observations. For our sample of companies we use monthly observations over five years with a decay factor equal to 0.97. This value is based on the analysis relating to optimal λ provided in RiskMetrics[TM] (1996).

The fourth and last method that we used is GARCH(1,1), which takes into account the fact that the variance of a time series returns tends to revert to its long-run average over time (see Bollerslev, 1986). We estimate the GARCH(1,1) model for daily data over a five-year interval in the form

$$\sigma_t^2 = b + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \;,\; \alpha_0 > 0, \alpha_1 \geq 0, \alpha_2 \geq 0$$

where $b = \alpha_0 \sigma_{LR}^2$, σ_{LR}^2 represents the long-run unconditional variance of the daily returns r and $\alpha_0, \alpha_1, \alpha_2$ are the weights, whose sum is equal to 1. Since we are concentrating on the long-run volatility, we use only the long-run average variance σ_{LR}^2 to which the process will convert in the future. The long-run volatility is therefore computed from the estimated parameters as

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[23] In the case of insufficiently long time series, we use the longest available one. This holds also for the other five-year estimates computed later in this section.

$$\sigma_E = \sqrt{\frac{b}{1-\alpha_1-\alpha_2}} \; .$$

However, for some companies we did not estimate the long-run GARCH volatility, since their return time series were not weakly stationary. Also, the GARCH is unstable when the fitted parameters $\hat{\alpha}_1 + \hat{\alpha}_2$ are close to 1. This leads to an integrated IGARCH(1,1) model with the additional constraint $\alpha_1 + \alpha_2 = 1$. However, the unconditional variance σ^2_{LR} is not defined in this case. Nonetheless, as can be found in Tsay (2005), this special IGARCH(1,1) model can be rewritten as the EWMA formula with which we have already estimated σ_E.

For most of the companies in our sample we estimated four types of daily equity volatility by the aforementioned methods. These still need to be scaled to obtain the annualized volatility used in later computations.

All estimates are presented in Table 1 in the Appendix. Since higher volatility of equity results in higher volatility of the firm's value and higher default risk, the choice of estimated σ_E can significantly influence the further results. As a rule of prudence, however, we consider it more desirable to provide overstated rather than understated values of LGD. Therefore, we use the average of the two highest σ_E estimates, $\sigma_E{*}$, as the parameter entering the system of two equations.

As the firm's expected rate of return, the system derived for obtaining the unobservable values of V and σ_V exploits the risk-free rate r_f, for which we used the yield of the five-year government bond. Therefore, the last parameter that must be estimated in order to solve the equations is the dividend rate δ. Nonetheless, to acquire δ, one needs to obtain the market value of the firm V. Hence, we use the approximate market value V' as the sum of the market value of equity E and the book value of debt.[24] Since we are estimating the five-year horizon, in the computations we will use the adjusted rate $\delta{*}$, capturing the dividend stream in the last five years, instead of the one-year dividend rate δ.[25]

We solved the two equations simultaneously using the iterative Newton search algorithm. The approximate value V' and the equity volatility were used as the starting values for V and σ_V, respectively. In almost all cases, the process converges within ten iterations. Note that the equation linking equity and asset volatility given by equation (10) holds only instantaneously, which causes bias in the V and σ_V estimates when the leverage changes. Crosbie and Bohn (2003) assert that a quick decrease in the leverage would lead to

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[24] This approach, as Wong and Li (2004) show, overestimates the true market value of the firm.

[25] We used the exponentially weighted average with decay factor $\lambda = 0.9$.

overestimation of asset volatility and that, conversely, a rapid increase would lead to underestimation.[26]

Note that the dynamics of the estimated σ_V follow the equity volatility $\sigma_E{*}$; nevertheless, σ_V is always lower than $\sigma_E{*}$. This is caused by the presence of leverage, since the debt is considered to be non-traded. With increasing leverage, the equity occupies a lower share in the overall value of the firm and therefore V is less volatile than E.

To estimate the expected LGD for the risk-neutral measure we already know all the necessary parameters. However, as the risk-free rate can significantly differ from the firm's real rate of return, we also estimate the expected market return on assets, μ_V, as the return on assets during the previous year. We can easily use the estimated values of the firm's market value V and obtain the one-year return μ_V as

$$\mu_V(t) = \frac{V(t) + Div(t) - V(t-1)}{V(t-1)}$$

where *V(t)* is the firm's market value at the end of year *t* and *Div(t)* denotes the sum of the common and preferred dividends declared during this year. Since the five-year expected return will not be based solely on a one-year observation only, in our calculations we use the adjusted $\mu_V{*}$ as the five-year weighted average, in which recent years carry more weight to react faster to current information.

5. Estimate of LGD in the Czech Republic

We implement the aforementioned methods on a sample of the most liquid firms listed on the Prague Stock Exchange (PSE) and present the dynamics of the five-year expected LGD for each company between 1999 and 2008. We restrict our sample to non-financial firms so that the leverage ratios are comparable across them. In addition, we exclude enterprises that became listed after 2007 to obtain the long time series of share prices necessary to estimate asset volatility. The 15 companies analyzed account for around 7% of the corporate sector's total assets.

Income statements and balance-sheet items for our set of PSE corporations were obtained from the Magnus (2009) database, and for some of them the information was supplemented with data from company annual reports. Share prices, dividend yields, and the number of shares outstanding are available on the PSE website.[27] We use the time series of share prices from the beginning of 1999 to the end of 2008 and accounting information reported

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[26] The impact of a change in the firm's leverage on ELGD is presented later, in the sensitivity analysis section.

[27] The information is also available for the Czech companies in the Magnus (2009) database.

at the end of the fiscal year. The series of five-year risk-free interest rates comes from the ARAD database of the Czech National Bank (CNB).

The non-existence of dividend payouts in the seminal Merton model was modified in the last section. Still, one should also incorporate the costs of bankruptcy, which result in debt holders receiving less than the total firm value in the event of default. Additional default costs also arise from deviations in APR where equity holders gain at the expense of bondholders. While Betker (1997) estimated the direct administration costs relating to bankruptcy at around 5% of firm value, a study by Andrade and Kaplan (1998) indicates higher costs of financial distress, in the range of 15–20%. Based on those empirical studies we consider exogenous common bankruptcy costs $(1 - \varphi)$ equal to 10%.[28]

The final formula for the five-year expected LGD at the beginning of year t for the physical measure, including both dividend payouts and bankruptcy costs, is then

$$ELGD_t = 1 - \varphi \frac{V_t}{F_t} \exp[(\mu_{V,t}^* - \delta_t^*)T] \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} \tag{11}$$

$$d_1^* = \frac{\ln(V_t / F_t) + \left((\mu_{V,t}^* - \delta_t^*) + 0{,}5\sigma_{V,t}^2\right)T}{\sigma_{V,t}\sqrt{T}}, \text{ and } d_2^* = d_1^* - \sigma_{V,t}\sqrt{T}$$

where the time indexes represent particular values at the beginning of year t (the end of the previous year), and $\mu_{V,t}^*$, δ_t^* denotes adjusted rates considering five-year historical observations. One can get the expected risk-neutral LGD by replacing $\mu_{V,t}^*$ by r_f.

The results are given in Table 2, which presents the expected LGD for each company estimated at the end of every year during the period 1999–2008 for both the risk-neutral and physical measure.[29] All the parameters used for the computations are given in Table 1 in the Appendix.

In the theoretical framework the risk-neutral LGD is always an upper bound to its physical counterpart. Nevertheless, this holds only if asset drift μ_V is greater than the risk-free rate. In the conventional analysis rate r_f is supposed to be always less than drift μ_V. For example, Hillegeist et al. (2004) compute μ_V for PD estimates and use r_f as a minimum bound for μ_V, since they claim that lower expected growth rates than r_f are inconsistent with asset pricing theory. Allowing μ_V to be lower than the risk-free rate may therefore seem to be an

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[28] However, there is quite high uncertainty about the value of this parameter, which may be country specific and depend on the legal system of the particular country.

[29] The estimates for the physical measure begin from the year 2000 since we lost one observation for acquiring the firm's growth rate.

arbitrage-free opportunity. However, we try to evaluate the possible expected value of the company from the viewpoint of the creditor, whose recovery rate will depend also on the negative evolution of the company's market value. As a result, letting the risk free rate be the minimum bound for μ_V can result in highly underestimated values of LGD if the real growth rate is lower than r_f. This can be demonstrated using the given results.

Table 2: The five-year expected LGD in the period 1999–2008

Company	Expected LGD (%) – risk neutral measure										Expected LGD (%) – physical measure								
	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2000	2001	2002	2003	2004	2005	2006	2007	2008
CETV	–	–	–	–	–	–	18.0	22.5	21.4	52.7	-	-	-	-	-	-	23.1	18.0	62.7
ČEZ	24.1	27.7	34.4	35.7	35.3	30.7	29.3	29.2	24.1	40.3	32.7	47.1	39.3	29.6	21.2	18.1	18.7	16.7	27.0
ECM	–	–	–	–	–	–	–	13.8	27.7	47.3	-	-	-	-	-	-	-	18.8	43.6
JČ PAPÍRNY VĚT.	29.2	23.7	26.3	26.5	21.3	32.4	23.1	23.0	33.6	38.9	30.3	52.6	33.2	57.9	33.2	13.0	14.1	36.2	22.3
ORCO	–	–	–	–	–	–	21.3	22.5	29.5	62.8	-	-	-	-	-	-	13.2	16.7	73.8
PARAMO	30.4	17.6	16.2	20.5	19.5	23.8	25.0	21.4	22.5	25.1	78.4	65.4	44.3	16.5	20.6	19.1	18.7	19.6	26.0
PEGAS	–	–	–	–	–	–	–	28.4	19.0	47.2	-	-	-	-	-	-	-	20.4	78.2
PHILIP MORRIS	–	17.0	25.4	36.9	32.1	31.1	28.9	32.5	43.5	46.0	-	15.8	21.7	18.8	20.8	21.0	29.5	44.5	51.2
PR. ENERGETIKA	51.5	40.8	42.5	44.0	35.9	28.8	25.1	22.9	21.9	25.0	52.7	53.5	40.4	28.5	22.0	18.5	17.4	15.9	16.7
SPOL. CH.H. VÝR.	20.0	16.2	23.0	23.4	24.9	22.4	25.5	22.0	33.5	36.3	70.1	37.8	28.1	23.9	15.8	14.5	13.7	21.1	21.8
SPOLANA	33.3	33.5	36.1	34.2	35.0	34.9	27.8	27.5	26.6	22.8	42.9	76.6	58.5	44.3	45.0	28.9	27.1	30.0	25.6
TELEFÓNICA	23.9	32.5	36.7	36.0	33.4	33.3	26.3	22.9	43.4	39.1	40.2	49.5	51.7	35.4	32.7	23.0	20.9	37.1	33.5
TOMA	29.9	29.1	23.0	23.5	21.0	19.7	23.5	21.4	18.7	19.1	67.5	24.2	29.6	18.4	15.6	16.5	15.8	13.4	13.5
UNIPETROL	36.1	30.1	26.5	24.8	26.4	29.8	35.0	36.3	34.1	59.6	24.0	25.3	23.4	22.1	27.0	18.8	22.3	23.2	49.9
ZENTIVA	–	–	–	–	–	18.6	22.6	22.9	24.6	25.3	-	-	-	-	-	15.3	18.7	19.6	22.8
Mean (%)	30.9	26.8	29.0	30.6	28.5	27.8	25.5	24.6	28.3	39.2	48.8	44.8	37.0	29.6	25.4	18.8	19.5	23.4	37.9
Std. Dev. (%)	9.2	8.1	8.1	7.8	6.5	5.7	4.2	5.4	7.9	13.6	19.4	19.1	12.1	13.2	9.2	4.4	5.0	9.2	20.9

Source: computed from eq. (11)

Paramo ended 2000 with a loss of more than CZK 430 million and an almost 24% drop in its market value. This negative result has no impact on the expected risk-neutral LGD at the end of 2000 and its value is even below average for that year. However, the physical estimate captures the huge deterioration in the firm's asset value, which leads to a more than four times higher expected LGD. Moreover, Spolana recorded losses of about CZK 700 million in 2001 as a result of a downswing in the plastics market. The subsequent year it was negatively affected by floods, which led to further losses. While the risk-neutral LGDs in these years do not incorporate any problem compared to the estimates for other years, the physical measure counterparts indicate the company's poor performance quite well. The same situation can be found in the case of Papírny Větřní in 2001 and 2003. By contrast, when the growth rate of a firm's assets μ_V is higher than r_f, the risk-neutral estimates overstate ELGD.

 The relatively high ELGD for both measures for ČEZ at the end of 2001 might seem contradictory, since ČEZ ended 2001 successfully with an increase in net profit of over 26% to more than CZK 9 billion. However, its share price dropped from an initial CZK 101 at the end of 2000 to CZK 77.5 at the end of 2001, which led to a more than 23% decrease in the market value of its equity. This, together with a high dividend rate, was reflected in an

almost 14% deterioration in asset value and led to a significant increase in ELGD. Similarly, a large decrease in the market value of equity caused the predictions for Telefónica to worsen in 2001 and 2002. Nonetheless, the sharp rise of ELGD in 2007 is due solely to a sharp increase in asset volatility.

The expected downswing in economic activity due to global and domestic factors was not incorporated enough into share prices at the end of 2007. Therefore, the average ELGD at the end of 2007 is relatively small, still capturing the good economic trend in recent years. For some of the companies analyzed, however, the expected slowdown in economic growth resulted in a drop in the market prices of equity. As a result, the average ELGD estimate at the end of 2008 rose to 38%, indicating a considerable increase in credit risk in the non-financial corporations sector.[30] However, while some companies showed only moderate LGD growth differing little from the previous years' values (ČEZ, Pr. Energetika, and Zentiva), or even the same or decreasing values of ELGD (Spolana, Toma, and Telefónica), some companies recorded sharp increases several times higher than the historical values (CETV, ECM, ORCO, and PEGAS). The latter were mostly companies that had been listed on the PSE for a short time only and property developers, which were one of the sectors hardest hit by the crisis, as the housing market was declining significantly. The unfavorable situation on the market was reflected in negative market sentiment, drops in companies' share prices, and consequent declines in the market values of companies. Also, equity volatility increased in 2008 for almost all companies, although for newly listed companies it reached very high levels (see Table 1 in the Appendix).

The comparison of our estimates with the realized LGDs is not straightforward, since the literature about historical LGDs concentrates on different facilities in different countries and is based on diverse sample sizes across different time periods. What is more, our sample of companies comprises better rated companies with rare occurrence of defaults, so a historical database is not available. Grunert and Weber (2005) summarized 25 empirical studies regarding historical values of LGD and found an average LGD of about 30%, which corresponds to our results. CNB (2008) gives LGDs for large companies of around 34% for secured claims and 48% for unsecured claims. Also, the aforementioned studies by Altman and Kishore (1996), Castle and Keisman (1999), and Keenan et al. (2000) give average LGDs of around 50%. However, since the average indebtedness of our sample is lower than

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[30] Seidler and Jakubík (2009) present only preliminary expected LGDs for 2008, which are still based on the accounting information from the previous year. Still, since the results do not differ significantly for most of the companies (e.g. CETV 74 vs. 63%, Orco 65 vs. 73.8, and Pegas 70 vs. 78.2%) we can conclude that the stock market was the main factor influencing the estimates of LGD in 2008, and that financial statements (mainly indebtedness) played a relatively minor role.

the indebtedness of the whole non-financial corporate sector, the average ELGDs of our sample under analysis should be lower than the aforementioned values.[31]

The risk-neutral estimates are based on the same company structural values relating to credit risk as the physical estimates, except for different assumptions about expected growth of company assets. Nevertheless, as was demonstrated, the risk-neutral estimates do not properly characterize the company's actual riskiness. The more μ_V differs from r_f, the more inaccurate results they provide compared to their physical counterpart. Therefore, creditors trying to appraise their possible recovered amounts in the event of an obligor defaulting should consider the real future growth rate of the firm's assets μ_V as the main determinant of the future LGD,[32] even if the average values of the physical and risk-neutral measures are almost identical (Table 2). From this point of view, it is more desirable to use real physical estimates.

6. Sensitivity Analysis

The sensitivity analysis relating to Merton's initial model discussed in the theoretical section assumed that all the necessary structural variables are known. However, as already said, the value of a firm's assets and its volatility are not directly observable and they have to be estimated through a system of two equations which hold only at a given time. Therefore, the following analysis concentrates on the sensitivity of ELGD due to potential changes in the structural variables of a company influencing also the estimates of σ_V and V. Emphasis is put on leverage, defined as the ratio of total liabilities to the market value of all assets (F/V).

Before we present the ELGD sensitivity for the individual companies in the sample analyzed, we provide a general theoretical discussion based on different input parameter scenarios. The main difference between the current analysis and the previous one illustrated in Figure 2 is due to the fact that a change in leverage influences the estimate of the firm's asset volatility σ_V. Thus, if the leverage increases, the weight of equity in the firm's value declines and the volatility decreases. The rate of decline for a given set of parameters is presented in the first part of Figure 3.

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[31] The comparison is based on the economic results of non-financial corporations with more than 100 employees provided by the Czech Statistical Office.

[32] Also, the risk-neutral estimates consider changes in the market value of a company's assets through the leverage ratio. Still, as we saw, it does not seem to be sufficient.

Figure 3: Sensitivity analysis for ELGD – part 1
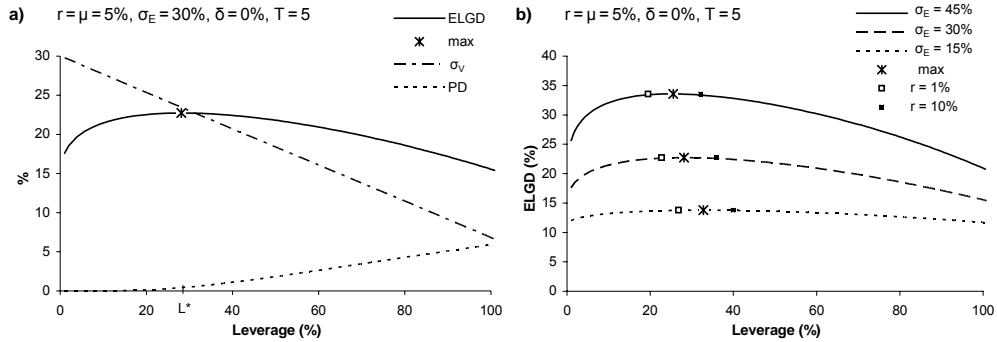
***Source*:** computed from eq. (11) and system (9) and (10)

This figure also illustrates the impact of an increase in the firm's leverage on PD and ELGD. However, while growth in leverage has a positive unambiguous effect on PD, ELGD peaks for a particular leverage ratio and then starts to decrease.

The negative relation between ELGD and leverage may look counterintuitive; however, it is caused by decreasing asset volatility σ_V.[33] Although PD increases with increasing leverage, the expected value of the firm's assets at maturity *T*, conditioned by default ($V_T < F$), increases with respect to the given leverage. In other words, due to lower volatility σ_V it is less likely that the firm's expected value will be excessively below the default barrier *F* at time *T* and therefore the expected recovery ratio (V_T / F) in the case of default has increased.

The result is that by leaving the initial volatility of equity constant,[34] an increase in leverage causes a decline in asset volatility, which generates a negative correlation between PD and ELGD starting from a particular leverage ratio (L* – the breakpoint). Nevertheless, for all the scenarios presented the increase in PD outweighs the decline in LGD and the expected loss for a unit of exposure (PD.ELGD) is therefore strictly increasing with leverage.

Pursuing the issue further, we analyze the changes in breakpoints with respect to other parameters. The maximum ELGD points are presented for three different values of r_f and σ_E. As can be seen, a decline in the risk-free interest rate shifts the max ELGD points to the left, similarly as an increase in equity volatility (Figure 3b). It is evident that any increase in

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[33] The previous analysis reported in Figure 2 shows a strictly positive correlation between ELGD and leverage. However, σ_V was taken as a constant and did not change with leverage.

[34] A change in leverage will also affect the equity volatility. However, since we use the long-run volatility σ_E^* for the computation, in which sudden short-term changes do not take effect, the assumption of constant σ_E in the sensitivity analysis is maintainable.

σ_E will lead (because of higher uncertainty) ceteris paribus to higher values of ELGD. However, the figure also presents the variability of potential ELGDs along the whole range of leverages. While for $\sigma_E = 45\%$ the ELGDs vary from 22 to 33 percent, the volatility for $\sigma_E = 30\%$ is only 7 percentage points, and in the case of $\sigma_E = 15\%$ the variability of possible ELGDs is minimal. This further highlights the importance of volatility as a crucial variable for LGD predictions and indicates that companies with identical leverage ratios can have substantially different ELGD sensitivity.

The existence of the dividend rate in the system of equations lowers the estimated market value of the company V, since part of its value is paid out to the equity holders. Supposing the same value of equity, the presence of dividends increases the estimated asset volatility compared to the state with a zero dividend rate. Thus, dividends offset the initial lowering of σ_V given by an increase in leverage, which results in higher ELGD and consequently a lower ELGD decrease behind the breakpoint. Moreover, the increase in asset volatility given by a sufficiently high dividend rate outweighs the decline in volatility after the breakpoint and the overall effect of increase in leverage on ELGD is positive (see Figure 4c).

Figure 4: Sensitivity analysis for ELGD – part 2
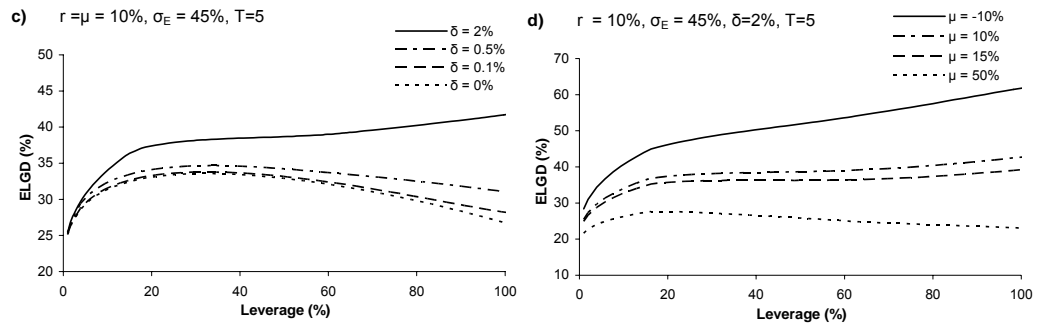
Source: computed from eq. (11) and system (9) and (10)

Until now we have not considered any differences between the physical and risk-neutral measures in the analysis of the sensitivity of ELGD to leverage. Since real asset growth μ_V does not figure in the estimation of V and σ_V, it may seem that the physical ELGD will differ for a given set of parameters only in absolute terms, keeping the same rate of change with respect to leverage. The right-hand side of Figure 4 displays the evolution of ELGD for various growth rates relating to the increasing ELGD sensitivity curve from the previous figure (2% dividend rate). As we can see, μ_V also affects the slope of the ELGD curve and not only its parallel shift. Bad company performance, represented by small and negative μ_V, will raise the rate of growth of ELGD, while good performance will offset the presence of

the dividend payout and the curve will become downward-sloping from the breakpoint again.

The result is that the ELGD under the physical measure has a lower growth rate in leverage for $\mu_V > r_f$, and for sufficiently high values of μ_V the initial growth rate may from some point even invert from increasing to decreasing (see Figure 4d, $\mu_V = 50\%$). This also holds in the opposite direction for low and negative values of μ_V.

The empirical results for the sample analyzed are reported in the following table, which shows the leverage elasticity of ELGD for both measures at the beginning of 2008.

Table 3: Elasticity of ELGD with respect to leverage

Company	$\varepsilon_{Leverage}^{ELGD\,Q}$	$\varepsilon_{Leverage}^{ELGD}$	Company	$\varepsilon_{Leverage}^{ELGD\,Q}$	$\varepsilon_{Leverage}^{ELGD}$	Company	$\varepsilon_{Leverage}^{ELGD\,Q}$	$\varepsilon_{Leverage}^{ELGD}$
CETV	0.071	0.022	PARAMO	-0.393	-0.498	SPOLANA	-0.647	-0.477
ČEZ	0.078	-0.034	PEGAS	0.341	0.405	TELEFÓNICA	0.175	0.150
ECM	-0.607	-0.643	PHILIP MORRIS	0.403	0.403	TOMA	-0.093	-0.179
JČ PAPÍRNY VĚTŘNÍ	0.116	0.129	PR. ENERGETIKA	0.268	0.128	UNIPETROL	-0.025	-0.148
ORCO	0.344	-0.128	SPOL. CH.HUT.VÝR.	-1.072	-1.095	ZENTIVA	0.012	-0.109

Source: computed from eq. (11) and system (9) and (10)

As can be seen, most of the companies analyzed have inelastic ELGD with respect to leverage. Only Spolek pro chem. a hut. výrobu has a negative elasticity, slightly exceeding 1. Based on our previous discussion we can analyze the differences in risk-neutral (ε^Q) and physical (ε^P) elasticity with respect to other parameters. For example, CET and Pr. Služby, companies with a zero dividend rate and low leverage at the beginning of 2008, are located on the rising parts of their ELGD sensitivity curves. However, because μ_V lowers the ELGD growth rate and the expected asset rate μ_V is higher than r_f for both companies, their "physical" elasticity is lower than ε^Q. By contrast, Č. Nám. Plavba and JČ Papírny show an inverse inequality between ε^P and ε^Q since their $\mu_V < r_f$.[35]

The sensitivity analysis further illustrates the differences already pointed out between the risk-neutral and physical measures. However, a more important finding seems to be that ELGD is quite inelastic with respect to leverage and sudden changes in it do not incur significantly large turns in the expected LGD. Possible inaccuracies in the estimation of V and σ_V, as mentioned by Crosbie and Bohn (2003), caused by change in leverage might be more relevant to the PD estimate, but should not cause important changes in the predictions of ELGD.

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[35] The values of leverage and expected asset growth are reported in Table 1 in the Appendix.

Another sensitivity analysis presented here concerns debt maturity, which was arbitrary set at five years for all companies, as already mentioned in the section on model implementation. The following table compares ELGDs for three different debt maturities estimated in one particular year, where the values for five-year maturity (5Y) are identical to the estimates from Table 2 in 2008. As we can see, the estimates of ELGD increase significantly with time to debt maturity, as the uncertainty about the firm's future value increases with longer time horizons. The sensitivity of ELGD with regard to maturity T is rather high, especially for increases in T from low initial values. However, the relationship is not linear and the elasticity decreases with higher T.[36]

Table 4: ELGD for the physical measure for different debt maturities

ELGD in 2008 – physical measure

Company	ELGD (%) for maturity:			Company	ELGD (%) for maturity:			Company	ELGD (%) for maturity:		
	1Y	5Y	10Y		1Y	5Y	10Y		1Y	5Y	10Y
CETV	35.0	62.7	77.4	PARAMO	16.5	26.0	32.3	SPOLANA	15.7	25.6	31.4
ČEZ	21.0	27.0	45.1	PEGAS	45.3	78.2	83.0	TELEFÓNICA	24.2	33.5	46.5
ECM	19.3	43.6	49.4	PHILIP MORRIS	25.1	51.2	68.4	TOMA	12.1	13.5	14.2
JČ PAPÍRNY VĚTŘNÍ	14.4	22.3	32.1	PR. ENERGETIKA	14.2	16.7	18.1	UNIPETROL	30.9	49.9	60.1
ORCO	44.0	73.8	85.3	SPOL.CH.HUT.VÝR.	16.7	21.8	22.9	ZENTIVA	15.4	22.8	28.3

Source: computed from eq. (11) and system (9) and (10)

Even if the assumption of five-year debt maturity is rather strong, we set it arbitrarily for all companies to have comparable ELGD results across the whole sample. For most firms the average debt maturity is shorter in reality (Jakubík and Seidler, 2009b, p. 624). However, the longer time period was chosen also for conservative prudential reasons in order to ensure that the LGD estimates obtained were slightly overestimated. The other limits and shortcomings of the estimates presented are discussed in more detail in the next section.

7. Criticism and Limitations

The first implementation of Merton's model, applied by Jones et al. (1984), Ogden (1987), and Franks and Torous (1989), suggested that the model generates lower credit spreads than those observed on the market. Similarly, more recent studies by Lyden and Saraniti (2001) and Helwege et al. (2004) showed that the basic Mertonian contingent claim model underpredicts the actual bond spread, especially for low-leveraged and low-volatility companies. Based on these findings, our ELGD estimates would be undervalued. However, considering that bond spreads also reflect market risk, tax, and liquidity effects, the

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[36] This may be caused by the fact that the process of modeling the firm's asset value dynamics is a poor diffusion process with no possible jumps and low maturity does not enable significant fluctuations in the firm's asset value.

aforementioned studies only confirmed Merton's inability to capture other components of debt spread, saying nothing about the model's ability to reveal default and recovery risk.

This issue is confirmed by Longstaff (2000), who has argued that corporate bond markets are much more illiquid than government bond and stock markets, so it seems likely that credit spread is only partly due to default risk. In spite of these well-known complications and imperfections, the majority of the literature empirically testing the structural models has presumed that credit spread is primarily due to default risk, since the other components are hardly tractable.[37] Sarig and Warga (1989) compared not the absolute values of theoretical corporate bond spreads, but only their rates of change with respect to change in the bond's actual default riskiness and praised the good predictive power of Merton's model. Furthermore, Dalianedis and Geske (2001) termed the difference between the observed and modeled spread the residual spread and empirically confirmed that the spreads estimated by the Merton approach correctly evaluate the default risk and that the residual spread is driven by liquidity, tax, and other effects.[38] These conclusions suggest that our LGD estimates are correct, since the accuracy of the ELGDs is based on capturing the company's default risk.

If we assume that share prices reflect all relevant information regarding the future development of the company as well as the expected conditions for the given industry or economy, these expectations are also incorporated into our ELGDs, since they are dependent on the development of the stock market. Thus, ELGDs based on the market value of equity are forward-looking estimates which may be used to instantaneously monitor a company's riskiness and can serve as an early-warning indicator. Nevertheless, the stock market dependence of ELGDs can also embody excessive movements in share prices caused by market bubbles. Also, the stock market may not efficiently incorporate all publicly available information about the default probability, especially in the case of a young market such as the Czech one.[39]

The model treats default as an event that cannot occur before debt maturity. In practice, liabilities are repaid more frequently and default can be observed anytime before maturity of the debt. Allowing default to occur before maturity would hedge debt holders against high losses in the event of the borrower's assets continuing to decrease. In that case, the

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[37] This idea stems from the theoretical assumption that corporate bond markets are perfect and complete and trading takes place continuously (see Dalianedis and Geske, 2001).

[38] Structural models may also understate spreads in the short run, since the pure diffusion process is not able to capture unpredicted extreme changes in a firm's asset value given by a shock. Therefore, it is also possible to add a jump process to Brownian motion or to model asset value as a discontinuous Lévy process.

[39] We are also aware of possible sample bias, as a company with very bad performance approaching default would probably be withdrawn from the stock market.

remaining value of the company would be higher at the time of default than at debt maturity, which implies lower LGD. The simplifying assumption of no default occurrence before maturity therefore overstates the expected LGD. However, as a rule of prudence, we prefer to provide overstated rather than understated values of LGD.

Furthermore, the definition of default used in the model corresponds more to the state of bankruptcy than to the obligor's ninety days past due obligation defined under Basel II. Thus, the model's definition of default also leads to overstated ELGD; however, the companies analyzed should have a high ability to raise funds. So, if a company is past due more than 90 days on its obligation, it has probably exhausted all means to raise the funds and bankruptcy will follow.

The computations also do not consider any debt priority, therefore ELGDs for secured and more senior claims should be lower than the presented estimates and, conversely, those for subordinated debt should be higher. However, the distribution of the value of a bankrupt firm also depends on violation of the APR, which is difficult to predict for single cases. The bankruptcy costs were determined by using other empirical studies, but bankruptcy laws and other procedures differ substantially by country and may therefore differ in the Czech Republic. Calibration on an empirical sample would be needed to obtain more accurate estimates, but no appropriate data sample is available owing to a low number of defaults of comparable companies.

The computed ELGDs also suffer from other shortcomings, such as the assumption of a constant interest rate and no tax shield, and other simplifications arising from the seminal Mertonian approach. On the other hand, more sophisticated models require a higher number of parameters, which have to be estimated. This increases the computational complexity and might therefore produce higher errors. Also, some amendments relating, for example, to stochastic interest rates have unambiguous effects and sometimes have only little impact on the results (Lyden and Saraniti, 2000). Nevertheless, the empirical application of more complex models will be the goal of further research.

In spite of all the aforementioned limitations, the presented results are the first estimates of expected LGD based on market information for single companies listed on the Prague Stock Exchange. However, because of the many exogenous and simplifying assumptions, the presented estimates should serve more as a stepping stone for further improvements or as some kind of warning indicator and cannot substitute for estimated LGD values based on historical data as required under Basel II.

8. Conclusion

The intensively studied topics in quantitative finance currently include the concept of Loss Given Default, which is rather unexplored territory in the credit risk area. Especially with the implementation of the New Capital Accord, LGD has received increased attention and has become a frequent object of empirical and theoretical research. The goal of this paper was to present the basic knowledge concerning this key input parameter of credit risk analysis and primarily to introduce a modeling technique which enables estimation of forward-looking LGDs from market-observable data.

We exploited the information embedded in the stock market and used the Mertonian structural approach based on contingent claim analysis, which considers the remaining value of a firm's assets as the recovered amount in the case of default. This demonstrates that LGD is stochastic even in Merton's initial framework, since it depends on the uncertain development of asset value. We also pointed out the joint dependence between PD and LGD, which implies that those parameters should not be treated as independent in credit risk modeling.

We analyzed 15 companies listed on the Prague Stock Exchange in the 1999–2008 period and computed the expected LGD for every single company in a given year. The average LGD of the sample across time was estimated in the range of around 20–50%. We also described estimation procedures exploiting prices of equity and their volatility and showed that LGD is relatively inelastic with respect to leverage of the company. By contrast, the LGD estimates are highly elastic with respect to debt maturity, which was arbitrarily set at five years for all companies in the sample analyzed. The presented approach is based on some simplifying assumptions, hence we are aware of the uncertainty regarding the precise values of the LGD estimates presented. Still, the computed estimates can serve as an indicator of the evolution of a company's riskiness over time and should be taken as the first attempt to estimate LGD using the Mertonian approach for companies listed on the Prague Stock Exchange. These estimates can be further developed and improved.

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Appendix

Table 1: All relevant parameters for the sample of companies analyzed

Company	End of year	Estimates of equity volatility					Parameters used for ELGD computation						Other parameters used				
		σE MA(5y) (%)	σE MA(1y) (%)	σE EWMA (%)	σE GARCH (%)	σ*E (%)	rf (%)	μ* (%)	δ* (%)	σV (%)	V (bill. CZK)	F (bill. CZK)	δ (%)	μ (%)	Leverage (%)	V' (bill. CZK)	Equity (bill. CZK)
CETV	2005	22.7	22.7	21.9	22.8	22.7	3.1	-	0.0	17.5	62.94	16.99	0.0	-	27.0	65.35	48.36
	2006	28.2	30.7	27.5	28.7	29.7	3.3	7.2	0.0	24.2	73.04	15.91	0.0	7.2	21.8	75.45	59.54
	2007	28.4	28.6	27.5	28.7	28.7	4.0	26.3	0.0	24.9	102.67	16.52	0.0	40.6	16.1	105.63	89.12
	2008	57.6	97.5	22.5	42.8	77.5	3.7	-13.7	0.0	41.9	34.73	23.96	0.0	-66.2	69.0	40.13	16.17
ČEZ	1999	35.6	35.6	31.2	39.0	37.3	6.7	-	0.0	17.9	113.76	84.34	0.0	-	74.1	136.83	52.49
	2000	36.0	36.4	33.1	36.7	36.5	6.8	-0.2	0.5	21.1	112.36	81.61	0.8	-0.2	72.6	141.49	59.87
	2001	38.1	41.7	35.2	41.9	41.8	4.8	-8.0	0.8	24.6	95.30	76.45	1.2	-13.9	80.2	122.37	45.92
	2002	36.9	33.3	34.0	39.9	38.4	3.2	-1.3	1.3	26.2	99.83	69.54	2.1	7.5	69.7	124.30	54.76
	2003	34.2	20.9	29.1	41.1	37.7	3.8	14.6	1.8	28.4	137.20	78.22	2.9	42.2	57.0	164.51	86.29
	2004	32.8	27.6	28.1	38.3	35.5	3.4	41.5	2.0	30.2	264.57	79.22	1.9	96.7	29.9	280.99	201.77
	2005	31.9	32.1	28.4	36.4	34.2	3.1	67.1	1.9	30.3	546.00	132.92	1.6	109.7	24.3	568.96	436.04
	2006	29.0	29.6	27.4	39.6	34.6	3.3	61.5	1.9	30.8	701.40	161.00	1.6	30.6	23.0	729.52	568.52
	2007	27.7	27.0	26.7	29.4	28.6	4.0	57.9	2.0	26.9	942.99	169.56	0.0	37.8	18.0	976.15	806.59
	2008	37.1	58.6	40.5	35.1	49.6	3.7	29.3	2.6	32.1	602.50	287.77	0.0	-33.3	47.8	709.98	422.21
ECM	2006	16.0	16.0	16.0	15.0	16.0	3.3	-	0.0	8.7	11.16	5.98	0.0	-	53.6	12.07	6.09
	2007	25.8	26.3	23.6	33.5	29.9	4.0	26.6	0.0	19.1	14.13	10.91	0.0	26.6	77.3	16.03	5.12
	2008	52.5	70.1	56.9	0.0	63.5	3.7	8.1	0.0	31.7	13.30	14.54	0.0	-5.8	109.3	16.33	1.79
JČ PAPÍRNY VĚTŘNÍ	1999	44.8	44.8	41.6	45.2	45.0	6.7	-	0.0	22.9	0.43	0.30	0.0	-	71.3	0.51	0.20
	2000	41.1	37.0	35.3	40.7	40.9	6.8	-0.9	0.0	15.7	0.42	0.38	0.0	-0.9	89.1	0.53	0.15
	2001	43.4	47.8	39.0	44.5	46.1	4.8	-15.6	0.0	17.7	0.31	0.25	0.0	-26.7	81.9	0.36	0.11
	2002	40.0	27.2	31.4	40.9	40.4	3.2	-6.1	0.0	20.4	0.33	0.20	0.0	6.4	59.6	0.35	0.16
	2003	39.8	38.6	30.2	42.0	40.9	3.8	-18.0	0.0	12.3	0.20	0.18	0.0	-38.6	86.8	0.23	0.06
	2004	42.0	53.5	35.7	45.7	49.6	3.4	2.2	0.0	26.1	0.29	0.17	0.0	43.5	59.1	0.31	0.14
	2005	45.6	54.2	41.7	49.3	51.8	3.1	43.0	0.0	12.5	0.65	0.61	0.0	126.1	92.5	0.74	0.13
	2006	45.8	48.7	42.9	51.8	50.2	3.3	30.6	0.0	12.7	0.58	0.53	0.0	-11.5	92.2	0.66	0.12
	2007	50.9	56.7	47.6	51.3	54.0	4.0	-8.0	0.0	53.1	0.04	0.001	0.0	-92.8	2.0	0.04	0.04
	2008	54.0	55.9	51.9	53.5	54.9	3.7	-21.4	0.0	53.2	0.02	0.001	0.0	-56.2	3.1	0.02	0.02
ORCO	2005	21.0	21.0	19.6	35.9	28.5	3.1	-	0.0	18.9	29.57	11.58	0.0	-	39.2	31.18	19.60
	2006	25.8	29.6	24.6	31.7	30.7	3.3	76.4	0.2	18.1	51.91	26.72	0.4	76.4	51.5	56.57	29.86
	2007	26.7	28.1	25.6	28.8	28.4	4.0	34.4	0.4	17.0	53.07	52.69	0.0	3.0	99.3	76.15	23.46
	2008	54.6	96.8	63.8	64.8	80.8	3.7	-5.9	0.6	28.1	21.45	50.87	0.0	-58.9	237.1	52.75	1.89
PARAMO	1999	50.1	50.1	48.8	49.9	50.1	6.7	-	0.0	22.4	2.68	2.18	0.0	-	81.5	3.27	1.09
	2000	46.1	40.9	40.3	51.3	48.7	6.8	-24.3	0.0	7.0	2.03	2.51	0.0	-24.3	123.7	2.74	0.24
	2001	40.9	27.4	32.7	42.7	41.8	4.8	-17.1	0.0	6.1	1.79	1.98	0.0	-11.6	110.7	2.21	0.23
	2002	39.1	33.1	28.3	44.0	41.5	3.2	-11.7	0.0	11.1	1.71	1.51	0.0	-4.7	88.2	1.91	0.41
	2003	37.1	27.0	24.6	39.7	38.4	3.8	9.6	0.0	10.5	2.50	2.24	0.0	46.5	89.6	2.87	0.63
	2004	32.2	28.2	22.8	39.3	35.8	3.4	11.3	0.0	18.2	2.87	1.69	0.0	58.9	53.2	3.12	1.42
	2005	33.0	45.5	29.6	39.2	42.4	3.1	16.7	0.0	17.0	3.42	2.47	0.0	19.1	72.0	3.74	1.28
	2006	34.2	33.8	30.6	35.8	35.0	3.3	9.5	0.0	14.2	3.01	2.14	0.0	-12.0	71.1	3.32	1.18
	2007	33.2	27.6	28.8	34.8	34.0	4.0	11.2	0.0	16.6	3.35	2.12	0.0	11.2	63.2	3.71	1.60
	2008	32.8	24.2	24.5	37.5	35.1	3.7	-0.2	0.0	17.3	2.85	1.85	0.0	-15.0	64.9	3.19	1.35
PEGAS	2006	28.6	28.6	28.6	25.3	28.6	3.3	-	1.7	23.3	9.63	4.78	1.7	-	49.6	11.73	6.95
	2007	20.9	20.6	20.2	20.6	20.7	4.0	0.3	0.7	15.4	9.66	4.47	0.0	0.3	46.3	11.40	6.93
	2008	40.5	53.4	41.1	37.5	47.3	3.7	-29.3	1.8	24.1	4.49	3.72	0.0	-51.5	82.8	5.87	2.15
PHILIP MORRIS	2000	13.8	13.8	10.5	12.5	13.8	6.8	-	12.4	13.8	12.74	3.46	12.4	-	27.2	14.47	11.01
	2001	23.3	24.0	20.3	n.a.	23.7	4.8	63.8	15.5	23.7	17.46	3.21	17.9	63.8	18.4	19.06	15.85
	2002	29.2	34.5	26.8	33.3	33.9	3.2	60.7	15.4	33.9	23.69	4.69	15.3	58.5	19.8	26.03	21.34
	2003	28.3	26.2	25.8	29.5	28.9	3.8	61.0	14.0	28.9	33.90	7.61	11.5	61.4	22.4	37.70	30.10
	2004	28.9	31.0	26.9	29.4	30.2	3.4	44.9	13.2	30.2	35.24	6.27	11.5	16.9	17.8	38.37	32.10
	2005	29.0	28.4	26.9	29.1	29.1	3.1	35.7	11.3	29.1	38.14	6.42	7.4	16.9	16.8	41.34	34.93
	2006	30.6	32.0	27.6	29.9	31.3	3.3	10.6	9.2	31.3	23.39	5.28	6.3	-34.4	22.6	26.03	20.74
	2007	28.6	25.0	26.0	29.9	29.3	4.0	2.8	8.6	29.3	18.55	12.38	0.1	-3.4	66.8	27.56	15.18
	2008	32.0	41.2	33.0	32.7	37.1	3.7	-9.4	7.7	37.1	13.61	4.96	0.1	-20.8	36.5	16.50	11.53
PR. ENERGETIKA	1999	50.1	50.1	44.0	48.3	50.1	6.7	-	4.7	47.8	6.77	3.24	4.7	-	47.9	7.88	4.64
	2000	38.1	19.5	26.5	28.5	33.3	6.8	-4.4	4.7	33.3	5.86	3.52	4.7	-4.4	60.0	7.86	4.35
	2001	33.0	19.0	20.9	26.7	29.9	4.8	-0.3	5.0	29.9	5.12	3.87	5.5	2.9	75.6	7.79	3.92
	2002	33.5	35.0	21.0	31.0	34.3	3.2	7.0	6.2	34.3	5.92	3.22	8.3	16.6	54.4	7.79	4.57
	2003	30.6	14.3	17.2	25.3	28.0	3.8	21.6	6.4	28.0	7.27	3.56	6.8	46.8	49.0	10.16	6.60
	2004	21.8	14.7	12.7	22.9	22.4	3.4	22.5	6.5	22.4	8.23	3.64	6.3	24.5	44.2	11.52	7.88
	2005	21.8	19.3	13.8	24.7	23.2	3.1	30.5	6.5	23.2	11.31	3.08	5.9	40.4	27.3	14.32	11.24
	2006	21.1	14.7	12.6	18.9	20.0	3.3	25.4	7.6	20.0	10.91	3.20	9.8	8.3	29.4	14.06	10.86
	2007	16.7	19.8	13.5	20.1	20.0	4.0	37.4	9.3	20.0	15.07	3.67	0.1	60.1	24.3	18.71	15.04
	2008	16.6	13.5	10.2	19.1	17.9	3.7	26.9	6.4	17.9	15.78	6.49	0.0	4.7	41.1	21.94	15.45
SPOL.CH.HUT.VÝR.	1999	47.3	47.3	44.4	46.6	47.3	6.7	-	0.0	9.7	1.43	1.63	0.0	-	114.6	1.88	0.25
	2000	41.3	34.2	36.9	41.0	41.2	6.8	-18.1	0.0	6.1	1.17	1.42	0.0	-18.1	121.7	1.57	0.15
	2001	41.5	41.5	38.0	41.4	41.5	4.8	-8.0	0.0	14.5	1.16	0.99	0.0	-0.4	85.5	1.37	0.37
	2002	41.7	42.3	37.2	42.5	42.4	3.2	-2.4	0.0	14.8	1.22	0.96	0.0	5.0	78.8	1.35	0.39
	2003	39.3	28.0	31.5	40.4	39.9	3.8	5.5	0.0	17.9	1.45	0.99	0.0	19.1	68.2	1.61	0.62
	2004	35.7	30.1	28.3	40.9	38.3	3.4	23.6	0.0	14.5	2.34	1.76	0.0	60.8	75.1	2.59	0.83
	2005	39.8	52.3	35.4	43.8	48.1	3.1	45.1	0.0	16.2	4.19	3.39	0.0	79.3	80.9	4.64	1.25
	2006	39.1	37.8	35.5	46.4	42.7	3.3	34.8	0.0	12.8	4.31	3.67	0.0	2.8	85.2	4.84	1.16
	2007	36.7	30.1	32.9	40.4	38.5	4.0	24.0	0.3	18.8	4.03	4.38	0.0	-5.3	108.7	5.68	1.30
	2008	34.6	0.0	13.5	0.3	24.0	3.7	18.8	0.2	15.6	4.31	5.87	0.0	6.9	136.4	7.17	1.30
SPOLANA	1999	44.0	44.0	39.4	44.4	44.2	6.7	-	0.0	18.4	5.29	6.79	0.0	-	128.5	7.20	0.40
	2000	39.4	34.2	31.8	40.1	39.8	6.8	-0.7	0.0	19.0	5.25	6.59	0.0	-0.7	125.5	7.13	0.54
	2001	40.2	41.7	34.2	40.5	41.1	4.8	-26.3	0.0	23.0	2.86	2.94	0.0	-45.5	102.6	3.48	0.54
	2002	40.5	41.7	32.9	43.3	42.5	3.2	-13.8	0.0	19.8	2.94	3.05	0.0	2.7	103.7	3.37	0.33
	2003	37.3	19.2	24.8	39.9	38.6	3.8	-4.5	0.0	21.7	3.28	3.26	0.0	11.4	99.4	3.82	0.56
	2004	33.7	25.9	21.7	38.9	36.3	3.4	-5.8	0.0	22.2	3.00	2.82	0.0	-8.4	94.0	3.42	0.60
	2005	37.8	50.9	29.7	51.7	51.3	3.1	1.6	0.0	18.3	3.54	2.80	0.0	18.0	79.2	3.91	1.11
	2006	36.2	34.4	28.9	62.2	49.2	3.3	3.9	0.0	18.6	3.45	2.67	0.0	-2.5	77.3	3.83	1.16
	2007	32.5	21.9	24.9	57.0	44.8	4.0	-0.5	0.0	18.7	3.11	2.29	0.0	-10.0	73.8	3.49	1.20
	2008	32.6	19.9	19.7	36.2	34.4	3.7	-1.5	0.0	14.1	3.09	2.33	0.0	-0.7	75.5	3.53	1.20
TELEFÓNICA	1999	31.9	31.9	28.7	32.0	32.0	6.7	-	0.0	26.8	221.55	49.96	0.0	-	22.6	235.65	185.68
	2000	38.2	43.8	36.4	39.9	41.9	6.8	-10.4	0.7	36.2	196.08	45.58	1.2	-10.4	23.2	208.95	163.36
	2001	43.2	51.8	42.9	44.4	48.1	4.8	-18.5	0.0	39.1	147.99	38.95	0.0	-24.5	26.3	155.71	116.76
	2002	42.8	41.7	41.6	43.3	43.1	3.2	-18.0	6.2	32.5	103.75	34.19	16.4	-17.4	33.0	113.01	78.82
	2003	41.8	37.6	39.1	42.3	42.0	3.8	0.5	5.4	28.1	132.06	55.46	3.7	32.6	42.0	149.28	93.82
	2004	40.7	24.0	34.0	40.3	40.5	3.4	4.8	0.0	33.3	149.91	38.74	0.0	13.5	25.8	157.65	118.92
	2005	36.6	17.8	26.6	n.a.	31.6	3.1	16.4	5.3	29.1	190.22	29.24	7.3	36.6	15.4	198.17	168.94
	2006	30.1	22.0	22.8	n.a.	26.4	3.3	13.1	6.9	23.5	168.01	29.40	8.8	-3.2	17.5	182.71	153.31
	2007	25.0	18.2	19.9	80.4	52.7	4.0	22.3	6.5	48.6	200.88	30.76	0.1	38.0	15.3	206.23	175.47
	2008	26.1	41.2	28.4	45.0	43.1	3.7	9.2	7.8	38.2	154.37	25.46	0.1	-15.1	16.5	162.05	136.60
TOMA	1999	28.1	28.1	22.7	27.6	28.1	6.7	-	0.0	20.1	0.22	0.20	0.0	-	94.9	0.27	0.07
	2000	26.3	24.4	21.7	25.9	26.1	6.8	-21.1	0.0	19.5	0.17	0.16	0.0	-21.1	95.0	0.22	0.05
	2001	32.1	41.4	28.9	31.7	36.8	4.8	2.9	0.0	16.2	0.21	0.15	0.0	20.9	72.8	0.24	0.09
	2002	30.9	27.0	24.9	31.6	31.3	3.2	-13.3	0.0	25.8	0.13	0.03	0.0	-34.5	20.6	0.14	0.11
	2003	29.1	20.0	20.5	29.9	29.5	3.8	24.4	0.0	27.3	0.26	0.02	0.0	89.7	8.9	0.26	0.24
	2004	29.3	28.8	21.6	31.2	30.3	3.4	64.0	0.0	29.4	0.63	0.02	0.0	145.3	3.3	0.63	0.61
	2005	29.6	26.4	20.8	32.8	31.2	3.1	54.4	0.0	25.8	0.72	0.15	0.0	14.4	20.3	0.74	0.59
	2006	24.6	18.4	16.9	31.4	28.0	3.3	41.4	0.0	21.7	0.76	0.20	0.0	6.8	26.7	0.80	0.59
	2007	22.9	18.4	16.3	25.8	24.4	4.0	48.6	0.0	16.1	1.11	0.46	0.0	45.5	41.6	1.20	0.73
	2008	22.7	18.6	13.0	22.3	22.8	3.7	39.6	0.0	15.3	1.46	0.60	0.0	30.9	41.3	1.58	0.97
UNIPETROL	1999	47.5	47.5	42.7	55.0	51.3	6.7	-	0.0	32.7	15.23	8.09	0.0	-	53.1	17.36	9.27
	2000	41.4	34.1	36.6	44.1	42.7	6.8	23.2	0.0	25.6	18.76	10.32	0.0	23.2	55.0	21.59	11.27
	2001	40.2	37.7	36.2	42.4	41.3	4.8	7.0	0.0	20.0	17.80	12.06	0.0	-5.1	67.8	20.23	8.17
	2002	41.1	44.0	37.9	43.5	43.8	3.2	5.4	0.0	16.4	18.38	13.99	0.0	3.3	76.1	20.26	6.27
	2003	38.5	25.2	32.9	40.7	39.6	3.8	14.1	0.0	20.9	23.73	13.88	0.0	29.1	58.5	25.93	12.05
	2004	33.8	23.2	27.1	47.0	40.4	3.4	11.0	0.0	29.3	24.88	8.23	0.0	4.8	33.1	26.04	17.81
	2005	38.6	53.7	33.8	47.9	50.8	3.1	71.9	0.0	30.4	74.49	36.75	0.0	199.4	49.3	78.91	42.16
	2006	37.8	33.8	31.6	64.8	51.3	3.3	52.0	0.0	33.0	69.26	30.75	0.0	-7.0	44.4	73.23	42.49
	2007	34.2	24.8	28.2	50.0	42.1	4.0	45.7	1.2	34.4	80.82	24.00	0.0	21.3	29.7	85.22	61.22
	2008	43.3	64.3	46.0	82.7	73.5	3.7	16.5	0.9	54.4	42.51	19.78	0.0	-47.4	46.5	46.97	27.19
ZENTIVA	2004	24.1	24.1	23.2	25.2	24.7	3.4	-	1.0	24.4	30.70	2.14	1.0	-	7.0	31.03	28.89
	2005	27.7	29.4	25.6	28.0	28.7	3.1	68.2	0.8	25.2	51.28	9.29	0.7	68.2	18.1	52.61	43.32
	2006	28.3	29.2	25.4	30.8	30.0	3.3	32.3	0.8	28.2	53.59	6.17	0.8	5.4	11.5	54.53	48.36
	2007	29.6	32.4	9.7	29.8	31.1	4.0	20.8	0.7	21.4	56.31	24.97	0.0	5.6	44.3	62.04	37.07
	2008	29.6	32.4	9.7	29.8	31.1	4.0	20.8	0.7	21.4	56.31	24.97	0.0	5.6	44.3	62.04	37.07

Source: author´s computation, Magnus (2009), Prague Stock Exchange

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