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Measures of Potential Output from an Estimated

DSGE Model of the United States

Michel Juillard, Ondřej Kameník, Michael Kumhof and Douglas Laxton *

Abstract

This paper develops a DSGE model for the United States that features rational inflation inertia and persistence. The model is estimated with Bayesian-estimation techniques and time-varying inflation objectives to account for movements between regimes. After showing that the model produces forecasts that are quite competitive with other methods we use the forecasts of the model to generate more robust Hodrick-Prescott filter end-of-sample estimates of the output gap.

JEL Codes: E31, E32, E52, C11.
Keywords: Bayesian estimation, inflation inertia, monetary policy, output gap.

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Nontechnical Summary

Dynamic general equilibrium models embedding nominal rigidities give rise to the so-called New Keynesian Phillips Curve (NKPC). These models have been quite successful in capturing many aspects of the dynamics of aggregate inflation and output. But some important problems remain. The most important is arguably the lack of inflation inertia and inflation persistence, and consequently the lack of significant real costs of disinflations. Inflation inertia refers to the delayed and gradual response of inflation to shocks, while inflation persistence refers to prolonged deviations of inflation from steady state following shocks. Another empirical problem in New Keynesian models is the very small contribution of technology shocks to macroeconomic dynamics.

We address both problems. Larger inflation inertia and persistence in our model is obtained by three interrelated mechanisms. First, the real marginal cost, the main driving force of inflation, is itself inertial. In addition to sticky wages, we propose pricing rigidities in the user cost of capital. Second, the sensitivity of inflation to marginal cost is low. This is achieved by our generalized upward-sloping marginal cost curve derived from a quadratic cost of deviations of an individual firm's output from industry-average output. Third, for a given marginal cost, firms' optimal pricing behavior, in which they set not only the initial price level but also their own updating rule, implies that past inflation is a very important determinant of current inflation. The second problem of the small contribution of technology shocks to macroeconomic dynamics is addressed by an innovative structure of the stochastic process for technology.

The model is estimated with Bayesian techniques. The paper evaluates the model's performance in several ways, including statistical fit, out-of-sample forecasts, comparison with other models, and forecast revisions as new information arrives. These results suggest that the model is quite competitive when compared to other methods, and we therefore have some confidence in the model's ability to fit the data.

We exploit the forecasting performance of the model to develop more reliable real-time measures of the output gap using the Hodrick-Prescott (HP) filter. We do this by using the model's forecasts for per capita GDP in each quarter to construct 2-sided measures of the output gap. We compare the performance of a traditional 1-sided HP filter with our 2-sided measures for the interesting downturn in periods 2000Q1 through 2002Q4. We find that our 2-sided measure of the output gap interprets the episode better.

1. Introduction

A large body of research in monetary theory uses the assumption of nominal rigidities embedded in dynamic general equilibrium models. This model class, which gives rise to the so-called New Keynesian Phillips Curve (NKPC), has been quite successful in capturing many aspects of the dynamics of aggregate inflation and output. But some important problems remain, and have recently been much discussed. The most important is arguably the lack of inflation inertia and inflation persistence, and consequently the lack of significant real costs of disinflations, in those versions of New Keynesian models that insist on rigorous microfoundations and rational expectations. Inflation inertia refers to the delayed and gradual response of inflation to shocks, while inflation persistence refers to prolonged deviations of inflation from steady state following shocks. We propose three interrelated ways in which a rational expectations model can address this problem, and subject their contribution to a Bayesian econometric evaluation. Another empirical problem in New Keynesian models is the very small contribution of technology shocks to macroeconomic dynamics. We motivate and introduce a way of modeling technology shocks that increases their contribution to the business cycle.

Given strong empirical evidence on inflation inertia[1] and on sizeable sacrifice ratios during disinflations[2], the inability of New Keynesian models to generate these effects is potentially a serious shortcoming. We survey the literature that has struggled with this problem, and then suggest a new approach. Ours is a structural, optimizing model with rational expectations. It relies neither on learning nor on ad hoc lagged terms in the Phillips curve.

The difficulties with the empirical performance of New Keynesian models have led different researchers to very different conclusions about the usefulness of structural modeling of the inflation process. On the one hand Rudd and Whelan (2005a/b/c) conclude that current versions of the NKPC fail to provide a useful empirical description of the inflation process, especially relative to traditional econometric Phillips curves of the sort commonly employed at central banks for policy analysis and forecasting. On the other hand we have papers like Cogley and Sbordone (2005) and Coenen and Levin (2004). The former conclude that the conventional NKPC provides a good representation of the empirical inflation process if a shifting trend in the inflation process is allowed for. However, the work of Paloviita (2004) suggests that a shifting inflation trend, while useful to improve the empirical fit of the NKPC, does not remove the need for an additional lagged inflation term. Coenen and Levin (2004) also find in favor of the conventional NKPC, in this case conditional on the presence of a stable and credible monetary policy regime and of significant real rigidities. But on the other hand, Altig, Christiano, Eichenbaum and Linde (2005), who employ similar real rigidities, continue to use indexation to lagged inflation to obtain a good fit for their model. The majority of the profession seems to hold an intermediate view, exemplified by Galí, Gertler and Lopez-Salido (2005), who find that backward-looking price setting behavior, of the sort that would generate high intrinsic inflation inertia, is quantitatively modest but nevertheless statistically significant.[3] The research program exemplified by Altig, Christiano, Eichenbaum and Linde (2005) and Eichenbaum and Fisher (2004) also falls into this category.

The view that there is significant structural inflation inertia left to be explained is our working hypothesis in this paper. In reviewing the currently dominant approaches that are based on

[1] Mankiw (2001), Fuhrer and Moore (1995).
[2] Gordon (1982, 1997).
[3] However, Rudd and Whelan (2005c) criticize that result on various empirical grounds.

the same working hypothesis, we find it useful to distinguish models that do or do not rely on rational expectations.

The latter category includes learning models such as Erceg and Levin (2003), and 'sticky information' as in Mankiw and Reis (2002). This literature mostly, although not exclusively, concentrates on private sector learning, or information acquisition, about monetary policy.[4] As such it has been successful in explaining inflation behavior observed during transitions between monetary regimes. But unless it is expanded to cover learning about all shocks in the model, it has less to say about the persistence observed during periods of stable monetary policy, meaning persistence in response to non-monetary shocks that affect the driving terms of pricing. Furthermore, learning is not the only candidate to explain persistence during transitions; structural inertia in a rational expectations model may be another. While we do feel that learning plays a very important role, the task we set ourselves in this paper is to see how far a rational expectations model alone, but one that features realistic pricing rigidities, can take us. But at the same time we want to take account of the results of Cogley and Sbordone (2005) concerning the importance of a shifting trend in the inflation target. As such, our model allows for a unit root in the central bank's inflation target and uses data on long-term inflation expectations to identify the shocks to that target.

A popular approach to introducing inflation inertia into rational expectations models is the 'hybrid' NKPC, introduced by Clarida, Galí and Gertler (1999) and Galí and Gertler (1999). This combines a rational forward-looking element with some dependence on lagged inflation. A similar role is played by indexation to past inflation in the work of Christiano, Eichenbaum and Evans (2005) and other more recent work. But Rudd and Whelan (2005c) make an important point concerning both of these approaches: At least as far as price setting is concerned, their microfoundations are quite weak, and they are as open to the Lucas critique as the traditional models they seek to replace. In our work we replace these pricing assumptions with rational, forward-looking optimization that is nevertheless capable of generating significant inertia. Moreover, in an important sense our price setting assumptions are less restrictive than even those of the conventional Calvo model.

Another area of active research within rational expectations models has been models of firm-specific capital.[5] A textbook treatment is contained in Woodford (2003). Often, as in the work of Altig, Christiano, Eichenbaum and Linde (2005) and Eichenbaum and Fisher (2004), this has been combined with indexation to generate inertia, and it is not always clear which of the two is the more important factor, but the work of Coenen and Levin (2004) suggests that firm-specific capital can be powerful even without indexation. The work of Bakshi, Burriel-Llombart, Khan and Rudolf (2003) shows why this is such an important idea. They demonstrate that conventional price-setting in a Calvo model without firm-specific capital has firms optimally choosing prices that imply a very large variability in demand and therefore in output. It is clear that in the real world such variability is very costly to firms, and one of the many reasons is the cost of adjusting firm-specific factors. If such factors are allowed for, an increase in the firm's price, by reducing demand, lowers marginal cost and thereby the amount by which the price optimally needs to be raised. Firm-specific factors need not be limited to capital, but can include labor adjustment costs, land, time delays to order intermediate goods, etc. In reality probably all of these are important, but modeling all of them may be too complex. We therefore adopt the same concept but simplify its modeling by way of a generalized upward-sloping short-run marginal

[4] An exception is Ehrmann and Smets (2003), who analyze cost-push shocks.
[5] Many authors have combined this with a non-constant elasticity of demand.

cost curve. Our analytical results are indistinguishable, in substantive terms, from a model with firm specific capital. We would also add that firm-specific factors may not be the main consideration for a firm in avoiding output/demand volatility. Instead, highly volatile output demand induced by frequent relative price changes is likely to damage customer relationships, and the induced volatility in intermediate inputs demand will also damage relationships with suppliers of those inputs. The recent ECB (2005) survey evidence on price setting suggests that firms do indeed cite customer relationships more frequently than input costs as reasons for avoiding large price changes. Such notions are encompassed in a generalized upward-sloping marginal cost curve.

Our work generates inflation inertia for three interrelated reasons. First, real marginal cost, the main driving force of inflation, is itself inertial. Second, the sensitivity of inflation to marginal cost is low. And third, for a given marginal cost, firms' optimal pricing behavior implies that past inflation is a very important determinant of current inflation.[6] We briefly explain each of these points in turn.

In realistic dynamic models it is common, and supported by independent empirical evidence, to introduce real rigidities that imply a delayed response of aggregate demand to shocks. This in turn implies a delayed response of marginal cost. Our own model follows this literature, in assuming habit persistence in consumption, investment adjustment costs, and variable capital utilization. But in addition we assume that *each* of the components of marginal cost is subject to pricing rigidities. Wage rigidities are commonly assumed, but if capital enters the production function, sticky wages alone may not be sufficient to make overall marginal cost inertial. We propose that user costs of capital are in fact also rigid. Interest rate margins on corporate bank loans and interest rates on corporate bonds change only infrequently, and so do dividend policies. As such, it seems doubtful that the prices firms pay for their capital services are as volatile as suggested by standard models. Of course we do not provide direct empirical evidence on this question in this paper, but we can and do assess the implications of this assumption for the statistical fit of our model.

The sensitivity of inflation to marginal cost is low, and it depends on the same factors as in models of firm-specific capital. Our generalized upward-sloping marginal cost curve is derived from a quadratic cost of deviations of an individual firm's output from industry-average output. The consequence is that the sensitivity of inflation to marginal cost is decreasing in the steepness of the marginal cost curve and in the price elasticity of demand. The same type of quadratic term also features in wage setting and in the setting of user costs by an individual provider of capital, referred to as an intermediary.

Firms' price setting behavior in our model is both optimizing and forward-looking, yet past inflation becomes an important determinant of current inflation. We think of a price setting firm as operating in an environment with positive trend inflation where collecting and responding to information about the macroeconomic environment is costly, which is documented as an important consideration for real world price setting in Zbaracki, Ritson, Levy, Dutta and Bergen (2004). This idea, which is different from the menu costs idea of Akerlof and Yellen (1985), can be formally modeled, see Devereux and Siu (2004). But more commonly, as in Christiano, Eichenbaum and Evans (2005) and a large literature that follows Yun (1996), it is used—without explicit modeling of the adjustment costs—as a rationale for models in which firms *change* prices every quarter but only *reoptimize* their pricing policies more infrequently.

[6] ECB (2005) refers to the first two factors as extrinsic persistence, and to the third as intrinsic persistence.

As such these models are not inconsistent with the recent empirical evidence for price setting of Bils and Klenow (2004), Klenow and Kryvtsov (2004), and Golosov and Lucas (2003), which points to an average frequency of price changes (in the US) of once every 1.5 quarters for consumer prices. We follow this literature, which therefore posits that in intervals between reoptimizations firms follow simple rules of thumb. The critical question is, what is a sensible rule of thumb? The Yun (1996) approach assumes that firms set their initial price and thereafter update at the steady state inflation rate. But of course this is the approach that has been found to give rise to almost no inflation inertia in New Keynesian models. The indexation approach of Christiano, Eichenbaum and Evans (2005) addresses that problem by assuming that non-optimizing firms index their price to past inflation. But in both cases firms can really only choose their initial price, while the rule of thumb itself is not a choice variable. This feature is what has been criticized by Rudd and Whelan (2005b) and some others as not consistent with the Lucas critique, or *ad hoc*.

We adopt a different approach—firms can *choose* both their initial price level and their rule of thumb, specifically the rate at which they update their own price, the ‘firm-specific inflation rate’.[7] Their objective is to keep them as close as possible to their steadily increasing flexible price optimum between the times at which price changing opportunities arrive. Furthermore, as mentioned above, their choice is subject to an increasing firm-specific marginal cost curve, which biases firms towards adjusting mainly their updating rate unless the shocks they face are transitory. They would otherwise experience excessive relative price and therefore output volatility throughout the duration of a pricing policy. At any point in time, this combination of firm-specific pricing policies and firm-specific marginal cost curves makes the historic pricing decisions of currently not optimizing firms an important determinant of current inflation. Or in other words, past inflation is an important determinant of current inflation. This is true even though firms that do optimize do so under both rational expectations and fully optimizing behavior. We emphasize that this modeling of price setting, by letting firms choose two instead of one pricing variable optimally, imposes fewer exogenous constraints on the firm’s profit maximization problem than either the Calvo-Yun model or a model with indexation. In this important sense the model is therefore less *ad hoc*.

Finally, note that if price setters behave as in our model, their behavior can be quite similar to that implied by learning or sticky information in that at any time a large share of firm specific inflation rates was chosen based on macroeconomic information available at the time of the last reoptimization. We expect this similarity to become an important factor once our model is applied to transitions between different monetary regimes.

In several previous attempts to estimate DSGE models it has been common either to detrend the data or to assume that total factor productivity follows a trend-stationary process—see Juillard, Karam, Laxton and Pesenti (2005) and Smets and Wouters (2004). We argue that both approaches impose limitations on the ability of DSGE models to explain key stylized facts at business cycle frequencies such as the strong comovement between hours worked and aggregate output. We allow for a more general stochastic process where there are both temporary changes in the growth rate of total factor productivity as well as highly autocorrelated deviations from an underlying steady-state growth rate. We show that the latter assumption helps the model to generate a larger contribution of technology shocks to business cycles.

[7] The approach was first introduced by Calvo, Celasun and Kumhof (2001, 2002).

The rest of the paper is organized as follows. Section 2 presents the model and section 3 discusses the estimation methodology. Section 4 presents some Bayesian estimation results and impulse response functions. Section 5 compares the fit and forecasts of the model with other methods and section 6 uses these forecasts to construct more reliable end-of-sample estimates of the output gap. The results in this paper should be encouraging for researchers that are attempting to build DSGE models for both forecasting and policy analysis, but there are a number of extensions that we are pursuing to improve the specification of the model. Section 7 provides some concluding thoughts about useful extensions.

2. The Model

The economy consists of a continuum of measure one of households indexed by $i \in [0,1]$, a continuum of firms indexed by $j \in [0,1]$, a continuum of financial intermediaries indexed by $z \in [0,1]$, and a government.

2.1 Households

Household i maximizes lifetime utility, which depends on his per capita consumption $C_t(i)$, leisure $1 - L_t(i)$ (where 1 is the fixed time endowment and $L_t(i)$ is labor supply), and real money balances $M_t(i)/P_t$ (where $M_t(i)$ is nominal money and P_t is the aggregate price index):

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ S_t^c (1-v) \log(H_t(i)) - S_t^L \psi \frac{L_t(i)^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} + \frac{a}{1-\epsilon} \left(\frac{M_t(i)}{P_t} \right)^{1-\epsilon} \right\} . \quad (2.1)$$

Throughout, shocks are denoted by S_t^x, where x is the variable subject to the shock. Households exhibit external habit persistence with respect to C_t^i, with habit parameter ν:

$$H_t(i) = C_t(i) - \nu C_{t-1} \ . \quad (2.2)$$

Consumption $C_t(i)$ is a CES aggregator over individual varieties $c_t(i,j)$, with time-varying elasticity of substitution $\sigma_t > 1$,

$$C_t(i) = \left(\int_0^1 c_t(i,j)^{\frac{\sigma_t - 1}{\sigma_t}} dj \right)^{\frac{\sigma_t}{\sigma_t - 1}} \quad , \quad (2.3)$$

and the aggregate price index P_t is the consumption based price index associated with this consumption aggregator,

$$P_t = \left(\int_0^1 P_t(j)^{1-\sigma_t} dj \right)^{\frac{1}{1-\sigma_t}} \quad . \quad (2.4)$$

Households accumulate capital according to

$$K_{t+1}(i) = (1-\Delta) K_t(i) + I_t(i) \ . \quad (2.5)$$

We assume that demand for investment goods takes the same CES form as demand for consumption goods, equation (2.3), which implies identical demand functions for goods varieties j.

In addition to capital, households accumulate money and one period nominal government bonds $B_t(i)$ with gross nominal return i_t.[8] Their income consists of nominal wage income $W_t(i)L_t(i)$,

[8] All financial interest rates and inflation rates, but not rates of return to capital, are expressed in gross terms.

nominal returns to utilized capital $R_t^k x_t K_t(i)$, where x_t is the rate of capital utilization, and lump-sum profit redistributions from firms and intermediaries $\int_0^1 \Pi_t(i,j)dj$ and $\int_0^1 \Pi_t(i,z)dz$. Expenditure consists of consumption spending $P_t C_t(i)$, investment spending $P_t I_t(i)(1+S_t^I)$, where S_t^I is an investment shock, the cost of utilizing capital at a rate different from 100% $P_t a(x_t) K_t(i)$, where $\bar{x}=1$ and $a(1)=0$, lump-sum taxation $P_t \tau_t$, quadratic capital and investment adjustment costs, and quadratic costs of deviating from the economywide average labor supply (more on this below). The budget constraint is therefore

$$\begin{aligned} B_t(i) \;=\;& (1+i_{t-1})B_{t-1}(i) + M_{t-1}(i) - M_t(i) \\ & +W_t(i)L_t(i) + R_t^k x_t K_t(i) - P_t a(x_t) K_t(i) \\ & + \int_0^1 \Pi_t(i,j)dj + \int_0^1 \Pi_t^z(i,z)dz - P_t \tau_t(i) \\ & -P_t C_t(i) - P_t I_t(i)(1+S_t^I) \\ & -P_t \frac{\theta_k}{2} K_t(i) \left(\frac{I_t(i)}{K_t(i)} - \Delta \right)^2 - P_t \frac{\theta_i}{2} K_t(i) \left(\frac{I_t(i)}{K_t(i)} - \frac{I_{t-1}}{K_{t-1}} \right)^2 \\ & -W_t \frac{\phi_w}{2} \frac{\left(L_t(i) - L_t\right)^2}{L_t} \quad . \end{aligned} \tag{2.6}$$

We assume complete contingent claims markets for labor income, and identical initial endowments of capital, bonds and money. Then all optimality conditions will be the same across households, except for labor supply. We therefore drop the index i. The multiplier for the budget constraint (2.6) is denoted by λ_t / P_t, and the multiplier of the capital accumulation equation (2.5) is $\lambda_t q_t$, where q_t is Tobin's q. Then the first-order conditions for $c_t(j)$, B_t, C_t, I_t, K_{t+1}, and x_t are as follows:

$$c_t(j) = C_t \left(\frac{P_t(j)}{P_t} \right)^{-\sigma_t} \quad , \tag{2.7}$$

$$\lambda_t = \beta i_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \quad , \tag{2.8}$$

$$\frac{S_t^c (1-v)}{H_t} = \lambda_t \quad , \tag{2.9}$$

$$q_t = 1 + \theta_k \left(\frac{I_t}{K_t} - \Delta \right) + \theta_i \left(\frac{I_t}{K_t} - \frac{I_{t-1}}{K_{t-1}} \right) + S_t^I \quad , \tag{2.10}$$

$$\begin{aligned} \lambda_t q_t \;=\;& \beta E_t \lambda_{t+1} \Big[q_{t+1}(1-\Delta) + r_{t+1}^k \\ & +\theta_k \left(\frac{I_{t+1}}{K_{t+1}} - \Delta S_{t+1}^I \right) \frac{I_{t+1}}{K_{t+1}} + \theta_i \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I_t}{K_t} \right) \frac{I_{t+1}}{K_{t+1}} \\ & -\frac{\theta_k}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \Delta S_t^I \right)^2 - \frac{\theta_i}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I_t}{K_t} \right)^2 \Big] \quad , \end{aligned} \tag{2.11}$$

$$r_t^k = a'(x_t) \quad . \tag{2.12}$$

We will return to the household's wage setting problem at a later point, as we will be able to exploit analogies with firms' price setting. Full derivations of all first-order conditions in the paper, their transformation into a stationary system through normalization by technology and the inflation target, and their linearization, are presented in a separate Technical Appendix (available on request).

2.2 Firms

Each firm j sells a distinct product variety. Heterogeneity in price setting decisions and therefore in demand for individual products arises because each firm receives its price changing opportunities at different, random points in time. We first describe the cost minimization problem and then move on to profit maximization.

2.2.1 Cost Minimization

The production function for variety j is Cobb-Douglas in labor $\ell_t(j)$ and capital $k_t(j)$:

$$y_t(j) = \left(S_t^y \ell_t(j)\right)^{1-\alpha} k_t(j)^{\alpha} \ , \tag{2.13}$$

where

$$\ell_t(j) = \left(\int_0^1 L_t(i,j)^{\frac{\sigma_t^w - 1}{\sigma_t^w}} di \right)^{\frac{\sigma_t^w}{\sigma_t^w - 1}} \ , \ k_t(j) = \left(\int_0^1 k_t(z,j)^{\frac{\sigma^k - 1}{\sigma^k}} dz \right)^{\frac{\sigma^k}{\sigma^k - 1}} \quad , \tag{2.14}$$

where the expressions in (2.14) state that each firm employs a CES aggregate of different labor and capital varieties. Let w_t be the aggregate real wage (the cost of hiring the aggregate $\ell_t(j)$), and u_t the aggregate user cost of capital (the cost of hiring the aggregate $k_t(j)$). These are determined in competitive factor markets and discussed in more detail below. Then the real marginal cost corresponding to (2.13) is

$$mc_t = A \left(\frac{w_t}{S_t^y} \right)^{1-\alpha} (u_t)^{\alpha} \quad , \tag{2.15}$$

where $A = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. Technology S_t^y is stochastic and consists of both i.i.d. shocks to the level of technology and of highly persistent shocks to the growth rate of technology:

$$S_t^y = S_{t-1}^y g_t \quad , \tag{2.16}$$

$$g_t = g_t^{gr} g_t^{iid} \quad ,$$

$$\ln g_t^{gr} = (1-\rho_g) \ln \bar{g} + \rho_g \ln g_{t-1}^{gr} + \hat{\varepsilon}_t^{gr} \quad ,$$

$$\ln g_t^{iid} = \hat{\varepsilon}_t^{iid} \quad .$$

Let $\tilde{Y}_t = \int_0^1 y_t(j)dj$, $\ell_t = \int_0^1 \ell_t(j)dj$, and $k_t = \int_0^1 k_t(j)dj$. Given that factor markets are competitive so that all firms face identical costs of hiring aggregates of capital and labor (2.14), we can derive the following aggregate input demand conditions:

$$\ell_t = (1-\alpha)\frac{mc_t}{w_t}\tilde{Y}_t \ , \tag{2.17}$$

$$k_t = \alpha\frac{mc_t}{u_t}\tilde{Y}_t \ . \tag{2.18}$$

2.2.2 Profit Maximization

Following Calvo (1983) it is assumed that each firm receives price changing opportunities that follow a geometric distribution. Therefore the probability $(1-\delta)$ of a firm's receiving a new opportunity is independent of how long ago it was last able to change its price. It is also independent across firms, so that it is straightforward to determine the aggregate distribution of prices. Each firm maximizes the present discounted value of real profits. The first two determinants of profits are real revenue $P_t(j)y_t(j)/P_t$ and real marginal cost $mc_t y_t(j)$. In each case demand is given by

$$y_t(j) = Y_t\left(\frac{P_t(j)}{P_t}\right)^{-\sigma_t} \ , \tag{2.19}$$

which follows directly from consumer demand functions (2.7) and identical demands from investors and government (see below). Two key features of our model concern first the manner in which firms set their prices when they receive an opportunity to do so, and the cost (through excessively large or small demand) of setting prices far away from prevailing average market prices P_t. To model the latter, we assume that firms face a small quadratic cost Φ_t of deviating from the output level of its average competitor, meaning the firm that charges the current market average price. The cost is therefore

$$\Phi_t = \frac{\phi}{2}Y_t\left(\frac{y_t(j)-Y_t}{Y_t}\right)^2 \ . \tag{2.20}$$

The term Y_t in front of the quadratic term serves as a scale factor. As for price setting, we assume that when a firm j gets an opportunity to decide on its pricing policy, it chooses both its current price level $V_t(j)$ and the gross rate $v_t(j)$ at which it will update its price from today onwards until the time it is next allowed to change its policy. At any time $t+k$ when the time t policy is still in force, its price is therefore

$$P_{t+k}(j) = V_t(j)(v_t(j))^k \ . \tag{2.21}$$

As for the possibility of introducing even more general price paths, it seems natural to focus on equilibria characterized by a constant expected long-run growth rate of the nominal anchor.[9] The model can then be solved by linearizing around that growth path, in which case it is sufficient to allow firms to specify their pricing policies up to the growth rate of their price path.

[9] This includes both a constant steady state growth rate of the nominal anchor and a unit root in that growth rate, as in this paper.

This permits the use of conventional solution methods, which makes quantitative analysis much more straightforward.

Firms discount profits expected in period $t+k$ by the k-period-ahead real intertemporal marginal rate of substitution and by δ^k, the probability that their period t pricing policy will still be in force k periods from t. They take into account aggregate demand for their output (2.19). The firm specific index j can be dropped in what follows because all firms that receive a price changing opportunity at time t will behave identically. Their profit maximization problem is therefore

$$\underset{V_t,v_t}{Max} E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t (v_t)^k}{P_{t+k}} \right)^{1-\sigma_t} Y_{t+k} \right. \tag{2.22}$$

$$\left. -mc_{t+k} \left(\frac{V_t (v_t)^k}{P_{t+k}} \right)^{-\sigma_t} Y_{t+k} - \frac{\phi}{2} Y_{t+k} \left(\frac{y_{t+k}(j) - Y_{t+k}}{Y_{t+k}} \right)^2 \right] \quad .$$

We define the ratio of a new price setter's first period price to the market average price as $p_t \equiv V_t/P_t$, cumulative aggregate inflation as $\Pi_{t,k} \equiv \prod_{j=1}^{k} \pi_{t+j}$ for $k \geq 1$ $(\equiv 1$ for $k = 0)$, and the mark-up term as $\mu_t = \frac{\sigma_t}{\sigma_t - 1}$. Then the firm's first order conditions for the choice of its initial price level V_t and its inflation updating rate v_t are

$$p_t = \mu_t \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(mc_{t+k} + \phi \left(\frac{y_{t+k}(j) - Y_{t+k}}{Y_{t+k}} \right) \right)}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{(v_t)^k}{\Pi_{t,k}} \right)} \quad , \tag{2.23}$$

$$p_t = \mu_t \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k k\lambda_{t+k} y_{t+k}(j) \left(mc_{t+k} + \phi \left(\frac{y_{t+k}(j) - Y_{t+k}}{Y_{t+k}} \right) \right)}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k k\lambda_{t+k} y_{t+k}(j) \left(\frac{(v_t)^k}{\Pi_{t,k}} \right)} \quad . \tag{2.24}$$

The intuition for this result becomes much clearer once these conditions are log-linearized and combined with the log-linearization of the aggregate price index. As this is algebraically very involved, the details are presented in the Technical Appendix. We discuss the key equations here. They replace the traditional one-equation New Keynesian Phillips curve with a three-equation system in $\hat{\pi}_t$, $\hat{v}_t$ and an inertial variable $\hat{\psi}_t$:

$$E_t \hat{\pi}_{t+1} = \hat{\pi}_t \left(\frac{2}{\beta} - \delta \right) + \hat{v}_t \left((1-\delta)(1+\delta) \right) + \hat{\psi}_t \left(\delta(1+\delta) - \frac{2}{\beta} \right) \tag{2.25}$$

$$- \frac{2(1-\delta)(1-\delta\beta)}{(\delta\beta)(1+\phi\bar{\mu}\bar{\sigma})} \left(\widehat{mc}_t + \hat{\mu}_t \right) + \frac{(1-\delta)}{(1+\phi\bar{\mu}\bar{\sigma})} \left(E_t \hat{\mu}_{t+1} - \hat{\mu}_t \right) \quad ,$$

$$E_t \hat{v}_{t+1} = \hat{v}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2} \frac{\delta}{1-\delta} \hat{\psi}_t - \frac{(1-\delta\beta)^2}{(\delta\beta)^2} \frac{\delta}{1-\delta} \hat{\pi}_t \tag{2.26}$$

$$+\frac{(1-\delta\beta)^2}{(\delta\beta)^2\,(1+\phi\bar{\mu}\bar{\sigma})}\left(\widehat{mc}_t+\hat{\mu}_t\right) \quad ,$$

$$\hat{\psi}_t = \delta\hat{\psi}_{t-1} + (1-\delta)\hat{v}_{t-1} - \hat{\varepsilon}_t^{\pi^*} \quad . \tag{2.27}$$

Equations (2.25) and (2.26) show the evolution of the two forward-looking variables, $\hat{\pi}_t$ and $\hat{v}_t$. The most notable feature is the presence of the term $(1+\phi\bar{\mu}\bar{\sigma})$ in the denominator of the terms multiplying marginal cost. It results from the upward-sloping firm-level marginal cost curve, and as long as $\phi > 0$ it makes prices less sensitive to changes in marginal cost. Note that both the steepness of the marginal cost curve ϕ and the elasticity of the demand curve $\bar{\sigma}$ affect this term. Equation (2.27) is, in deviation form and allowing for permanent changes in the inflation target $\hat{\varepsilon}_t^{\pi^*}$, the weighted average of all those past firm-specific inflation rates $\hat{v}_t$ that are still in force between periods $t-1$ and t, and which therefore enter into period t aggregate inflation. This term is inertial, and the degree of inertia depends directly on δ and therefore on the average contract length.

The following key equation follows from the differencing and log-linearization of the aggregate price index:

$$\hat{\pi}_t = \frac{1-\delta}{\delta}\hat{p}_t + \hat{\psi}_t \quad . \tag{2.28}$$

The two components of this equation reflect the two main sources of aggregate inflation inertia in response to shocks. The first term $\hat{p}_t$ represents inflation caused by significant instantaneous price changes (relative to the aggregate price level) of new price setters. Note that in a Calvo-Yun model this is the only term driving inflation. But in our case the quadratic cost term means that significant instantaneous price changes can be very costly, because it generally causes big deviations from industry average output during part of the duration of a pricing policy. New price setters will therefore respond as much as possible through changes in their updating rates $\hat{v}_t$. But these only slowly feed through to aggregate inflation via $\hat{\psi}_t$, which initially mainly reflects the continuing effects of price updating decisions made before the current realization of shocks. The result is that past inflation, by (2.28) and (2.27), becomes a key determinant of current inflation.

In our sensitivity analysis we will report not only the fit of our model, but also that of a Calvo (1983) model with Yun (1996) indexation to steady state inflation, augmented as in the baseline case by firm-specific marginal cost and sticky user costs. That model, in our case with markup shocks, gives rise to the following one-equation representation of the inflation process, the New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{((1-\delta\beta)\,(1-\delta))}{\delta(1+\phi\bar{\mu}\bar{\sigma})}\widehat{mc}_t + \frac{(1-\delta)}{\delta(1+\phi\bar{\mu}\bar{\sigma})}\left(\hat{\mu}_t - \delta\beta\hat{\mu}_{t+1}\right) \quad . \tag{2.29}$$

This equation can be directly derived from (2.25), (2.26) and (2.27) by setting $\hat{v}_t = \hat{\psi}_t = 0$. In other words, a firm in our model is always free to behave exactly like a Calvo-Yun price setter by loading all its price changes into the current price. However, this is generally far from optimal, especially if the processes driving inflation are highly persistent. And for aggregate inflation dynamics, as is well known, this kind of price setting implies very little inflation inertia and persistence.

2.3 Household Wage Setting

Every firm j must use composite labor as defined in (2.14), a CES aggregate with elasticity of substitution σ_t^w of the labor varieties supplied by different households. Firms' cost minimization, aggregated over all firms, yields demands

$$L_t(i) = L_t \left(\frac{W_t(i)}{W_t}\right)^{-\sigma_t^w} \quad , \tag{2.30}$$

where the aggregate nominal wage is given by

$$W_t = \left(\int_0^1 (W_t(i))^{1-\sigma_t^w} \, di\right)^{\frac{1}{1-\sigma_t^w}} \quad . \tag{2.31}$$

The term driving wage inflation is the log-difference between the marginal rate of substitution between consumption and leisure and the real wage. The marginal rate of substitution is given by

$$mrs_t = \frac{S_t^L \psi L_t(i)^{\frac{1}{\gamma}}}{\lambda_t} \quad . \tag{2.32}$$

Household nominal wage setting can then be shown to follow the same pattern as the price setting discussed in the previous subsection. With an appropriate change of notation, and after replacing $\widehat{mc_t}$ with $\widehat{mrs_t} - \hat{w}_t$, it leads to an identical set of equations to (2.25)-(2.28) above. The reader is referred to the Technical Appendix for details.

2.4 Financial Intermediaries

We assume that all capital is intermediated by a continuum of intermediaries indexed by $z \in [0, 1]$. These agents are competitive in their input market, renting capital K_t from households at rental rate r_t^k. On the other hand, they are monopolistically competitive in their output market, lending capital varieties $k_t(z)$ to firms at rental rates $u_t(z)$. This gives rise to sluggish user costs of capital, which interact in the model with sticky wages to produce stickiness in marginal cost. Sticky user costs imply that the output—capital—of intermediaries is demand determined. The assumption of variable capital utilization is therefore essential to allow the market for capital services to clear in the presence of sticky user costs.

Every firm j must use composite capital as defined in (2.14), a CES aggregate with elasticity of substitution σ^k of the varieties supplied by different intermediaries. Firms' cost minimization yields demands

$$k_t(z) = k_t \left(\frac{u_t(z)}{u_t}\right)^{-\sigma^k} \quad , \tag{2.33}$$

where the overall user cost to firms is given by

$$u_t = \left(\int_0^1 (u_t(z))^{1-\sigma^k} \, dz\right)^{\frac{1}{1-\sigma^k}} \quad . \tag{2.34}$$

The profit maximization problem of the intermediary follows the same pattern as firms' problem. We define the gross intermediation spread as $s_t = u_t / r_t^k$. With an appropriate change of notation and after replacing $\widehat{mc_t}$ with $-\hat{s}_t$, we obtain an identical set of equations to (2.25)-(2.28) above. The Technical Appendix contains the details.

2.5 Government

We assume that there is an exogenous stochastic process for government spending GOV_t

$$GOV_t = S_t^{gov}\overline{GOV} \quad , \tag{2.35}$$

with demands for individual varieties having the same form as consumption demand for varieties (2.7). The government's fiscal policy is assumed to be Ricardian, with the government budget balanced period by period through lump-sum taxes τ_t, and with an initial stock of government bonds of zero. The budget constraint is therefore

$$\tau_t + \frac{M_t - M_{t-1}}{P_t} = GOV_t \quad . \tag{2.36}$$

We assume that the central bank pursues an interest rate rule for its policy instrument i_t. Its quarterly inflation target π_t^* is assumed to follow a unit root process:

$$\pi_t^* = \pi_{t-1}^* \varepsilon_t^{\pi^*} \quad . \tag{2.37}$$

The year-on-year inflation rate is denoted as $\pi_{4,t} = \pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}$. The current year-on-year inflation target is simply the annualized quarter-on-quarter inflation target, $\pi_{4,t}^* = (\pi_t^*)^4$. Finally, the steady state gross real interest rate is given by $1/\beta_g$, where $\beta_g = \beta/\bar{g}$. Then we have

$$i_t^4 = \left[i_{t-1}^4\right]^{\xi^{int}} \left[\beta_g^{-4}\pi_{4,t+3}\right]^{1-\xi^{int}} \left[\frac{\pi_{4,t+3}}{\pi_{4,t}^*}\right]^{\xi^{\pi}} \left[\frac{Y_{t+3}}{Y_{t-1}}\right]^{\xi^{Y}} S_t^{int} \quad , \tag{2.38}$$

where S_t^{int} is an autocorrelated monetary policy shock. A *government policy* is defined as a set of stochastic processes $\{i_s, \tau_s\}_{s=t}^{\infty}$ such that, given stochastic processes $\{P_s, \pi_s^*, S_s^{int}, S_s^{gov}\}_{s=t}^{\infty}$, the conditions (2.36) and (2.38) hold for all $s \geq t$.

2.6 Equilibrium

An *allocation* is given by a list of stochastic processes $\{B_s, M_s, C_s, I_s, L_s, K_s, k_s, Y_s, L_t(i,j), k_t(z,j), i, j, z \in [0,1]\}_{s=t}^{\infty}$. A *price system* is a list of stochastic processes $\{P_s, W_s, R_s^k, U_s\}_{s=t}^{\infty}$. *Shock processes* are a list of stochastic processes $\{S_s^c, S_s^L, S_s^{inv}, S_s^{gov}, S_s^{int}, \mu_s, \mu_s^w, S_s^y, \pi_s^*\}_{s=t}^{\infty}$. Then the equilibrium is defined as follows:[10]

An equilibrium is an allocation, a price system, a government policy and shock processes such that

(a) given the government policy, the price system, shock processes, the restrictions on wage setting, and the process $\{L_s\}_{s=t}^{\infty}$, the allocation and the processes $\{V_s^w(i), v_s^w(i), i \in [0,1]\}_{s=t}^{\infty}$ solve households' utility maximization problem,

(b) given the government policy, the price system, shock processes, the restrictions on price setting, and the process $\{Y_s\}_{s=t}^{\infty}$, the allocation and the processes $\{V_s(j), v_s(j), j \in [0,1]\}_{s=t}^{\infty}$ solve firms' cost minimization and profit maximization problem,

[10] Except for bonds we only show log-linearized market clearing conditions. Their derivation from market clearing conditions in levels, including aggregation, is presented in the Technical Appendix.

(c) given the government policy, the price system, shock processes, the restrictions on setting user costs, and the process $\{k_s\}_{s=t}^{\infty}$, *the processes* $\{V_s^k(z), v_s^k(z), z \in [0,1]\}_{s=t}^{\infty}$ *solve intermediaries' profit maximization problem,*

(d) the goods market clears at all times,

$$\bar{Y}\hat{Y}_t = \bar{C}\hat{C}_t + \hat{I}_t + \widehat{\overline{GOV}GOV}_t \ , \tag{2.39}$$

(e) the labor market clears at all times,

$$\hat{\ell}_t = \int_0^1 \int_0^1 \hat{L}_t(i,j) di dj \ , \tag{2.40}$$

(f) the market for capital clears at all times,

$$\hat{k}_t = \hat{x}_t + \hat{K}_t \ , \tag{2.41}$$

(g) the bond market clears at all times,

$$B_t = 0 \quad . \tag{2.42}$$

3. Estimation Methodology, Priors, and Calibration

3.1 Estimation Methodology

The model above is log-linearized and then estimated in two steps in DYNARE-MATLAB. In the first step, we compute the posterior mode using an optimization routine (CSMINWEL) developed by Chris Sims. Using the mode as a starting point, we then use the Metropolis-Hasting (MH) algorithm to construct the posterior distributions of the model and the marginal likelihood.[11] We choose as our baseline case a particular combination of structural model features and priors for parameters, and use the parameter estimates for this case to construct impulse responses. Sensitivity analysis will be performed either by restricting certain parameters or shocks, or by removing some features of the structural model, and by comparing the marginal likelihood to that of the baseline case.

3.2 The Role of Unit Roots

Recent efforts at estimating DSGE models have been based mainly on data that were detrended either with linear time trends or with the Hodrick-Prescott filter—for examples see Smets and Wouters (2004) and Juillard and others (2005). More recently there have been attempts to use Bayesian methods to help identify more flexible stochastic processes that contain permanent, or unit-root components—see Adolfson and others (2005). This recent work is encouraging

[11] For one estimation run the whole process takes anywhere from 6 to 8 hours to complete using a Pentium 4 processor (3.0 GHz) on a personal computer with 1GB of RAM. DYNARE includes a number of debugging features to determine if the optimization routines have truly found the optimum and if enough draws have been executed for the posterior distributions to be accurate.

because it could potentially eliminate distortions in inference that can arise from prefiltering data.

Failing to account adequately for variation in the perceived underlying inflation objectives in DSGE models should be expected to seriously overstate the degree of structural inflation inertia and persistence if the model was estimated over a sample that had significant regime changes, with the central bank acting to change the underlying rate of inflation—see Erceg and Levin (2003). A similar argument applies to detrending inflation and interest rates with any procedure that removes too little or too much of the variation and persistence in the data.

Detrending productivity inappropriately could also bias key parameters that influence macroeconomic dynamics, as the behavioral responses of consumption, labor effort and investment will depend intricately on agents' forecasts of the future path of productivity. For example, under the assumption that productivity shocks are temporary deviations from a time trend standard models would predict a small rise in both consumption and leisure in the short run as the additional wealth generated by a productivity improvement would be consumed by distributing it over time. But an increase in leisure during periods of booms is at complete odds with the data at business cycle frequencies, which suggests clearly that GDP and hours worked are strongly and positively correlated. We show that if the model is extended to allow for shocks that result in highly persistent deviations of productivity growth from its long-term steady-state rate, it can generate a short-run positive correlation between output and hours, albeit only in the very short run. While the improvement is limited, we can nevertheless conclude that models which do not allow for a more flexible stochastic process for productivity run the risk of underestimating the importance of productivity shocks and producing significant bias in the model's key structural parameters.

For the reasons sketched out above we generally prefer to allow for unit roots in both underlying inflation objectives and the level of productivity, but we recognize that the case for the former in particular will obviously depend on the country and the sample that is being studied.[12] Over our sample with US data, which starts in the early 1990s, allowing for a unit root in inflation objectives is necessary because there is ample and convincing evidence that long-term inflation forecasts have declined significantly from values around 4 percent at the beginning of our sample to values around 2.5 percent at the end of the sample. Figure 7.1 plots three measures of long-term inflation expectations and the 10-year government bond yield, and all of them suggest that there was a gradual reduction in the perceived inflation target. A similar argument applies for productivity over this sample. Figure 7.2 reports measures of expected long-term growth from the same surveys and confirms that perceived long-term growth prospects for the United States have been revised up significantly over the last decade and have remained persistently higher than in the first half of the 1990s. Note that such revisions in growth prospects are completely inconsistent with a trend-stationary view of productivity, which predicts that periods of above-trend levels should be followed by slower medium-term growth as the level of productivity reverts back to trend.

To estimate the model with unit roots in both productivity and inflation it was necessary to normalize the model by both technology and the inflation target, and to then transform it into a

[12] For example, it may not be necessary to control for shifts in perceived inflation objectives in inflation-targeting countries over samples where the central bank has established a track record and managed to anchor long-term inflation expectations—see Levin, Natalucci and Piger (2004), Batini, Kuttner and Laxton (2005), Gürkaynak, Sack, and Swanson (2005).

linearized form. Unlike a previous version of this paper that expressed all growing observable variables in first differences, the model is now estimated directly in levels.

3.3 Data and Data Transformations

Our sample period covers 63 quarterly observations from 1990Q2 through 2005Q4. We employ the same 7 observable variables that have been employed in other studies (GDP, consumption, investment, hours, real wage, Fed funds rate, and inflation, as measured by the implicit GDP deflator), but we have added as an additional variable a measure of long-term inflation expectations to help identify perceived movements in the Fed's underlying inflation objectives. This measure is taken from a survey by Consensus Economics, which measures expected inflation between 6 and 10 years in the future, a period that is sufficiently far ahead for inflation to be expected to be equal to the target on average. The data for GDP, consumption, investment, and real wages are all measured on a per capita basis and the data for the Fed funds rate and the inflation rate (GDP deflator) are measured as annualized log first differences of the gross rate. The only variable that is measured in (de-meaned) log levels is hours worked per person.

Real GDP, investment, consumption and the GDP price deflator are taken from the US NIPA accounts. Hours worked are taken from the Labor Force Survey. The real wage is calculated by dividing labor income (from US NIPA) by hours and the GDP deflator.

3.4 Calibrated Parameters

The only parameters needed to be calibrated relate to time resources. These include a quarterly discount rate β set to 0.999 and steady state ratio of available working time set to $\frac{1}{3}$.The latter only pins down units of some unobservable variables in the model.

All the other parameters including those which determine the steady state are estimated.

3.5 Specification of the Stochastic Processes

Table 7.1 reports the specifications of the stochastic processes for the 10 structural shocks in the model.[13] Following Juillard and others (2005) we classify shocks as demand and supply shocks depending on the short-run covariance they generate between inflation and real GDP. Shocks that raise demand by more than supply and cause inflation to rise in the short run are classified as demand shocks, while shocks that produce a negative covariance between inflation and GDP are classified as supply shocks. Based on this classification system, shocks to government absorption, the Fed funds rate, the inflation target, consumption, and investment, [$\widehat{s_t^{gov}}$, $\widehat{s_t^{int}}$, $\hat{\pi}_t^*$, $\widehat{s_t^c}$, $\widehat{s_t^{inv}}$], are all classified as demand shocks. Shocks to wage and price markups as well as labor supply shocks, [$\widehat{\mu_t^w}$,$\widehat{\mu}_t$, $\widehat{s_t^L}$], are classified as supply shocks. Both markup shocks are assumed to have zero serial correlation, as otherwise the autocorrelation coefficients would pick up most of the observed inflation inertia, rather than the multiple and competing structural features of the model. The remaining two shocks determine the growth rate of productivity. The classification of the $\widehat{g}_t^{iid}$ shock is simple because increases in its value make output rise and inflation fall. However, the classification of the $\widehat{g}_t^{gr}$ shock as a demand or supply shock is more difficult. Interestingly, when shocks to this component are highly serially correlated they generate responses that share characteristics with what many professional forecasters would

[13] In their model of the US economy, Smets and Wouters (2004) also allow for ten structural shocks, six of which are specified as first-order stochastic processes and four of which are assumed to be white noise.

characterize as shocks to consumer and business confidence in that they result in sustained increases in aggregate demand and a temporary, but persistent, increase in inflation.

3.6 Prior Distributions

Our assumptions about the prior distributions can be grouped into two categories: (1) parameters for which we have relatively strong priors based on our reading of existing empirical evidence and their implications for macroeconomic dynamics, and (2) parameters where we have fairly diffuse priors. Broadly speaking, parameters in the former group include the core structural parameters that influence, for example, the lags in the monetary transmission mechanism, while parameters in the latter category include the parameters that characterize the stochastic processes (i.e., the variances of the shocks and the degree of persistence in the shock processes).

The first, fourth and fifth columns of Table 7.2 report our assumptions about the prior distributions for the 21 structural core parameters of the model. The second and third columns report the posterior mode estimates and standard errors of the parameters. The assumptions about and results for the remaining parameters are reported in a similar format in Tables 7.3 and 7.4.

3.6.1 Priors about Structural Parameters (Table 7.2)

Habit Persistence in Consumption $[v]$: We set the prior at 0.85 as high values are required to generate realistic lags in the monetary transmission mechanism and hump-shaped consumption dynamics—see Bayoumi, Laxton and Pesenti (2004) for a discussion of the role of habit persistence in generating hump-shaped consumption dynamics in response to interest rate shocks. This prior is somewhat higher than other studies such as Boldrin, Christiano and Fisher (2001), who use a value of 0.7.

Frisch Elasticity of Labor Supply $[\gamma]$: We set the prior at 0.50. Pencavel (1986) reports that most microeconomic estimates of the Frisch elasticity are between 0 and 0.45, and our calibration is at the upper end of that range, in line with much of the business cycle literature. [14]

Adjustment Costs on Changing Capital and Investment $[\theta_k, \theta_i]$: We set priors equal to 5 and 50 for θ_k and θ_i. These assumptions are based on analyzing the simulation properties of the model. The data do not seem to have much to say about these parameters other than that they cannot be zero or very large. This is not uncommon.

Duration of Pricing Policies $[\delta, \delta_w, \delta_k]$: The duration of pricing policies is $(1/(1-\delta))$. In the base case we set the prior equal to a three quarters duration for prices, wages and user costs, therefore the priors equal 0.66 for $[\delta, \delta_w, \delta_k]$. This is based on our reading of the empirical literature for the US and on the results cited in ECB (2005). In the US, consumer prices are re-set on average (slightly faster than) every two quarters, while the average for producer prices is four quarters. As our model does not distinguish between the two, it seems reasonable to choose an intermediate prior of three quarters. The priors for wages and user costs are set to the same value, but for user costs we will consider alternatives in the sensitivity analysis.

Steepness of Marginal Cost Curve $[\phi, \phi_w, \phi_k]$: Simulation experiments with the model suggest that plausible values for these parameters might fall between 0.50 and 2.0. In our base case we

[14] As discussed by Chang and Kim (2005), a very low Frisch elasticity makes it difficult to explain cyclical fluctuations in hours worked, and they present a heterogeneous agent model in which aggregate labor supply is considerably more elastic than individual labor supply.

set the prior at 1.0. Our sensitivity analysis includes a case where all three of these parameters are restricted to be zero. There are significant interactions between these adjustment cost parameters and the duration parameters that will be explained below.

Interest Rate Reaction Function [$\xi_{int}, \xi_{\pi}, \xi_y$]: We impose prior means of 0.75, 0.25, 0.50 to be consistent with previous work, but we make these priors diffuse to allow them to be influenced significantly by the data.

Steady State Parameters: We set the mean of the quarterly steady-state rate of productivity growth $\bar{g}$ to 1.0042, which implies an annual rate of 1.7 percent, the average over our sample. The rate of productivity growth and quarterly discount rate β together pin down the equilibrium real interest rate of 2.1 percent. The quarterly depreciation rate on capital Δ is assumed to be 0.025, implying an annual depreciation rate of 10 percent. The prior means of elasticities of substitution among goods, labor inputs and capital inputs $\sigma, \sigma^w, \sigma^k$ are assumed to be 5.35, 7.25 and 11.00 respectively, resulting in markups of 23%, 16% and 10%. The prior for the labor income share l_s is set to 0.58. This prior combined with the assumptions on elasticities of substitutions results in a share of capital in valued added of 0.28 and a capital-to-GDP ratio of 1.71. Further, the prior for government absorption g_s is set to 18 percent of GDP in steady state. These assumptions imply that 62 percent remains for consumption and 20 percent for investment. Most of these values are similar to what have been employed in other DSGE models of the US economy—see Juillard and others (2005) and Bayoumi, Laxton and Pesenti (2004). There are two exceptions. First, the share of capital of 0.28 looks lower than what is typically assumed, but this is the share in value added, not in output. Capital's share in output includes monopoly profits from three sectors, and is reasonable at 41 percent. Second, the mark-up in financial intermediation is a new concept in this literature. Our intuition is that this sector is more competitive than the goods and labor markets.

3.6.2 Priors about Structural Shocks (Tables 7.3- 7.4)

Persistence parameters for the structural shocks [ρ_{gov}, ρ_{inv}, ρ_c, ρ_{int}, ρ_{gr}, ρ_L, ρ_{μ}, ρ_{μ^w}]: Table 7.3 reports the assumptions about the priors for these parameters. With the exception of the shocks to the markups and the autocorrelated productivity shocks we set the prior means equal to 0.85 with a fairly diffuse prior standard deviation of 0.10. For the two markup shocks we impose zero serial correlation. These priors are consistent with other studies such as Smets and Wouters (2004) and Juillard, Karam, Laxton and Pesenti (2005).

We treat the prior on the serial correlation parameter for the productivity shocks differently. Here, we utilize a tight prior so that the model can generate highly persistent movements in the growth rate relative to its long-run steady state. As mentioned earlier, this is necessary to explain some facts in our sample (persistent upward revisions in expectations of medium-term growth prospects), but it is also more consistent with the data over the last century in the United States and other countries, where productivity growth has departed from its long-term average growth rate for as long as decades in many cases. Obviously, there will not be a lot of information in our short sample for estimating this parameter.[15] We are considering adding expectations of long-term productivity growth to the list of observable variables to help identify this parameter, but have not attempted to do so at this point.

[15] Provided the researcher can provide sensible priors, Bayesian techniques offer a major advantage over other system estimators such as maximum likelihood, which in small samples can often allow key parameters such as this one to wander off in nonsensical directions.

Structural shocks standard errors [$\sigma_{\hat{\varepsilon}^{gov}}$, $\sigma_{\hat{\varepsilon}^{inv}}$, $\sigma_{\hat{\varepsilon}^{c}}$, $\sigma_{\hat{\varepsilon}^{int}}$, $\sigma_{\hat{\varepsilon}^{\pi*}}$, $\sigma_{\hat{\varepsilon}^{iid}}$, $\sigma_{\hat{\varepsilon}^{gr}}$, $\sigma_{\hat{\varepsilon}^{L}}$, $\sigma_{\hat{\varepsilon}^{\mu}}$, $\sigma_{\hat{\varepsilon}^{\mu w}}$]: Table 7.4 reports our assumptions about the priors for these parameters. The strategy here was to develop rough priors of the means by looking at the model's impulse response functions, conditional on all the other priors, and then to form a diffuse prior around this mean in order to let the data adjust the parameters in a way that improves the overall fit of the model. The specific values for these priors are not intuitive, as they require a very detailed knowledge of the structure of the model.

4. Estimation Results

4.1 Parameter Estimates

The posterior mode for habit persistence is 0.73, which is above our prior of 0.85. The data and model seem to confirm our prior for the Frisch elasticity of labor supply and imply a slightly larger adjustment cost parameter estimate on investment changes (52.9 versus 50.0). The parameter estimates of the policy rule imply a slightly higher coefficient on the deviation of the inflation forecast from the perceived target (0.32 versus 0.25), a significantly higher estimate of the interest rate smoothing term (0.95 versus 0.75) and a lower estimate on the deviation of the output growth rate from the technology growth rate (0.50 versus 0.34).

The posterior estimates for the parameters that determine pricing duration are lower than the prior means for wages (0.59 versus 0.66), and higher for prices (0.71 versus 0.66). According to these estimates, the mean durations of pricing policies are 10.3 months in the goods market, 9.3 months in the capital market, and 7.3 months in the labor market. The parameters determining the steepness of the marginal cost curve change little in all three markets (1.03, 1.01 and 0.96 versus 1.00). Broadly speaking, the range of parameter estimates does not look implausible.

The parameter estimates for the structural shock processes are reported in Tables 7.3 and 7.4. The results for the standard errors in Table 7.4 are not easy to interpret without understanding the model's properties (IRFs and variance decompositions). The estimates of the serial correlation parameters in Table 7.3 are more interesting. Aside from the persistent productivity growth shocks, the shock with the highest degree of serial correlation is government spending (0.99). [16] Unsurprisingly, the data do not have very much of an influence over the parameter estimate of the growth shocks, producing a posterior mean that is nearly equal to the prior. What is most significant about these results is that our priors of a high degree of serial correlation for all processes are within the estimated 90% confidence intervals. This means among other things that the shocks driving pricing are highly persistent, and as such generally require an optimal pricing response that makes firms change their firm-specific inflation rates. A model that rules this out imposes strong restrictions on optimal behavior and on macroeconomic dynamics.

4.2 Impulse Response Functions

4.2.1 The IRFs for Demand Shocks

Figure 7.3 reports the impulse responses for a one-standard deviation increase in the Fed funds rate. The Fed funds rate increases by about 20 basis points and as a result output, consumption, investment, hours worked, and the real wage all fall in the short run and display hump-shaped dynamics that trough after about three to four quarters. There is a similar small reduction in year-on-year inflation (which lags behind output) reflecting the significant inertia in the infla-

[16] Note that in the data this variable is a residual that includes the current account.

tion process. Figure 7.4 reports the results for a permanent increase in the inflation target of .08 percentage points. As can be seen in the Figure this requires a temporary, but persistent, reduction in real and nominal interest rates, which results in a temporary boost to GDP, consumption, investment and hours worked. Figure 7.5 reports the results for a shock to government absorption. This shock is expansionary in the short run and induces higher output and work effort. However, to restrain inflationary forces, real interest rates rise and this crowds out consumption. Investment and work effort remain high for an extended period because this shock is estimated to be highly persistent. For the consumption shock in Figure 7.6, consumption rises in the short run and this eventually requires an increase in real interest rates to return inflation back to the inflation target. Inflation is highly persistent for this shock, and also for the (negative) investment shock in Figure 7.7. Here investment falls over the medium term and the fall in the real interest rate crowds in consumption sufficiently in the short run to generate the savings necessary to finance the higher level of investment. However, over time the lower level of capital requires a lower level of consumption. Finally, and as can be seen in all of these figures, inflation and output co-vary positively in the short run.

4.2.2 The IRFs for Supply Shocks

Figure 7.8 reports the results for a shock that reduces the wage markup and expands labor supply. In this case, the real wage falls and there is an expansion in output, hours worked, consumption and investment. Inflation falls and the Fed funds rate is reduced over time to gradually push inflation back up to its target. Figure 7.9 deals with a shock that reduces the price markup. This has very similar short-run qualitative effects to a wage-markup shock, except that the real wage rises in the short run. Figure 7.10 reports the results for a negative shock to labor supply. This induces an increase in the real wage and results in a reduction in output, consumption, investment and hours worked. Finally, we note that under all of these shocks, a negative covariance exists between output and inflation in the short run.

4.2.3 The IRFs for Productivity Shocks

Figure 7.11 reports the results for a temporary shock to the growth rate of productivity. While this results in an increase in output, consumption, investment and the real wage, there is a reduction in hours worked as workers consume more leisure. As pointed out by Gali (1999) and others, this feature severely constrains the potential role of productivity shocks in DSGE models as it implies a counterfactual strong negative correlation between hours worked and output.

Figure 7.12 shows that this problem is less severe with a persistent shock to the growth rate of productivity. GDP, consumption, investment, productivity and the real wage all trend up over time and have not converged to their new long-run values after a decade. Because it takes time to put capital into place, in the very short run the increase in output is accomplished partly through an increase in hours worked. However, as investment rises hours worked eventually decline and in the very long run return back to baseline. This last requirement is a condition for balanced growth. In the very short run inflation rises as demand increases by more than supply. Consequently, real interest rates rise in part to constrain these short-run inflationary forces, but they also rise persistently as the marginal product shifts upwards and then falls slowly over time until the level of the capital stock increases to its new steady-state level.

4.2.4 The Importance of Pricing Policies for Inflation Dynamics

Figure 7.13 illustrates the effect on inflation dynamics of the average contract length δ, δ^w, and $\delta^k$and the steepness of the marginal cost curve ϕ, ϕ^w, and ϕ^k. For the purpose of this exercise we maintain all parameters at those of our baseline experiment while allowing for different values of these six parameters. The shock we consider is a permanent increase in the inflation target by one percent per annum. We consider 16 cases, ranging from fast to slow price adjustment ($\delta = 0.25, 0.5, 0.75, 0.9$) and from flat to steep marginal cost curves ($\phi = 0.5, 1, 2, 5$). Two results stand out.

First, the most interesting difference between these parameter combinations concerns inflation inertia, rather than persistence. Inertia is dramatically lower for slower speeds of price adjustment, while higher speeds of price adjustment are characterized by an initial overshooting (by a factor of two) of inflation over its new target. Note that a standard New Keynesian model without indexation would exhibit no inertia whatsoever for a shock to the inflation target; inflation would immediately converge to the new target. In our model persistence would increase dramatically for very long contract lengths, as shown in the last row of plots. Contracts of such length are, however, clearly rejected by the data.

Second, the steepness of the marginal cost curve matters far less than contract length for this particular shock. In order for past inflation to become an important determinant of current inflation, historic pricing policies with their history of updating behavior must remain in force at least for some time. Otherwise even very steep marginal cost curves will not prevent firms from rapidly adjusting their prices, because they can do so in anticipation of soon being able to readjust their price again.[17]

5. Model Validation and Forecasting Performance

This section examines the model's ability to forecast. We begin by evaluating the model's fit by comparing the model's marginal data density with those from Bayesian VARs. Next, we examine the out-of-sample forecasting performance for inflation, output and the Fed funds rate by comparing the model's out-of-sample root-mean-square errors (RMSEs) with five alternative forecasting approaches. This includes the Fed's judgmental Greenbook forecasts, the Fed's MAQS DSGE model, as well as a BVAR, VAR and random walk model (RW).[18] Interestingly, we find that the model performs quite well when compared to these other methods, but our conclusions with respect to the Greenbook forecasts (GB) are tentative as we do not have a sample that includes any recessions and large turning points for the economy. In the last part of this section we show how the model interprets new data and decompose revisions in medium-term forecasts into demand and supply components.

5.1 Comparing the Model's Fit with BVARS

The marginal data density provides a very useful summary statistic of the overall fit of the model and can be compared directly with other DSGE models estimated on the same data set or less restricted models such as vector autoregressive models (VARS). In cases where researchers

[17] This also suggests that the empirical finding of a very short contract length in Altig, Christiano, Eichenbaum and Linde (2005) may have more to do with the non-rational price updating behavior of their firms than with their estimated steepness of their marginal cost curve.

[18] The Fed's MAQS DSGE model was developed principally by Rochelle Edge, Jean-Pierre Laforte and Michael Kiley in the Macroeconomic and Quantitative Studies Section at the Board of Governors of the Federal Reserve System—see Edge, Kiley and Laforte (2006a).

have not prefiltered the data with some detrending technique the marginal data density will also provide a direct measure of out-of-sample forecasting performance.[19]

In addition to comparing the fit of different DSGE models, it is also possible to compare their marginal data densities with the marginal data densities of Bayesian VARS—see Sims (2003) and Schorfheide (2004). [20] Table 7.5 reports the marginal likelihood of eight BVARs (1 to 8 lags) based on Sims and Zha (1998) priors.[21] The BVAR estimates were obtained by combining a specific type of the Minnesota prior with dummy observations. The prior decay and tightness parameters are set at 0.5 and 3, respectively. As in Smets and Wouters (2004), the parameter determining the weight on own-persistence (sum-of-coefficients on own lags) is set at 2 and the parameter determining the degree of co-persistence is set at 5. To obtain priors for the error terms we followed Smets and Wouters (2004) by using the residuals from an unconstrained VAR(1) estimated over a sample of observations that was extended back to 1980Q1.[22] The estimates reported in Table 7.5 suggest that the best fitting BVAR has 4 lags. As can be seen in the top row of Table 7.5 the estimates of the marginal data density obtained from 500,000 replications of the Metropolis-Hastings algorithm using the modified harmonic mean formula suggested by Geweke shows that the DSGE model provides a better fit than the best fitting BVAR over this sample. To test whether this was the result of the specific sample of observations that was used to develop priors for the error terms in the BVAR we considered two alternative shorter samples (1987:1-1990:2 and 1984:1-1990:2), but in both cases none of the BVARs produced a better fit than the DSGE model. We also considered the procedure suggested by Schorfheide (2004) for setting the priors on the error terms using the standard error of the endogenous variables on the presample and obtained the same basic findings. While the estimates of the marginal data density of each BVAR changed for each sample none of the BVARS fit as well as the DSGE model.

5.2 More Comparisons of Forecasting Performance

In a recent paper Edge, Kiley and Laforte (EKL: 2006b) compare the out-of-sample forecasting performance of a DSGE model they have developed at the Fed with the judgmental Greenbook

[19] One problem with prefiltering data such as output with filters such as the Hodrick-Prescott filter prior to estimation is that uncertainty in the estimates of the trend will not be accounted for by the estimates of the marginal data density of the estimated model. In other words, when researchers prefilter the data before estimation there will no longer be a direct correspondence between in-sample fit and out-of-sample forecasting performance. This problem with prefiltering data has not been limited to empirical work on DSGE models, but has plagued most of the empirical work on the generation of macro models that DSGE models are being developed to replace.

[20] It is well known that large dimensional unrestricted VAR models do not forecast very well without imposing some priors on the parameters and for that reason we compare the fit of the DSGE model with Bayesian VARs instead of unrestricted VARs. It is important to stress that we do not consider the BVARs as serious alternatives to a structural view about how the economy works because they offer little useful in this dimension, but they do provide a potentially useful metric for comparing the fit and out-of-sample forecasting performance of DSGE models when there is a paucity of alternative DSGE models readily available that can be used to assess any specific model. Because BVARs have been developed principally as forecasting models this approach might seem to suggest that the deck is being stacked against DSGE models, which in many cases impose serious cross-equation restrictions that could easily be rejected by the data.

[21] The marginal likelihood values for the BVAR were computed in DYNARE using a program developed by Chris Sims.

[22] The DSGE model was estimated over a sample from 1990Q3-2005Q2. This choice was based on available measures of long-term inflation expectations from Consensus Economics. To extend our measure of long-term inflation expectations back we used an alternative measure available from the Survey of Professional Forecasters. As can be seen in Figure 7.1 the measure of long-term inflation expectations from the Consensus Economics survey displays a similar pattern as the measure from the Survey of Professional Forecasters over the sample where both series exist.

forecasts prepared by Federal Reserve Board staff as well as forecasts from 3 simple reduced-form models, which included a VAR, a Bayesian VAR and a simple random walk (RW) model.[23] While the sample period that EKL study is limited to forecasts covering the period August 1996 to March 2000 and does not contain any significant turning points in the economy as a result of recessions, this period is still of some interest because the economy produced significantly higher output growth than was anticipated by most forecasters—see Figure 7.2, which shows that the Consensus forecasts of trend GDP growth were not revised upwards in response to higher actual growth in the mid-1990s until just before the recession in 2001.

Tables 7.6 to 7.8 report the RMSEs for per capita GDP, the GDP deflator and the Fed funds rate for the 5 models and the Greenbook forecasts.[24] Before presenting the estimates it is important to emphasize that while forecasting performance is the only objective criterion for evaluating alternative models, it is important to bear in mind when performing horse races of this type that sometimes even the worst horse can win a race if the track is short enough or through happenstance just happens to have the perfect conditions on a particular day and track.

5.2.1 Results for Per Capita GDP

Table 7.6 reports the RMSEs for GDP per capita over horizons of 1, 4 and 8 quarters. Our model, which is reported as DSGE-JKKL in the table, clearly wins the race at long horizons, producing an RMSE which is significantly better than all the models and 1/2 as large as the Greenbook forecasts. Note, however, that at horizons as short as 1 quarter the Greenbook forecasts and the Bayesian VAR do slightly better than the other models, suggesting that there may be an advantage to using a less-structured approach for near-term forecasting. As indicated earlier, the particular sample studied by EKL and here does not include any recessions and therefore may underestimate the value of the Greenbook forecasts. Obviously, we are looking forward to the release of the Greenbook forecasts that cover the 2001 recession and subsequent recovery to see if the benefits of the Greenbook forecasts become even larger when there are interesting turning points in the sample.

5.2.2 Results for the GDP Deflator

Table 7.7 reports the results for the GDP deflator. Now the Greenbook forecasts dominate the other forecasts at the 8-quarter horizon and the DSGE-JKKL is slightly better at the 1-quarter horizon. One interpretation for why the Greenbook forecasts may dominate at longer horizons is that Fed staff may have more information about the underlying preferences of Fed policymakers concerning medium-term inflation objectives than what is embodied explicitly in the 2 DSGE models, or implicitly in the assumptions of the reduced-form models. It remains to be seen if the favorable performance of DSGE-JKKL at the 1-quarter horizon is simply a result of ignoring data revisions to the national accounts. The DSGE-JKKL model also dominates the forecasts of DSGE-EKL at longer horizons, so it would be interesting to study in more detail what assumptions in the two models are capable of accounting for such differences. The superior performance of the DSGE-JKKL model may not be surprising since the major focus of developing this model has been to improve output-inflation dynamics, so it would have been somewhat disappointing if it did not compare favorably to more conventional models that have a

[23] We are indebted to Jean-Philippe Laforte for providing the RMSEs for the Greenbook forecasts and other models that we use here. For graphs of the RMSEs for GDP and the GDP deflator see Figure 3 of Edge, Kiley and Laforte (2006b).

[24] The tables do not include estimates for the Fed funds rate for the Greenbook forecasts because these data are not publicly available.

simpler specification for inflation dynamics.[25] It is also important to emphasize that the DSGE-EKL model offers other features by providing a more detailed decomposition of GDP into components, thereby providing a framework for addressing questions that cannot be adequately addressed with highly aggregated models.

5.2.3 Results for the Fed Funds Rate

Table 7.8 reports the results for the Fed funds rate. At the 1-quarter horizon DSGE-JKKL is the clear loser, producing an RMSE that is about double what is produced from the other models. However, at longer horizons DSGE-JKKL clearly wins the race, with the random walk model (RW) coming in second. Two points need to be mentioned here. First, in this particular sample the Fed funds rate was roughly stable so that even the RW model, which is a completely uninteresting model for policy analysis, beats the other models. There is obviously some luck involved in this race as the theoretical distributions of the DSGE-JKKL and RW models suggest significantly higher RMSEs at horizons as long as 4 or 8 quarters. It would be interesting to see why the DSGE-JKKL model performs poorly at short horizons, but then dominates at longer horizons. We plan to study these issues further by replacing the model's specification of inflation dynamics with the standard Calvo model with lagged indexation to see if this accounts for the difference in forecasting performance between DSGE-JKKL and DSGE-EKL over this period.

5.2.4 Some Caveats and Future Work

It is important to emphasize that the forecasts from the DSGE-JKKL model have an advantage over the models considered by EKL. First, EKL derive real-time forecasting errors by using the historical data that was available at the time, while our estimates are based on the revised data available now. [26] Second, because of data limitations for our measure of long-term inflation expectations, which only starts in 1990, we use estimates of parameter values that are based on the whole sample.[27] EKL, which do not use these measures of long-term inflation expectations to control for time variation in underlying inflation objectives, can be more careful by conducting a real-time forecasting exercise. In fact, they use parameter estimates that are only based on available data and then update them once a year after the first major revisions to the data from the national accounts. It is unclear how much this affects our results so we plan to explore these issues further in a separate paper, which will focus exclusively on the real-time forecasting accuracy of the model.

We are also interested in developing a better methodology for doing comparisons with the forecasts from DSGE models and other methods. While reduced-form methods or judgmental forecasts may have an advantage for near-term forecasting, the DSGE models are much more useful for policy simulations since there is an endogenous determination of the policy rate that is necessary to bring inflation back to the underlying target. For example, even if an RW model was

[25] The framework used by EKL is quite standard and involves augmenting the basic Calvo model with lagged indexation. We are in the process of estimating a version of our model with this specification for inflation dynamics to compare the marginal data densities from both models.

[26] Each Greenbook forecast database from August 1996 also contains the historical time series in a FAME database so it is straightforward for researchers at the Fed to control for historical revisions. This is an extension we hope to do in the future once we get access to the initial data. Unfortunately we just learned that these data are readily available from the web site of the St. Louis Fed. The problem associated with historical revisions may be significant for many series in the national accounts, but is not a problem for the Fed funds rate as these data are not revised. As far as we know researchers have not quantified the importance of historical revisions with the objective of showing how this additional source of uncertainty affects forecasting accuracy at different horizons in DSGE models.

[27] EKL's data sample started in 1984 providing a large enough sample where they could start rolling the estimation forward from 1996. We are in the process of trying to do the estimation starting in 1990, but obviously our initial sample will be quite short so we do not know how this is going to pan out.

found to forecast inflation better than a DSGE model over say an 8-quarter horizon, it would provide no useful information for policy deliberations that were aimed at deciding on the current and expected future path of the policy rate. However, less structured methods may be useful for very near-term forecasts for those variables where there is likely to be little feedback between the policy rate and the variable that is being predicted. For example, a real forecast of US GDP for the 1st quarter of a year that was being conducted in March can probably safely ignore the implications of a small rate hike in March. However, it is unclear what benefits forecasts over longer horizons have for policy deliberations unless they spell out clearly the linkages between the policy rate and the objectives of monetary policy. A method that combined judgmental near-term forecasts or atheoretical methods with the forecasts from DSGE models would be a very useful tool for inflation-targeting central banks as it would allow them to quantify the benefits of judgment and help them to distinguish between the predictable and unpredictable components of monetary policy.

5.3 DSGE-JKKL Forecasting Performance

The analysis above compared DSGE-JKKL with other forecasts over a limited sample period studied by EKL and only for GDP, the GDP deflator and the Fed funds rate. Next, we extend our analysis by examining the root mean square errors (RMSEs) over more horizons and for all the observable variables in the model. Table 7.9 reports RMSEs for 1, 4, and 12 quarters ahead. The first set of numbers in each column reports the estimates based on the complete historical sample that dates back to the early 1990s, while the estimates in parentheses are derived from the theoretical distributions of the model. It is important to emphasize that this analysis of the errors does not account for uncertainty in parameters, but focuses simply on uncertainty in the underlying shocks.[28]

The presence of a unit root in both inflation and productivity will imply that the RMSEs will become larger as the forecast horizon is extended. For example, the theoretical RMSEs for inflation rise from 0.72 for forecasts 1 quarter ahead to 1.54 for forecasts that are 12 quarters ahead. At short horizons (1 to 4 quarters ahead) the model does a reasonable job at forecasting inflation developments.[29] Interestingly, the RMSEs based on the historical data are smaller at longer horizons than what is suggested by a pure unit root specification for inflation objectives. This should not be surprising as this unit-root assumption for inflation objectives is designed to simply account for permanent reductions in inflation objectives from the higher values at the beginning of the sample. Obviously, given that the Fed is committed to maintaining low and stable inflation we would expect that the actual RMSEs at longer horizons would start to stabilize at some point. [30]

The RMSEs for the real variables also widen at longer forecast horizons and in most cases provide plausible estimates of forecast uncertainty, while in other cases they suggest weaknesses in the model's structure and properties. For example, while the RMSEs for GDP expand in a plausible way, the theoretical RMSEs particularly for investment seem to expand too quickly,

[28] We are in the process of extending the analysis to allow for uncertainty in parameters.

[29] Remember inflation is measured at annual rates, or more precisely 400 times the first difference of the GDP deflator. Considering the significant near-term volatility in the GDP deflator an RMSE of these magnitudes suggests that the model might be a serious contender among competing models of the US inflation process.

[30] While the Fed does not have explicit numerical objectives for inflation it is clear from its communications and actions that it intends to keep inflation in a range that is low enough to prevent costly distortions and high enough to guard against periods of deflation. The RMSEs for advanced inflation-targeting countries, which have explicit numerical objectives that are designed to anchor long-term inflation expectations, typically stabilize faster and at lower values.

and indeed in this case the RMSE based on the actual forecasts 3 years ahead is substantially lower than the RMSE based on the theoretical distributions of the model (8.81 versus 19.78). We think this may be related to a fundamental weakness in these types of models because they do not allow for sufficient positive correlation between consumption and investment. For example, Juillard and others (2005) address this problem by simply allowing the consumption and investment shocks in their model to be positively correlated.[31] We have not performed a formal comparison of these forecast errors for all variables with other structural models or judgmental methods, but at this point our forecast errors for the Fed funds rate seem significantly worse than from the futures market, suggesting that there may be valuable information in the futures market to help identify the parameters of the reaction function and the underlying shocks driving the US economy.

5.4 Accounting for Very Near-Term Forecasting Errors

Figures 7.16 to 7.24 report 1-quarter-ahead forecasts for output, inflation and the Fed funds rate, which are based on information that starts in 1990Q3 and ends in 2005Q4. The middle panel of these figures reports the forecast errors for each 1-quarter-ahead forecast and the bottom panel decomposes these forecast errors into the 10 underlying shocks. In the middle panel of each figure we have included lines that are equal to plus or minus 2 RMSEs to provide an indication if the forecasting error is particularly large. In the bottom panel we have color coded the shock contributions as follows. The five cold colors (black and the two shades of blue and green) signify demand shocks because they generate positive covariance between GDP and inflation. By contrast the four hot colors (yellow, orange, red, and brown) represent supply shocks because they generate negative covariance between GDP and inflation. For a simple example about how to interpret each set of the charts please turn to Figure 7.16, which reports data for per capita GDP over the sample from 1990Q3 to 1995Q1. It shows per capita GDP is falling at the very beginning of the sample and the forecasts based on data up to 1990Q3 overpredict it by over 3/4 of a percent—see the negative black bar in the middle panel. As can be seen in the bottom panel, this forecast error was a result of negative demand shocks (hot colors) that reduced output by considerably more than other shocks that raised it. From examining the bottom panels of Figures 7.16 to 7.18 and 7.22 to 7.24 it can be seen that most of the unpredictable short-run variation in GDP and the Fed funds rate is a result of demand shocks (the hot colors) while for inflation it is supply shocks—see the cold colors at the bottom of Figures 7.19 to 7.21.

Turning to the figures on GDP it can be seen that there can be pretty significant forecasting errors, particularly around turning points such as the two NBER-dated recessions (1990Q4-1991Q1 and 2001Q2-2001Q4), as well as in other periods when output growth either significantly exceeded or fell below trend growth. To summarize, while DSGE models can provide a useful framework for better understanding macro dynamics and the types of shocks driving the economy they seem to forecast the economy equally as badly as other mechanical techniques during turning points. It will be interesting to compare these particular forecasts with the Greenbook forecasts when the figures for 2001 are released early next year.

5.4.1 What Quarters Had Large Errors?

For per capita GDP there were four forecasting errors that were greater than 2 RMSEs and three of them occurred near the beginning of the sample. First, the forecast for GDP growth in 1990Q4 was expected to be positive and instead it fell quite significantly, producing an error

[31] We are in the process of reestimating the model to see if this would work here, but at this point do not have any results to report.

of -1.3 percent. The forecast for the following quarter was almost spot on, but then the model forecasts a continued fall in GDP that was not realized. It is this transition from recessions to recoveries that the model seems to miss completely. The next large forecasting error was for 1993Q1, which also marked a fall in per capital GDP, while the model would have suggested a fairly strong quarter. The only other really large forecasting error was for 2000Q3, when growth was significantly lower than projected.

There were three large forecasting errors for inflation where inflation was above 2 RMSEs. This occurred towards the end of the sample in 2003Q1, 2004Q1 and 2005Q2. In all three cases there were significant increases in the prices of nondurables and the government deflator. Again it will be interesting to see if the Greenbook forecasts, which are based on more detailed and timely information, were more accurate at forecasting inflation in these quarters.

There were three large forecasting errors for the Fed funds rate that were below 2 RMSEs. This was in 1998Q4, when the model predicts an increase, but the Fed funds rate was cut. Second, the model does not predict the cuts in interest rates in 2001, when there were two large forecasting errors in 2001Q1 and 2001Q4, but it also overpredicted interest rates significantly in the two other quarters in 2001. In addition, there is a string of negative errors over the next 2 years. Obviously, the model does not adequately capture concerns that Fed policymakers had about deflation and it is unclear at this point how to extend a linear DSGE model to capture such effects without producing significant computational costs and complexity.

5.5 Revisions in Medium-Term Forecasts

To understand how the model's medium-term forecasts are revised in response to new data we have included Figures 7.25 to 7.33, which show each forecast up to 12 quarters into the future for GDP per capita, inflation and the Fed funds rate. However, rather than reporting forecasting errors and their decomposition, the middle panel of these figures reports how much the forecasts 12 quarters ahead are revised based on one quarter of new data and the bottom panel reports what shocks account for these revisions. To do the comparison we create a forecast 12 quarters ahead, update the information set by one quarter and then compare the difference between the forecast 11 quarters ahead with the previous forecast that was made 12 quarters ahead. The bottom and middle panels are lined up by the values at the end of the forecast, but are based on the arrival of new data 12 quarters preceding each of these quarters.

6. Using the Model's Forecasts to Create Trend-Cycle Decompositions

In this section we exploit the forecasting performance of the model to develop more reliable real-time measures of the output gap using the Hodrick-Prescott (HP) filter. We do this by using the model's forecasts for per capita GDP in each quarter to construct 2-sided measures of the output gap. After presenting the estimates of this procedure over the last 5 years we then show the benefits of this procedure by comparing how its measures at the end of each historical sample period would be revised after the real data are released and the model's forecasts are replaced with actual observations.[32]

[32] It is well known that univariate filters such as the HP filter give very imprecise estimates of the output gap at the end of the sample—see for example Laxton and Tetlow (1992).

6.1 Estimates of Output Gaps

Figure 7.34 presents the gaps under six values of the HP smoothing parameter that range from 600 to 5600 in increments of 1000. These estimates were constructed by taking the model's forecasts for GDP per capita for the next 5 years and then detrending the data based on an endpoint of 2010Q4. The figure also includes estimates of the output gap from the FRBUS model as a set of reference values. The series all suggest positive values of the output gap in early 2000 that then decline and become negative shortly after the economy is hit by a recession in the first quarter of 2001. The gaps then trough between -1 and -2 percent before starting to recover in 2003Q3. After this period the output gap gradually closes as the economy recovers from the recession and by the end of the sample all estimates suggest that the output gap has turned slightly positive. The estimates from the FRBUS models suggest much larger positive values of the output gap in 2000, but then the FRBUS estimates are broadly similar over the last three years.

6.2 Historical Revisions in Output Gaps

The first column of table 7.10 shows the RMSE difference between the end-of-sample one-sided estimates from the HP filter and the final 2-sided estimates, which have the benefit of hindsight in that ”future values” of the GDP per capita are used in the calculations. The second column of table 7.10 reports similar estimates, but in this case the end-of-sample estimates from the extended HP filter are based on forecasts from the model. As can be seen there is a significant improvement using the forecasts of the DSGE model to help condition the estimates, with the RMSEs falling between .35 and .62 depending on what HP smoothing parameter is chosen.

As indicated earlier the forecasting performance of the DSGE model is weaker during turning points so while we might expect its trend-cycle decomposition to be somewhat better during recessions and recoveries it will be anything but a panacea. Figure 7.35 shows the real-time updating of the trend estimates from the HP filter (assuming a smoothing parameter equal to 1600) between 2000Q1 and 2002Q4 and then compares the gaps with the final gaps that are based on information up to 2005Q4. This charts shows clearly how the trend estimates from the HP filter are adjusted during a business cycle, starting at high levels during a peak and then being revised down systematically over time only to be revised up again if the economy recovers strongly from a trough. Figure 7.36, which reports the extended HP estimates based on the forecasts of the DSGE model, shows the benefits of using forecasts from the DSGE model. In this case the downward revisions in the estimates occur faster and stabilize faster, suggesting the providing of a better signal that the previous peak in activity was not sustainable.

7. Conclusions

In this paper we have proposed a New Keynesian DSGE model that, based on our preliminary Bayesian estimation results, looks promising for addressing some key problems of this model class. The fit and out-of-sample forecasting properties of the model are quite competitive when compared to other methods, and we therefore have some confidence in the model's ability to fit the data. After evaluating the model's ability to forecast we exploit the model's forecasting performance to develop measures of output gaps that are more reliable at the end of the sample. However, more work needs to be done to distinguish what features contribute to the overall fit of the model and what features are nonessential. In future work we aim to expand further on this analysis in a number of directions and then extend it to include open-economy and multi-country dimensions.

References

ADOLFSON, M., LASEEN, S., LINDE, J. AND VILLANI, M. (2005): "Estimating and Testing a New Keynesian Small Open Economy Model." Paper prepared for the IMF-FED-ECB Conference in Washington, D.C.

AKERLOF G. AND YELLEN, J. (1985): "A Near Rational Model of the Business Cycle with Wage and Price Inertia." *Quarterly Journal of Economics.*

ALTIG, D., CHRISTIANO, L.J., EICHENBAUM, M. AND LINDE, J. (2005): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle." Working Paper.

BAKSHI, H., BURRIEL-LLOMBART, P., KHAN, H. AND RUDOLF, B. (2003): "Endogenous Price Stickiness, Trend Inflation, and the New Keynesian Phillips Curve." Working Paper, Bank of England and Swiss National Bank.

BATINI, N., KUTTNER, K. AND LAXTON, D. (2005): *Does Inflation Targeting Work in Emerging Markets?* chapter IV, pages 161–186. World Economic Outlook.

BAYOUMI, T., LAXTON, D. AND PESENTI, P. (2004): "Benefits and Spillovers of Greater Competition in Europe: A Macroeconomic Assessment." NBER Working Paper No. 10416.

BILS, M. AND KLENOW, P. (2004): "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy,* 112:947–985.

BOLDRIN, M., CHRISTIANO, L.J. AND FISHER, J. (2001): "Habit Persistence, Asset Returns and the Business Cycle." *American Economic Review,* 91(1):149–166.

CALVO, G.A. (1983): "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics,* 12:383–398.

CALVO, G.A., CELASUN, O. AND KUMHOF, M. (2001): "A Theory of Rational Inflationary Inertia." In P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, editors, *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps,* Princeton. Princeton University Press.

CALVO, G.A., CELASUN, O. AND KUMHOF, M. (2002): "Inflation Inertia and Credible Disinflation: The Open Economy Case." Working Paper, IADB and Stanford University.

CHANG, Y. AND KIM, S.-B. (2005): "On the Aggregate Labor Supply." *Federal Reserve Bank of Richmond Economic Quarterly,* 91(1):21–37.

CHRISTIANO, L.J., EICHENBAUM, M. AND EVANS, C. (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy,* 113(1): 1–45.

CLARIDA, R., GALÍ, J. AND GERTLER, M. (1999): "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature,* 37:1661–1707.

COENEN, G. AND LEVIN, A. (2004): "Identifying the Influences of Nominal and Real Rigidities in Aggregate Price-Setting Behavior." ECB Working Paper, No. 418.

COGLEY, T. AND SBORDONE, A.M. (2005): "A Search for a Structural Phillips Curve." Federal Reserve Bank of New York Staff Reports, No. 203.

DEVEREUX, M. AND SIU, H. (2005): "State-Dependent Pricing and Business Cycle Asymmetries." *International Economic Review* (forthcoming).

EDGE, R., M. KILEY AND J.P LAFORTE (2006A): "Documentation of the February 2006 Version of the MAQS Dynamic Stochastic General Equilibrium Model of the U.S Economy." draft available from authors.

EDGE, R., M. KILEY AND J.P LAFORTE (2006B): "A Comparison of Forecast Performance Between Federal Reserve Forecasts, Simple Reduced-Form Models, and a DSGE Model." draft available from authors.

EHRMANN, M. AND SMETS, F. (2003): "Uncertain Potential Output: Implications for Monetary Policy." *Journal of Economic Dynamics and Control,* 27:1611–1638.

EICHENBAUM, M. AND FISHER, J. (2004): "Evaluating the Calvo Model of Sticky Prices." Working Paper, Northwestern University.

ERCEG, C.J. AND LEVIN, A.T. (2003): "Imperfect Credibility and Inflation Persistence." *Journal of Monetary Economics,* 50(4):915–944.

ECB (2005): "Inflation Persistence and Price Setting Behaviour in the Euro Area." European Central Bank, Inflation Persistence Network.

FUHRER, J. AND MOORE, G. (1995): "Inflation Persistence." *Quarterly Journal of Economics,* 110:127–159.

GALÍ, J. AND GERTLER, M. (1999): "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics,* 44(2):195–222.

GALÍ, J., GERTLER, M. AND LOPEZ-SALIDO, D. (2005): "Robustness of Estimates of the Hybrid New Keynesian Phillips Curve." *Journal of Monetary Economics,* 52.

GOLOSOV, M. AND LUCAS, R.E. (2003): "Menu Costs and Phillips Curves." NBER Working Paper No. 10187.

GORDON, R.J. (1982): "Why Stopping Inflation May Be Costly: Evidence from Fourteen Historical Episodes." In Hall, R.E., editors, *Inflation: Causes and Consequences,* Chicago. University of Chicago Press.

GORDON, R.J. (1997): "The Time-Varying NAIRU and its Implications for Economic Policy." *Journal of Economic Perspectives,* 11–32.

GÜRKAYNAK, R., SACK, B. AND SWANSON, E. (2005): "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models." *American Economic Review,* 95:425–436.

JUILLARD, M., KARAM, P., LAXTON, D. AND PESENTI, P. (2005): "Welfare-Based Monetary Policy Rules in an Estimated DSGE Model of the US Economy." Working Paper.

KLENOW, P. AND KRYVTSOV, O. (2004): "State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?" Working Paper.

LAXTON, DOUGLAS, AND ROBERT TETLOW (1992): "A Simple Multivariate Filter for the Measurement of Potential Output." Technical Report 59, Ottawa: Bank of Canada

LEVIN, A., NATALUCCI, F. AND PIGER, J. (2004): "The Macroeconomic Effects of Inflation Targeting." *Federal Reserve Bank of St. Louis Review,* 86(4):51–80.

MANKIW, G. N. (2001): "The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment." *Economic Journal,* 111:45–61.

MANKIW, G. N. AND REIS, R. (2002): "Sticky Information or Sticky Prices: A Proposal to replace the New-Keynesian Phillips Curve." *Quarterly Journal of Economics,* 117: 1295–1328.

PALOVIITA, M. (2004): "Inflation Dynamics in the Euro Area and the Role of Expectations: Further Results." Bank of Finland Discussion Paper, No. 21.

PENCAVEL, J. (1986): "Labor Supply of Men." In O. Ashenfelter and R. Layards, editors, *Handbook of Labor Economics, Vol. 1,* Amsterdam: North Holland.

RUDD, J. AND WHELAN, K. (2005a): "Does Labor's Share Drive Inflation?" *Journal of Money, Credit, and Banking,* 37:297–312.

RUDD, J. AND WHELAN, K. (2005b): "New Tests of the New Keynesian Phillips Curve." *Journal of Monetary Economics.*

RUDD, J. AND WHELAN, K. (2005c): "Modelling Inflation Dynamics: A Critical Survey of Recent Research." Working Paper.

SCHORFHEIDE, F. (2004): "Notes on Model Evaluation." Department of Economics, University of Pennsylvania.

SIMS, C. (2002): "The Role of Models and Probabilities in the Monetary Policy." *Brookings Papers on Economic Activity,* 2.

SIMS, C. (2003): "Matlab Procedures to Compute Marginal Data Densities for VARs with Minnesota and Training Sample Priors." Department of Economics, Princeton University.

SIMS, C. AND ZHA, T. (1998): "Bayesian Methods for Dynamic Multivariate Models." *International Economic Review,* 49:949–968.

SMETS, F. AND WOUTERS, R. (2004): "Shocks and Frictions in Business Cycles: A Bayesian DSGE Approach." Working Paper, European Central Bank and the National Bank of Belgium.

YUN, T. (1996): "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics,* 37:345–370.

ZBARACKI, M.J., RITSON, M., LEVY, D., DUTTA, S. AND BERGEN, M. (2004): "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets." *Review of Economics and Statistics,* 86(2):514–533.

Table 7.1: Specification of the Stochastic Processes

Assumptions about the Shocks	Stochastic Processes
Total Factor Productivity	$\widehat{g}_t = \widehat{g}_t^{gr} + \widehat{g}_t^{iid}$
Demand Shocks	
Government Absorption	$\widehat{s_t^{gov}} = \rho_{gov} \widehat{s_{t-1}^{gov}} + \hat{\varepsilon}_t^{gov}$
Investment	$\widehat{s_t^{inv}} = \rho_{inv} \widehat{s_{t-1}^{inv}} + \hat{\varepsilon}_t^{inv}$
Marginal Utility of Consumption	$\widehat{s_t^c} = \rho_c \widehat{s_{t-1}^c} + \hat{\varepsilon}_t^c$
Monetary Policy Reaction Function	$\widehat{s_t^{int}} = \rho_{int} \widehat{s_{t-1}^{int}} + \hat{\varepsilon}_t^{int}$
Inflation Target	$\hat{\pi}_t^* = \hat{\pi}_{t-1}^* + \hat{\varepsilon}_t^{\pi^*}$
Autocorrelated Growth Shocks	$\widehat{g}_t^{gr} = \rho_{gr} \widehat{g}_{t-1}^{gr} + \hat{\varepsilon}_t^{gr}$
Supply Shocks	
Price Markup	$\widehat{\mu}_t = \hat{\varepsilon}_t^{\mu}$
Wage Markup	$\widehat{\mu_t^w} = \hat{\varepsilon}_t^{\mu^w}$
Marginal Disutility of Labor	$\widehat{s_t^L} = \rho_L \widehat{s_{t-1}^L} + \hat{\varepsilon}_t^L$
I.i.d. Growth Shocks	$\widehat{g}_t^{iid} = \hat{\varepsilon}_t^{iid}$

Table 7.2: Estimation Results

	Parameters				
	Prior	Estimate	Std	Density	Prior Std
υ	0.85	0.73	0.058	*Beta*	0.10
γ	0.50	0.50	0.089	*Normal*	0.10
δ	0.66	0.71	0.033	*Beta*	0.10
δ_w	0.66	0.59	0.046	*Beta*	0.10
δ_k	0.66	0.68	0.10	*Beta*	0.10
ϕ	1.00	1.03	0.19	*Normal*	0.20
ϕ_w	1.00	0.96	0.20	*Normal*	0.20
ϕ_k	1.00	1.00	0.19	*Normal*	0.20
θ_k	5.00	5.31	0.97	*Normal*	1.00
θ_i	50.00	52.95	8.74	*Normal*	10.00
ξ_{int}	0.75	0.95	0.043	*Normal*	0.10
ξ_π	0.25	0.32	0.066	*Normal*	0.10
ξ_y	0.50	0.34	0.072	*Normal*	0.20
ϵ	0.50	0.48	0.23	*Gamma*	0.25
$\bar{g}$	1.0042	1.0058	0.0018	*Normal*	0.008
g_s	0.18	0.23	0.029	*Beta*	0.04
σ	5.35	5.28	0.95	*Gamma*	1.00
σ^k	11.0	10.79	1.98	*Gamma*	2.00
σ^w	7.25	6.30	1.30	*Gamma*	1.50
Δ	0.025	0.027	0.0076	*Gamma*	0.01
l_s	0.58	0.64	0.036	*Beta*	0.20

Table 7.3: Estimation Results Continued

Parameters					
	Prior	Estimate	Std	Density	Prior Std
ρ_{gov}	0.85	0.99	0.037	*Beta*	0.10
ρ_{inv}	0.85	0.75	0.068	*Beta*	0.10
ρ_c	0.85	0.92	0.032	*Beta*	0.10
ρ_{int}	0.85	0.53	0.083	*Beta*	0.10
ρ_{gr}	0.95	0.95	0.01	*Beta*	0.01
ρ_L	0.85	0.99	0.0053	*Beta*	0.10

Table 7.4: Estimation Results Continued

Standard Deviation of Shocks					
	Prior	Estimate	Std	Density	Prior Std
$\sigma_{\hat{\varepsilon}^{gov}}$	0.025	0.0089	0.015	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{inv}}$	0.2000	0.058	0.010	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{c}}$	0.0250	0.0227	0.0062	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{int}}$	0.0100	0.0036	0.0004	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{\pi*}}$	0.1000	0.020	0.0019	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{iid}}$	0.0010	0.0058	0.0006	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{gr}}$	0.2000	0.0824	0.0261	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{L}}$	0.0050	0.0227	0.0063	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{\mu}}$	0.0250	0.0298	0.0064	*invg*	*inf*
$\sigma_{\hat{\varepsilon}^{\mu^w}}$	0.0250	0.1387	0.0304	*invg*	*inf*

Table 7.5: Comparison of Marginal Likelihoods with BVARs

	Marginal Likelihood
Base Case Model (MH-500,000 Draws)	-352.20
BVAR (1 lag)	-388.99
BVAR (2 lag)	-362.14
BVAR (3 lag)	-358.94
BVAR (4 lag)	-355.16
BVAR (5 lag)	-360.39
BVAR (6 lag)	-365.56
BVAR (7 lag)	-370.07
BVAR (8 lag)	-375.46

Table 7.6: GDP RMSEs for Alternative Models

	Forecast Horizon		
	1 Quarter	4 Quarters	8 Quarters
DSGE-JKKL	0.49	0.89	2.00
DSGE-EKL	0.53	1.50	3.55
BVAR	0.37	1.30	3.03
VAR	0.42	1.23	2.85
RW	0.49	1.75	3.92
GB	0.44	2.13	4.13

Table 7.7: GDP Deflator RMSEs for Alternative Models

	Forecast Horizon		
	1 Quarter	4 Quarters	8 Quarters
DSGE-JKKL	0.18	0.66	1.55
DSGE-EKL	0.24	1.19	2.54
BVAR	0.23	0.79	1.67
VAR	0.23	0.71	1.67
RW	0.24	0.96	1.43
GB	0.20	0.62	1.29

Table 7.8: Fed Funds Rate RMSEs for Alternative Models

	Forecast Horizon		
	1 Quarter	4 Quarters	8 Quarters
DSGE-JKKL	0.25	0.57	0.45
DSGE-EKL	0.12	0.57	1.25
BVAR	0.10	0.65	1.27
VAR	0.11	0.82	1.72
RW	0.11	0.61	0.76
GB	na	na	na

Table 7.9: RMSEs at Different Forecast Horizons

	Historical and Theoretical Values (in parentheses)		
	1 Quarter	4 Quarters	12 Quarters
Inflation	0.80 (0.72)	0.84 (0.99)	1.31 (1.54)
Fed Funds Rate	0.33 (0.34)	1.22 (0.99)	2.28 (1.80)
Hours	0.41 (0.51)	1.37 (1.52)	3.44 (2.99)
GDP	0.53 (0.51)	1.48 (1.73)	2.98 (3.98)
Consumption	0.57 (0.59)	1.36 (2.25)	2.72 (5.18)
Investment	2.58 (2.69)	5.68 (9.81)	8.81 (19.78)
Real Wage	0.59 (0.59)	1.50 (1.45)	3.12 (3.14)
Long-Term Inflation Expectations	0.07 (0.08)	0.20 (0.17)	0.42 (0.28)

Table 7.10: RMSE Differences Between Real Time HP Filter Gaps and Final Estimates

	HP Filter	Extended HP Filter
600	1.02	0.67
1600	1.27	0.78
2600	1.38	0.81
3600	1.38	0.81
4600	1.44	0.83
5600	1.47	0.85

Figure 7.1: Measures of Long-Term Inflation Expectations and Interest Rates

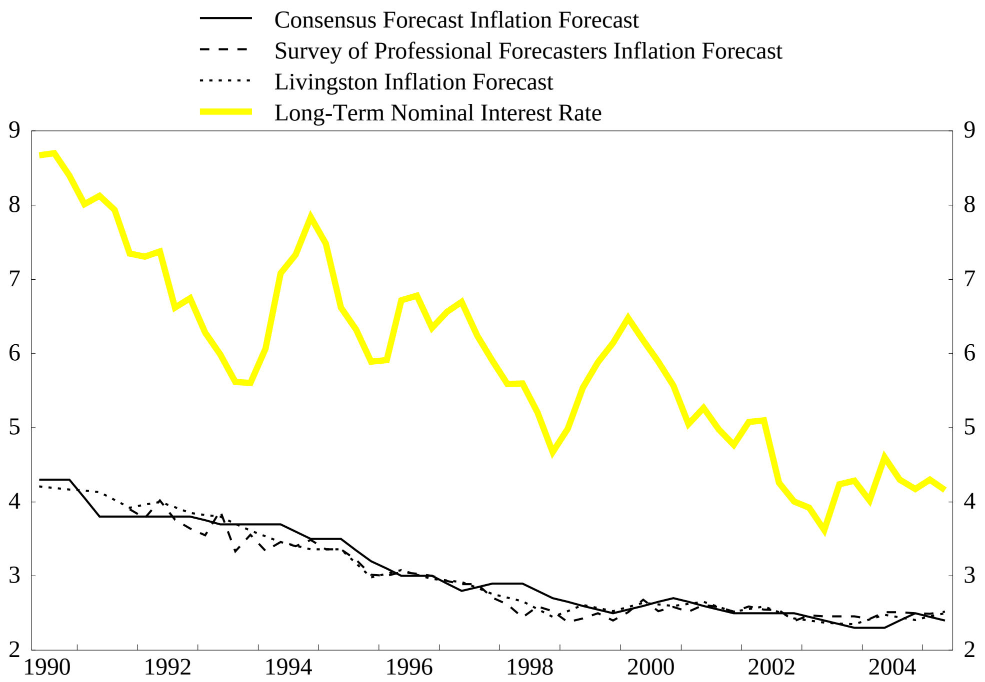


Figure 7.2: Measures of Expected Long-Term Growth

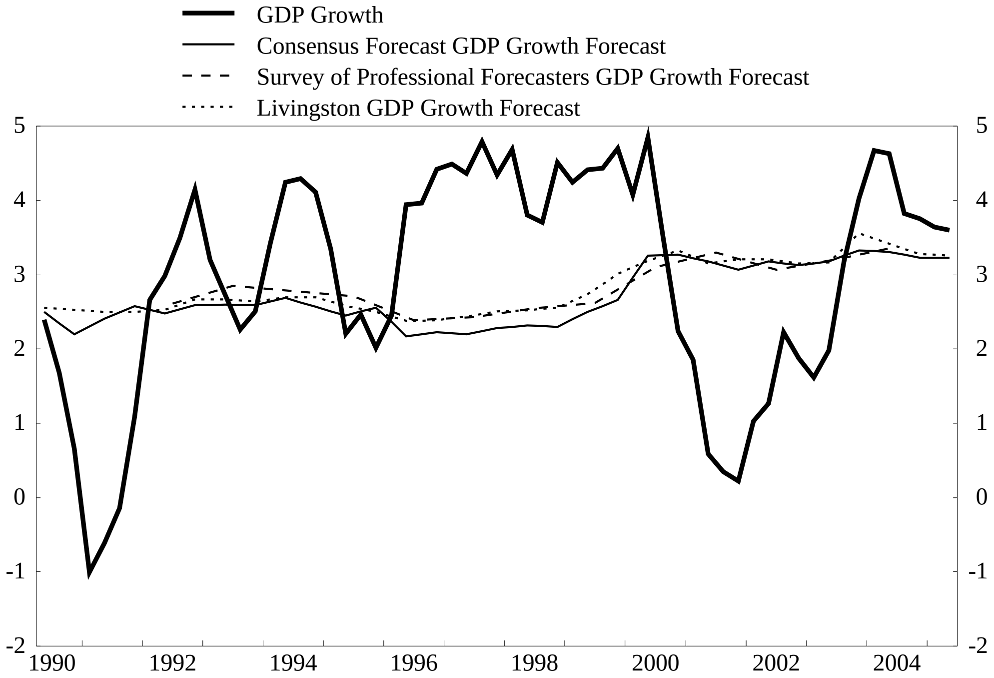


Figure 7.3: Shock to the Fed Funds Rate (Demand)

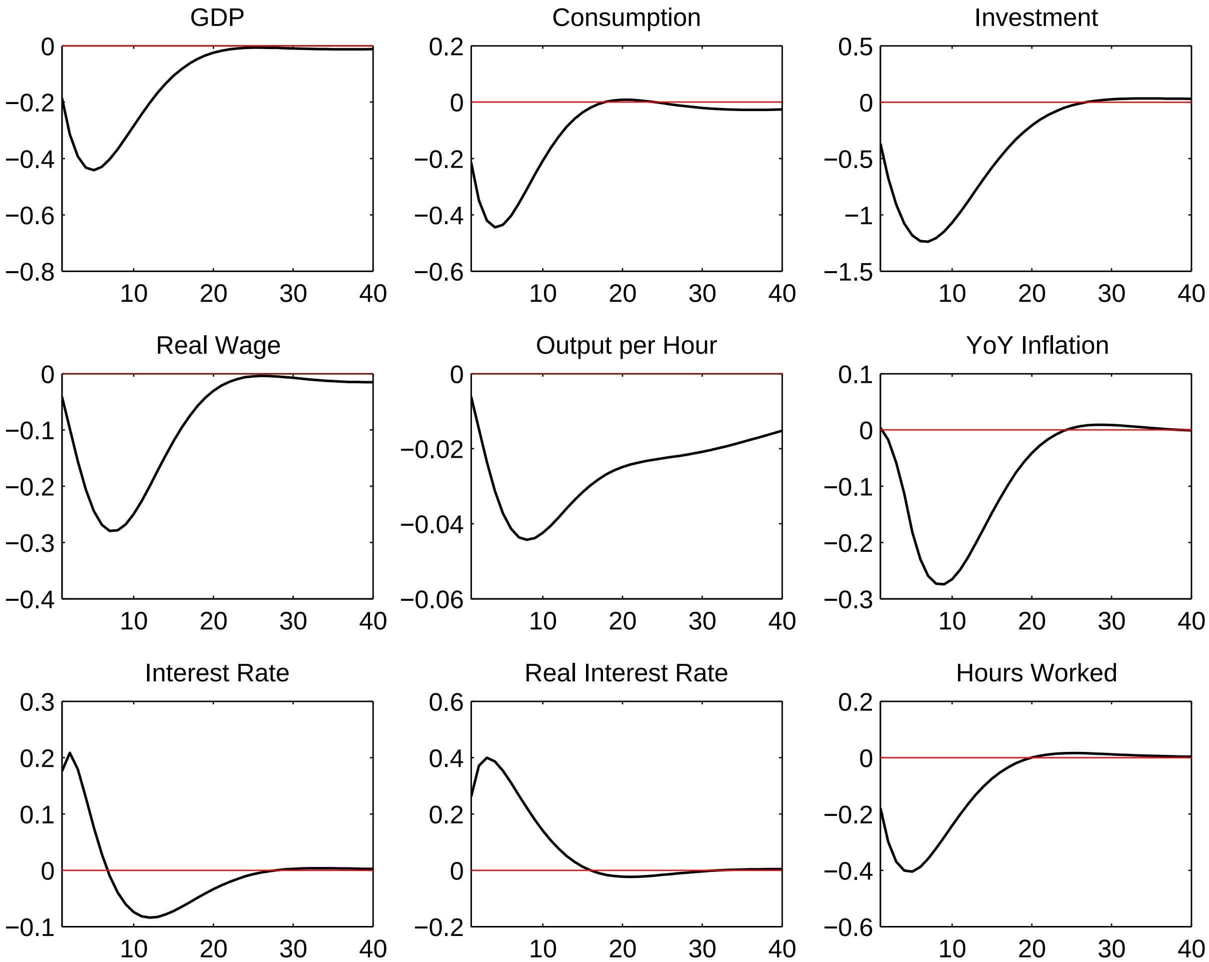


Figure 7.4: Shock to the Inflation Objective (Demand)

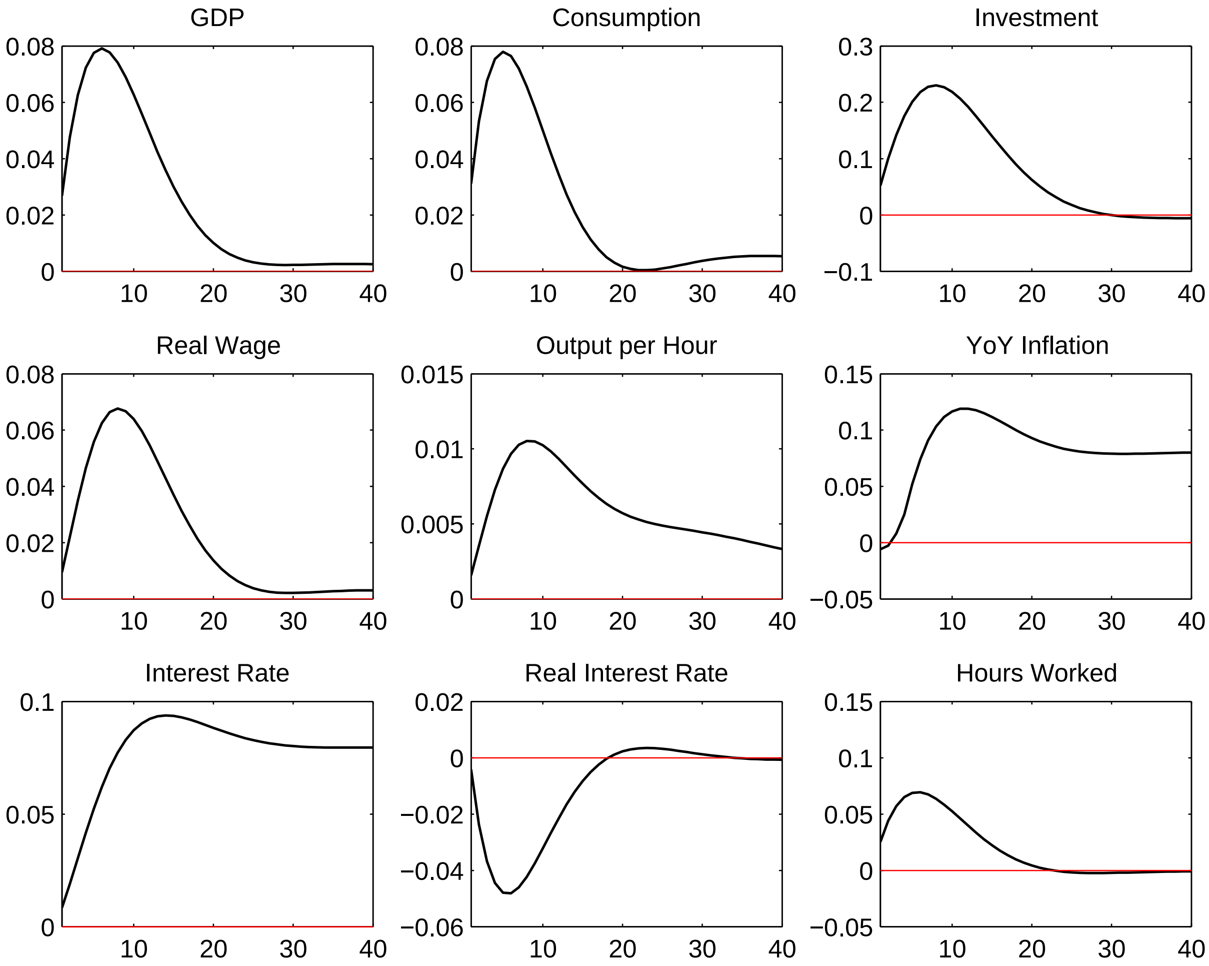


Figure 7.5: Shock to Government Absorption (Demand)

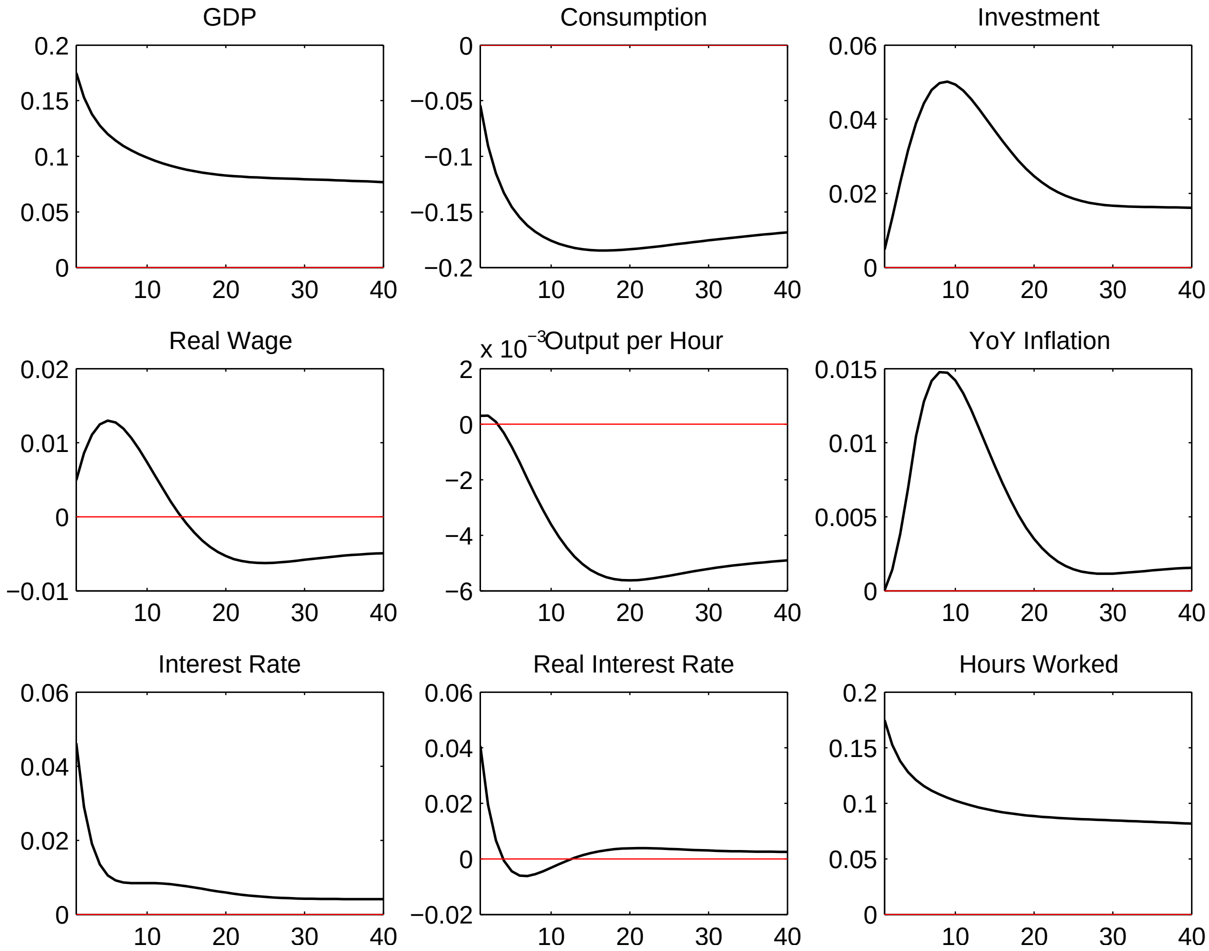


Figure 7.6: Shock to Consumption (Demand)

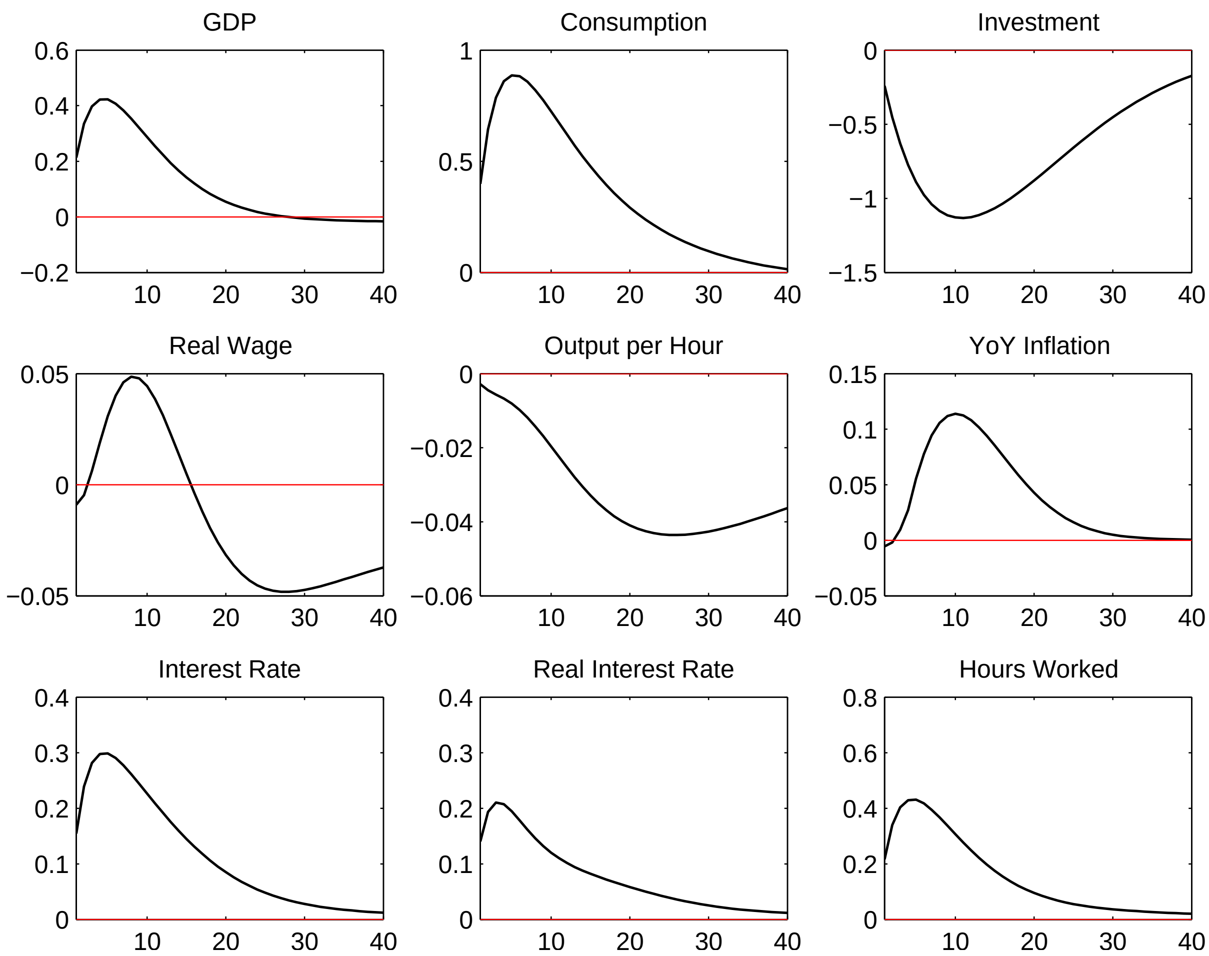


Figure 7.7: Shock to Investment (Demand)

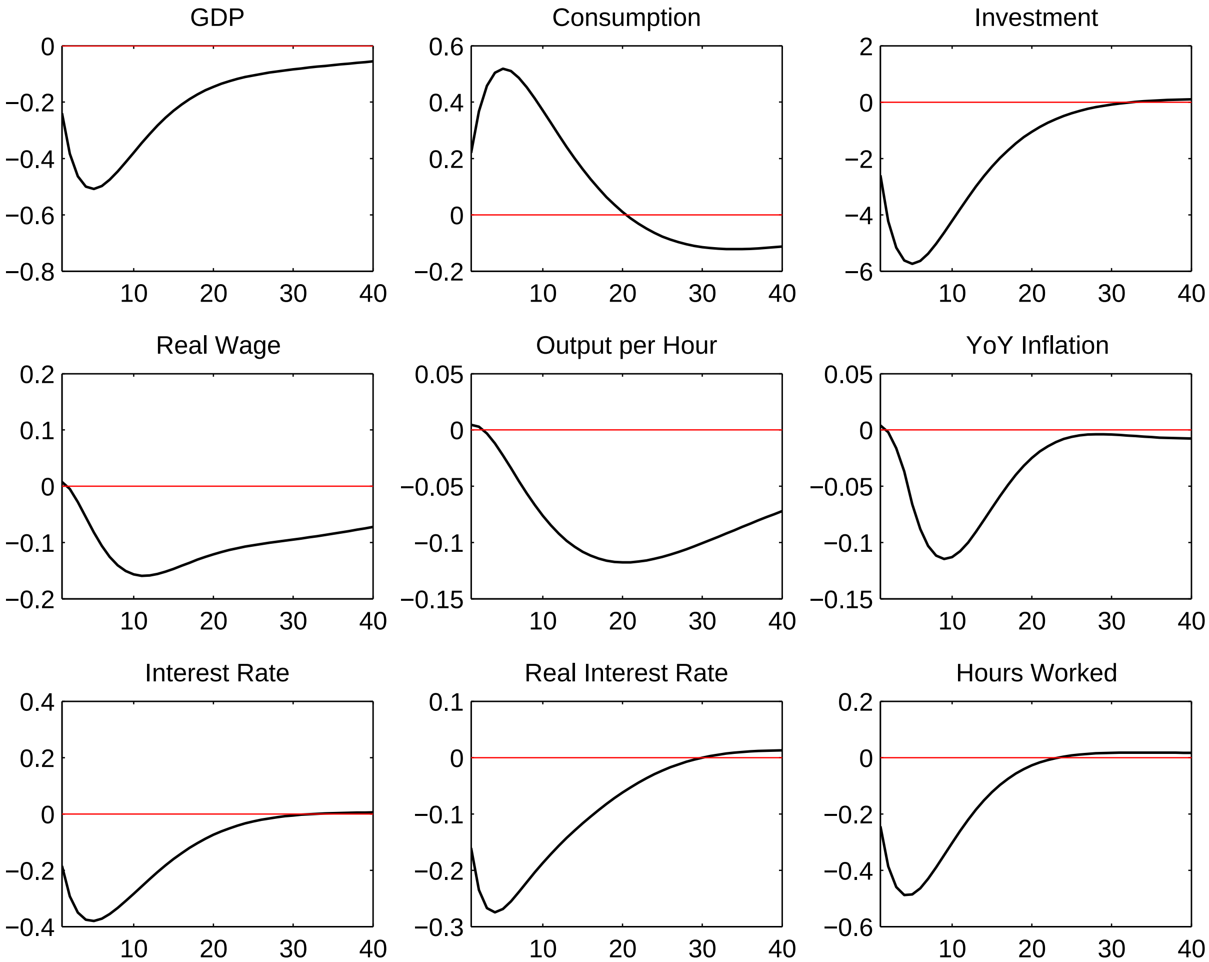


Figure 7.8: Shock to Wage Markup (Supply)

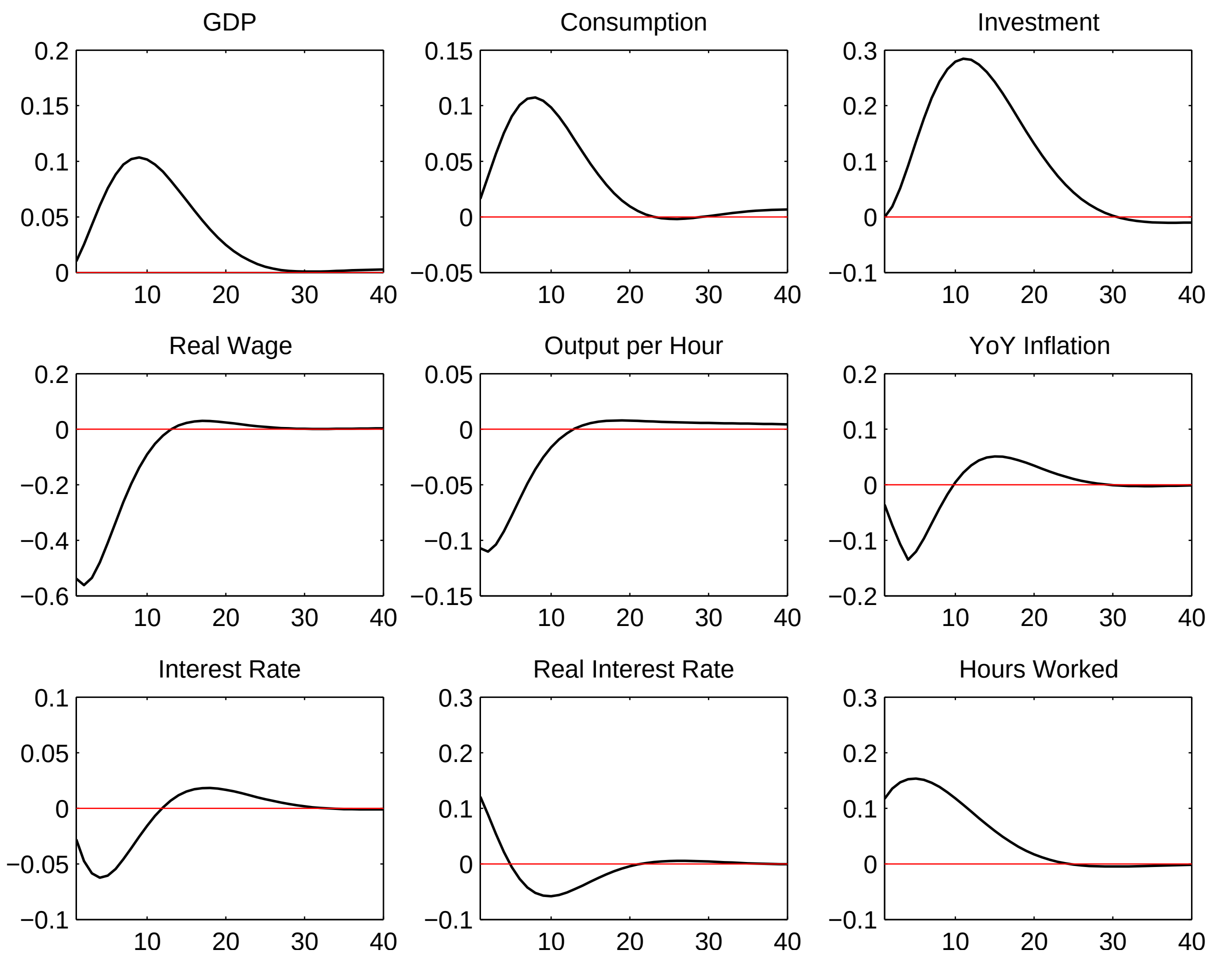


Figure 7.9: Shock to Price Markup (Supply)

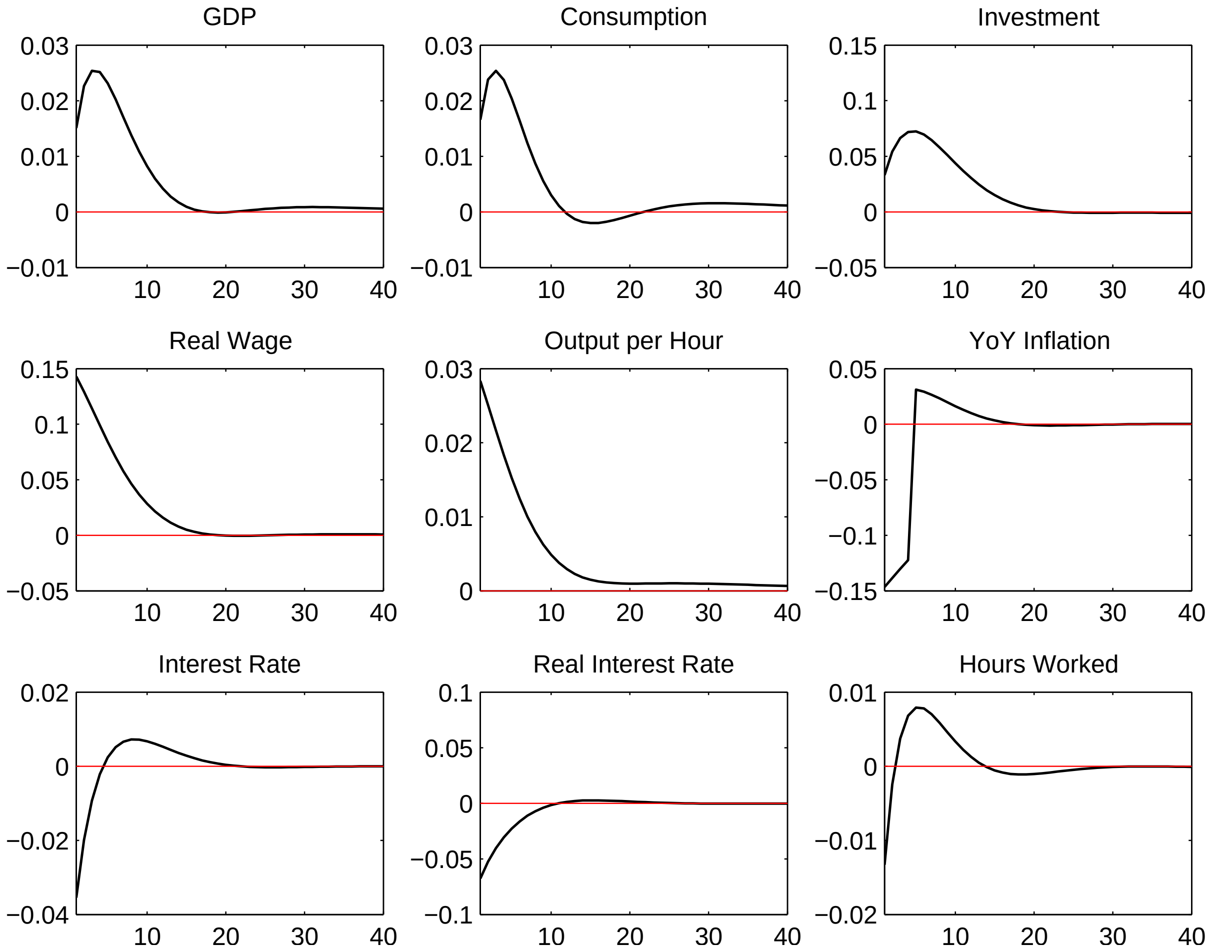


Figure 7.10: Shock to Labor Effort (Supply)

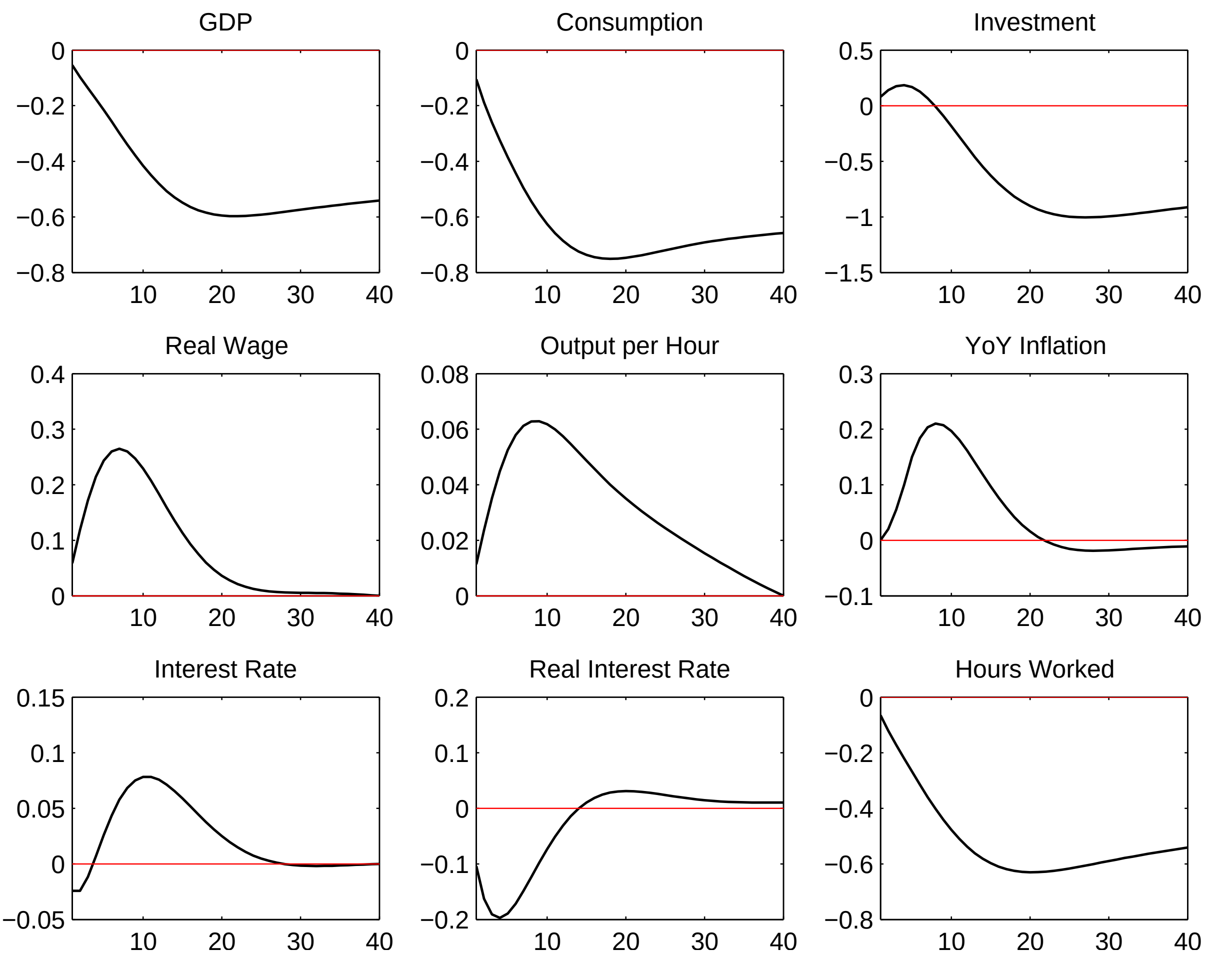


Figure 7.11: Shock to Productivity Level (Supply)

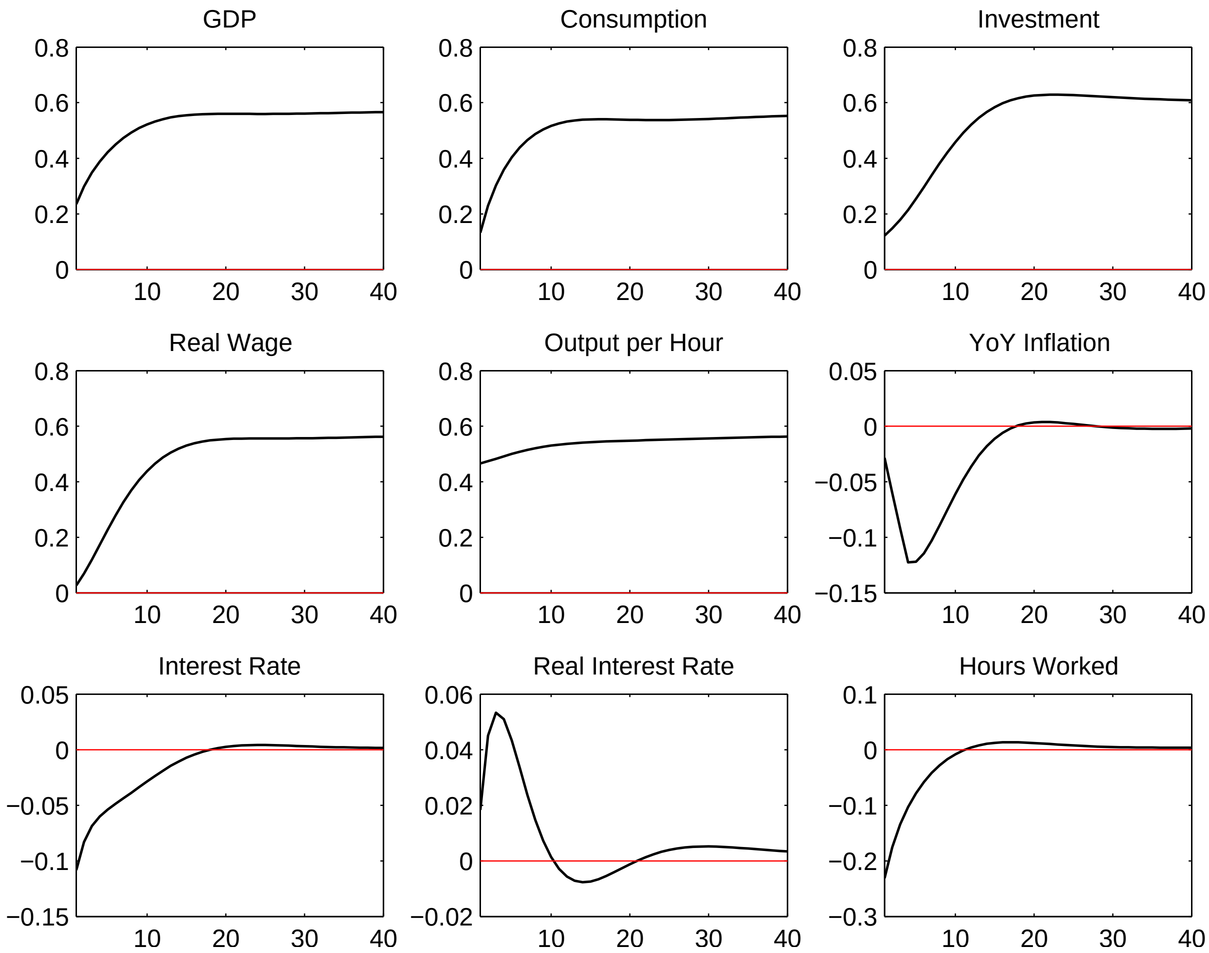


Figure 7.12: Shock to Productivity Growth (Demand)

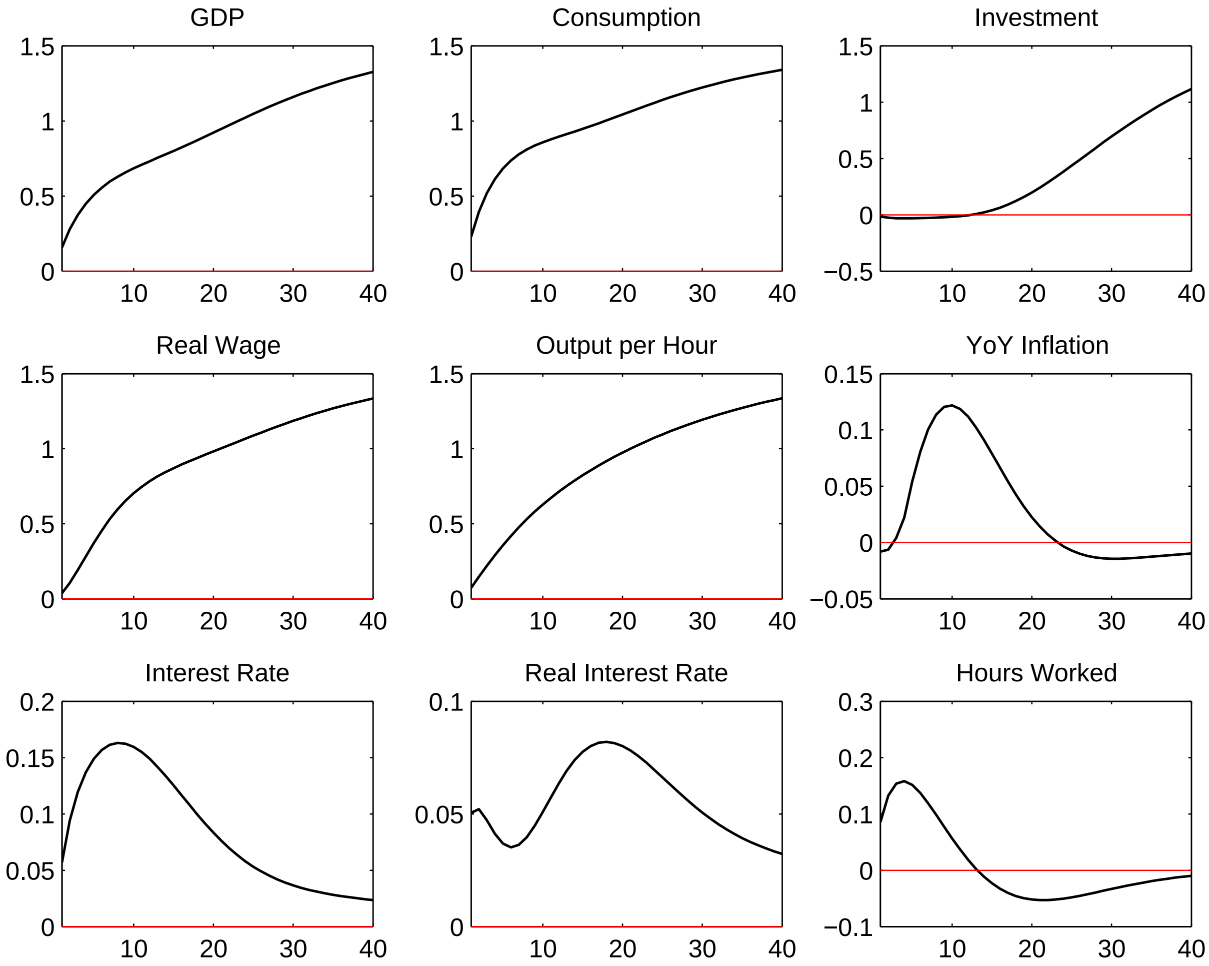


Figure 7.13: Inflation Target Shock and Inflation Dynamics

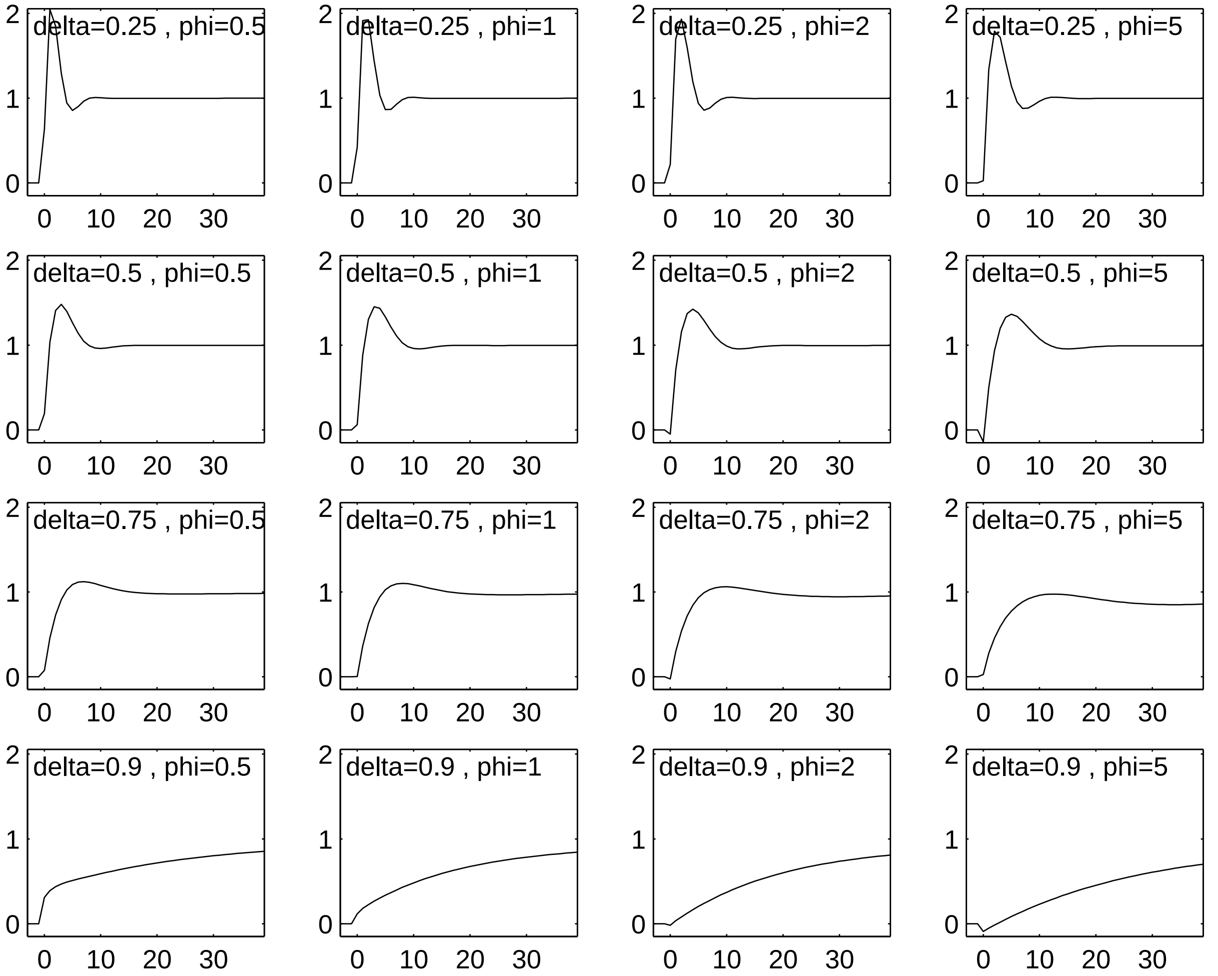


Figure 7.14: Estimated Structural Shocks

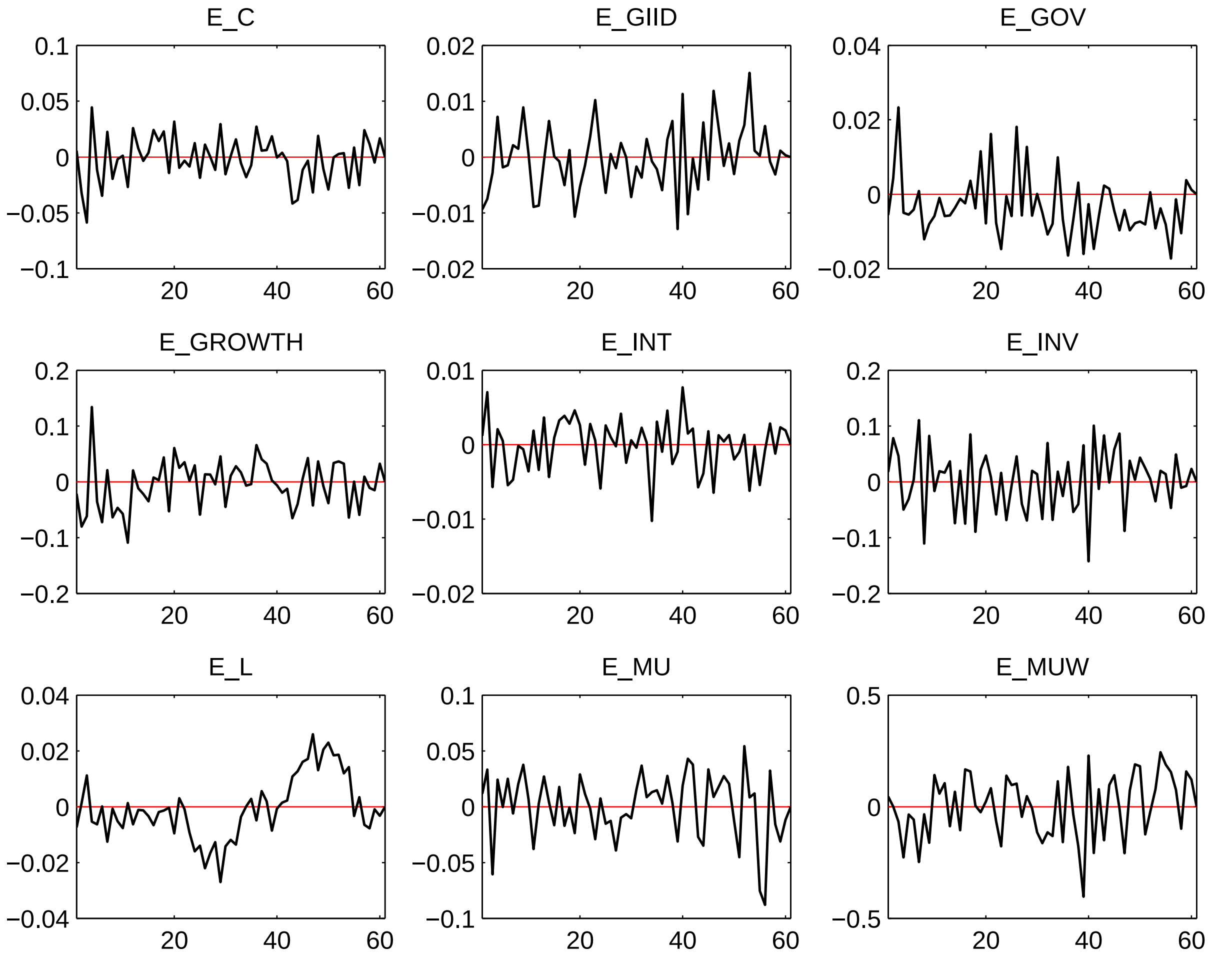


Figure 7.15: Estimated Shocks to the Inflation Target

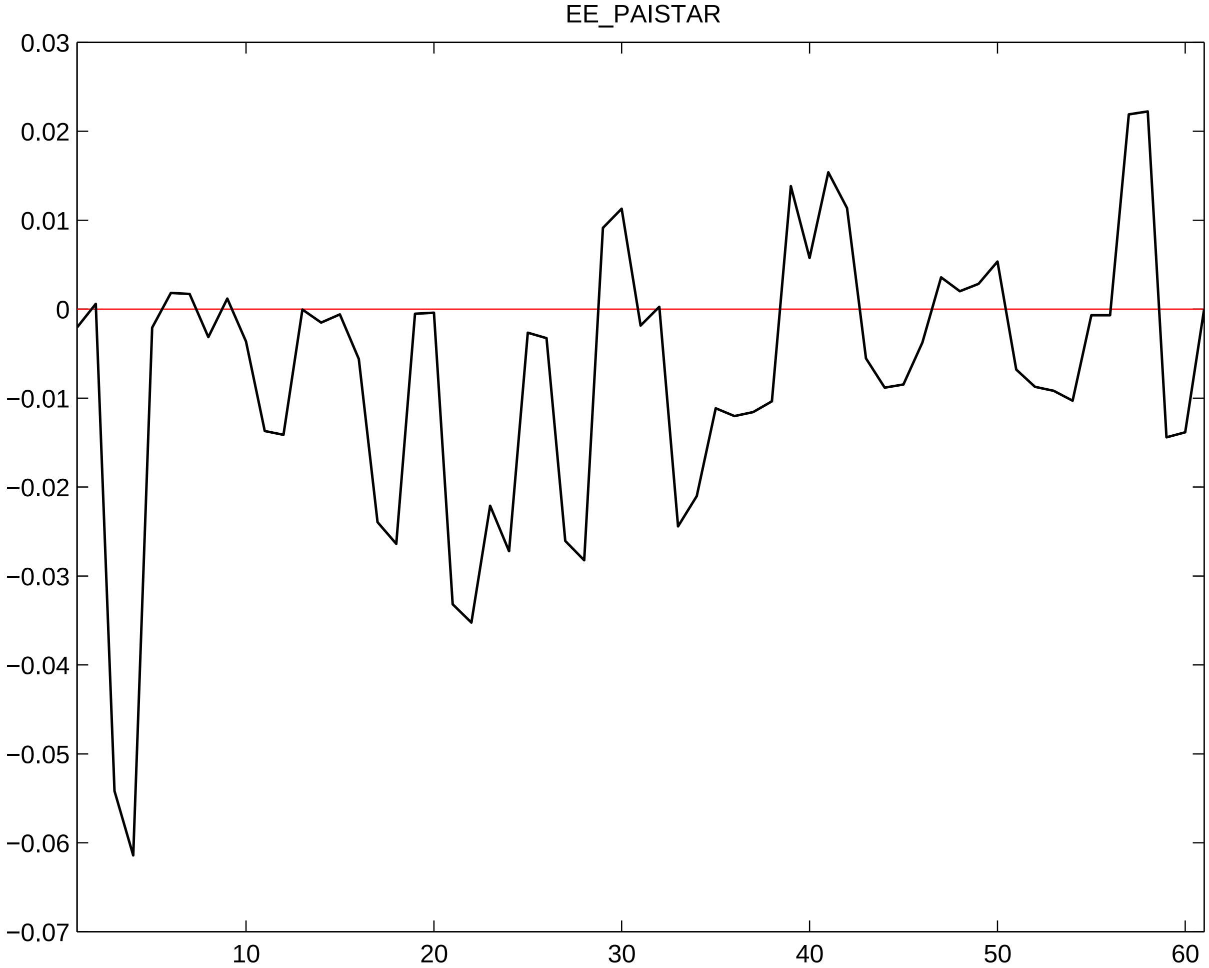


Figure 7.16: GDP Forecast Errors 1990Q3-1995Q2

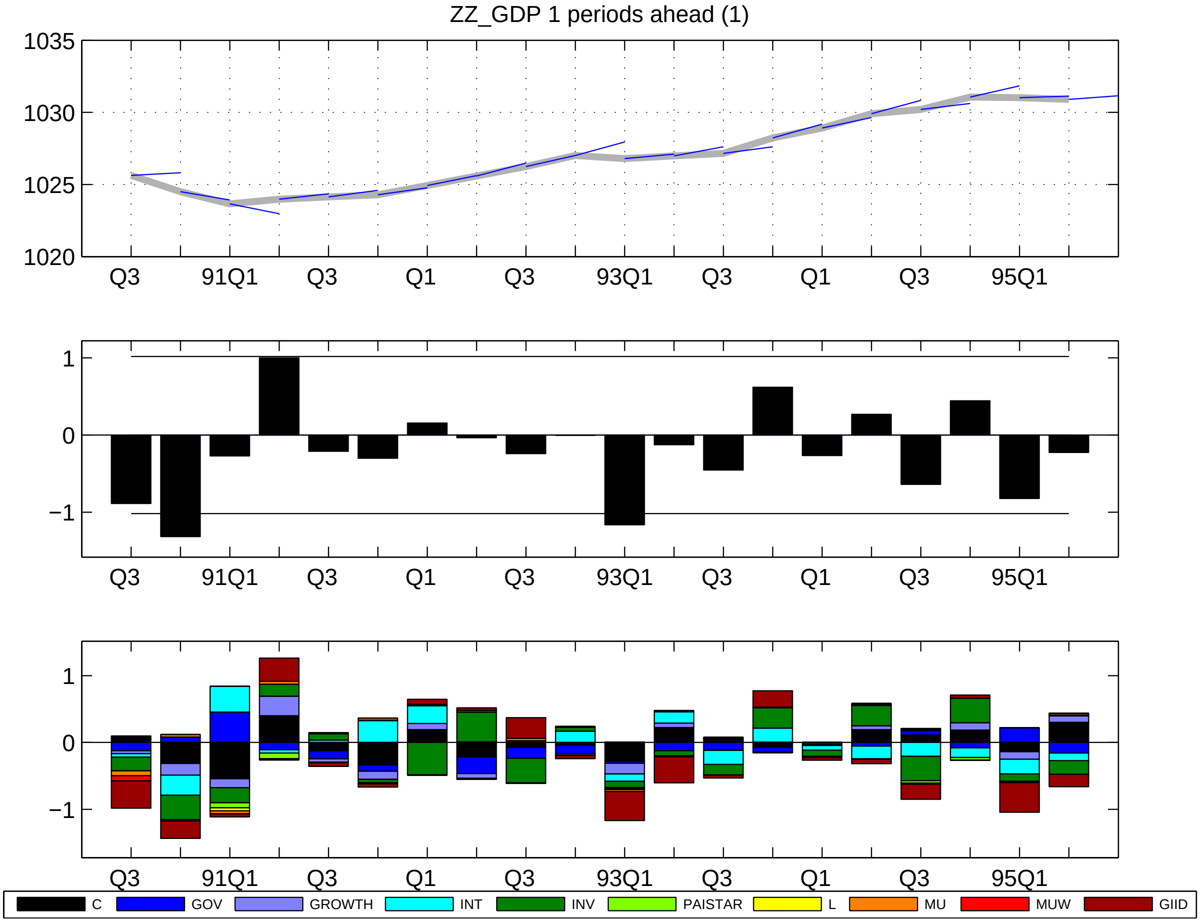


Figure 7.17: GDP Forecast Errors 1995Q3-2000Q2

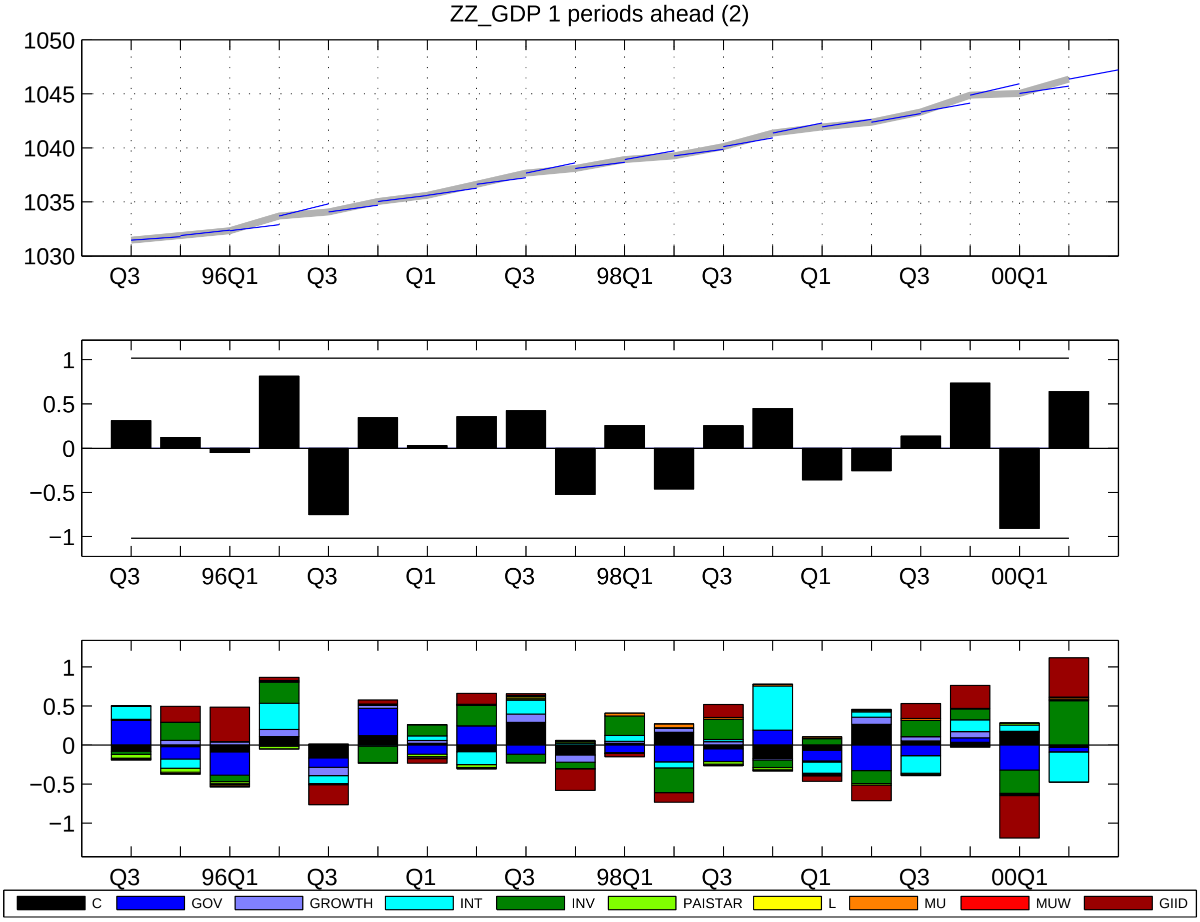


Figure 7.18: GDP Forecast Errors 2000Q3-2005Q4

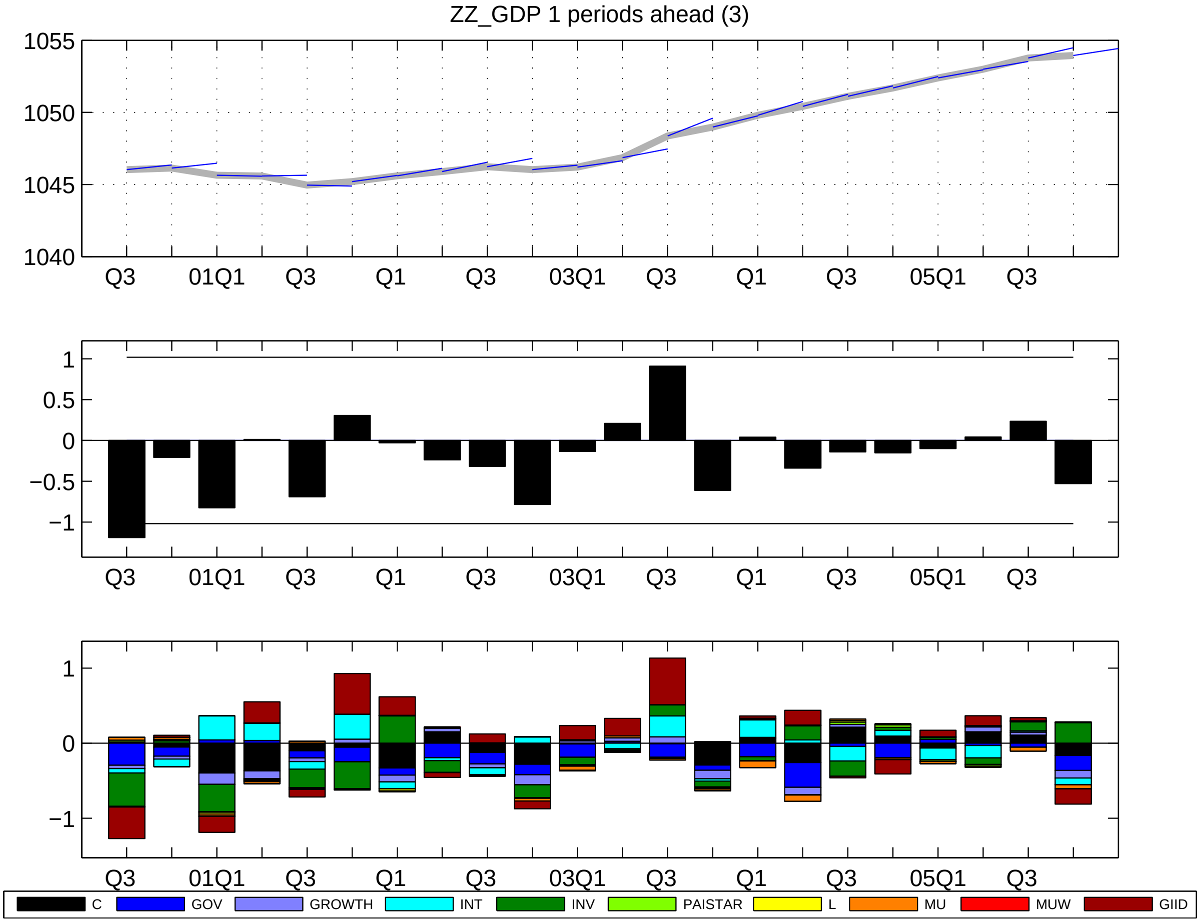


Figure 7.19: Inflation Forecast Errors 1990Q3-1995Q2

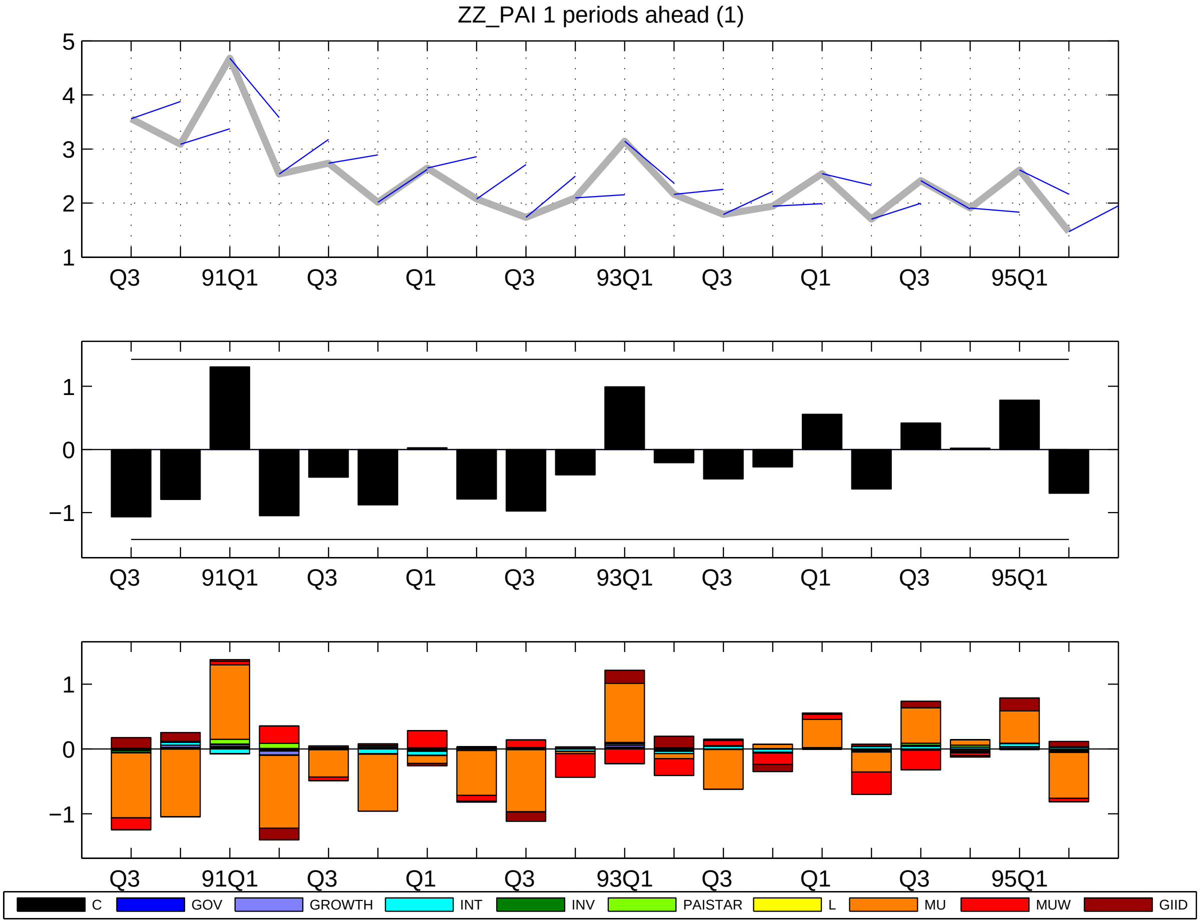


Figure 7.20: Inflation Forecast Errors 1995Q3-2000Q2

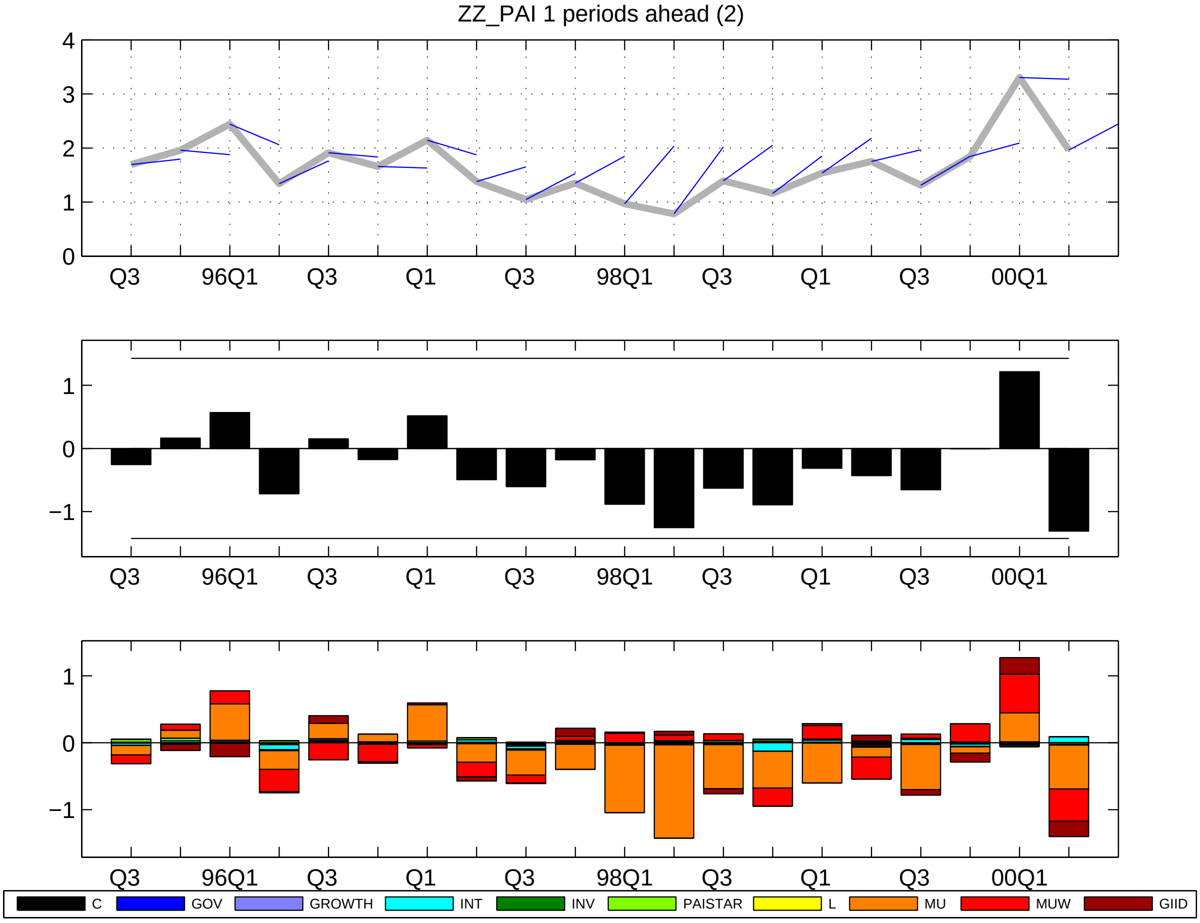


Figure 7.21: Inflation Forecast Errors 2000Q3-2006Q1

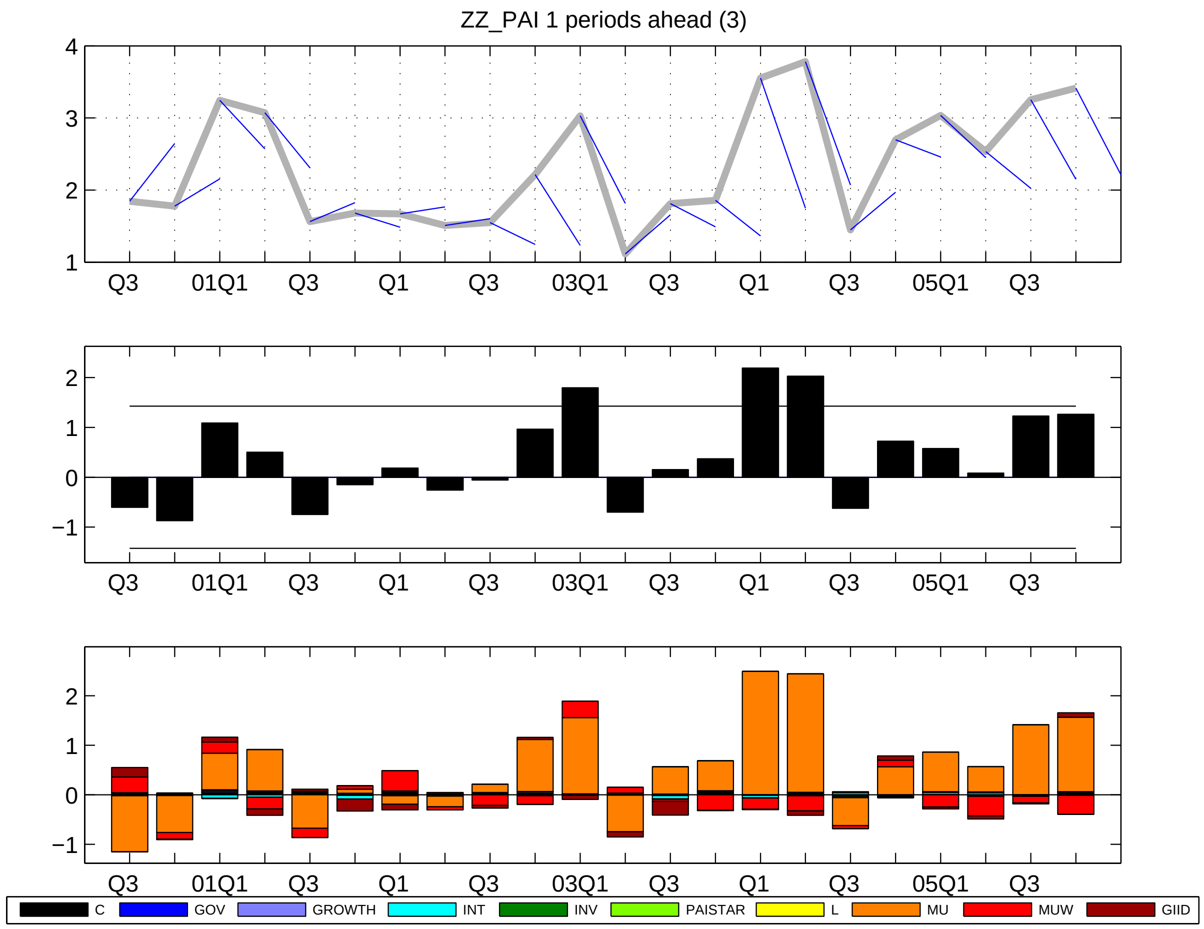


Figure 7.22: Fed Funds Rate Forecast Errors 1990Q3-1995Q2

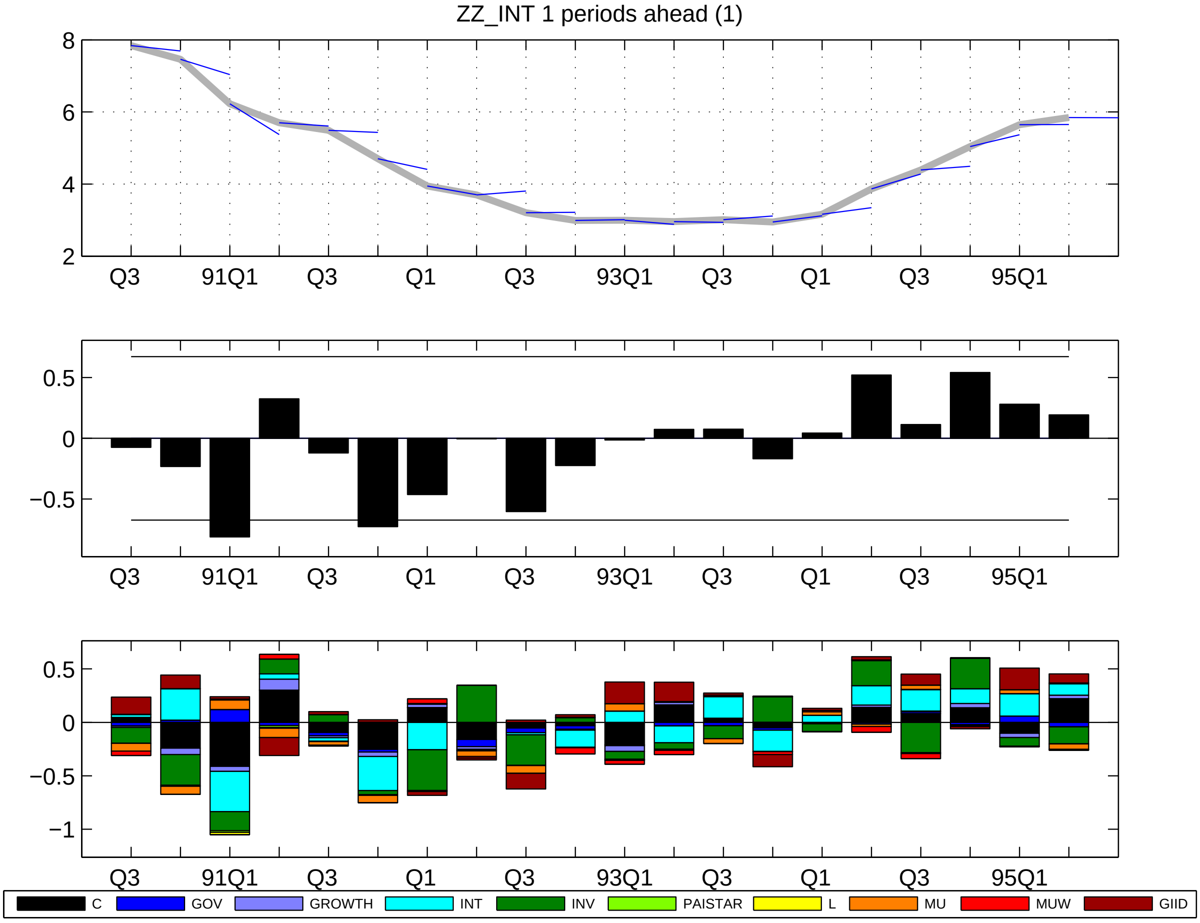


Figure 7.23: Fed Funds Rate Forecast Errors 1995Q3-2000Q2

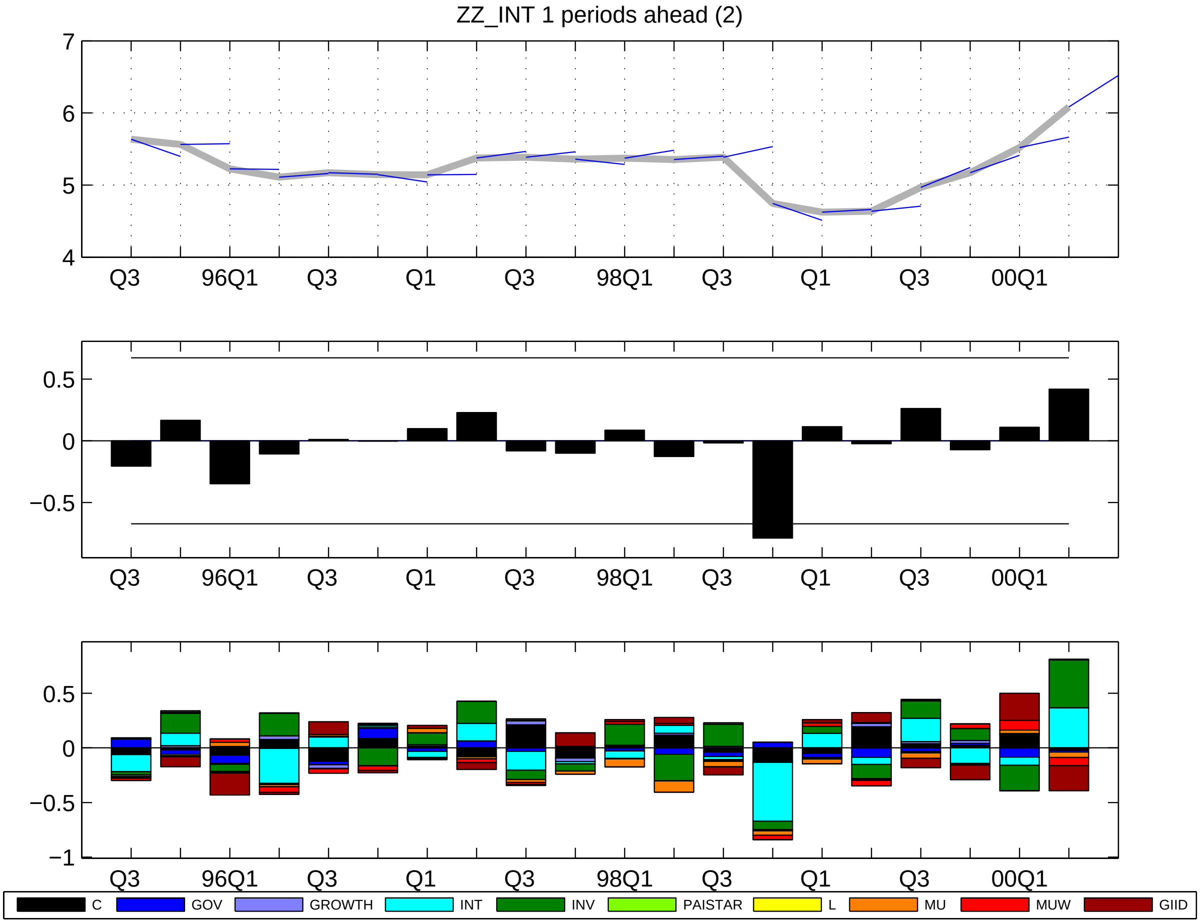


Figure 7.24: Fed Funds Rate Forecast Errors 2000Q3-2005Q4

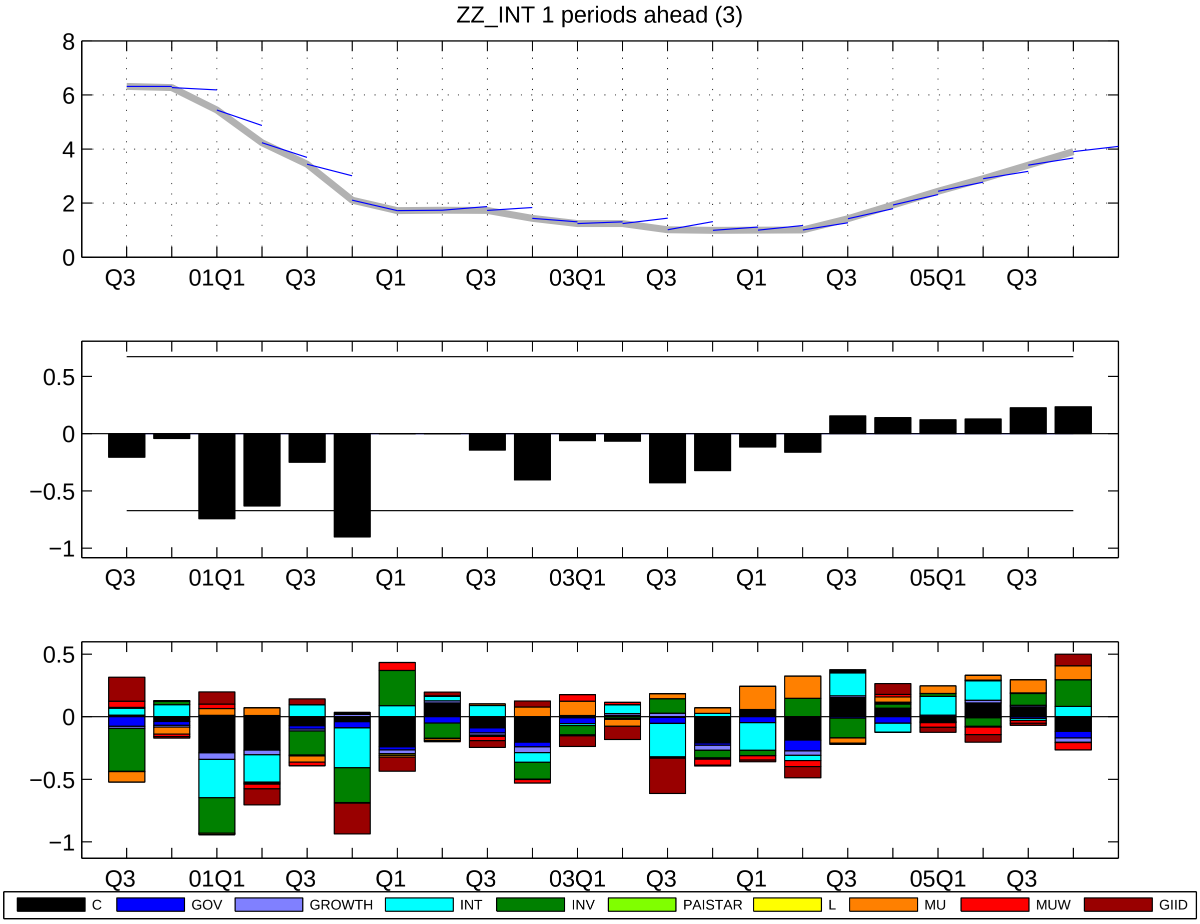


Figure 7.25: GDP Medium-Term Forecast Revisions 1990Q3-1998Q1

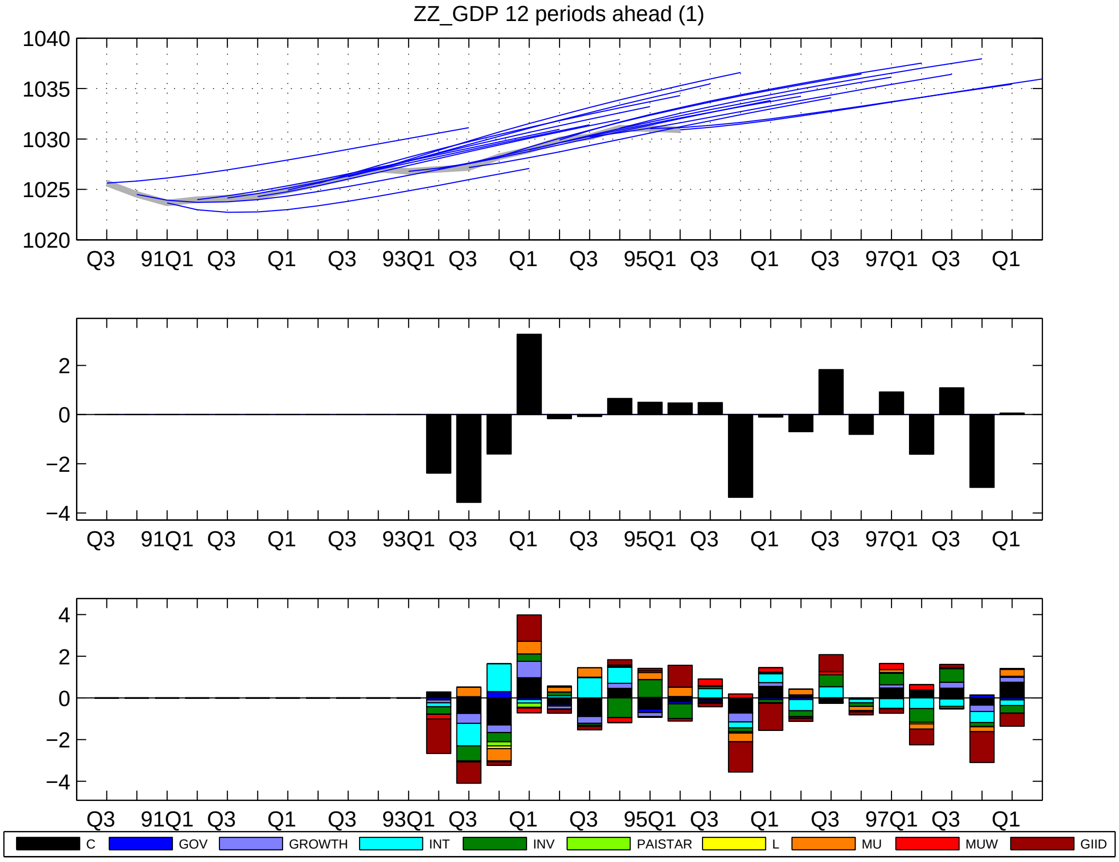


Figure 7.26: GDP Medium-Term Forecast Revisions 1995Q3-2003Q1

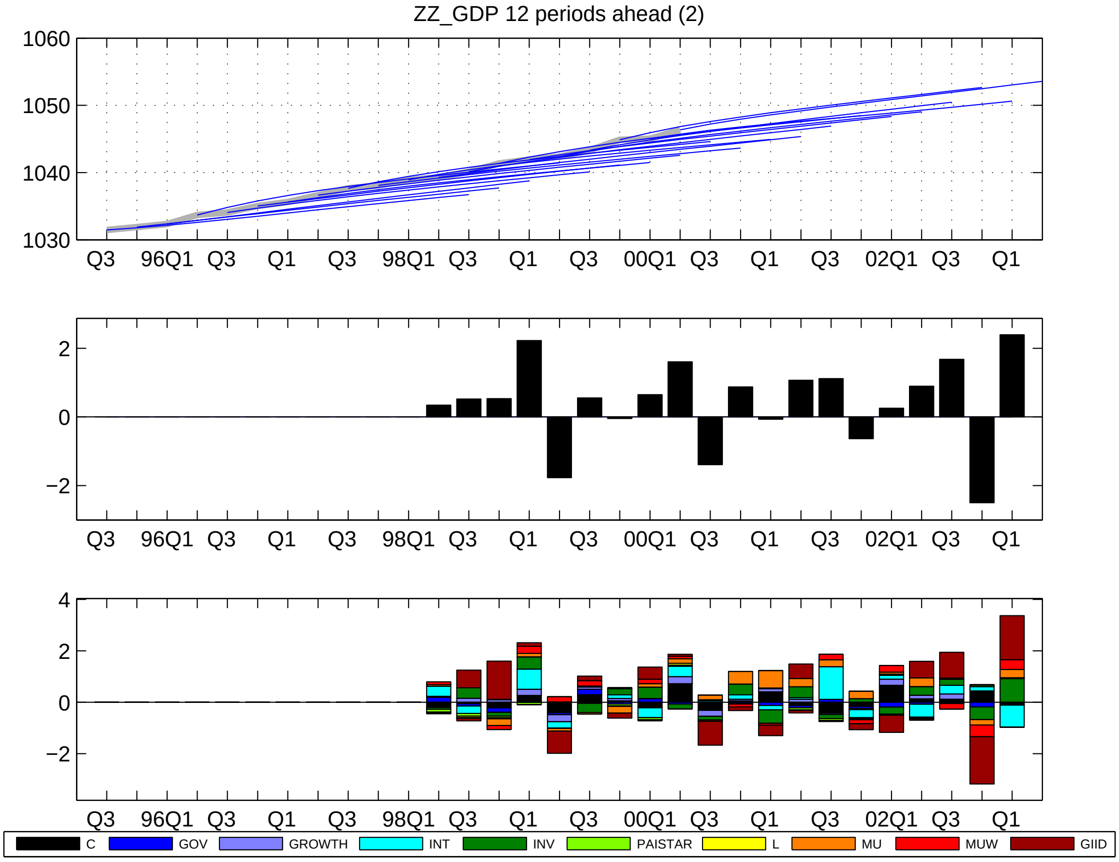


Figure 7.27: GDP Medium-Term Forecast Revisions 2000Q3-2008Q3

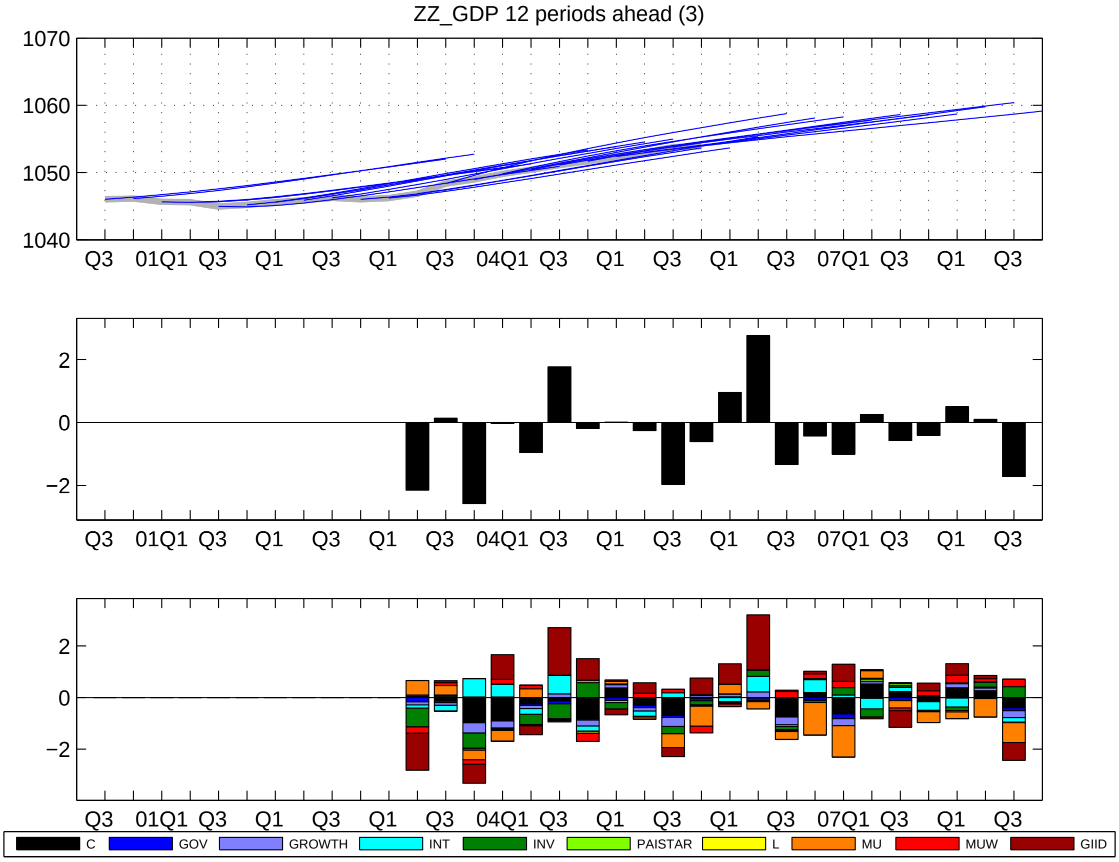


Figure 7.28: Inflation Medium-Term Forecast Revisions 1990Q3-1998Q1

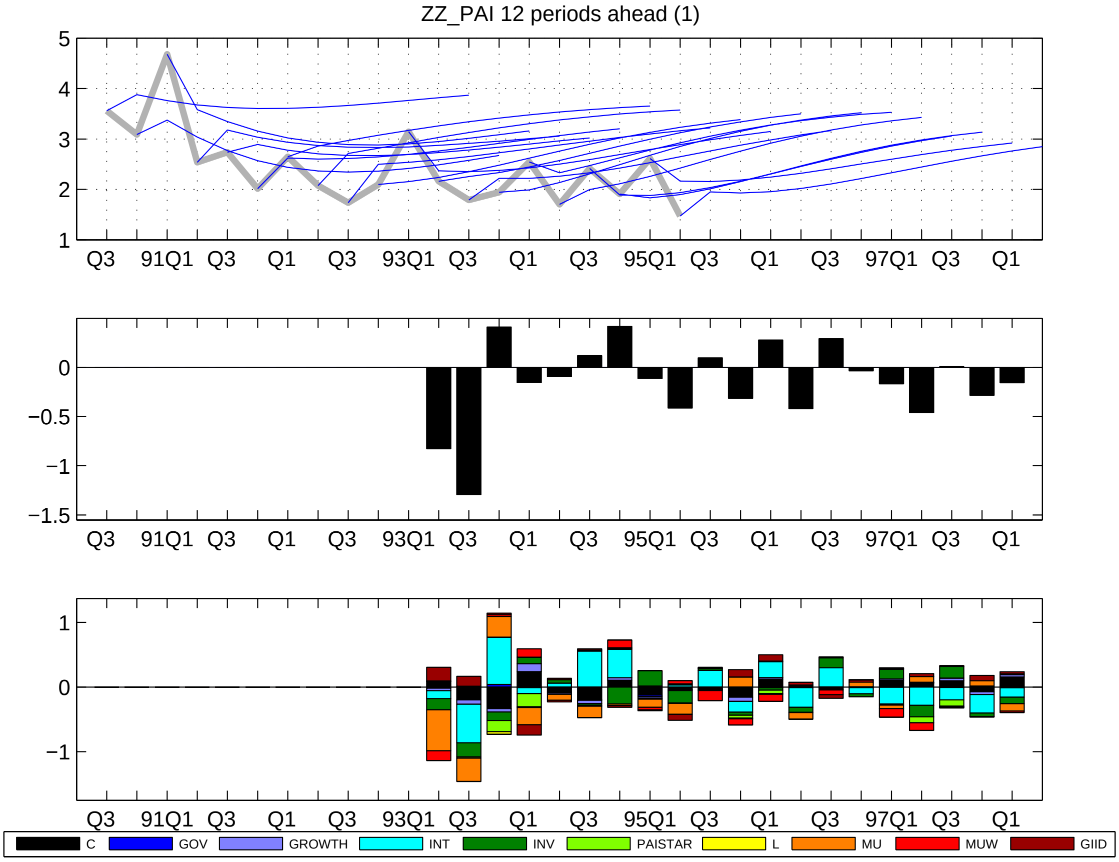


Figure 7.29: Inflation Medium-Term Forecast Revisions 1995Q3-2003Q1

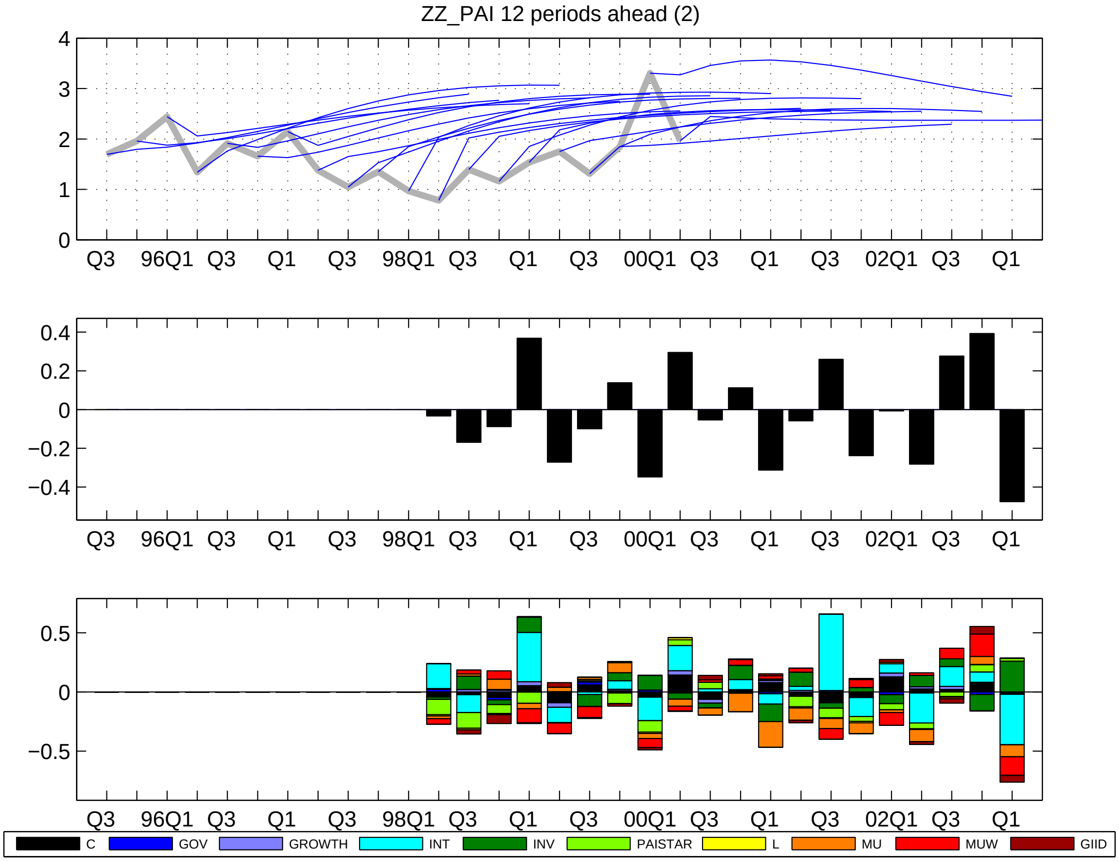


Figure 7.30: Inflation Medium-Term Forecast Revisions 2000Q3-2008Q3

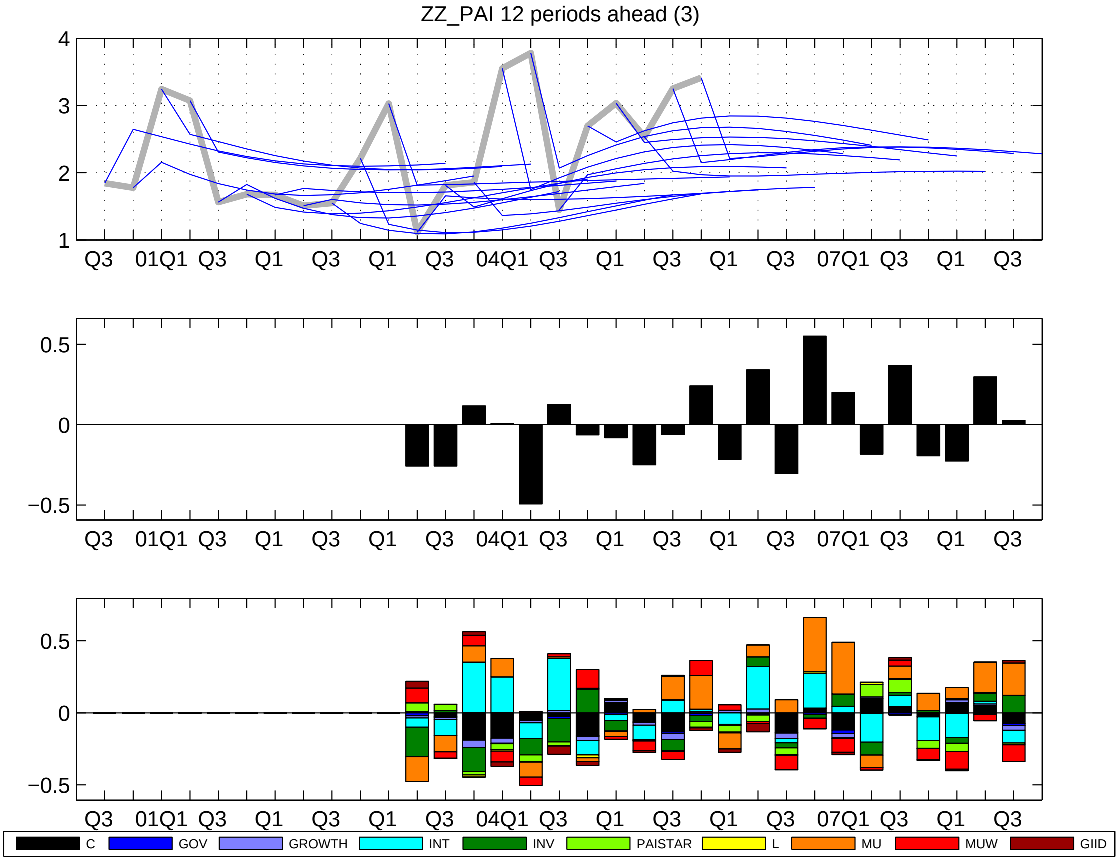


Figure 7.31: Fed Funds Rate Medium-Term Forecast Revisions 1990Q3-1998Q1

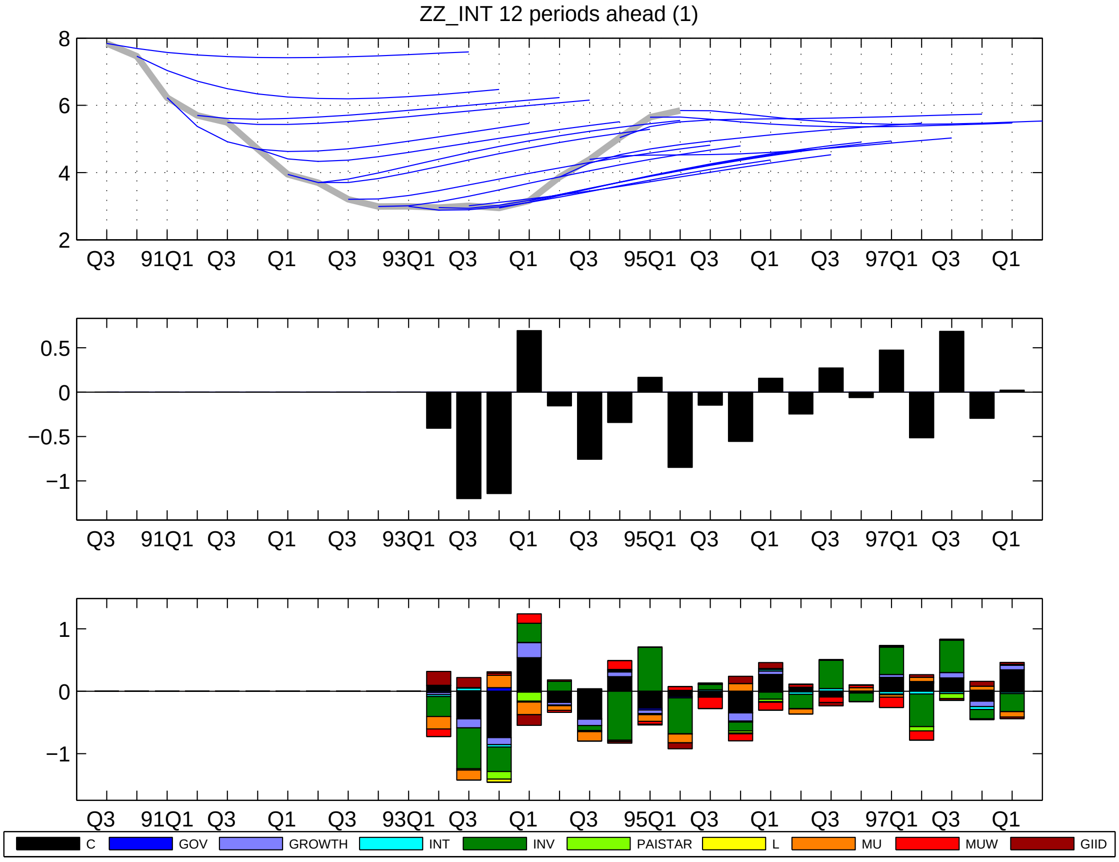


Figure 7.32: Fed Funds Rate Medium-Term Forecast Revisions 1995Q3-2003Q1

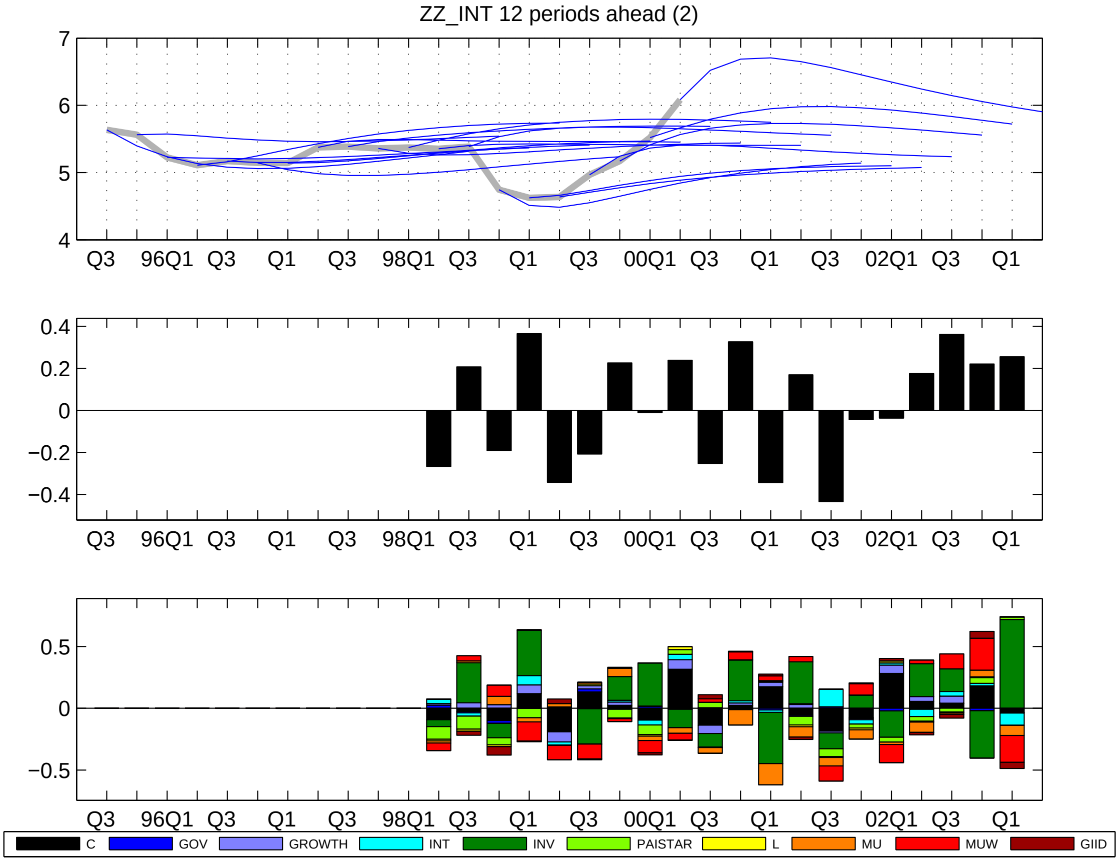


Figure 7.33: Fed Funds Rate Medium-Term Forecast Revisions 2000Q3-2008Q3

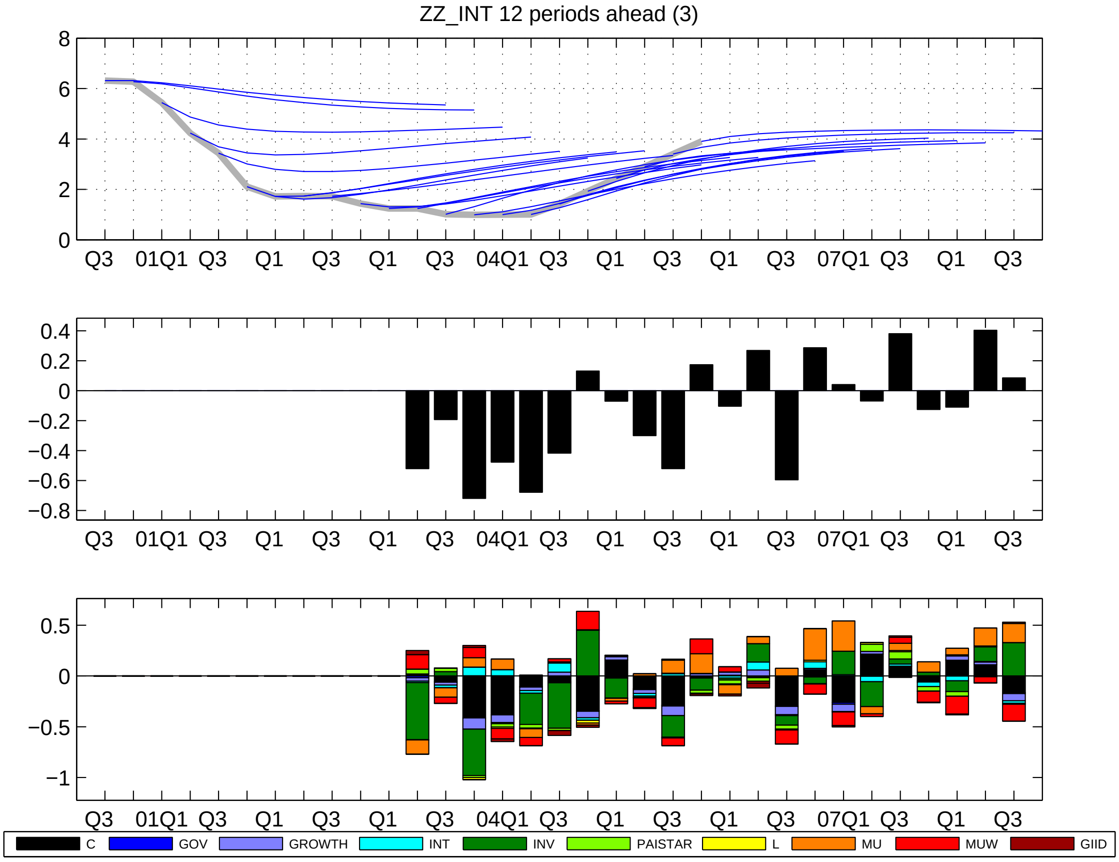


Figure 7.34: Extended HP Output Gaps Under Different Smoothing Parameters

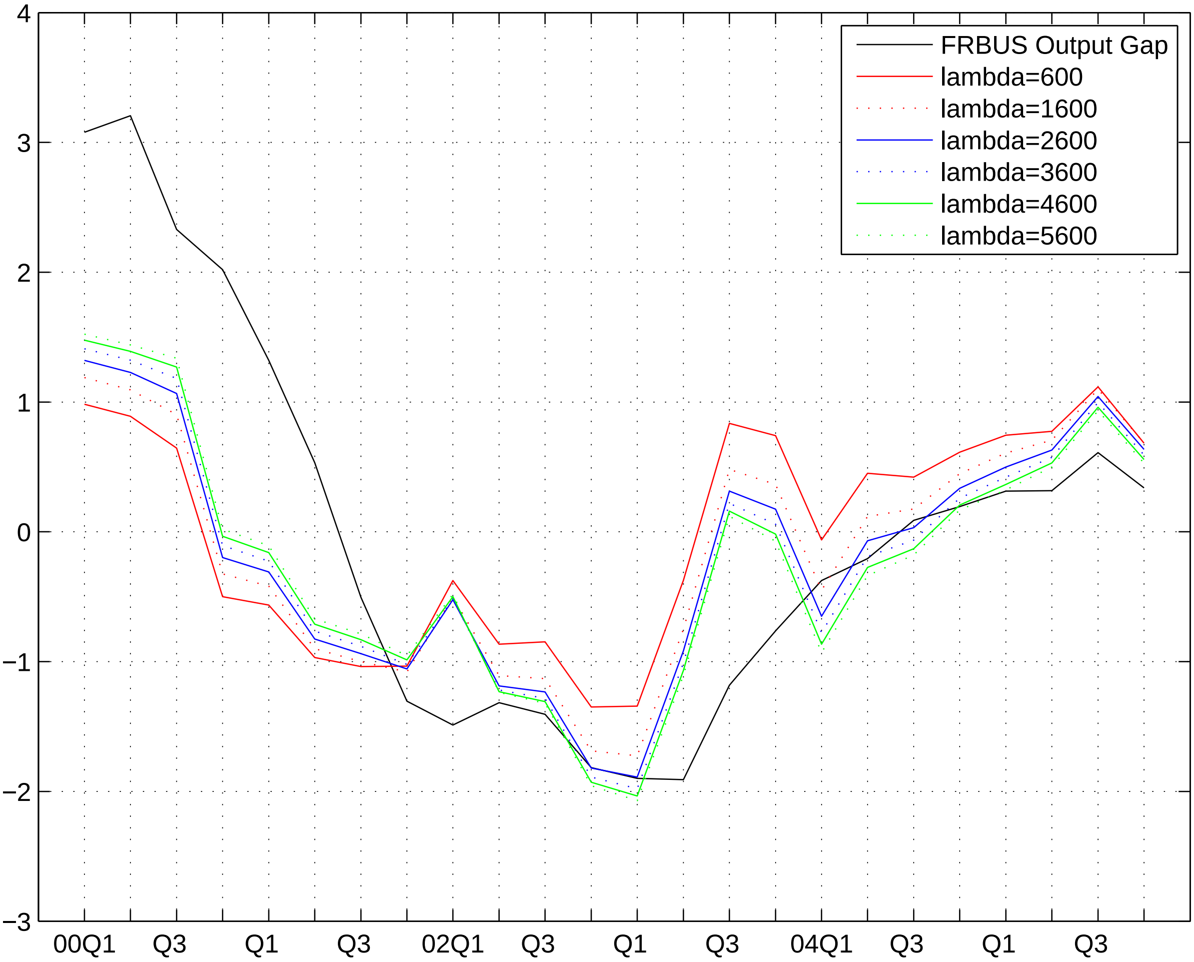


Figure 7.35: Real-Time HP Trends and Output Gaps (2000Q1-2002Q4)

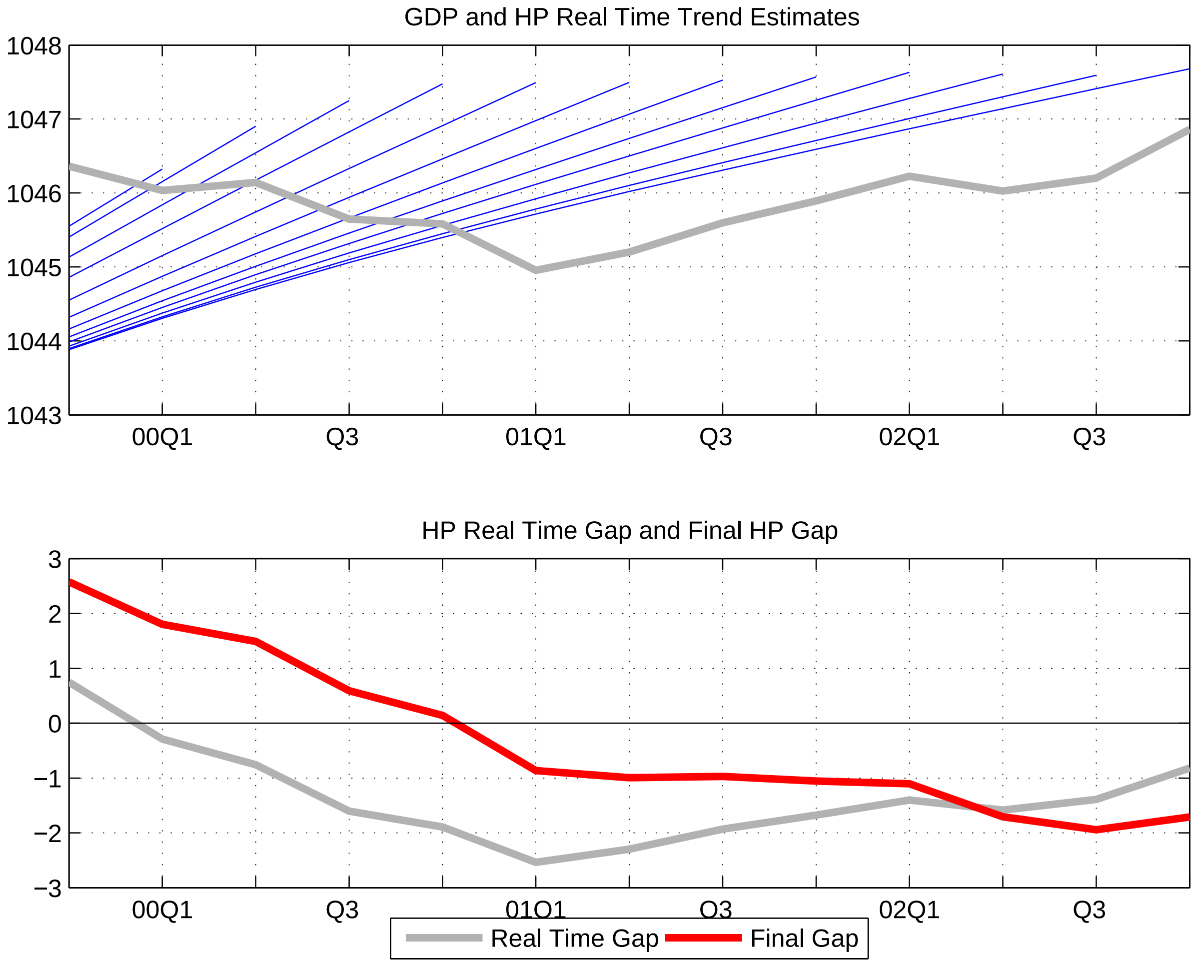
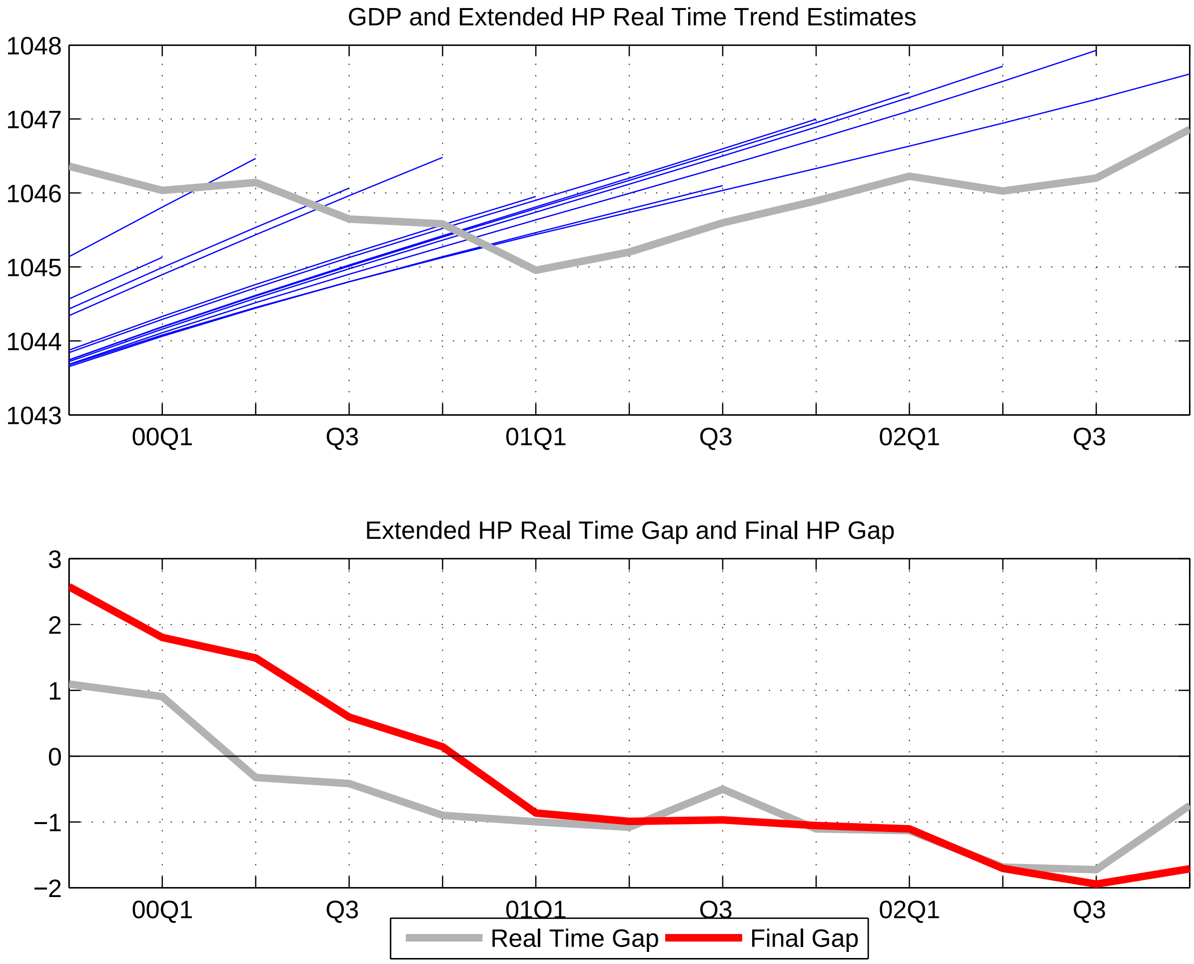


Figure 7.36: Real-Time Extended HP Trends and Output Gaps (2000Q1-2002Q4)



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