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The Convergence of a Transition Economy: The Case of the Czech Republic

Jan Brůha, Jiří Podpiera and Stanislav Polák *

Abstract

In this paper we develop a two-country dynamic general equilibrium model by means of which we seek to explain the long-run paths of a converging emerging market economy. The model's novel feature is the inclusion of quality investment to the standard framework of applied general equilibrium two-country models. This extension proves crucial ingredient for explanation of the trend in real exchange rate. Using a case study calibration of productivity and deep parameters for the Czech economy we demonstrate the ability of the model to consistently explain dynamics in key macroeconomic variables that are essential inputs for commonly used 'gap models' in monetary policy practice.

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Nontechnical Summary

Monetary policy practice is often based on model simulations. However, since the widely used models are so called 'gap' models, there is a need for setting equilibrium trajectories. This becomes a nontrivial task especially in a transition economy that is integrating into an economic and monetary union and thus is converging to a more developed counterpart. The convergence means that the equilibrium trajectories necessarily exhibit dynamics, which complicates their derivations and in turn the application of the entire models.

In order to derive the convergence trajectories for the key monetary policy relevant variables, we develop a two-country model based on the New International Macroeconomics (NIM). We model initially asymmetrically developed countries in a general equilibrium framework, where the less developed transition country converges to its developed counterpart. In contrast to the usual SDGE models' solution of symmetric countries and simulations around a steady state (business cycle perspective), we focus on the transition path in key macroeconomic variables, thus our model is a perfect foresight DGE.

Since we explain the convergence paths of a transition economy, among them the significant trend real appreciation observed in reality, we propose an extension to the NIM framework to address this specific feature. Namely, we introduce explicit investment in quality as an additional production factor and thus recognize the produced final good as a mixture of quality and quantity. This amendment allows us to theoretically-consistently motivate the real exchange rate appreciation otherwise unexplained in the existing literature.

We provide a calibration for the Czech Republic as a typical Central and Eastern European Transition Country that has gone through the process of integration into the European Union. Conditional on the calibration, we demonstrate the ability of the model to consistently replicate the trend real exchange rate appreciation of the local currency, the differential in return on assets, and the dynamics in output and balance of payments components. In addition, we provide long-run simulations for the per capita GDP convergence and real exchange rate path as the two key equilibrium trajectories in 'gap' models.

1. Introduction

One of the most challenging tasks for policy makers in an emerging market converging open economy is to correctly judge and predict the dynamic development of endogenously determined key policy-relevant variables, since the long-term trajectories of these variables define the 'equilibrium' trends and thereby anchor the monetary policy models in practice. These so called 'gap models' are based on deviations from trends in endogenous variables and are currently used by many inflation targeting countries, among them, for instance, the Czech Republic, Norway, and Romania (see Coats et al., 2003). For this purpose, a coherent explanation of the trends in the theoretically founded framework of New International Trade Theory and simulations of those trends into the future is of utmost topical importance for correct policy implementation. To contribute to this task, the paper analyzes the potential of two-country dynamic general equilibrium modeling initiated by the so-called New International Macroeconomics (henceforth *NIM*). The paper offers a promising extension to the canonical NIM framework which may be useful for the assessment of convergence of emerging market economies.

NIM models have become increasingly popular in the recent past. The reason is that they are able to provide a rigorous microfoundation for many observations which are puzzling from the perspective of the standard DSGE models (such as persistent deviations from the PPP or low volatility in the relative price of nontraded goods). Thus, this type of model may be a suitable tool not only for academic curiosity to explain certain puzzling phenomena, but also for policy purposes. Typical features of the NIM framework include monopolistic competition, heterogeneity of production entities and trade self-selectivness, as in Melitz (2003). The framework is used, for example, by Ghironi and Melitz (2005) to explain international business-cycle dynamics, by Naknoi (2006) to decompose real exchange rate movements, by Bergin and Glick (2005) to study the behavior of price dispersion during episodes of international economic integration, and by Bergin and Glick (2006) to explain the low degree of volatility in the relative price of nontraded goods. Since the NIM framework seems to be better microfounded than standard open-economy dynamic general equilibrium models, it seems to be more promising as a tool for welfare evaluation of policy regimes. Naknoi et al. (2005) use the NIM framework to compare the benefits and costs of fixed versus flexible exchange rate regimes and Baldwin and Okubo (2005) integrate the NIM approach into a New Economic Geography model and derive a set of useful normative assessments and positive political-economy predictions of economic integration.

Recently, Bayoumi et al. (2004) construct a DSGE model with NIM features and calibrate it for a transition economy (the Czech Republic). This is an important step, since the macroeconomic dynamics of transition economies are even more puzzling from the perspectives of standard DSGE models than in the case of advanced economies. Unfortunately, the model of Bayoumi et al. (2004) does not address any specific transition feature and thus its applicability for convergence projections or policy prescriptions may be limited[1]. Nevertheless, the NIM framework may still be a promising tool for explaining the pace of transition countries if the framework is married with structural issues relevant for transition economies. Structural stories are better suited for understanding important phenomena of the external position of emerging

[1] Thus, it is not surprising that the model is not able to replicate the significant observed pace of real exchange rate appreciation in Central and Eastern European countries.

market economies and can provide a more solid basis for understanding, explaining, and possibly forecasting the real exchange rate path.

Recently, many authors have suggested that quality improvements might play a role among the determinants of real exchange rate appreciation of transition economies and the symptoms of quality investments in transition economies are found in empirical studies. Studies appealing to a quality-driven real exchange rate for tradables, such as Broeck and Slok (2006) or Égert and Lommatzsch (2004), find that quality improvements of tradable goods in catching-up economies are a source of real exchange rate appreciation. Also, on the example of the Czech Republic, Podpiera (2005) shows that the large gains in physical quantities exported were concurrently observed with improving terms of trade, which mirrors quality improvements. At the same time, quality improvements are not accounted for by the statistical offices in transition economies, such as the Czech Republic, Hungary, Poland, Slovakia, or Slovenia (see Ahnert and Kenny (2004) for a comprehensive survey). In addition, according to assessments of the quality bias of the consumer price index in the Czech Republic, see for instance Hanousek and Filer (2004), the inflation overstatement could have been as high as 5 percentage points a year in the first decade of economic transformation. Therefore, quality-unadjusted price indexes might well be responsible for a substantial part of the pace of real exchange rate development in a transition economy.

In order to capture the key features of an emerging market economy and simulate the transition dynamics in the key macroeconomic variables in the consistent framework of general equilibrium we use a deterministic model in aggregate variables. We build our model on postulates developed by Ghironi and Melitz (2005) and extend the framework. Canonical NIM models, such as by Ghironi and Melitz (2005), can give only a limited insight into the understanding of the external position of emerging market economies. The reason is that the production side operates with one production factor (labor) only. This feature does not address additional important factors of production capacities. In particular, in this paper we argue that for successful replication of the pace of relative prices of goods produced in an emerging, converging, economy, in terms of goods prices of an advanced economy, it is necessary to enrich the production structure by an additional factor, which we interpret as investments in quality. In addition, the model allows for non-trivial cross-border asset ownership, i.e., modeling of foreign direct and portfolio investment. Our model is solved for the transition dynamics of a transition country which is converging to its more advanced counterpart. Thus, it contrasts with the standard DSGE models, which aim at explaining deviations from exogenously given long-run trends.

In an example model calibration for the Czech economy, involving a (continuous) drop in fixed exporting costs and a drop in portfolio adjustment costs in 2000 following financial liberalization, we succeed in replicating the trends of all the endogenously modeled variables, such as the real exchange rate, the consumption and investment to GDP ratios, foreign direct and other investment balances, the export, import, and trade balance to GDP ratio, and real return on assets. We conclude that, conditional on the projections, a policy tightening in the Czech real interest rate compared to the EU15 is expected in the future in order to align the Czech excess real return with the trend trajectory.

The rest of the paper is organized as follows: Section 2 describes some relevant stylized facts and Section 3 presents the two-country model. Section 4 contains the calibration and explains the dynamics of some of the endogenous variables, and Section 5 concludes. The Appendix contains a detailed derivation of the model, reformulates the model using a recursive form and discusses the numerical techniques used to solve the model.

2. Some Stylized Facts

An open emerging market economy passes important milestones on its transition to a developed fully functioning market economy. Among them, undoubtedly the most crucial one is the build-up of the political, legal, and institutional infrastructure. From the economic point of view, the prime policy interest is often focused on privatization and full liberalization, i.e., price, current account, and financial account liberalization. These measures are meant to foster economic development and facilitate speed-up in the economic convergence process. The evidence on the positive effects of current account liberalization is largely documented in the literature. Fischer et al. (1996), who used the De Melo et al. (1996) liberalization index, which comprises the degree of liberalization of internal markets, of external markets, and of private sector entry, established a positive link between cumulative liberalization and output dynamics in a panel of twenty transition economies. Similarly, Sachs (1996) confirms the aforementioned relation by employing an index of reform constructed by the EBRD. Kaminski et al. (1996) also report that among other factors, liberalization and openness to international trade were the key factors underpinning the export performance in a large sample of transition economies.

In relation to income differential elimination among less and more developed countries, liberalization is often cited as a prominent factor. For instance Ben-David (1993) studied the income differentials within the European Economic Community and concludes that the income disparities started to diminish only after removal of the trade barriers among member countries. Similar empirical support can be found in the literature in the case of the financial account. Henry (2003) provides sample evidence on eighteen emerging markets and shows that following capital (financial) account liberalization, the cost of capital declines and both the capital stock growth and output growth per worker accelerate.

In a small emerging market economy external liberalization generates various effects during its economic development, enables and promotes flows of capital, boosts capital accumulation in the home economy, affects selectivity to trade in goods, and creates pressures on the terms of trade and real exchange rate. These aspects remain, however, largely unaddressed in traditional models of Open Economy Macroeconomics. Most importantly, the assumption of the purchasing power parity condition in tradable goods, see for instance Edison and Pauls (1993) or Obstfeld and Rogoff (1995), renders these types of models inapplicable for the explanation of transition economy dynamics, as the empirical evidence for emerging market economies documents significant violation of this assumption. For evidence of the trend development of the real exchange rate for tradables in Central and Eastern European transition economies, see Cincibuch and Podpiera (2006) and for evidence of small Harrod-Balassa-Samuelson type convergence, see for instance Mihaljek and Klau (2006).

The trend real exchange rate appreciation (also in tradables) observed in the majority of CEET economies, see Cincibuch and Podpiera (2006) for recent empirical evidence, constitutes a puzzle and renders the standard models incomplete for explanation of the transition economy dynamics. Indeed, the observed inconstancy of the real exchange rate for tradables seems to be in contradiction with the view of the traditional models of Open Economy Macroeconomics, where the purchasing power parity condition in tradable goods is a standard assumption, see for instance Edison and Pauls (1993) or Obstfeld and Rogoff (1995).

The New Open Economy Macroeconomics of two-country models, such as by Ghironi and Melitz (2005), provides a solid base for tackling some of the issues, for instance that of inconstancy of the real exchange rate for tradables and endogenously determined foreign trade. It basically allows for an endogenous short-run, and possibly persistent, deviation from purchasing power parity, i.e., for an endogenously generated Harrod-Balassa-Samuelson (HBS) effect. However, the permanent, equilibrium, trend in the real exchange rate remains unaddressed. Besides, the empirical evidence of small HBS type convergence dominates the recent literature, see for instance Mihaljek and Klau (2006) or Flek et al. (2003). As already noted, the trend equilibrium in the real exchange rate is also a puzzle for the alternative stream of two-country modeling in recent literature. A standard DSGE model, even if applied to a transition country with various real and nominal rigidities, see Bayoumi et al. (2004), does not predict a long-run appreciation of the real exchange rate, despite its relatively rich structure.

3. The Two-country Model

This section presents the core of the two-country model. A more detailed discussion is provided by Brůha and Podpiera (2007a).

The two countries are modeled in a discrete time that runs from zero to infinity. The home country is populated by a representative competitive household which has recursive preferences over discounted streams of period utilities. The period utility is derived from consumption. A similar household inhabits the foreign country. Production takes place in heterogeneous production entities called firms.[2]

3.1 Firms

There is a continuum of firms in the domestic country. In each period there is an unbounded mass of potential, ex-ante identical, entrants. Firms ex-post differ by the total factor productivity: upon entry, a firm draws a shock z from a distribution $G(z)$, which has the support on $[z_L, z_U)$ with $0 \leq z_L < z_U < \infty$. This shock determines the idiosyncratic part of the firm productivity. At the end of each period, there is an exogenous probability δ that a firm is hit by an exit shock, which is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The production function maps two inputs into two outputs. One of the inputs is sunk and we label it as 'capital', while the other input is variable and is labeled as 'labor'. The variable input is available in inelastic supply in each country and is immobile between countries. Since symbols used through the paper may seem to be cumbersome, we endow the paper with two tables: Table 1 summarizes main parameters, while Table 2 summarizes endogenous variables.

[2] The production entities are called firms; however, since we aim at understanding equilibrium convergence of a transition economy, which is likely to experience a significant change in its production structure, it would be appropriate to associate production entities with *production projects*.

The first output is quality h and if the firm j uses k_j units of capital, then the quality of its product is given simply as $h_j = k_j$. Capital investment can be thus considered as an improvement in quality. The second output is the physical quantity of produced goods x. The production function is assumed to be as follows: $x_{jt}=z_jA_t\, \ell(l_{jt},k_j)$. The production function ℓ is strictly increasing in the first argument (labor), but strictly decreasing in the second argument.[3] This implies that investments into quality increase the needed labor inputs to produce physical quantities. One may think that the production of a better good requires more labor or more skilled labor. Thus, quality investment is costly for two reasons: first, it requires input k_j, and second more labor is required to produce better goods.

The productivity of a firm A_tz_j has two components: (a) idiosyncratic component z_j, which is i.i.d. across firms and which follows distribution $G(z)$ introduced above, and (b) the common component A_t. The total factor productivity (TFP) A_t pertains to the ownerships: firms owned by the domestic household enjoy at time t the productivity A_t, while firms owned by the foreign household enjoy the productivity A_t^*. This is an important assumption since if the productivity depended on the location, then the FDI flows would be perverse: from a poor to a rich country (see Brůha and Podpiera, 2007b, for more discussion). The productivity does not depend on the location of production or on the time of entry of firms (the time of entry is henceforth called *vintage*).

We assume that the final output of the firm is given by the product of quality and quantity: $q_{jt}=h_jx_{jt}$ and that this final quality-quantity bundle is what is sold at the market. This assumption reflects the nowadays standard approach of growth theoreticians, for example Young (1998). Thus, the production of the final bundle can be described as $q_{jt}=z_jA_tf(k_j,l_{jt})$, where f is given as $f(k_j,l_{jt})\equiv k_j\ell(l_{jt},k_j)$. We assume that the final bundle production function is increasing in both arguments and is homogenous of degree one. This places some restrictions on the quantity production function ℓ; the most important restriction is that ℓ should be homogenous of degree zero. The need to distinguish the quality-quantity bundle from the physical quantity (and thus to distinguish the production functions f and ℓ) is due to a need of distinguishing quality-adjusted and quality unadjusted prices (see Section 3.4 for more details). The dichotomy is crucial for explaining the observed trend real exchange rate appreciation.

The quality investment is a sunk factor, set at the time of entry, while labor can be freely adjusted. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by the stream of discounted profits. The expected value of the firm then determines the number of new entrants (see Section 3.2 for the exposition).

Since the presented model involves several kinds of goods and firms, we will use indexes to distinguish among them. To make reading of the paper easier, we introduce the following convention. Firms differ by location, ownerships, and vintage. Location of firms is distinguished by superscripts d and f, where the former stands for the *domestic* and the latter for the *foreign* country. Firms owned by household from the foreign country are denoted by the superscript *,

[3] We require that the function ℓ is strictly decreasing in the capital. If the function ℓ does not depend on the capital, the linearity of h_j in k_j will imply endogenous growth, as in Young (1998) or Baldwin, Forslid (2000). Although it may be interesting to investigate the model under the endogenous growth paradigm, this draft avoids this issue to concentrate on the potential of the introduced extension to explain convergence experience of some emerging economies.

while to the ownership of domestic household is given no special superscript. The vintage is denoted by Greek letters τ, σ, while the real time is denoted by the Latin character t, υ.

Firms produce differentiated goods, which are labeled as follows: the good produced by the firm located in the country in which the good is also sold is denoted by the superscript d, while goods imported (produced in the non-resident country) are denoted by the superscript m. The sale market is denoted as follows: goods consumed by the domestic household are without superscript, while goods consumed by the foreign household are distinguished by the superscript *.

Similarly, p_{jt}^{d} will denote the price of a good produced by a firm j located in the domestic country at time t sold to the domestic market, p_{jt}^{m} is the price of a good j imported to the domestic market from the foreign country, while p_{jt}^{m*} would be the price of a good from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market of sale.

According to the introduced convention, $\Pi_{jt\tau}^{d}$ denotes a t-period profit of the firm located in the domestic country of vintage τ and owned by the domestic household[4]. The nominal profit $\Pi_{jt\tau}^{d}$ is given as follows:

$$\Pi_{jt\tau}^{d} = \left[\kappa_{jt} p_{jt}^{d} + (1-\kappa_{jt}) \frac{s_t}{1+\mathbf{t}} p_{jt}^{m*}\right] A_t z_j f(k_j, l_{jt}) - w_t l_{jt},$$

where $0 \leq \kappa_{jt} \leq 1$ is a share of product q_{jt} sold in the domestic market, s_t being a nominal *foreign exchange rate*, and $\mathbf{t} \geq 0$ represents unit iceberg exporting costs, nominal wage is denoted by w_t and l_t is labor hired at time t. Firms of different vintages and different ownership have different levels of investment into quality; that is why $\Pi_{jt\tau}^{d}$ will be naturally different along these dimensions. Similar definitions apply to the remaining types of firms as well.

Firms – at the time of entry – decide whether to pay low sunk costs c^n and such a firm will not be eligible to export or whether to pay sunk costs c^e, $c^e > c^n$. Only firms paying c^e become eligible to export[5]. The fixed export eligibility costs may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting standards of the foreign market.
Exporting decisions of eligible firms are taken on a period-by-period basis. Therefore total nominal investment costs take the following form: $P_t(k+c^{\xi})$, $\xi \in \{e, n\}$, where P_t represents the 'ideal' price index, which is the price of both consumption and investment goods. The investments take the form of final-good demand and thus are part of the domestic absorption.

Since $c^e > c^n$, eligible firms pay larger fixed costs. This implies – as in Melitz (2003) – that in equilibrium there is an endogenous cut-off productivity value $\bar{z}$, such that firms with lower idiosyncratic productivity $z_j < \bar{z}$ will not invest to become eligible, while firms with a sufficiently high productivity level $z_j \geq \bar{z}$ will do.

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[4] Here we will use the subscript j to denote both an idiosyncratic productivity z_j and a firm, which experiences such productivity. The exact meaning should be clear from the context.

[5] The superscript on c refers to eligibility, i.e. e – *eligible* or n – *noneligible*

We assume that firm's manager maximizes the expected stream of discounted profits. The discounting respects the ownerships. Thus the value of the profit stream of the firm of vintage τ, enjoying the idiosyncratic productivity level z_j and owned by the domestic household in real terms is:

$$V_\tau^d(z_j) = \max_{\xi,k,\{l_t\}} \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t \frac{\Pi_{j\pi}^d}{P_t} - (c^\xi + k) \qquad (1)$$

where $\frac{\Pi_{j\pi}^d}{P_t}$ is the t-time real profit of a firm of vintage τ, enjoying the productivity level z_j under the optimal production plan (derived below), and the effective discount factor is given as $(1-\delta)^{\tau-t}\mu_\tau^t$, where μ_τ^t is the marginal rate of intertemporal substitution between dates τ and t. The rate of the intertemporal substitution is defined in Subsection 3.2.

The value of the firm owned by the foreign household is defined analogously with the exception that the marginal rate of the intertemporal substitution is taken from the perspective of the foreign household.

To summarize the sequencing, the timing proceeds first with the domestic and foreign households' decision about a number of new entrants in both countries. This decision is based on the expected value of firms and on expected investment costs and is described in more details in Section 3.2 below (see equations 11 and 12). Then each new entrant draws a productivity level from the distribution G and the owner decides the amount of investment into quality and whether to invest for export eligibility. Then labor demand and production (of both entrants and incumbents) take place.[6] At the end of the period, some firms experience the exit shock and shut down.

Firms differ across various dimensions: ownerships, idiosyncratic productivity variance and vintage. The ownership within each country affects the amount of investment into quality, since both households have different rates of the intertemporal substitution along the transition path. Since our computational experiment will investigate the effect of the domestic TFP A convergence to A^*, the vintage affects incentives to invest. This is because firms entering at different times have a different macroeconomic perspective of the domestic TFP and therefore firms of different vintages and ownership will invest different amounts into quality, even if they experience the same idiosyncratic productivity level. Therefore we shall define the time-varying distribution measure over firms: $\Gamma_t^d(j,\tau)$ for the firms in the home country owned by the domestic household and the star version $\Gamma_t^{d*}(j,\tau)$ will denote the analogous measure for the firms owned by the foreign household. The counterparts of firms located in the foreign country are denoted by $\Gamma_t^f(j,\tau)$, and $\Gamma_t^{f*}(j,\tau)$. The superscript convention applied to the distributions follows the one applied to firms.

3.1.1 Market Structure

The final good Q in home country[7] is composed of a continuum of intermediate goods, some of them are produced in the home country and some are imported. There is an imperfect substitution

[6] The capital is firm specific and the model lacks the usual one-lag time-to-build assumption. The time-to-build is not needed in our model since we aim at long-run dynamics, not at short-run fluctuations.

[7] The final good is consumption as well as investment good, so that Q can be interpreted as domestic absorption.

among these goods. The parameter $\theta > 1$ measures substitution among goods. The aggregate good in the domestic country is defined as:

$$Q_t = \left(\sum_{\xi \in \{d,d^*\}} \int_{\Omega^\xi} (q_{jt}^d)^{\frac{\theta-1}{\theta}} d\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f,f^*\}} \int_{\Omega_e^\xi} (q_{jt}^m)^{\frac{\theta-1}{\theta}} d\Gamma_t^\xi(j,\tau) \right)^{\frac{\theta}{\theta-1}}, \quad (2)$$

where, q_j is the output of the firm j, Ω^d denotes the set of products of firms located in the domestic country and owned by the domestic household, and Ω^{d^*} denotes the set of products of firms located in the domestic country and owned by the foreign household. Analogously, for sets of firms located in the foreign country we have: Ω^f, Ω^{f^*}. If a set is labeled by the subscript e, it reads as a subset of eligible firms: thus $\Omega_e^{f^*} \subset \Omega^{f^*}$ is the subset of goods produced by *eligible* firms owned by the foreign household located in the foreign country.[8] The final good in the foreign country is defined similarly. The market structure implies the following definition of the aggregate price index:

$$P_t = \left(\sum_{\xi \in \{d,d^*\}} \int_{\Omega^\xi} (p_{jt}^d)^{1-\theta} d\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f,f^*\}} \int_{\Omega_e^\xi} (p_{jt}^m)^{1-\theta} d\Gamma_t^\xi(j,\tau) \right)^{\frac{1}{1-\theta}},$$

where p_{jt} is the price of products of firm j at time t.

The CES market structure implies that the demand for individual producer's products in the domestic market satisfies:

$$q_{jt}^d = \left(\frac{p_{jt}^d}{P_t} \right)^{-\theta} Q_t,$$

$$q_{jt}^m = \left(\frac{p_{jt}^m}{P_t} \right)^{-\theta} Q_t.$$

Analogous formulae apply to the demand for the products in the foreign market as well.

3.1.2 Optimal Plans

The optimal production and investment plans are derived using backward induction. We present the derivation for a firm located in the domestic country and owned by the domestic household. The reader can then similarly derive optimal plans for other types of firms.

Thus, let us assume the problem of maximizing the value of a firm, under given location, ownership, and sunk investments. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. Standard results of monopolistically competitive pricing under the CES market structure suggest that prices are set as a mark-up over marginal costs.

[8] It holds that $q_j^d \in \Omega^d$ or $p_j^d \in \Omega^{d^*}$ and $q_j^{m^*} \in \Omega_e^d$, $q_j^{m^*} \in \Omega_e^{d^*}$, but $q_j^{m^*} \notin \Omega^d \setminus \Omega_e^d$ nor $q_j^{m^*} \notin \Omega^{d^{*}} \setminus \Omega_e^{d^*}$.

Nevertheless, an important issue here is that the standard assumption of symmetric equilibrium is given up: firms enjoying identical productivity levels z_j and identical capital levels k_j are supposed to price identically, but firms with different characteristics charge different prices $\{p_{jt}{}^{d}, p_{jt}{}^{m*}\}$, and obviously produce different output q_{jt}.

Simultaneously with prices, firms also decide κ_j. Brůha and Podpiera (2007b) show that - for a general neoclassical production function f - eligible firms would produce goods for both markets, i.e., $0 < \kappa < 1$ for an eligible firm. This part of the paper derives the optimal production plan for such a general production function. See Appendix A. for the derivation of the model for the specific parameterization used in calibration and policy scenario. We denote *real* quantities by the *Monotype Corsiva* scripts: $\mathcal{P}^{d}_{j\tau t} \equiv \Pi^{d}_{j\tau t} / P_t$ is the real profit of a domestic firm and $\mathcal{W}_t \equiv w_t / P_t$ is the real domestic wage. The real exchange rate will be denoted as η_t and is defined as $\eta_t = s_t P_t^* / P_t$.

Now, let us take the perspective of a non-eligible firm of vintage τ and productivity level A_t. Its real profit $\mathcal{P}^{dn}_{j\tau t}$ in a period t is given, conditional on non-eligibility status, aggregate productivity, idiosyncratic productivity z_j, as a solution to the following:

$$\mathcal{P}^{dn}_{j\tau t} = \max_{l_{jt}}\{\tfrac{p_{jt}}{P_t} A_t z_j f(k_j, l_{jt}) - \mathcal{W}_t l_{jt}\} = \max_{l_{jt}}\{[A_t z_j f(k_j, l_{jt})]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathcal{W}_t l_{jt}\}. \tag{3}$$

Similarly, the real profit of an eligible firm $\mathcal{P}^{de}_{j\tau t}$ of vintage τ in a period t is given by:

$$\begin{aligned}\mathcal{P}^{de}_{j\tau t} &= \max_{l_{jt}}\{[\kappa_{jt} \tfrac{p_{jt}}{P_t} + (1-\kappa_{jt}) \tfrac{\eta_t}{1+\mathrm{t}} \tfrac{p^*_{jt}}{P^*_t}] A_t z_j f(k_j, l_{jt}) - \mathcal{W}_t l_{jt}\} = \\ &\max_{l_{jt}}\{[\kappa_{jt} Q_t^{\frac{1}{\theta}} + (1-\kappa_{jt}) \tfrac{\eta_t}{1+\mathrm{t}} Q_t^{*\frac{1}{\theta}}][A_t z_j f(k_j, l_{jt})]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathcal{W}_t l_{jt}\}\end{aligned} \tag{4}$$

The optimal investment decision of an eligible firm located in the domestic country and owned by the domestic household, which enjoys a productivity level z_j, maximizes the value of the firm, which is given as

$$V_\tau^{de}(k_j \mid z_j) = \sum_{t=\tau}^{\infty} \mu_\tau^t (1-\delta)^{t-\tau} \mathcal{P}^{de}_{j\tau t} - (c^e + k_j), \tag{5}$$

and similarly for a non-eligible firm:

$$V_\tau^{dn}(k_j \mid z_j) = \sum_{t=\tau}^{\infty} \mu_\tau^t (1-\delta)^{t-\tau} \mathcal{P}^{dn}_{j\tau t} - (c^n + k_j). \tag{6}$$

Maximization of $V_\tau^{de}(k_j \mid z_j)$ (resp. $V_\tau^{dn}(k_j \mid z_j)$) yields the optimal demand for quality investment (capital) for eligible (resp. non-eligible) firms, and the value of a firm is:[9]

$$\mathbf{V}_\tau^{d\xi}(z_j) = \max_{k_j \geq 0} V_\tau^{d\xi}(k_j \mid z_j),$$

where $\xi \in \{n, e\}$. The value functions $\mathbf{V}_\tau^{dn}(z_j)$, $\mathbf{V}_\tau^{de}(z_j)$ implicitly define the cut-off value $\bar{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable.10

Thus it is defined as:

$$\bar{z}_\tau^d = \min_{z_j}(\mathbf{V}_\tau^{de}(z_j) \geq \mathbf{V}_\tau^{dn}(z_j)).$$

The value of a firm is given by:

$$\mathbf{V}_\tau^d(z_j) = \max_{\xi \in \{n,e\}} \mathbf{V}_\tau^{d\xi}(z_j) = \begin{cases} \mathbf{V}_\tau^{de}(z_j) & if\ z_j \geq \bar{z}_\tau^d \\ \mathbf{V}_\tau^{dn}(z_j) & if\ z_j < \bar{z}_\tau^d \end{cases},$$

and the expected value of a new entrant, owned by the domestic household, of vintage τ, V_τ^d is:

$$\mathrm{v}_\tau^d = \int_{z_L}^{z_U} \mathbf{V}_\tau^d(z) G(dz). \quad (7)$$

This completes the backward induction.

The, just derived, optimal production plan naturally induces a measure over firms. We denote $\varPhi_{\tau,t}^d$ as the t-time expected profit of a domestically-owned firm, which enters in time τ, expectation being taken with respect to that measure $\widetilde{\varPhi}_{\tau t}^d = \int_{z_L}^{z_U} \varPhi_{\tau t j}^d G(dz_j)$ and $\widetilde{c}_\tau^d$ the expected investment costs under such measure. Thus $\mathbf{V}_\tau^d = \sum_{\sigma \geq 0} \mu_\tau^{\tau+\sigma}(1-\delta)^\sigma \widetilde{\varPhi}_{\tau,\tau+\sigma}^d - \widetilde{c}_\tau^d$.

Similarly, one can express the expected real investment costs as:

$$\widetilde{c}_\tau^d = G(\bar{z}_\tau^d)c^n + (1 - G(\bar{z}_\tau^d))c^e + \int_{z_L}^{\bar{z}_\tau^d} k_j^{opt,n} G(dz) + \int_{\bar{z}_\tau^d}^{z_U} k_j^{opt,e} G(dz).$$

The first two terms correspond to the expected fixed costs, while the last two terms correspond to the expected costs of capital investment. The expected investment costs differ across locations,

[9] There are two distinct value functions, the one with the arguments V(kj | zj) and the other with zj. The first function denotes the expected value of a firm, which enjoys the productivity level z_s and invest k_j, the second function is the value under the optimal investment and therefore depends on the productivity z_j only. The functions are distinguished by fonts: the second function is typed using the bold font.

[10] It is worth to mention that the cut-off value differs across locations and vintages (since firms located in different location or firms appeared in different times face different relative prices) and across ownership (because the marginal rates of substitution in the two countries are - in general - different).

vintages, and ownerships and this is because (i) the cut-off values differ across these dimensions too (as was already described) and (ii) these dimensions also vary the optimal amount of invested capital kjopt,e and kjopt,n. Therefore – in accordance to the convention introduced above – we will denote expected investment costs in the domestic country from the perspective of the domestic household as $\tilde{c}_t^d$ and from the perspective of the foreign household as $\tilde{c}_t^{d*}$. The counterpart of these costs in the foreign country will be denoted as $\tilde{c}_t^f$ (from the perspective of the domestic household) and as $\tilde{c}_t^{f*}$ (when foreign household's perspective is taken).

3.2 Households

The home country is populated by a representative competitive household who has recursive preferences over discounted streams of period utilities. The period utilities are derived from consumption of the aggregate good. Leisure does not enter the utility and so labor is supplied inelastically. The aggregate labor supply in the domestic country is L, while L^* is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$B_t = (1 + r_{t-1}^*)B_{t-1} + \frac{1}{\eta_t}(-C_t + \mathcal{W}_t L) + \frac{1}{\eta_t}\left(\Xi_t^d - \tilde{\chi}(n_t^d)\right) + \left(\Xi_t^f - \tilde{\chi}(n_t^f)\right) - \frac{\Psi_B}{2}B_t^2 + \mathcal{T}_t \quad (8)$$

where B_t is the real bond holding[11] of the domestic household, C_t denotes consumption and r_{t-1}^* is the real interest rate of the internationally traded bond. $\Xi_t^d = \sum_{\sigma \le t}(1-\delta)^{t-\sigma} n_\sigma^d \tilde{\mathcal{P}}_{\sigma,t}^d$ is the flow of profits from the domestic firms owned by the domestic household (and Ξ_t^f is the analogous profit flow from firms located in the foreign country and owned by the domestic household), Ψ_B represents adjustment portfolio costs as in Schmitt-Grohe and Uribe (2003) to stabilize the model[12] and $\mathcal{T}_t$ is the rebate of these costs in a lump-sum fashion to the household.

The momentary utility function $u(C)$ is assumed to take the conventional constant-relative-risk-aversion form: $u(C) = \frac{C^{1-\varepsilon}}{1-\varepsilon}$, with the parameter of intertemporal substitution ϵ. As usually, the case of $\epsilon = 1$ is interpreted as $\log(C)$.

[11] Bonds are denominated in the foreign currency by our convention; however, since the model is deterministic, this assumption is completely innocent.

[12] In a strict sense, the model is stable even without adjustment costs (i.e. under $\Psi_B = 0$). The model is deterministic and therefore it would not exhibit the unit-root behavior even under $\Psi_B = 0$. On the other hand, if $\Psi_B = 0$, then the model would exhibit the steady state dependence on the initial asset holding and we do not like such a model property. Therefore we use the nontrivial adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

The number of new domestically located entrants owned by the domestic household in time t is n_t^d, while $\tilde{\chi}(n_t^d)$ represents the investment cost associated with entry of n_t^d entrants. These costs are given as follows:

$$\tilde{\chi}(n_t^d) = \tilde{c}_t^d n_t^d + \frac{\Psi_d}{2}(n_t^d)^2.$$

The first term is obvious – it is the expected[13] investment cost (where the expectation is taken with respect to the measure induced by the optimal production plan). The second term may be interpreted as adjustment costs (e.g. due to limited supply of skills needed to run firms, such as legal experts), and its purpose is to mitigate knife-edge conditions on household investments[14]. These adjustment costs are assumed to be rebated by the lump-sum fashion to households (they are included in $\mathcal{T}_t$).

Similarly, n_t^f denotes number of new entrants in the foreign country owned by the domestic household. The associated costs are given as:

$$\hat{\chi}(n_t^f) = \tilde{c}_t^f n_t^f + \frac{\Psi_f}{2}(n_t^f).$$

The two functions $\tilde{\chi}, \hat{\chi}$ differ by the terms Ψ_d, Ψ_f only. The parameter Ψ_d is the adjustment cost of investing in the resident country (i.e., in the domestic country for the domestic household and in the foreign country for the foreign household), while the parameter Ψ_f is the adjustment cost of investing in the non-resident country.

The first order conditions for the domestic household are as follows:

$$u'(C_t)(1+\Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t}(1+r_t^*)\beta u'(C_{t+1}), \quad (9)$$

$$\lim_{t\to\infty} B_{t+1} = 0, \quad (10)$$

$$\tilde{\chi}'(n_t^d)u'(C_t) = \sum_{v\geq 0}(1-\delta)^v \beta^v u'(C_{t+v})\tilde{\mathcal{P}}_{t,t+v}^d,$$

$$\eta_t \hat{\chi}'(n_t^d)u'(C_t) = \sum_{v\geq 0}(1-\delta)^v \eta_{t+v}\beta^v u'(C_{t+v})\tilde{\mathrm{P}}_{t,t+v}^f.$$

In a strict sense, the equation (10) should read as $\lim_{t\to\infty} B_t u'(C_t)\beta^t = 0$, as a combination of the transversality condition and the non-Ponzi game conditions. However, because of nontrivial bond adjustment costs $\Psi_B > 0$, such a condition reduces to a simpler form of (10). The last two optimality conditions determine the number of new entrants and read as:

$$\tilde{c}_t^d + \Psi_d n_t^d = \sum_{v\geq 0}(1-\delta)^v \mu_t^{t+v}\tilde{\mathcal{P}}_{t,t+v}^d, \quad (11)$$

$$\eta_t(\tilde{c}_t^f + \Psi_f n_t^f) = \sum_{v\geq 0}(1-\delta)^v \eta_{t+v}\mu_t^{t+v}\tilde{\mathcal{P}}_{t,t+v}^f. \quad (12)$$

[13] Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with *ex-post* realizations.
[14] If these costs were not introduce, then each agent would invest only in one country, and therefore the FDI modeling would not be possible.

The marginal rate of substitution between times t1 and t2 is defined as $\mu_{t_1}^{t_2} \equiv \beta^{t_2-t_1} \frac{u'(C_{t_2})}{u'(C_{t_1})}$.

Although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore the marginal rate of substitution does not involve the expectation operator.

The part of the model related to the foreign household is defined analogously and details of the derivations are given in Brůha and Podpiera (2007b).

3.3 General Equilibrium

The general equilibrium is defined as a time profile of prices and quantities such that all households optimize and all markets clear. Since there is no price stickiness, nominal prices are indeterminate. Therefore, only the relative prices matter. The general equilibrium requires that the market-clearing conditions hold.

The aggregate resources constraint is given as follows:

$$C_t + n_t^d \tilde{c}_t^d + n_t^{d*} \tilde{c}_t^{d*} = Q_t, \tag{13}$$

$$C_t^* + n_t^f \tilde{c}_t^f + n_t^{f*} \tilde{c}_t^{f*} = Q_t^*. \tag{14}$$

Similarly, the labor market equilibrium requires:

$$\int l_{jt} d\Gamma_t^d(j,k) + \int l_{jt} d\Gamma_t^{d*}(j,k) = L, \tag{15}$$

where L is the aggregate, inelastic, domestic labor supply.

Analogous market clearing conditions hold in the foreign country. The international bond market equilibrium requires that:

$$B_t + B_t^* = 0. \tag{16}$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that:

$$B_{t+1} = (1 + r_t^*)B_t + \eta_t X_t + (\Xi_t^f - \hat{\chi}(n_t^f)) - \frac{1}{\eta_t}(\Xi_t^{d*} - \hat{\chi}(n_t^{d*})), \tag{17}$$

where X_t is the value of *net* real exports of the domestic country expressed in the domestic currency.

The definition of the general equilibrium is standard. A more complicated task is to simulate the dynamic path, because the model is effectively a vintage type model. However, the model can be rewritten in the recursive (first-order) form, and the recursive form makes it convenient for application of a variety of efficient numerical methods. It seems that the domain-truncation approach could be the most efficient approach. The full set of equations of the model in the recursive form and a detailed discussion on methods are available in Appendix B and C.

3.4 Note on the Real Exchange Rate

The prices p_{jt} and the corresponding price indexes P_t and P_t^* are quality-adjusted prices. Therefore, the real exchange rate η_t is measured in the terms of qualities. These measures correspond to real-world price indexes only if the latter are quality-adjusted perhaps using a hedonic approach, which is rarely the case for transition countries, see Ahnert and Kenny (2004) for a survey of quality adjustments in prices. It is a fact that price indexes in transition economies are not adjusted for quality changes.

Thus, in order to obtain indexes closer to real-world measures, we have to define aggregate indexes over prices pertaining to physical quantities. Let us denote such indexes as $\wp_t$ and $\wp_t^*$. Ideally, one can compute these indexes based on theoretical-consistent aggregation. We use a simpler approximation instead and set:

$$\wp_t = \mathcal{K}_t P_t,$$

where $\mathcal{K}_t$ is the total amount of quality investment by firms selling its products in the domestic country:

$$\mathcal{K}_t = \sum_{\xi \in \{d,d^*\}} \int_{\Omega^{\xi}} k_{j\tau} d\Gamma_t^{\xi}(j,\tau) + \sum_{\xi \in \{f,f^*\}} \int_{\Omega_e^{\xi}} k_{j\tau} d\Gamma_t^{\xi}(j,\tau).$$

Nevertheless, $\wp_t$ might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect. This means that the welfare-theoretical price indexes differ from the 'average' price by the term $n^{\frac{1}{\theta-1}}$, where n is the number of available varieties and θ is the parameter of substitution in the CES function (see Melitz, 2003 for rigorous definition and derivation of the average price). Hence, quality-unadjusted CPI-based real exchange rate (empirical real exchange rate) is the correct model counterpart of the *measured real exchange rate in reality* and is defined as:

$$\eta_t^e = \left(\frac{n_t^*}{n_t}\right)^{\frac{1}{\theta-1}} \frac{\mathcal{K}_t^*}{\mathcal{K}_t} \eta_t.$$

The reader is referred to Brůha and Podpiera (2007b) for a more detailed discussion on real exchange rate measurements.

4. Calibration and Projections

This section discusses model's calibration for the Czech economy. The investigated time span runs from 1995 to 2005. The choice of the start date is motivated by the fact that by 1995 the full external (trade and financial) and price liberalization has been completed – see Roland (2004) for a comparison of transition EBRD indexes of liberalization and reforms.

We choose $L^*/L = 6$, which only says that in the model this ratio is enough to ensure that the converging economy does not significantly influence the large developed economy. When we calibrate the model, we use the iso-elastic production function $\ell(l,k) \equiv (l/k)^{1-\alpha}$ for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ϵ and was calibrated at standard vale of 2.09. The distribution G is calibrated to be uniform[15] on the interval [0,1].

The model is calibrated to replicate the observed trends in the Czech data during 1995-2005 in a set of major variables. The data for the Czech and EU15 economies has been taken from various sources: the Czech Statistical Office, the Czech National Bank, Bloomberg, and the Eurostat. Calibration respects microeconomic evidence summarized by Brůha and Podpiera (2007b). The summary of the parameters can be found in Table 1.

The fixed parameters take the respective value as can be seen in Table 1. In the case of the one transitory parameter (c^e/c^n), a path during the convergence is allowed (initial and terminal value is specified). The direction its change appears intuitive. While the productivity of the domestic firms increases, the export eligibility cost decreases along the convergence path (effects of integration). In the steady state, the transitory parameter reaches the *SS value* (terminal value).

One of the important factors for convergence is the exit rate, where the firms do exit and new firms enter. This is a typical feature of a transition economy, where the closure and start-ups of firms is relatively high. Therefore, we have chosen the value of $\delta = 0.46$. And the discount factor β takes the conventional vale of 0.95.

4.1 Czech Policy Relevant Variables

Key monetary policy variables for a small open economy such as the Czech Republic are: output, real exchange rate, and interest rate. Deriving their 'equilibrium' paths plays an instrumental role for 'gap models' which drive actual monetary policy actions. Also, understanding the long-term interaction of these variables is essential for the country's monetary integration into the European Monetary Union (participation in the ERM II and adoption of euro). Since our model is designed to simultaneously deliver all three convergence trajectories endogenously and interdependently, it can naturally be of use.

First, we interpret the convergence of the *output per capita* to the average of the EU 15. Starting with the Czech GDP per capita at the 60 % of the EU15 average in mid-1990s, and remaining at that level for the rest of the 1990s, in the early-2000s, the Czech economy started to converge more apparently, standing at roughly 70 % in 2005. The model's outcome along with the data (and forecast of the Ministry of Finance, which predicts this ratio) is displayed in the Figure 1.1. The calibration (as summarized in Table 1) of the logistic curve assumes an average growth

[15] Microeconomists usually use other distributions than uniform for modeling the distribution of productivities across firms. The usual choice is the Pareto distribution. However, since we aim at calibrating the long-run trajectories, the uniform distribution is sufficient for that purpose.

(1995-2005) in the total factor productivity of 3 % p.a. This is roughly in congruence with the other empirically found values.[16]

Second, we aim at replicating *the real exchange rate* appreciation, which has reflected the economic convergence. The real exchange rate has been appreciating and stood approximately 30 % stronger in 2005 compared to base of 1997. Figure 1.2 compares the actual real exchange rate and the model's trajectory. The series are rebased such that the value of the average of the years 1997 and 1998 of the original data equals to the model's outcome. Although this is an arbitrary normalization, the reason behind is that in order to facilitate comparison of price indexes, we need to choose a benchmark equilibrium year. Since all available estimates of the equilibrium or parity of the real exchange rate falls into these two years – a summary of the evidence is provided by Babetskii and Égert (2005) - we choose it as a benchmark equilibrium year.

The model can explain the real exchange rate appreciation due to the presence of two factors. The first is the fact that the CES aggregation implies the *love for variety*, which means that the expansion of the number of domestic production varieties can be considered a quality improvement of the domestic goods basket. This is the effect which is responsible for results of Ghironi and Melitz (2005). However, Brůha and Podpiera (2007) show by simulations that it is unlikely that solely this effect can achieve the real exchange rate appreciation observed in Visegrad-4 countries[17]. This is the reason why we introduce the second feature: *quality investment*. Our calibration implies that the quality input is quite intensive in production (α = 0.32) that causes increase in the portion of the quality in the produced quality-quantity bundle. The accumulation of the quality brings about the empirically observed exchange rate appreciation (see section 3.4).

In a strict sense, the quality of goods basket increases in both countries. However, this effect is much stronger in the converging country and it is amplified by trade and financial openness,[18] therefore the perceived quality of domestic goods increases relatively more.

The quality improvements of the domestic composite basket is the very explanation why the converging country is able to sell more and - at the same time - for relatively higher price as its total factor productivity increases. The value of the parameter θ = 4.7 that was chosen for calibration and falls into the range of parameters in the literature to replicate the empirically observed mark-ups, for instance see Ghironi and Melitz (2005) who used the value 3.8 and claimed that this value implies reasonable mark-up over average costs. Indeed, the value around 4.7 delivers the mark-up over average costs close to the observed mark-up in the Czech manufacturing industry (20-25 % on average over 1995-2005, see Podpiera and Raková, 2006).

It is worth noting that the pace of real exchange rate appreciation in the model is obtained without any explicit assumption of exogenous productivity differential in tradable and non-tradable

[16] The Czech Ministry of Finance (2006) for instance found the growth in TFP between 1-3 % during the period 1995-2005.

[17] Our calibration exercises suggest that the love-for-variety effect can account for only about one third of the observed real exchange-rate appreciation in the Czech Republic. A similar pattern holds also for the Slovak Republic, Hungary and Poland, see Brůha and Podpiera (2007a,b).

[18] That is why our model implies welfare gains from trade and financial liberalization in both countries. It also implies that these welfare gains will be more significant in a smaller (less developed) country, since such a country will benefit more from the variety expansion.

sectors (although the model displays endogenous productivity differential between traded and non-traded goods). In fact, the reason for the appreciation comes from the improvement of the domestic composite good through the variety expansion and explicit investment into quality. Moreover, hypotheses explaining real exchange rate appreciation based on exogenous productivity differential (Harrod-Balassa-Samuleson hypothesis) are empirically flawed (Mihaljek and Klau 2006 and Flek et al. 2003). Indeed, models with exogenous productivity differential imply that the terms-of-trade will remain constant.

The third crucial information for the monetary policy implementation is the *implicit 'equilibrium' trajectory of return on domestic relative to foreign assets*. Since a small open emerging market economy exhibits convergence in the output, the corresponding (neutral) level of the real interest rate is hard to judge based on the historical averages of output growth (a standard approach in developed economies). This stands in contrast to the developed foreign country in the model, where the neutral interest rate is easily set to the long run average of the output growth.

Nevertheless, deriving the neutral interest-rate level from the output growth seems intuitive as any economy can pay return on assets equal to the growth in value added. Therefore, the interest rate trajectory can be derived from the excess growth of the domestic long-term output growth over the long-term growth in the foreign country. It is apparent that as the domestic economy develops and converges to the foreign one, the real output per capita increases and the domestic country gets richer. The convergence-implied neutral, cumulative return on investment made at the beginning of the convergence process is derived from the speed of convergence.

In a small open economy, there are two channels through which the foreign investor can gain from the economic convergence. The first is the traditional channel, i.e., the real interest rate differential *vis à vis* the developed foreign country, while the second is the domestic currency real appreciation channel. The mechanics of the former channel is very standard and apparent. Since the economy growth is higher than that of the developed country, the interest received on the investment (portfolio or direct) in converging country is higher accordingly. The latter channel is mainly due to improvement in quality of the products (variety or explicit quality investment) of the converging country.

In the calibration exercise we compare model's outcome of the excess return received by the foreign investor with the actual data on excess return from the 1Y governmental bonds. The relative 1Y return in both countries (the Czech Republic vs. the Euro area, prior 1999 German governmental Bonds) is tightly linked to the monetary policy settings. The cumulative actual and modeled yield differential is shown in Figure 1.4.

In summary, our analysis suggests that while the exchange rate followed the convergence trajectory with limited deviations, somewhat more pronounced deviations can be observed in the case of the remaining two variables. The output has recorded substantial deviation from its convergence trajectory starting 1998 and has never returned fully on the trajectory, albeit getting closer to it at the end of 2005. Similarly, the cumulative excess return departed from the implied convergence trajectory starting 1999 and the distance from the trajectory even increased after 2002. These concurrent observations are in our opinion intuitive. The return on 1Y bonds has decreased as a reaction to lowering monetary policy rates (since 1999). This stimulated the economic activity and stopped the economy from departing from its convergence trajectory. Further decrease in the real policy rate from 2002 on has stimulated economic activity even more and helped to close the gap between output and its convergence trajectory. Nevertheless, as the closure of this gap is underway and the discrepancy in excess real return is high, the policy is expected to gradually react by tightening of the real policy rate.

4.2 Czech GDP Components

Consumption and investment have been steadily proportional to gross domestic product over the entire period 1995-2005. The calibrated parameters of the production function and the investment costs are chosen such that to replicate the observed shares of consumption and investment on the domestic absorption. For the Czech Republic, these shares are 72 % and 28 % (these numbers add up to 100 % since we divided government spending into consumption and investments), which complies with the hypothesis of consumption smoothing and low frictions in financing investment in the Czech economy. The actual data as well as model's outcome are presented in Figure 1.3.

Exports as well as *imports* have been gaining on importance over the studied period. The model[19] implies an increasing involvement in trade and thus replicates the tendency observed in the data. Figures 2.1 and 2.2 present the actual data and model's outcome. Slight excess in imports share of GDP over exports at the beginning on 1990s implies a negative trade balance with gradual improvement towards positive numbers at the end of the sample period, which is also observed in the data. This is elaborated more in the next subsection.

4.3 Czech Balance of Payments

The *financial account* recorded a net inflow of investment. We set portfolio adjustment costs Ψ_B and Ψ_f so as to replicate the net investment in 1995-2005. The development of financial account is shown in Figure 2.4 (positive values denote Czech net debit). As a consequence of increasing net inflow of investment, the foreign owned companies have increased their share quite rapidly. Based on the financial survey of the Czech Statistical Office among non-financial companies, in 1998, the foreign owned companies represented only one tenth of the total number of firms, while in 2004 it exceeded one forth by large margin (28 %).

Also, the real wage paid by the foreign-owned sector attained 112 % of the average wage in the economy in 2004, thus concentrating a higher productivity then the rest of the economy. The excess productivity is also apparent from the share of the value added produced in this sector, which reached roughly half of the produced total value added (46.2 %) in 2004. In addition, the share of exported value added by this sector to the total value added exported was at a considerable 45.3 % in 2004 as well.

The model responds to an improving productivity (investment driven) and decreasing export costs of domestic and foreign firms in domestic country (c^E, see Table 1), which leads to an increasing number of firms that are exporting and thus it is able to explain the increase in export dynamics that exceeded the import dynamics and in 2005 led to *trade balance* surplus. As a logical consequence of the development on the *current account*, the direct investment has produced deepening deficit in *income balance*.

Thus, the initial smoothing of consumption represented by an excess in imports of goods and services (goods for final consumption in early stages, later moderated by increasing share of

[19] The iceberg transportation cost was calibrated at 4%.

investment goods import) over exports, was replaced with stronger exporting performance and excess of export over import. This is in line with the intuition about the phases of convergence in an open transition economy as represented by the model's projection; see Figure 2.3 for the trade balance actual and simulated values.

4.4 Long-run Convergence Projection

We carried out projection of the Czech economy convergence using the calibrated model. We present scenario for the two policy relevant variables, i.e., the output convergence and the real exchange rate path. The scenario, showed in Figures 3.1 and 3.2, assumes that the Czech GDP per capita will reach the EU15 average in 2020. The path of the 'equilibrium' output suggests its fast growth in an upcoming decade (in excess of the EU15 long-term growth). Around 2015 it is anticipated to moderate towards the EU15 growth. As for the real exchange rate, the projected trend appreciation by the model is slowly moderating and stabilizing around 2010 at a level, which is roughly 45 % more appreciated than the exchange rate in 1997.

5. Conclusion

In this paper, we aim at providing an essential input for the Czech monetary policy makers - the long-run trend in the key policy relevant variables. Unlike a developed economy, which exhibits standard and settled characteristics for sufficiently long period of time and long run values (sometimes called *equilibrium*) can be obtained by averaging past observations, every emerging market economy falls short in this respect. In order to find and assess these variables for an emerging market economy, one needs a specific model that would deliver simultaneously determined their long-term trajectories. We present a two-country model, in which there is an underdeveloped economy converging to its large and developed counterpart. The presented model adds the vertical investment margin, which seems to be the crucial ingredient for successful simultaneous replication of the GDP per capita convergence to the EU and the real exchange rate appreciation.

The model calibrated for the Czech economy and EU15 shows that the symptoms of the convergence in selected policy relevant variables can be explained by decreasing export costs (direct investment enhanced) and by growing productivity in the converging country. The development of the economy is described by the endogenously determined trajectories for a large set of variables starting with gross domestic product, consumption, investment, exports and imports, direct foreign investment, and ending up with real exchange rate, and excess real return on domestic assets.

The presented modeling framework can be used to answer a number of policy questions, since the derived trends can be used for assessing the size of the medium term deviations of the output gap, real exchange rate gap, and the gap in the excess return on Czech assets. In particular, the real monetary policy conditions (excess return on assets in the converging economy) speak directly to the monetary policy. In addition, the long-run trajectories might be of high importance when considering the timing of the monetary integration of the Czech Republic and other new EU member states. For such an application, see Brůha and Podpiera (2007a).

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Appendix: Detailed Derivation of the Model

A. Model Equations under Particular Functional Form

In this part of the paper, we derive the main model equation for particular functional forms of the production function, utility function and investment cost functions. In particular, as a benchmark calibration, we use the iso-elastic production function $\ell(l,k) \equiv (l/k)^{1-\alpha}$ for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ϵ. As usually, the case of $\varepsilon = 1$ is interpreted as log (C). The distribution G of idiosyncratic shocks is uniform on the interval [0,1].

The real cost function associated with the Cobb-Douglas production function is given as follows:[20] $\mathcal{C}(q,\mathcal{W}_t,A_t,z_j,k_j) = \mathcal{W}_t \left[\frac{q}{A_t z_j k_j^{\alpha}}\right]^{\frac{1}{1-\alpha}}$.

First, we derive the optimal investment decision, and the present value of profit flows for a non-eligible firm.[21] Such a firm will supply the following quantity-quality bundle q_{jt}^d to the domestic market (at time t):

$$q_{jt}^d = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\mathcal{W}_t^{-1}[A_t z_j k_j^{\alpha}]^{\frac{1}{1-\alpha}}\right]^{\theta} Q_t\right)^{\frac{(1-\alpha)}{\alpha\theta+(1-\alpha)}},$$

the real turnover is:

$$\frac{p_{jt}^d}{P_t} q_{jt}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathcal{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

And the real profit is given by:

$$\mathcal{P}_{j\pi}^d = \frac{p_{jt}^d}{P_t} q_{jt}^d - \mathcal{C}(q_{jt}^d,\mathcal{W}_t,A_t,z_j,k_j) = \lambda z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} A_t^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

where we define:

$$\lambda \equiv \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} - \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}} = \frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)}\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}},$$

which is obviously positive.

[20] Recall that *Monotype Corsiva* fonts, such as $\mathcal{W}_t$ and $\mathcal{P}_t$ denote real variables such as real wage and real profits. Following that convention, the *Monotype Corsiva* $\mathcal{C}$ denotes real cost function.

[21] Also, in this part of the paper, we derive expression only for firms located in the domestic country and owned by the domestic agent. The expressions for other types of firms are easily derived then.

Second, we derive optimal production decisions of eligible firms. The optimal production decision implies that $q_{jt}^{d}=\left[\frac{\theta-1}{\theta}\left(\frac{MC_{jt}}{P_t}\right)^{-1}\right]^{\theta}Q_t$, and $q_{jt}^{m*}=\left[\frac{\theta-1}{\theta}\frac{\eta_t}{1+t}\left(\frac{MC_{jt}}{P_t}\right)^{-1}\right]^{\theta}Q_t^{*}$.

Some simple, but tedious, algebraic manipulations yield:

$$\kappa_{jt}q_{jt}\equiv q_{jt}^{d}=\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathcal{W}_t^{\ -1}\left(A_t z_j k_j^{\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{\theta}\frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

$$(1-\kappa_{jt})q_{jt}\equiv q_{jt}^{m*}=\left[\frac{\theta-1}{\theta}(1-\alpha)\frac{\eta_t}{1+t}\,\mathcal{W}_t^{\ -1}\left(A_t z_j k_j^{\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{\theta}\frac{Q_t^{*}}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}}.$$

This implies that $\kappa_{jt}=\dfrac{Q_t}{Q_t+Q_t^{*}\left(\frac{\eta_t}{1+t}\right)^{\theta}}$, observe that κ_{jt} does not depend on individual characteristics of firms: z_j and k_j; it depends only on relative tightness of both markets and on the real exchange rate corrected for transport costs t. Therefore, all eligible firms will sell the same share of its products to the domestic, respectively foreign markets. Thus henceforth we will simply write κ_t for κ_{jt}.

Define $\xi_t\equiv Q_t+Q_t^{*}\left(\frac{\eta_t}{1+t}\right)^{\theta}=\frac{Q_t}{\kappa_t}$. The total production of eligible firms can be written as follows:

$$q_{jt}=\left(z_j^{\theta}k_j^{\alpha\theta}\right)^{\frac{1}{(1-\alpha)+\alpha\theta}}\left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathcal{W}_t^{\ -1}\left[A_t\right]^{\frac{1}{1-\alpha}}\right]^{\theta}\xi_t\right\}^{\frac{(1-\alpha)}{(1-\alpha)+\alpha\theta}},$$

and the real turnover on the domestic and the foreign markets, respectively are given by:

$$\frac{p_{jt}^{d}}{P_t}q_{jt}^{d}=z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}}k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\kappa_t^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathcal{W}_t^{\ -1}\left[A_t\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}}Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

$$\frac{\eta_t}{1+t}\frac{p_{jt}^{m*}}{P_t^{*}}q_{jt}^{m*}=z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}}k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}(1-\kappa_t)^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{(1-\alpha)+\alpha\theta}}\left[\frac{\theta-1}{\theta}(1-\alpha)\mathcal{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}}Q_t^{*\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Real production costs of eligible firms read as follows:

$$C_{jt}=z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}}k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}A_t^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}}\mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}}\left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\theta}\xi_t\right\}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

thus, the real profit in a period t is given as:

$$\mathcal{P}^{d}_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[A_t\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \mathscr{W}_1 \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an eligible firm $P^{de}_{j\tau}$, the expected present value satisfies: $\mathcal{P}^{de}_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^{de}_{\tau}$, where we denote $\varpi^{de}_{\tau} = \lambda \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^{t}_{\tau} A_t^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}$. Similarly, the expected present value $\mathcal{P}^{dn}_{j\tau}$ of a non-eligible firm satisfies: $\mathcal{P}^{dn}_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^{nd}_{\tau}$, where we denote $\varpi^{nd}_{\tau} = \lambda \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^{t}_{\tau} A_t^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}}$.

The value of an eligible firm located in the domestic country and owned by the domestic household – which enjoys a productivity level z_j – is determined by capital investment:

$$V^{de}_{\tau}(k_j \mid z_j) = \mathcal{P}^{de}_{j\tau} - (c^e + k_j) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^{e}_{\tau} - (c^e + k_j);$$

and similarly for a non-eligible firm

$$V^{dn}_{\tau}(k_j \mid z_j) = \mathcal{P}^{dn}_{j\tau} - (c^n + k_j) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^{n}_{\tau} - (c^n + k_j).$$

If firms' managers maximize the value of firms, they choose the following capital level:

$$k_j^{opt,e} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi^{e}_{\tau}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1},$$

and the value of an eligible firm is

$$\mathbf{V}^{de}_{\tau}(z_j) = \max_{k_j \geq 0} V^{de}_{\tau}(k_j \mid z_j) = z_j^{(\theta-1)} \left[\varpi^{e}_{\tau}\right]^{\alpha(\theta-1)+1} \Im - c^e,$$

where $\Im \equiv \left[\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)} - \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)+1} \right] = \frac{1}{\alpha(\theta-1)+1} \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)}.$

Similarly, the value of a non-eligible firm is

$$\mathbf{V}^{dn}_{\tau}(z_j) = \max_{k_j \geq 0} V^{dn}_{\tau}(k_j \mid z_j) = z_j^{(\theta-1)} \left[\varpi^{n}_{\tau}\right]^{\alpha(\theta-1)+1} \Im - c^n,$$

and the optimal capital investment into quality is

$$k_j^{opt,n} = z_j^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi_\tau^n}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}. \qquad \text{(A.1)}$$

The value functions $\mathbf{V}_\tau^{dn}(z_j)$, $\mathbf{V}_\tau^{de}(z_j)$ implicitly define the cut-off value $\bar{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable. Thus it is defined as $\bar{z}_\tau^d = \min_{z_j}(\mathbf{V}_\tau^{de}(z_j) \geq \mathbf{V}_\tau^{dn}(z_j))$.

Also for the chosen parameterization of the production function, one can derive the labor demand. The formula is complicated, since it involves summation over labor demands of firms of various vintages, and is given in the next section, see (A.8), (A.9), and (A.10) below.

B. Model in the Recursive Form

In this part of the paper, we show how to transform the model into the recursive (first-order) form, which is suitable for numerical evaluation. We do it for parameterization used in Appendix A. Although it is in principle possible to apply selected (but not all) numerical techniques directly to the vintage-formulation of the model, such a strategy would be very inefficient: numerical experiments suggest that the computation time is substantially reduced when the numerical techniques are applied to the recursive formulation of the model.

The first-order form consists of dynamic and static equations. These are listed below.

B.1 Dynamic Equations

Intertemporal marginal rate of substitution:

$$\mu_t^{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{\varepsilon}, \qquad \mu_t^{*t+1} = \beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{\varepsilon}. \qquad \text{(A.2)}$$

To define *profit flows* we need to introduce the following variables

$$
\begin{aligned}
\varpi_t^{nd} &= \lambda\left(A_t^{\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{nd}, \\
\varpi_t^{ed} &= \lambda\left(A_t^{\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ed}, \\
\varpi_t^{nf} &= \lambda\left(A_t^{*\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{nf}, \\
\varpi_t^{ef} &= \lambda\left(A_t^{*\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{ef}, \\
\varpi_t^{nf*} &= \lambda\left(A_t^{*\theta-1} \mathcal{W}_t^{*-(\theta-1)(1-\alpha)} Q^*{}_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{nf*}, \\
\varpi_t^{ef*} &= \lambda\left(A_t^{*\theta-1} \mathcal{W}_t^{*-(\theta-1)(1-\alpha)} \xi^*{}_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{*t+1}\varpi_{t+1}^{ef*}, \\
\varpi_t^{nd*} &= \lambda\left(A_t^{\theta-1} \mathcal{W}_t^{*-(\theta-1)(1-\alpha)} Q^*{}_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{nd*}, \\
\varpi_t^{ed*} &= \lambda\left(A_t^{\theta-1} \mathcal{W}_t^{*-(\theta-1)(1-\alpha)} \xi^*{}_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ed*},
\end{aligned}
\tag{A.3}
$$

where $\xi_t = Q_t + Q_t^*\left(\frac{\eta_t}{1+t}\right)^\theta$, and $\xi_t^* = Q_t^* + Q_t\left(\frac{\eta_t^{-1}}{1+t}\right)^\theta$.

Expected value of the stream of future profits (from the one unit of investment now) Ω_t^o are given as the sum of weighted expected values from eligible and non-eligible profits:[22] $\Omega_t^{x_1x_2} = \Omega_t^{nx_1x_2} + \Omega_t^{ex_1x_2}$, (with $x_i \in \{d, d^*, f, f^*\}$). These are given as follows:

[22] Henceforth, in order to diminish the notational burden, we use A^o in lieu of $\{A^{nd}, \ldots, A^{ef*}\}$. I.e. the bullet stands as a formal argument for ownership and location.

$$
\begin{aligned}
\Omega_t^{ed} &= \widetilde{\mathcal{P}}_t^{ed} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{ed}\left(\frac{\varpi_t^{ed}}{\varpi_{t+1}^{ed}}\right)\frac{\int_{\bar{z}_t^{d}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{d}}^{z_U} z^{\theta-1}G(z)}, \\
\Omega_t^{nd} &= \widetilde{\mathcal{P}}_t^{nd} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{nd}\left(\frac{\varpi_t^{nd}}{\varpi_{t+1}^{nd}}\right)\frac{\int_{z_L}^{\bar{z}_t^{d}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_t^{d}} z^{\theta-1}G(z)}, \\
\Omega_t^{ef} &= \frac{\widetilde{\mathcal{P}}_t^{ef}}{\eta_t} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{ef}\left(\frac{\varpi_t^{ef}}{\varpi_{t+1}^{ef}}\right)\frac{\int_{\bar{z}_t^{f}}^{z_U} z^{\theta-1}G(z)}{\int_{z}^{z_U} z^{\theta-1}G(z)}, \\
\Omega_t^{nf} &= \frac{\widetilde{\mathcal{P}}_t^{nf}}{\eta_t} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{nf}\left(\frac{\varpi_t^{nf}}{\varpi_{t+1}^{nf}}\right)\frac{\int_{z_L}^{\bar{z}_t^{f}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_t^{f}} z^{\theta-1}G(z)}, \\
\Omega_t^{ef*} &= \widetilde{\mathcal{P}}_t^{ef*} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{ef*}\left(\frac{\varpi_t^{ef*}}{\varpi_{t+1}^{ef*}}\right)\frac{\int_{\bar{z}_t^{f*}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{f*}}^{z_U} z^{\theta-1}G(z)}, \\
\Omega_t^{nf*} &= \widetilde{\mathcal{P}}_t^{nf*} + \mu_t^{*t+1}(1-\delta)\Omega_{t+1}^{nf*}\left(\frac{\varpi_t^{nf*}}{\varpi_{t+1}^{nf*}}\right)\frac{\int_{z_L}^{\bar{z}_t^{f*}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_t^{f*}} z^{\theta-1}G(z)}, \\
\Omega_t^{ed*} &= \eta_t\widetilde{\mathcal{P}}_t^{ed*} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{ed*}\left(\frac{\varpi_t^{ed*}}{\varpi_{t+1}^{ed*}}\right)\frac{\int_{\bar{z}_t^{d*}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{d*}}^{z_U} z^{\theta-1}G(z)}, \qquad \text{(A.4)} \\
\Omega_t^{nd*} &= \eta_t\widetilde{\mathcal{P}}_t^{nd*} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{nd*}\left(\frac{\varpi_t^{nd*}}{\varpi_{t+1}^{nd*}}\right)\frac{\int_{z_L}^{\bar{z}_t^{d*}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_t^{d*}} z^{\theta-1}G(z)},
\end{aligned}
$$

where first-order expressions for profits $\mathcal{P}_t^{\circ}$ and cut-off values will be given in the next subsection.

To get equations for actual realized profits Ξ_t°, we have to split it into two parts (according to eligibility): $\Xi_t^{\circ} = \Xi_t^{e\circ} + \Xi_t^{n\circ}$. The first-order equations are then:

$$
\begin{aligned}
\Xi_{t+1}^{ed} &= (1-\delta)\left(\frac{A_{t+1}^{\theta-1}\xi_{t+1}}{A_t^{\theta-1}\xi_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W_{t+1}}{W_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{ed} + n_{t+1}^{ed}\widetilde{\varphi}_t^{ed}, \\
\Xi_{t+1}^{nd} &= (1-\delta)\left(\frac{A_{t+1}^{\theta-1}Q_{t+1}}{A_t^{\theta-1}Q_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W_{t+1}}{W_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{nd} + n_{t+1}^{nd}\widetilde{\varphi}_t^{nd}, \\
\Xi_{t+1}^{ef} &= (1-\delta)\left(\frac{A_{t+1}^{*\theta-1}\xi_{t+1}}{A_t^{*\theta-1}\xi_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W_{t+1}}{W_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{ef} + n_{t+1}^{ef}\widetilde{\varphi}_t^{ef}, \\
\Xi_{t+1}^{nf} &= (1-\delta)\left(\frac{A_{t+1}^{*\theta-1}Q_{t+1}}{A_t^{*\theta-1}Q_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W_{t+1}}{W_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{nf} + n_{t+1}^{nf}\widetilde{\varphi}_t^{nf}, \\
\Xi_{t+1}^{ef*} &= (1-\delta)\left(\frac{A_{t+1}^{*\theta-1}\xi^*_{t+1}}{A_t^{*\theta-1}\xi^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W^*_{t+1}}{W^*_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{ef*} + n_{t+1}^{ef*}\widetilde{\varphi}_t^{ef*}, \\
\Xi_{t+1}^{nf*} &= (1-\delta)\left(\frac{A_{t+1}^{*\theta-1}Q^*_{t+1}}{A_t^{*\theta-1}Q^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W^*_{t+1}}{W^*_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{nf*} + n_{t+1}^{nf*}\widetilde{\varphi}_t^{nf*}, \\
\Xi_{t+1}^{ed*} &= (1-\delta)\left(\frac{A_{t+1}^{\theta-1}\xi^*_{t+1}}{A_t^{\theta-1}\xi^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W^*_{t+1}}{W^*_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{ed*} + n_{t+1}^{ed*}\widetilde{\varphi}_t^{ed*}, \qquad \text{(A.5)} \\
\Xi_{t+1}^{nd*} &= (1-\delta)\left(\frac{A_{t+1}^{\theta-1}Q^*_{t+1}}{A_t^{\theta-1}Q^*_t}\right)^{\frac{1}{\alpha(\theta-1)+1}}\left(\frac{W^*_{t+1}}{W^*_t}\right)^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}}\Xi_t^{nd*} + n_{t+1}^{nd*}\widetilde{\varphi}_t^{nd*},
\end{aligned}
$$

where the numbers of eligible and non-eligible firms distinguished by location and ownerships (i.e. n_t^o) is given in the next subsection.

Net exports used in the balance-of-payment equation (17) are given as

$$X_t = \left(X_t^d + X_t^f\right) - \eta_t^{-1}\left(X_t^{d*} + X_t^{f*}\right),$$

where we use the convention that exports are denominated in the currency of location (thus X_t^d, X_t^f are in the domestic currency). The exports of the respective types of firms satisfy the following recursive relations:

$$X_t^d = \hat{n}_t^{ed}(1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)W_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^f = \hat{n}_t^{ef}(1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)W_t^{-1}A_t^{*\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{f*} = \hat{n}_t^{ef*}(1-\kappa^*_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left(\frac{\eta_t^{-1}}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)W_t^{*-1}A_t^{*\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{d*} = \hat{n}_t^{ed*}(1-\kappa^*_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left(\frac{\eta_t^{-1}}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)W_t^{*-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where $\hat{n}_t^{e\circ}$ are 'weighted' numbers of eligible firms, which obeys the following recursive relation:

$$\hat{n}_{t+1}^{e\circ} = (1-\delta)\hat{n}_t^{e\circ_2} + n_{t+1}^{e\circ}\left[\frac{\alpha(\theta-1)\varpi_{t+1}^{e\circ}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\underline{z}_{t+1}^{\circ}}^{z_U} z^{\theta-1}G(dz)\ .$$

A similar recursive equation holds for non-eligible firms:

$$\hat{n}_{t+1}^{e\circ} = (1-\delta)\hat{n}_t^{e\circ} + n_{t+1}^{e\circ}\left[\frac{\alpha(\theta-1)\varpi_{t+1}^{n\circ}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\bar{z}_{t+1}^{\circ}} z^{\theta-1}G(dz)\,.$$

These recursive schemes are used in the next subsubsection too (when deriving the labor demand).

The rest of model dynamic equations are *balance-of-payment equation* (17), households' *budget constraint* (8), households' *Euler equations* (9), households' *equations*, which determines the *asset holdings*: (11), (12), plus the corresponding equations for the foreign household. Equation describing optimal asset holding are not in the recursive first-order form, but we can easily convert them into it (for sake of clarity, we put the equations for both agents):

$$\begin{aligned} \tilde{c}_t^d + \Psi_d n_t^d &= \Omega_t^d, \\ \tilde{c}_t^f + \Psi_f n_t^f &= \Omega_t^f, \\ \tilde{c}_t^{d*} + \Psi_d n_t^{d*} &= \Omega_t^{d*}, \\ \tilde{c}_t^{f*} + \Psi_f n_t^{f*} &= \Omega_t^{f*}. \end{aligned} \qquad \text{(A.6)}$$

where expected investment costs obey:

$$\begin{aligned}\widetilde{c}_t^{\circ} = \quad & G(\bar{z}_t^{\circ})c^n + \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{nx_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\bar{z}_{t+1}^{\circ}} z^{\theta-1}G(dz) + \\ + \quad & (1-G(\bar{z}_t^{\circ}))c^e + \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{e\circ}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\bar{z}_{t+1}^{\circ}}^{z_{tU}} z^{\theta-1}G(dz).\end{aligned} \tag{A.7}$$

B.2 Static Equations

The model has static equations too. These are mainly market clearing conditions and definitions. The market clearing conditions include the clearing of the goods markets (13), (14), international bond market clearing (16), and labor market clearing conditions. We now show how the labor market conditions look like: define $\hbar_t^o$ as

$$\begin{aligned}
\hbar_t^{nd} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{\theta-1}W_t^{-(\theta-1)}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{ed} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{\theta-1}W_t^{-(\theta-1)}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{nf} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{*\theta-1}W_t^{-(\theta-1)}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{ef} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{*\theta-1}W_t^{-(\theta-1)}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{nf*} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{*\theta-1}W_t^{*-(\theta-1)}Q^*_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{ef*} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{*\theta-1}W_t^{*-(\theta-1)}\xi^*_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{nd*} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{\theta-1}W_t^{*-(\theta-1)}Q^*_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\
\hbar_t^{nf*} &= \left(\frac{\theta-1}{\theta}(1-\alpha)A_t^{\theta-1}W_t^{*-(\theta-1)}\xi^*_t\right)^{\frac{1}{\alpha(\theta-1)+1}}.
\end{aligned} \tag{A.8}$$

Then the domestic labor demand is given as

$$L_t = \sum_{\xi\in\{e,n\}} \sum_{x\in\{d,f\}} \hbar_t^{\xi x}\hat{n}_t^{\xi x}, \tag{A.9}$$

and the foreign labor demand is given by

$$L_t^* = \sum_{\xi\in\{e,n\}} \sum_{x\in\{d,f\}} \hbar_t^{\xi x*}\hat{n}_t^{\xi x*}. \tag{A.10}$$

The labor demands should be equal to inelastic labor supply.

The only remaining definitions are those of average profits and expected cut-offs. They follow:

$$\bar{z}_t^{\circ} = \left(\frac{c^e - c^n}{\Im\left[\left[\varpi_t^{e\circ}\right]^{\alpha(\theta-1)+1} - \left[\varpi_t^{n\circ}\right]^{\alpha(\theta-1)+1}\right]} \right)^{\frac{1}{\theta-1}}, \tag{A.11}$$

for the formal argument $\circ \in \{d, d^*, f, f^*\}$, and

$$\tilde{\mathcal{P}}_t^{nd} = \lambda \int_{z_L}^{\bar{z}_{t+1}^{d}} z^{\theta-1} G(dz) \left(A_t^{\theta-1} W_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{nd}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{ed} = \lambda \int_{\bar{z}_{t+1}^{d}}^{z_U} z^{\theta-1} G(dz) \left(A_t^{\theta-1} W_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{ed}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{nf} = \lambda \int_{z_L}^{\bar{z}_{t+1}^{f}} z^{\theta-1} G(dz) \left(A_t^{*\theta-1} W_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{nf}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{ef} = \lambda \int_{\bar{z}_{t+1}^{f}}^{z_U} z^{\theta-1} G(dz) \left(A_t^{*\theta-1} W_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{ef}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{nf*} = \lambda \int_{z_L}^{\bar{z}_{t+1}^{f*}} z^{\theta-1} G(dz) \left(A_t^{*\theta-1} W_t^{*-(\theta-1)(1-\alpha)} Q^*_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{nf*}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{ef*} = \lambda \int_{\bar{z}_{t+1}^{f*}}^{z_U} z^{\theta-1} G(dz) \left(A_t^{*\theta-1} W_t^{*-(\theta-1)(1-\alpha)} \xi^*_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{ef*}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{nd*} = \lambda \int_{z_L}^{\bar{z}_{t+1}^{d*}} z^{\theta-1} G(dz) \left(A_t^{\theta-1} W_t^{-*(\theta-1)(1-\alpha)} Q^*_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{nd*}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)},$$

$$\tilde{\mathcal{P}}_t^{ed*} = \lambda \int_{\bar{z}_{t+1}^{d*}}^{z_U} z^{\theta-1} G(dz) \left(A_t^{\theta-1} W_t^{*-(\theta-1)(1-\alpha)} \xi^*_t \right)^{\frac{1}{\alpha(\theta-1)+1}} \left[\frac{\alpha(\theta-1)\varpi_t^{ed*}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)}.$$

C. Numerical Methods

This part of the appendix discusses numerical methods used to simulate the model. Basically, we have experimented with two classes of methods: (i) projection-based methods and (ii) domain-truncation methods.

Before discussing these methods, it is worth to realize a fact, which we use when applying both methods: If one can guess the time profile of the following six variables: domestic output $\{Q_t\}_{t=0}^{\infty}$, domestic real wage $\{W_t\}_{t=0}^{\infty}$, domestic consumption $\{C_t\}_{t=0}^{\infty}$, their foreign counterparts: $\{Q_t^*\}_{t=0}^{\infty}$, $\{W_t^*\}_{t=0}^{\infty}$, and $\{C_t^*\}_{t=0}^{\infty}$ the real exchange rate $\{\eta_t\}_{t=0}^{\infty}$, one can easily compute the time profile of all other endogenous variables (given exogenous and policy variables). Indeed, the algorithm is the following:

1. Given $\{C_t\}_{t=0}^{\infty}$, $\{C_t^*\}_{t=0}^{\infty}$ compute the marginal rate of substitutions $\{\mu_t^{t+1}\}_{t=0}^{\infty}$, $\{\mu^{*t+1}_t\}_{t=0}^{\infty}$ using (A.2).
2. Given $\{Q_t\}_{t=0}^{\infty}$, $\{W_t\}_{t=0}^{\infty}$, $\{Q_t^*\}_{t=0}^{\infty}$, $\{W_t^*\}_{t=0}^{\infty}$, and $\{\mu_t^{t+1}\}_{t=0}^{\infty}$, $\{\mu^{*t+1}_t\}_{t=0}^{\infty}$, it is possible to solve for $\{\varpi_t^{o}\}_{t=0}^{\infty}$, and therefore for $\{\bar{z}_t^{o}\}_{t=0}^{\infty}$; use (A.3) and (A.11).

3. Then, use backward difference equations (A.4) to compute $\{\Omega_t^o\}_{t=0}^{\infty}$, (A.7) to compute expected investment costs $\{\tilde{c}_t^o\}_{t=0}^{\infty}$ and first-order conditions (A.6) to compute the numbers of new entrants.

4. Then use the forward difference equation (A.5) to solve for profit flows $\{\Xi_{t+1}^o\}_{t=0}^{\infty}$ and (A.8), (A.9) and (A.10) to find labor demand in both countries.

5. One can use households' Euler equations to derive the optimal bond holding and from the international-bond market clearing condition (16) to derive the equilibrium interest rate $\{r_t\}_{t=0}^{\infty}$;

Now, one guesses the time profile and verifies the guess. The guess should be verified as follows:

1. Budget constraints for both households have to be satisfied: (8) and similarly for the foreign household.
2. Labor markets in both countries have to be cleared: (15) and similarly for the foreign country.
3. The balance of payment condition has to be satisfied: (17).
4. Goods markets have to be cleared as well: (13), (14).

Denote the guess of the seven variables as

$$\vec{\Re} = \{\{Q_t\}_{t=0}^{\infty}, \{w_t\}_{t=0}^{\infty}, \{C_t\}_{t=0}^{\infty}, \{Q_t^*\}_{t=0}^{\infty}, \{w_t^*\}_{t=0}^{\infty}, \{C_t^*\}_{t=0}^{\infty}, \{\eta\}_{t=0}^{\infty}\},$$

and the seven equilibrium conditions as $\left\{\zeta_t(\vec{\Re})\right\}_{t=0}^{\infty}$, where we interpret $\zeta_t(\vec{\Re}^o) = 0$ as the fulfillment of these conditions at time t for a guess $\vec{\Re}^o$. Note that the fulfillment of equilibrium condition at time t, $\zeta_t = 0$ does not depend on the value of the seven variables at time t only: it depends on their entire time profiles. It depends on future values because of expectations of profits, e.g. today's investment decisions depend on future streams of profits, cf. (11), (12), and it depends on past values because of predetermined variables in budget constraints.

In any case, the equilibrium candidate $\vec{\Re}$ is an infinite-dimensional object and for practical simulations, we have to approximate it by a finite-dimensional representation. The projection and domain-truncation methods do that in different ways.

The strategy of the projection method is the following: approximate the time profiles using an object parameterized by a low number of parameters (such as polynomials, splines, neutral networks, or wavelets). Thus approximate

$$\vec{\Re} \approx \tilde{\Re}(\Theta),$$

where Θ is a finite vector of parameters. Then the problem is to find such a vector of parameters $\vec{\Theta}$, such that the equilibrium conditions $\zeta_t(\tilde{\Re}(\vec{\Theta})) = 0$ nearly holds for all t. Judd (2002) discusses applications of the projection methods in the context of perfect foresight discrete-time models.

Another approach (called domain truncation approach) to reduce dimensionality of $\vec{\Re}$ is to set $\{Q_t\}_{t=0}^{\infty} \approx \hat{Q} = \{Q_1, \ldots, Q_N, Q_+, Q_+, \ldots, Q_+\}$, where Q_+ is the steady state of the variable Q_t (and similarly for other variables too) and to set

$\hat{\Re} = \{\hat{Q}, \hat{W}, \hat{C}, \hat{Q}^{*}, \hat{W}^{*}, \hat{C}^{*}, \hat{\eta}\}$,

and solve the system

$$\begin{aligned} \zeta_1(\hat{\Re}) &= 0, \\ \zeta_2(\hat{\Re}) &= 0, \\ &\vdots \\ \zeta_M(\hat{\Re}) &= 0, \end{aligned} \qquad (A.12)$$

for $M > N$. This is a system of $7M$ unknowns. Lafargue (1990) proposed this approach, and Boucekkine (1995) and Juillard et al. (1998) exploited the sparseness of the system to apply an efficient algorithm. Hence, the approach uses to be called as L-B-J approach (see also Gilli and Pauletto, 1998 or Armstrong et al., 1998 for further discussions about efficient implementation). The stacked system (A.12) is usually solved using Newton-based iterations. When applied to the model presented in this paper, we cannot use efficient algorithms for sparse systems unless $\delta = 1$. The case of $\delta = 1$ is the only case, when the Jacobian of (A.12) is sparse.

Table 1: Summary of Model Parameters

Parameter	Description	Value
θ	elasticity of substitution	4.71
β	discount factor	0.95
α	capital share	0.32
δ	exit rate	0.46
t	iceberg transportation cost	0.04
ϵ	elasticity of intertemporal substitution	2.09
m	auxiliary parameter for A^H_t	9.19
n	auxiliary parameter for A^H_t	12.03
τ	auxiliary parameter for A^H_t	4.89
A^H_{SS}	terminal value of domestic productivity	10.00
A^F	foreign productivity	10.00
c^N	fixed cost (non-eligible firms)	4.56
c^E_{SS}/c^N_{SS}	terminal ratio of fixed cost	2.10
c^E_{ini}/c^N_{ini}	initial ratio of fixed cost	3.48
$1/\psi_d$	adjustment cost parameter (domestic investment)	0.22
$1/\psi_f$	adjustment cost parameter (cross-country investment)	0.01
ψ_B	adjustment costs parameter (bond holding)	0.01
L^*/L	relative size of labor force	6.00

Note: A^H_t evolves according to the logistic curve: $A^H_t = A^H_{ss}*(1+m*\exp(-t/\tau))/(1+n*\exp(-t/\tau))$

Table 2: Summary of Endogenous Variables

Firm Decision	ℓ_{jt}	labor demand at time t
	k_j	capital investment
	κ_t	share of output sold to the domestic market (eligible firms)
	p_{jt}, $\wp_{jt}$	quality -adjusted and -unadjusted prices charged by a firm
	$\bar{z}_t^d$	eligibility cut-off
Prices	s_t, η_t	nominal / real exchange rate
	P_t, P^*_t	domestic / foreign price level
	w_t, $\mathcal{W}_t$, w^*_t, $\mathcal{W}^*_t$	domestic and foreign nominal / real wage
	r^*_t	interest rate on international bonds
Profits and Values of Firms	$\Pi^d_{j\pi}$, $\mathcal{P}^d_{j\pi}$	nominal / real period profit
	$V_\tau^{de}(k_j \mid z_j)$	the net present value of real period profits of a firm with productivity z_j and invested capital k_j under the optimal production plan
	$V_\tau^{de}(z_j)$	the value of a firm with productivity z_j, i.e. $V_\tau^{de}(k_j \mid z_j)$ under the optimal decision about capital investment
	$\mathbf{V}^d_\tau$	the expected value of the firm
Macro aggregates	Q_t, Q^*_t	domestic / foreign absorption (real)
	C_t, C^*_t	domestic / foreign consumption (real)
	B_t	bonds holding by domestic household
	Ξ^d_t, Ξ^f_t	profit flows from domestic (foreign) firms to the domestic household
	Ξ^{d*}_t, Ξ^{f*}_t	profit flows from domestic (foreign) firms to the foreign household
	$\mathcal{K}_t$, $\mathcal{K}^*_t$	capital (quality inputs) available in the domestic / foreign country
	n_t, n^*_t	number of varieties available in domestic / foreign country
	$\mathcal{T}_t$, $\mathcal{T}^*_t$	rebate of adjustment costs (in lump-sum manner) to the domestic / foreign household
Household Decisions	μ_t^{t+1} $\mu^{*\,t+1}_t$	marginal rate of intertemporal substitution of domestic / foreign household
	n^d_t, n^f_t	New entrants (owned by the domestic agent) located at the domestic / foreign market
	n^{d*}_t, n^{f*}_t	New entrants (owned by the foreign agent) located at the domestic / foreign market
	$\tilde{c}_t^d$, $\tilde{c}_t^f$	expected real investment costs from the perspective of the domestic household (per firm located at the domestic / foreign market)
	$\tilde{c}_t^{d*}$, $\tilde{c}_t^{f*}$	expected real investment costs from the perspective of the foreign household (per firm located at the domestic / foreign market)

Note: for firm decisions and value functions, only variables linked to the domestic household and firms located in the domestic country are displayed. Analogous symbols apply to remaining types of firms.

Figure 1: Czech Policy Relevant Variables

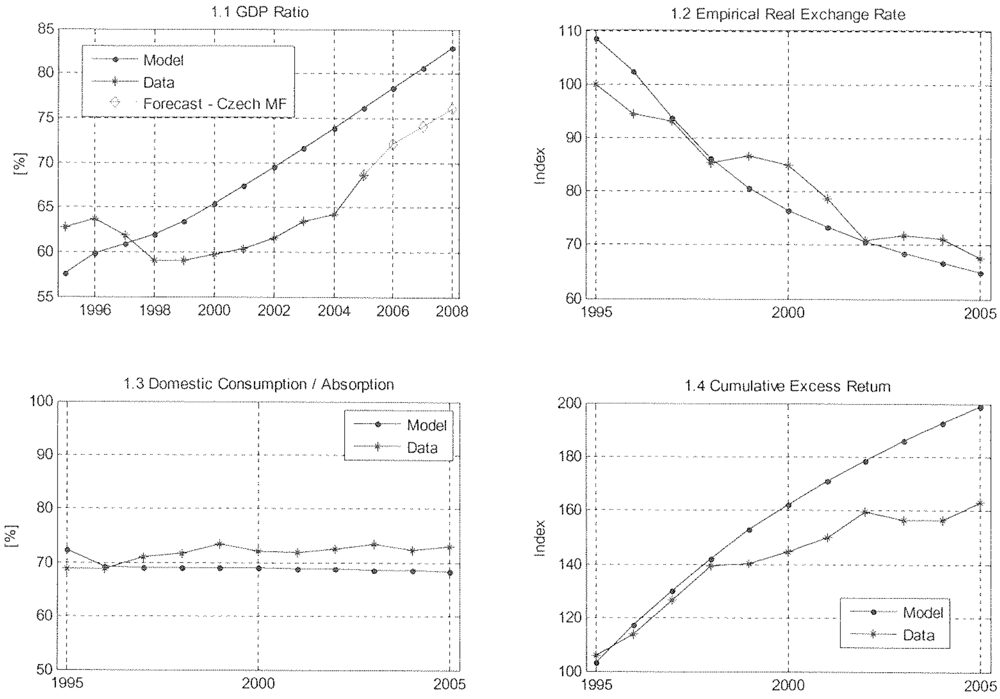


Figure 2: Czech Balance of Payments

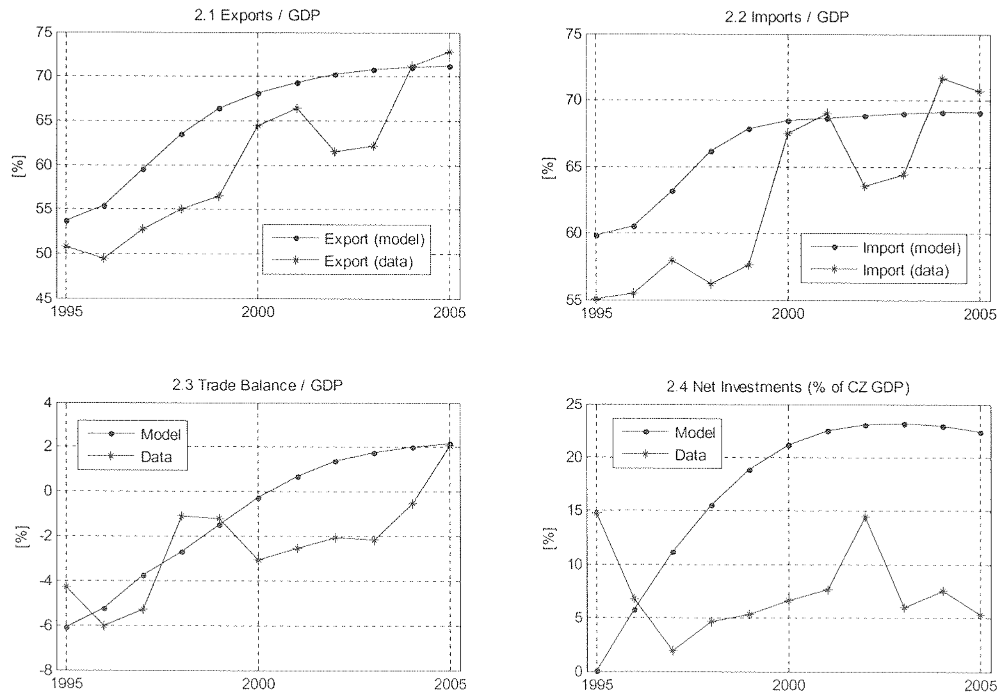
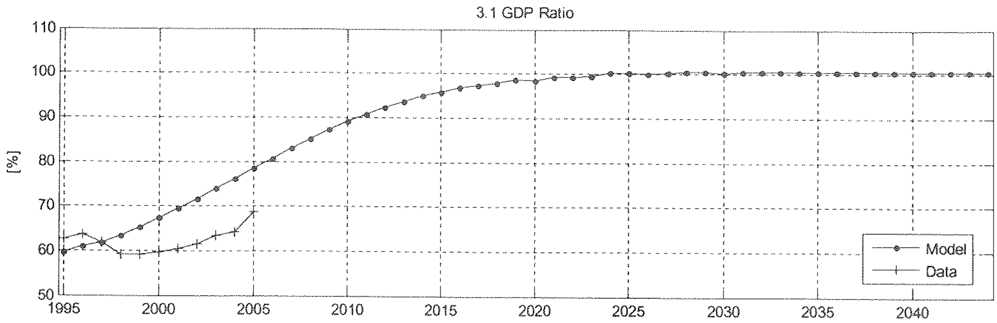
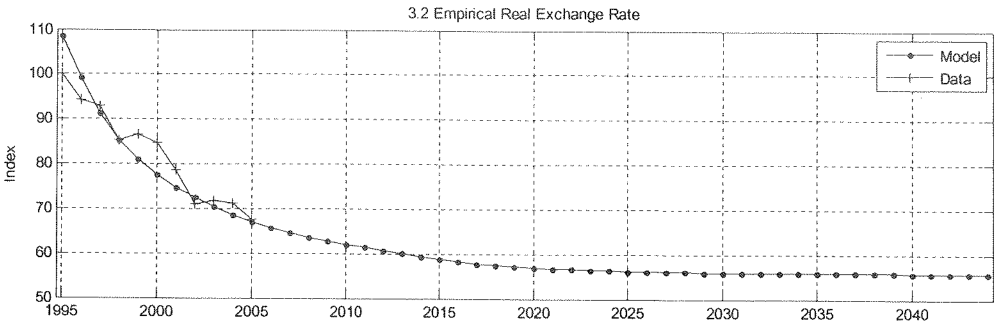


Figure 3: Long-run Convergence Projection





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