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New Keynesian Model Dynamics under Heterogeneous Expectations and Adaptive Learning

Martin Fukač*

Abstract

We analyze the economic dynamics in a basic New Keynesian model adjusted for imperfect, heterogeneous knowledge and adaptive learning. The policy, represented by a forward-looking Taylor rule, is driven by the central bank's own internal forecasts, whereas the core economic dynamics are driven by private agents' expectations. We study the implications of disagreement between those two. We find that if there is expectations heterogeneity, monetary policy should be less active in its actions in order to be short-run stability improving, and to affect positively the speed of convergence towards the first best equilibrium in the long run. This is in contrast to the homogeneous incomplete knowledge literature, which predicts the opposite. We also find that the homogeneous expectations economy is easier to operate in for monetary policy, and that policy can be more effective than in the heterogeneous expectations economy. From the perspective of incomplete, heterogeneous knowledge and adaptive learning methodology, we can thus see the importance of good communication policy and monetary policy credibility.

JEL Codes: E52.

Keywords: Imperfect and heterogeneous knowledge, adaptive learning, monetary policy.

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Nontechnical Summary

In standard monetary models used for policy analysis, economic agents are assumed to share the same, complete knowledge, that is, a representative agent knows perfectly the model structure and behavior. The mainstream monetary theory is based on this simplification of reality. Recently, however, there has been a growing interest in relaxing that assumption. Agents still share the same knowledge, but, it is not complete any more. Only a part of the theoretical, complete economic knowledge is available to economic agents, and this knowledge might also vary among them.A natural extension is for heterogeneous, incomplete knowledge and expectations.

Our objective is to contribute to the discussion on incomplete, heterogeneous knowledge and its consequences for monetary policy. Our particular interest is in the relation between policy activity, willingness to learn, economic variability and the speed of convergence to the first best equilibrium. That is, if economic agents do not share the same expectations, can a central bank effectively improve the short-run economic stability as predicted by the incomplete, homogeneous knowledge literature? By stability we mean minimization of the deviations from the first best, rational complete expectations, in terms of both amplitude and time.

If knowledge is homogeneous, inflation hawkiness helps to decrease inflation variability and speed up learning. If knowledge and beliefs are heterogeneous, the results suggest that policy ought not to be an inflation hawk as variability increases and the speed of convergence slows. For the central bank to play its role effectively in the heterogeneous information world and help the economy converge to the first best equilibrium, policy ought to be conservative and focus on information and knowledge homogenization in the economy.

This finding is crucial for monetary policy based on calibrated models. If monetary policy relies on a calibrated model which is not updated with respect to new information too much or too often, it may in theory be harmful to economic stability. This is the case, especially, if other economic agents use, for instance, simple statistical models.

1. Introduction

Unlike in the commonly considered representative agent macroeconomic models, in reality there exists a diversity among economic agents (consumers, businessmen, bankers or stock market brokers, monetary policy authorities etc.). The diversity is mirrored in the agents' economic knowledge and how they perceive current and expect future economic development. In Mankiw and Wolfers (2003), we can find some empirical evidence on inflation expectations in the US which documents heterogeneity in agents' expectations. Similar observations can also be made in other economies.

In standard monetary models used for policy analysis, economic agents are assumed to share the same, complete knowledge, that is, a representative agent knows perfectly the model structure and behavior. The mainstream monetary theory is based on this simplification of reality. Recently, however, there has been a growing interest in relaxing that assumption. Agents still share the same knowledge, but, it is not complete any more. Only a part of the theoretical, complete economic knowledge is available to economic agents, and this knowledge might also vary among them. Examples of this stream in the adaptive learning literature with application to monetary policy issues include Orphanides and Williams (2003), Bullard and Mitra (2002), and many others. A natural extension is for heterogeneous, incomplete knowledge and expectations. Examples of such literature include Evans and Honkapohja (2003a), Dennis and Ravenna (2005), and others. This stream of literature particularly focuses on economic system stability under incomplete, heterogeneous knowledge. There is a lack of literature studying implied economic dynamics under heterogeneous expectations.

Our objective is to contribute to the discussion on incomplete, heterogeneous knowledge and its consequences for monetary policy. Our particular interest is in the relation between policy activity, willingness to learn, economic variability and the speed of convergence to the first best equilibrium. That is, if economic agents do not share the same expectations, can a central bank effectively improve the short-run economic stability as predicted by the incomplete, homogeneous knowledge literature? By stability we mean minimization of the deviations from the first best, rational complete expectations, in terms of both amplitude and time.

In contrast to the rational and complete-knowledge world, in the incomplete-knowledge world the economy is not in its first best equilibrium. Knowledge improvement is thus welfare improving. Attaining the first best (rational expectations) equilibrium is linked to the agents' willingness to learn. In the homogeneous-knowledge case, monetary policy can contribute to knowledge improvement (learning). Orphanides and Williams (2003) find that monetary policy ought to be inflation vigilant, favoring policy activity as short-run stability improving. Ferrero (2003) qualifies this conclusion. It holds only in a simple structured model (agents form expectations about only one variable). The answer complexity grows with model complexity. For instance, if agents already form expectations about two variables, a too active policy does not need to be necessarily welfare-improving.

The paper by Orphanides and Williams (2003) is one of the first to investigate the impact of imperfect knowledge and perpetual adaptive learning on macroeconomic dynamics and the conduct of optimal monetary policy. The authors find two basic results: (i) ”policies that would be efficient under rational expectations can perform poorly when knowledge is imperfect”, Orphanides and Williams (2003, p.26), and (ii) ”policy should respond more aggressively to inflation under imperfect knowledge than under perfect knowledge... in order to anchor inflation expectations and foster macroeconomic stability”, Orphanides and Williams (2003, p.26).

The results are obtained with a very basic model consisting of the Lucas supply curve and a simple inflation targeting rule. In the light of the simplicity of the model, Evans (2003) questions the second result above. For him, there is no clear answer as to whether the policy maker should be biased towards inflation vigilance under imperfect knowledge.

Evans and Honkapohja (2003a) provide a review and extension of the recent work on monetary policy under learning. They also investigate, among other things, the consequences of different beliefs between private agents and policy-makers about the true structure of the economy. They show that expectations-based policy rules allow for E-stability and determinacy, even if the beliefs of private agents and the central bank differ. E-stability and determinacy also exist if the central bank adopts the private agents' beliefs when setting its instruments. The same result is found in Bullard and Mitra (2002).

The authors use the concept of (finite-horizon) Euler-equation learning. But it should be mentioned that there is also another view on the issue. Preston (2004) has addressed the problem from the perspective of infinite learning and produced different results. In Preston's approach, if both agents and policy-makers are learning about the model structure, and the central bank adopts the private agents' expectations for its decisions without considering how they are formed, it may result in a self-fulfilling expectation problem and macroeconomic instability. Preston argues in favor of policy rules based on the bank's own forecasts.

Honkapohja et al. (2003) react by showing that the approach of infinite learning in Preston (2004) does not invalidate the results based on Euler equation learning and demonstrate that Preston's approach can be replicated under plausible assumptions in the Euler-equation learning approach.

Our paper is structured as follows. We first introduce the model environment and define the basic terminology to be used throughout the paper. In the next, second, section we also analyze the equilibrium properties of the workhorse model under both rational expectations and adaptive learning. In the third section we study the model dynamic behavior. First, we set up a benchmark by presenting impulse responses for the homogeneous-knowledge case, we then consequently move our attention to the heterogeneous knowledge case. We describe our simulation results and provide a basic economic intuition for them. We conclude with a fourth section discussing the results and drawing possible implications for monetary policy.

2. The Model Environment

2.1 The Model

Our workhorse model follows the standard New Keynesian business cycle model scheme. On the one hand, there are households who make decisions about consumption, labour, and money holdings in order to maximize and smooth their lifetime welfare. On the other hand, there is a monopolistically competitive production sector that maximizes profits by controlling output, output prices and labour demand. The firms use Calvo's pricing mechanism to set prices. The central bank's objective seeks to anchor the nominal side of the economy and stabilize output variability. The model is derived from first principles in Appendix A.

The linearized model characterizing the aggregate economic dynamics is given by the IS curve (2.1), which comes from the households' Euler equation linearization, and the Phillips curve (2.2), which is the linearized firms' oligopolistic pricing rule. The central bank's policy rule is given by (2.3). In the complete-knowledge environment, the aggregated sticky-price model takes the form

$$x_t = E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + v_t, \tag{2.1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t, \tag{2.2}$$

$$i_t = \theta_0 + \theta_\pi E_t \pi_{t+1} + \theta_x E_t x_{t+1}. \tag{2.3}$$

x_t is the output gap, defined as the deviation of actual output from the output arising in a friction-less environment. π_t is the inflation rate, and i_t is the interest rate set by the central bank. v_t and u_t are demand and cost-push shocks, respectively, assumed to follow AR(1) processes. β, σ, λ, θ_π and θ_x are households' time preference parameter, risk aversion parameter, inflation-elasticity-with-respect-to-output-gap parameter, and the weights in the policy rule on inflation and the output gap, respectively. The model (2.1)-(2.3) assumes that all economic agents have complete knowledge about the structure of the economy and all expectations operators $E_t(.) = E_t(.|\Omega_t)$ stand for complete knowledge rational expectations, with $\Omega_t = \{(2.1) - (2.3), v_t, u_t, ...\}$.

The expectations-based policy rule (2.3) represents optimal discretionary policy. Evans and Honkapohja (2003b) derive an optimal policy rule when a central bank employs an internal forecast. We slightly deviate from their rule by assuming that the central bank cannot observe the shocks $\{v_t, u_t\}$ when making policy decisions. The rule then takes the form of (2.3) with parameters $\{\theta_\pi, \theta_x\} = \{1 + ((1-\alpha)\lambda^2 + \alpha)^{-1}\sigma^{-1}(1-\alpha)\lambda\beta, \sigma^{-1}\}$, where $\alpha \in (0,1)$ is the relative preference for output stabilization in the central bank's quadratic objective function:

$$\min \quad E_t[\alpha x_t^2 + (1-\alpha)(\pi_t - \pi^*)^2],$$

where the central bank minimizes output gap fluctuations and deviations of inflation from the desired rate π^*. For simplicity, we will assume $\pi^* = 0$, which implies $\theta_0 = 0$. The complete derivation of the optimal weights can be found in Appendix D.

In our analysis, the assumption that the complete information set Ω_t is available to all agents is relaxed. Instead, we will assume agents have imperfect and, moreover, heterogeneous knowledge, which will affect the way agents form their expectations. We will distinguish two groups of agents: (i) private agents - households, firms, and (ii) the central bank. In the following analysis we will distinguish between the expectation operators which these two groups form. We will assume expectations homogeneity within each group but heterogeneity between them, that is, all households and firms will share the same set of information and beliefs, but, this set will differ from the information set and beliefs of the central bank. This is a significant relaxation of the original, homogeneous, complete knowledge set-up. On the other hand we make a simplifying assumption that each group of agents ignores the expectations of the other group by assuming that the world is homogeneous and they all have the same information. This will, however, be relaxed in future research.

Finally, the workhorse model in this paper takes the form

$$x_t = \hat{E}_t^P x_{t+1} - \sigma \left(\hat{E}_{t-1}^P i_t - \hat{E}_t^P \pi_{t+1} \right) + v_t, \tag{2.4}$$

$$\pi_t = \beta \hat{E}_t^P \pi_{t+1} + \lambda x_t + u_t, \tag{2.5}$$

$$i_t = \theta_0 + \theta_\pi \hat{E}_t^{CB} \pi_{t+1} + \theta_x \hat{E}_t^{CB} x_{t+1}, \tag{2.6}$$

where we specifically distinguish between the form of expectations formed by private agents, $\hat{E}_t^P(.) = E_t(.|\Omega_t^P)$, and by the central bank $\hat{E}_t^{CB}(.) = E_t(.|\Omega_t^{CB})$, where $\Omega_t^P, \Omega_t^{CB} \subset \Omega_t$. Honkapohja et al. (2003) show that the move from the complete knowledge model to the imperfect and heterogeneous knowledge model is possible under Euler-equation learning. If all agents are adaptively learning (using recursive least squares, and the E-stability conditions hold), the originally heterogeneous knowledge Ω_t^P and Ω_t^{CB} enriches over time so that it converges to the complete knowledge set Ω_t.

To complete the model, we have to describe the learning mechanism. Besides the incomplete knowledge and heterogeneity between the private agents' and central bank's expectations, we assume agents are adaptively learning, i.e., they are improving their knowledge about the economy over time, and based upon the past mistakes they made in the anticipation of economic developments. As mentioned above, under certain conditions, if all agents are improving their knowledge over time, the economy converges to the complete knowledge case eventually. The complete knowledge case, *the rational expectations equilibrium* (REE), is a limiting case of the incomplete-knowledge case.

To introduce the adaptive learning methodology, we assume that agents are learning the reduced form of the model. The minimum-state representation to the structural model (2.4)-(2.6) can be shown to be

$$Y_t = a + b s_t.$$

Y_t is the vector of endogenous variables, s_t is the vector of exogenous shocks, and $\{a, b\}$ are the matrices of the structural parameters.

If we say that the agents have imperfect and heterogeneous knowledge, we assume that the agents' perception of the economy does not correspond to the complete knowledge case and, further, knowledge differs between the agents. It is assumed that the private agents' *perceived law of motion* (PLM) for the economy (4)-(6) takes the form

$$Y_t = \hat{a}_t^P + \hat{b}_t^P s_t,$$

and the central bank's PLM is

$$Y_t = \hat{a}_t^{CB} + \hat{b}_t^{CB} s_t,$$

where $\{\hat{a}_t^i, \hat{b}_t^i\} \in \Omega_t^i$ for $i = \{P, CB\}$ are the time-varying matrices of the model primitives. We implicitly assume here that agents have complete knowledge about the structure of the economy but they have incomplete knowledge about the true values of some model primitives. However, they are learning about the structural matrices $\{a, b\}$ over time. The learning mechanism is based on recursive least squares. The mechanism is formalized as

$$\begin{aligned} \xi_t^i &= \xi_{t-1}^i + \kappa_t^i R_t^{-1} X_t (Y_t - X_t' \xi_{t-1}^i), && (2.7) \\ R_t &= R_{t-1} + \kappa_t (X_t X_t' - R_{t-1}). && (2.8) \end{aligned}$$

where $i = \{P, CB\}$, $\xi_t^i = [vec(\hat{a}^i)' vec(\hat{b}^i)']'$ is the vector of the perceived-law-of-motion parameters, X_t is the matrix of appropriately stacked exogenous shocks s_t, and κ_t^i is the information gain.[1] Later in the text we pay close attention to the gain specification, since it will be the primary and only source of heterogeneity.

Before we proceed, it is useful to formalize some terminology to be used throughout the paper.

Definition 1 *Economic agents have complete knowledge if an information set Ω_t is available at time t, where*

$$\Omega_t = \{(2.4) - (2.6), u_t, v_t, ...\}.$$

(2.4)-(2.6) denote the agents' knowledge of the structural relations. The information also contains true steady-state values $\bar{a}_\pi$, $\bar{a}_x$ and current and past exogenous shocks u_t and v_t.

Definition 2 *Economic agents have incomplete, homogeneous knowledge if all agents share the same and incomplete information set $\hat{\Omega}_t$ at time t, where*

$$\hat{\Omega}_t = \{(2.4) - (2.6), \hat{a}_{\pi,t}, \hat{x}_{x,t}, \kappa_t, u_t, v_t, ...\}.$$

$\hat{a}_{\pi,t}$, $\hat{x}_{x,t}$, and κ are the incomplete beliefs about inflation and output-gap steady states and willingness to learn, respectively. The incomplete knowledge $\hat{\Omega}_t$ is a subset of the complete knowledge, $\hat{\Omega}_t \subset \Omega_t$, and as $t \longrightarrow \infty$, $\hat{\Omega}_t \longrightarrow \Omega_t$.

[1] In the text below we also call this as willingness to learn or sensitivity to new information. These terms are used interchangeably.

Definition 3 *We distinguish two groups of agents: (P) private agents, and (CB) the central bank. The private agents and central bank have incomplete, heterogeneous knowledge if the information they have differs and it is not complete, i.e.,* $\hat{\Omega}_t^P \neq \hat{\Omega}_t^{CB} \subset \Omega_t$*. The individual information sets are*

$$\hat{\Omega}_t^P = \{(2.4) - (2.6), \hat{a}_{\pi,t}^P, \hat{x}_{x,t}^P, \kappa_t^P, u_t, v_t, ...\},$$

$$\hat{\Omega}_t^{CB} = \{(2.4) - (2.6), \hat{a}_{\pi,t}^{CB}, \hat{x}_{x,t}^{CB}, \kappa_t^{CB}, u_t, v_t, ...\}.$$

In the next section we briefly outline the adaptive learning methodology. Specifically, we analyze under what conditions the model has a unique and stable equilibrium, and under what conditions such an equilibrium is learnable with a recursive least squares mechanism.

2.2 Model Analysis Under Adaptive Learning

To analyze the conditions under which the incomplete knowledge model (2.4)-(2.7) converges to the true model, REE form, the methodology developed by Evans and Honkapohja (2001) is employed. In principle the methodology consists of two parts. First, the rational expectation equilibrium of the model is examined. We look for conditions under which the REE is *determined*. The REE is said to be determined if it is found to be unique. The second part of the methodology is a check for the learnability of the REE. The question is, if economic agents have incomplete knowledge, can they learn, given a learning mechanism, the true RE dynamics? The conditions that guarantee the REE is attainable under the adaptive learning mechanism are called the *E-stability conditions*. For technical details on the methodology we refer to Evans and Honkapohja (2001) and Evans and Honkapohja (2003a), where adaptive learning in a homogeneous environment is explained, and to Honkapohja and Mitra (2003) for an extension to heterogeneous learning.

REE Determinacy

To examine the rational expectation equilibrium of the model (2.4)-(2.6) we begin by rewriting the model in a matrix *reduced form*

$$Y_t = M_1 \hat{E}_t^P Y_{t+1} + M_2 \hat{E}_t^{CB} Y_{t+1} + P s_t, \tag{2.9}$$

where $Y_t = [x_t, \pi_t]$, $s_t = [v_t, u_t]$,

$$M_1 = \begin{bmatrix} 1 & \phi \\ \lambda & \beta + \lambda\phi \end{bmatrix}, M_2 = \begin{bmatrix} -\phi\theta_x & -\phi\theta_\pi \\ -\lambda\phi\theta_x & -\lambda\phi\theta_\pi \end{bmatrix}, P = \begin{bmatrix} -\phi\theta_u & 1-\phi\theta_g \\ 1-\lambda\phi\theta_u & \lambda(1-\phi\theta_g) \end{bmatrix}.$$

To analyze the REE determinacy, we will assume for now a complete knowledge environment, $\hat{E}_t^P(.) = \hat{E}_t^{CB}(.) = E_t(.)$. Given that, rearranging the reduced form one obtains

$$Y_t = M E_t Y_{t+1} + P s_t, \tag{2.10}$$

where $M = M_1 + M_2$.

Proposition 4 *The model (2.4)-(2.6) has a unique and stable rational expectations equilibrium if the modulus of the eigenvalues of matrix M in (2.10) lies inside the unit circle.*

Proof follows from the properties of the stable FODE system.

E-Stability

The second important issue is to analyze the conditions under which the REE is learnable. We know that the REE exists and is unique. We are now interested in whether, having incomplete knowledge, we can learn the REE eventually. We will follow the methodology by Evans and Honkapohja (2003a) for heterogeneous adaptive learning based on recursive least squares. If the REE is determined, the model has the *minimum state variable* (MSV) representation

$$Y_t = a + bs_t. \tag{2.11}$$

a, and b are the (3x1) and (3x3) matrices of the model primitives. Their exact form is derived in Appendix B.

We recall that the private agents' PLM is

$$Y_t = \hat{a}_t^P + \hat{b}_t^P s_t, \tag{2.12}$$

and the central bank's PLM is

$$Y_t = \hat{a}_t^{CB} + \hat{b}_t^{CB} s_t. \tag{2.13}$$

The subscript t on the matrices indicates the time dependence of the matrices as the agents learn using (2.7) and (2.8). The private agents and central bank use their PLMs to form expectations

$$\hat{E}_t^P Y_{t+1} = \hat{a}_t^P + \hat{b}_t^P F s_t, \tag{2.14}$$

$$\hat{E}_t^{CB} Y_{t+1} = \hat{a}_t^{CB} + \hat{b}_t^{CB} F s_t. \tag{2.15}$$

Substituting (2.13)-(2.14) back into the reduced form (2.9), one obtains the economy's *actual law of motion* (ALM)

$$Y_t = \left(M_1 \hat{a}_t^P + M_2 \hat{a}_t^{CB}\right) + \left(P + M_1 \hat{b}_t^P F + M_2 \hat{b}_t^{CB} F\right) s_t. \tag{2.16}$$

The mapping from PLM to ALM is formalized to

$$T[a, b] = [M_1 \hat{a}_t^P + M_2 \hat{a}_t^{CB}, P + M_1 \hat{b}_t^P F + M_2 \hat{b}_t^{CB} F]. \tag{2.17}$$

Finally, E-stability is achieved if the steady state in the following differential equation is locally stable

$$\frac{d}{d\tau}(a,b) = T[a,b] - (a,b). \tag{2.18}$$

Honkapohja and Mitra (2003) and Evans and Honkapohja (2003a) show that the map (2.17) can be simplified. They show that the E-stability conditions in the case of heterogeneous expectations are equivalent (under least squares learning) to the homogeneous expectations case. Assuming $\hat{j}_t^P = \hat{j}_t^{CB} = \hat{j}_t$ for $j = \{a, b\}$, then (2.17) simplifies to

$$T[a,b] = \left[(M_1 + M_2)\hat{a}_t, P + (M_1 + M_2)\hat{b}_t F\right]. \tag{2.19}$$

Proposition 5 *The REE of the model (2.4)-(2.7) is E-stable under heterogeneous expectations if and only if the corresponding model with homogeneous expectations is E-stable. Hence the modulus of the eigenvalues of*

$$\begin{aligned} DT_a(a) &= I \otimes (M_1 + M_2) \\ DT_b(b) &= F' \otimes (M_1 + M_2) \end{aligned}$$

must lie inside the unit circle.

Proof see Evans and Honkapohja (2003a) for the proof of the first statement and Appendix C for the derivation of $DT_a(a)$, and $DT_b(b)$.

3. Model Dynamics and the Implications of Monetary Policy

Having described the model and its equilibrium, we can turn our attention to its dynamic properties. The goal of this paper is to investigate what new expectations heterogeneity brings to the model dynamics and how the monetary policy implications are or may be affected.

To address our objective we take the strategy of analyzing the model impulse responses. We begin with the homogeneous case where both private agents and the central bank evaluate innovations in their forecasts in the same manner. They have the same sensitivity to new information ($\kappa_t^P = \kappa_t^{CB} = \kappa_t$). The simulation begins from the REE, i.e. complete knowledge. A temporary shock hits the economy. It is a one-period shock and, for the baseline simulation, it is assumed to be of unit magnitude. The shock is either a cost-push shock (u_t) or a demand shock (g_t), or a combination of the two.

The strategy is the following. First, we describe the results from the point of view of the effect of monetary policy on inflation and output gap responses and their deviations from the RE dynamics, and the effect of policy on the speed of convergence. Second, we do the same and examine the effect of new-information sensitivity. We describe the results first, and we then provide an economic intuition.

Model calibration The model is calibrated using Clarida et al. (2000) as a benchmark, i.e. $\sigma = 1$, $\beta = 0.99$, and $\lambda = 0.3$. To derive the optimal weights for (6), it is assumed that the central bank's preference parameter is $\alpha = 0.3$. This value is also assumed when evaluating the central bank's loss with changing θ_π (deviating from its optimal value, which is implied under homogeneous expectations). In all the simulations, we assume an econometric learning algorithm which is consistent with $\kappa_t^i = c_i t^{-1}$, where t denotes time, $i = \{CB, PA\}$, and c_i is a positive constant and represents a bias in the gain. If $c_i = 1$, κ_t^i represents standard econometric learning. If $c_i > 1$, it implies a greater willingness to update the model parameters than under standard econometric learning. If, for instance, $c_{CB} = 1$ and $c_{PA} = 1.5$, we say that private agents are more willing to update their forecasting models than a central bank is. As $\kappa_t^i \to 0$ as $t \to \infty$, the effect of $c_i \neq 1$ is relevant only shortly after the shock hits and does not affect the REE.

Before we proceed with the incomplete knowledge cases, let us first outline the complete knowledge model dynamics and monetary policy transmission mechanism.

Rational expectations (RE) dynamics Assuming no persistence in exogenous shocks, if a positive demand shock hits the economy, causing excess demand, then output rises above its equilibrium level. The excess demand pushes prices higher and thus inflation increases. The effect of the demand shock on inflation depends on the output gap elasticity. Since shocks are assumed not to be persistent and given the forward-looking Taylor type policy rule, policy does not react to the shock. If the shock is persistent, monetary policy anticipates its future value and its effect on output, and raises the interest rate. Given that the Taylor principle must hold, the real interest rate increases. With the the opportunity costs of not-consuming falling, agents lower their demand for consumption goods and output decreases.

A cost-push shock has a different transition. If it is not persistent, it only affects the inflation rate without spreading further across the economy. There is a one-time change in the price level. If the shock is persistent, monetary policy now reacts. Fighting a cost-push shock, monetary policy has to affect the real economy, which is the only transmission channel to fight inflation. If a positive cost-push shock hits, the central bank raises the interest rate so that real output drops below its equilibrium value and thus creates disinflationary pressure to counteract the cost-push shock's inflationary pressure.

3.1 Homogeneous Learning Case

We can finally concentrate our attention on the economy with incomplete knowledge and adaptive learning. First, we describe and provide an interpretation of the homogeneous environment results and then we turn to the heterogeneous environment results, where the model has more degrees of freedom, making the model dynamics richer.

We are going to summarize the results using two measures to characterize the properties of an impulse response function. We report the maximum deviation of the adaptive learning (AL) dynamics from the RE dynamics, which is denoted in the summary tables as max. If max is a positive number, the AL economy responds more to a shock than the RE economy. If *max* is a negative number, the AL economy is less responsive to a shock than the RE economy. It could be interpreted as an improvement in the

RE economy's dynamics. Taking an absolute value of *max*, we capture the absolute difference between the impulse response in the RE economy and the AL economy. The second measure we employ to characterize the speed of convergence of AL to the RE dynamics is the shock half-life measure (HL). For this paper, HL is the number of periods between *max* and *max*/2, i.e. how many periods are needed to halve the original maximum amplitude.

The results for the homogeneous knowledge case are summarized in Table 3.1. We report the percentage deviations from the RE dynamics caused by an individual shock. As said above, a positive number refers to dynamics which are more volatile (in terms of a higher output gap or inflation) than in the RE case. A negative value indicates a better outcome than under RE.

Table 3.1: Impulse response functions - Inflation

	RE dynamics				$(\lvert AL\rvert - \lvert RE\rvert)$ dynamics											
					κ, $\theta_\pi = 1.3$						κ, $\theta_\pi = 2.5$					
	$\theta_\pi = 1.3$		$\theta = 2.5$		0.8		1		1.2		0.8		1		1.2	
	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL
$g_0 = 0, u_0 = 1$	1.7487	7	1.3387	7	0.6537	1000+	0.8634	463	1.1794	37	0.2539	39	0.3260	23	0.4482	14
$g_0 = 1, u_0 = 0$	0.5245	7	0.4016	7	0.1961	1000+	0.2590	463	0.3538	37	0.0762	39	0.0978	23	0.1345	14
$g_0 = 1, u_0 = 1$	2.2732	7	1.7403	7	1.0390	139	1.4775	25	2.3122	12	0.4016	22	0.5789	13	0.8841	7

Table 3.2: Output gap - IRF in RE, and difference between IRF in adaptive learning (AL) and RE environment,

	RE dynamics				$(\lvert AL\rvert - \lvert RE\rvert)$ dynamics											
					κ, $\theta_\pi = 1.3$						κ, $\theta_\pi = 2.5$					
	$\theta\pi = 1.3$		$\theta_\pi = 2.5$		0.8		1		1.2		0.8		1		1.2	
	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL
$g_0 = 0, u_0 = 1$	-0.2246	7	-0.9146	7	0.1961	1000+	0.2590	463	0.3538	37	0.6856	39	0.8803	23	1.2101	14
$g_0 = 1, u_0 = 0$	0.9326	7	0.7257	7	-0.0588	979	-0.0777	442	-0.1062	25	-0.2057	15	-0.2641	13	-0.3631	11
$g_0 = 1, u_0 = 1$	0.7080	7	-0.1889	7	-0.3117	111	-0.4433	7	-0.4534	4	1.0842	22	1.5630	13	2.3870	7

Demand shock The transmission of a demand shock in an adaptive learning environment has very similar implications for inflation and the output gap as under rational expectations. The economy becomes, however, more responsive to the shock. Deviations of inflation and the output gap from the RE dynamics take positive values. Monetary policy influences the dynamics. In terms of inflation, if monetary policy is inflation averse, it helps to lower the difference between the RE and AL dynamics. It also speeds up the convergence of AL to RE.

The output gap under AL is less responsive to the demand shock than under RE. Inflation vigilant policy increases this deviation and with increasing inflation aversion the response of the output gap lowers. Similarly as in the case of inflation, inflation vigilant policy helps to speed up the convergence.

The loss function summarizes the total effect of the demand shock from the point of view of a central bank. As can be anticipated from the inflation and output gap dynamics, inflation averse monetary policy improves the bank's loss function and helps to close the gap between the AL and RE dynamics. It also positively contributes to the speed of convergence.

The effect of the sensitivity to new information is very similar to the effect of monetary policy. As can also be observed in Table 3.1, it contributes to higher inflation sensitivity, which is on the other hand offset by faster convergence to the RE dynamics and to the REE, respectively. For the output gap, the implications are also similar. Higher sensitivity to new information yields a higher deviation from the RED. In fact, it causes an improvement in the dynamics. It makes the output gap less responsive to the shock. In summary, from the loss function perspective summarized in Table 3.3, the more the system is new-information sensitive, the higher the overall system response, which is, however, followed by faster convergence.

The intuition for the observed results might be as follows. The demand shock first hits the output gap by pushing real output above its equilibrium level and then transmits to inflation by raising the price of consumption goods. If the shock is not persistent the interest rate does not react because inflation and output gap expectations are not affected further. We can, however, observe a bias in the dynamics because of learning. The shock is not anticipated and creates a wedge between what is expected and the reality. Agents partially interpret the higher-than-expected inflation and output gap as a mistake of their forecasting model. They will adjust it. A positive demand shock raises inflation and the output gap above their expectations, and agents translate this as their model underestimating the variables. They adjust/improve the model so that it forecasts higher values in the future. The adjustment raises expectations and thus interest rates, which temporarily increases the real rate and lowers the impact of the demand shock. In the end, the response of the output gap to the demand shock is lower than under RE. The indirect effect of interest rates on inflation via the lower output gap (lower than in the RE environment), outweighs the effect of the higher inflation rate expectations, which, under the current calibration, affects inflation in a one-to-one ratio but the output gap in a one-to-one-third ratio (since $\lambda = 0.3$). If monetary policy becomes more responsive to inflation, it helps to eliminate the shock and the effect of learning more effectively.

Table 3.3: Impulse response functions - Loss Function

	RE dynamics				$(\|AL\| - \|RE\|)$ dynamics											
					κ, $\theta_\pi = 1.3$						κ, $\theta_\pi = 2.5$					
	$\theta_\pi = 1.3$		$\theta_\pi = 2.5$		0.8		1		1.2		0.8		1		1.2	
	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL
$g_0 = 0$, $u_0 = 1$	0.9866	7	1.2963	7	0.4249	1000+	0.5612	463	0.7666	37	0.2971	39	0.3815	23	0.5244	14
$g_0 = 1$, $u_0 = 0$	0.7286	7	0.4340	7	0.0920	1000+	0.1020	1000+	0.1238	1000+	0.0480	170	0.0616	58	0.0847	22
$g_0 = 1$, $u_0 = 1$	1.4906	7	1.5852	7	0.3805	1000+	0.5171	1000+	0.9294	22	0.4698	22	0.6773	13	1.0343	7

Cost-push shock A cost-push shock directly influences inflation and temporarily transmits to the output gap via inflation expectations and the real interest rate. If the shock is persistent, a surprise in inflation today affects future expectations. If monetary policy is effective, it will act to bring inflation back to the equilibrium/targeted value. By changing the interest rate, the central bank may neutralize the effect of the higher expected inflation on the ex ante real return and consequently on output. The policy responsiveness to the cost-push shock affects the speed at which the economy adjusts back to the REE. This is under the assumption that all agents in the economy know the true structure of the economy and they recognize that any surprise in expectations is caused by exogenous shocks and not by a forecasting model mis-specification.

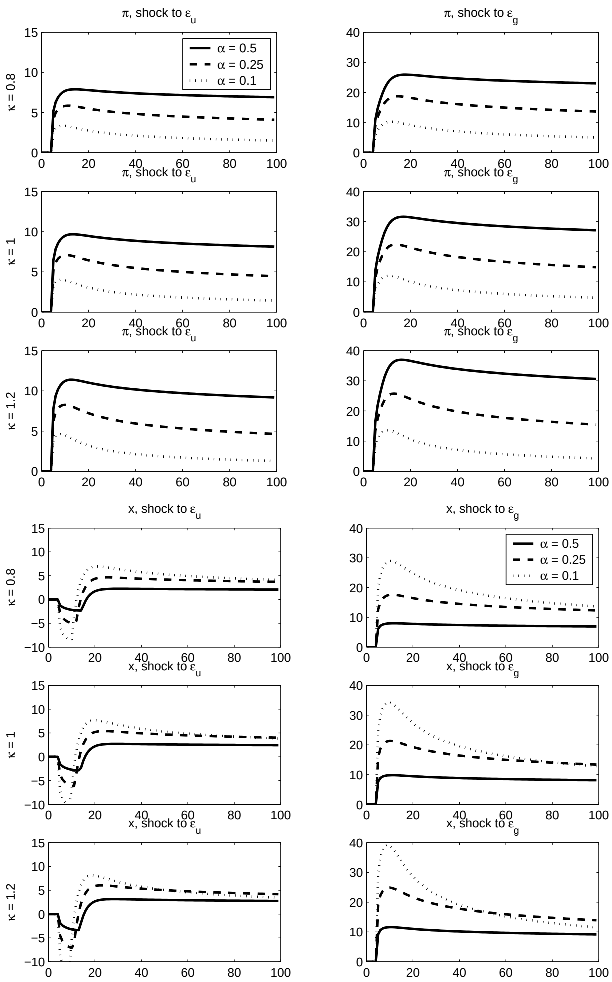
If agents put some weight on the model mis-specification possibility, they would favor the doubt by adjusting their model parameters. In such a case, the economy deviates from the RE dynamics. At time t a cost-push shock hits the economy and inflation increases. The increase has two impacts. The first one is the standard effect on expectations, and the second one is due to learning. Since inflation is higher than what was expected, agents are tempted to update their models, which they believe have been underestimating actual inflation. This triggers a mechanism which brings inflation higher than in a complete (RE) knowledge case. Updating their model, agents will form higher expectations about inflation in the future, which positively affects actual inflation. Higher inflation expectations drive monetary policy to change the interest rate more than the RE would require and thus decrease output more than in the optimum case. In the IRF of inflation and output gap in Figure 3.1 we can observe an interesting phenomenon. This is a period of time (about 5 periods) in which inflation/the output gap continues diverging from the REE path before it begins to converge back. In the case of the output gap it means a better outcome in the initial periods, and later, because under the RE dynamics the equilibrium is reached faster, the output gap is bigger than in the optimum case.

In Table 3.1, we can also observe, as monetary policy is becomes inflation averse relative to the output gap, that inflation variability decreases and the output gap increases. In this case the convergence to the RE is faster.

The effect of sensitivity to new information is summarized in Figure 3.2 and Table 3.3, where the central bank loss function is evaluated. In general, the more the model updates are dependent on innovations, the higher the volatility of the variables. This can be partially offset by monetary policy. On the other hand, a higher sensitivity to innovations due to a cost-push shock helps to improve the speed of convergence to the REE, which is important for central bank loss minimization.

We offer the following explanation of the transmission mechanism of the cost-push shock. We observe that monetary policy improves the inflation dynamics while worsening the output gap dynamics. On the other hand, it improves the speed of convergence of both. Let's interpret the transition again by starting with a non-persistent cost shock. A positive cost-push shock pushes inflation immediately up, which is not anticipated. There is no immediate effect on the output gap, though. The mistake in the inflation expectations is translated to a model update. Agents will interpret that a higher inflation rate

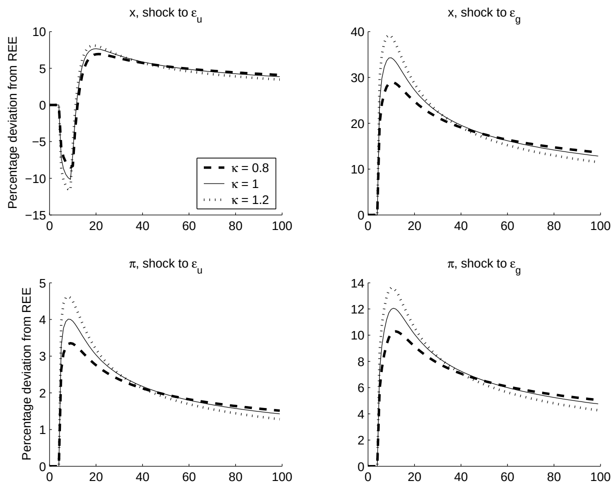
Figure 3.1: Impulse responses of inflation and the output gap to a cost-push shock under adaptive learning - homogeneous sensitivity to innovations



today implies higher inflation tomorrow and adjust their inflation forecasts upwards. This makes the policy respond, and the higher interest rate will drive the output gap and inflation back towards the equilibrium. The response of inflation and the output gap is greater in comparison to the RE case. This is a consequence of learning and expectations formation. As the agents update their models, they forecast higher inflation than it actually is/will be. This makes monetary policy react more than it should (under RE) and thus the output gap is lower, and inflation higher, than it could be.

In summary, in the homogeneous case, monetary policy can influence both economic variability and the speed of convergence to the optimum. Inflation aversion is paid for by higher output gap variability. However, the speed of convergence is in all such instances faster. This observation is in line with the basic finding in Ferrero (2004). Innovation sensitivity also positively contributes to a faster convergence. However, in the first periods (which are relevant for our analysis), it magnifies the variables' responses to shocks.

Figure 3.2: The effect of sensitivity to innovations on the impulse responses of inflation and the output gap ($\alpha = 0.1$ ***)***



This completes our observations on the incomplete homogeneous knowledge economy under adaptive learning. The purpose of the presented exercise was to provide a benchmark for the coming model extension. It should be mentioned that all the observations and presented results are in line with the results in the literature and are standard.

3.2 Heterogeneous Learning Case

Finally, we are getting to the paper's objective. In this section, we perform an exercise which should reveal the New Keynesian model behavior under heterogeneous expectations and shed some light on the implications of monetary policy in such an environment. Particularly, we want to identify the differences that arise from expectation heterogeneity. The system under consideration now will have one more degree of freedom, allowing private agents and the central bank to form differing expectations. The dynamics now become richer and more complicated. We will observe that some features of the homogeneous learning economy are perceived, but in more cases the monetary policy implications are different and may be counterintuitive. First we focus on the technical description of the results, and then we try to provide an economic explanation. The model response functions under incomplete heterogeneous knowledge are summarized in Tables 3.3, 3.4 and 3.5.

Demand shock From the summary tables it appears that the demand shock has very similar implications for inflation dynamics as in the homogeneous case. An inflation averse policy helps to improve the AL dynamics and makes the deviations from the RE smaller. A difference can be found, however, in the implications of new-information sensitivity. Using only rough measures from the numerical simulations we can identify two patterns. With an increasing sensitivity to new information of private agents, inflation becomes more sensitive/responsive to a shock, which is, however, followed by faster convergence. On the other hand, inflation becomes less sensitive to a demand shock if the central bank is more new-information sensitive. But this implies slower convergence (higher inflation persistence). Further, we can also observe that if the private sector's sensitivity is higher than the central bank's one, inflation is less responsive to a shock. This, however, implies a convergence speed which is slower. If the private sector is less sensitive than the central bank, inflation responds slightly more to a shock, but the convergence is faster.

To describe the dynamics of the output gap is not that straightforward and the response to a demand shock seems ambiguous. A big role is now played by the effect of monetary policy. An inflation-averse policy makes the output gap respond less to the demand shock than under RE. The implications for the speed of convergence are not clear. When $\kappa_{PA} < 1.2$, the inflation-averse policy speeds up the convergence. If $\kappa_{PA} = 1.2$, the effect is inverse. A higher information sensitivity of the central bank makes the output gap respond less to a demand shock than under the REE, but again the implications for the speed are not monotonous. The effect of private sector information sensitivity has more complicated implications. If the central bank is less inflation vigilant than under the optimal RE rule ($\theta_\pi = 1.3$), with growing private sector information sensitivity, the output gap reacts more to the shock, and from being less responsive than in RE, it ends up with a higher response than in RE. On the other hand, if policy is more inflation averse ($\theta_\pi = 2.5$), the output gap always reacts less than in RE, and with higher private sector information sensitivity, the reaction becomes smaller. For the speed of convergence it holds that if the central bank's new-information sensitivity is less than that of the private sector, the convergence speed suffers. If private agents update their model more in reaction to an exogenous shock than the central bank, the output gap converges to the RE dynamics faster.

Even though the demand shock transmission is very complex in the case of inflation and the output gap, if we evaluate it from the central bank's loss function perspective, the picture becomes sharper and allows for simpler conclusions about the policy's implications, as the effect of inflation dominates. Simply put, with an inflation-vigilant policy the central bank's loss decreases and the speed of convergence to the REE increases. This is what we observed in the homogeneous case. The loss also decreases if the central bank is more sensitive to new information. This is accompanied by slower convergence. The private sector's behavior acts in the opposite direction. If it is more information sensitive, it has a positive effect on the central bank loss, but it also implies faster convergence. From the perspective of the loss function response to the demand shock, the best state is if monetary policy is inflation vigilant ($\theta_\pi = 2.5$), and the central bank and the private sector both share the same, low sensitivity to new information ($\kappa_{CB} = \kappa_{PA} = 0.8$). From the perspective of the speed of convergence, it is still a better configuration if the central bank is inflation averse and has a lower sensitivity to new information than the private sector.

Cost-push shock Monetary policy has the same implications for inflation as in the previous case. It lowers the deviations from the RE dynamics and speeds the convergence. Sensitivity to new information also has the same implications as above. With increasing sensitivity of the central bank we observe smaller deviations in inflation and a slower convergence speed. On the other hand, with increasing sensitivity to new information on the part of the private sector, inflation variability increases and so does the convergence speed.

On the other hand, the output gap becomes more responsive to a demand shock as monetary policy becomes more inflation averse. Such a policy, however, contributes to faster convergence. The implication is not monotonous, however, as can be seen in the case when $\kappa_{PA} = 0.8$. We can observe faster convergence under the optimal RE policy setting $\theta_\pi = 1.3$. In contrast to the private sector's new-information sensitivity, the central bank's sensitivity to new information contributes to a higher output gap responsiveness and a higher convergence speed. The implications of the private sector's sensitivity are not that straightforward. If $\theta_\pi = 1.3$, the implications for the output gap responsiveness and the speed of convergence are ambiguous. The result depends on the combination of $\{\kappa_{CB}, \kappa_{PA}\}$. If $\theta_\pi = 2.5$ the picture becomes clearer. Increasing κ_{PA} increases the output gap responsiveness and increases the convergence speed.

From the perspective of the central bank's loss function, an inflation-vigilant policy (more reactive than the optimal policy) yields a better outcome in terms of a decrease in the total loss. The policy also positively contributes to the convergence speed. The implications of κ_{CB} can be divided into two dimensions: (i) $\alpha = 0.5$ and (ii) $\kappa_{PA} = \{0.8, 1\}$. Higher central bank new-information sensitivity yields higher losses for the central bank, which are offset by a faster convergence speed. If κ_{CB} is high (here 1.2), the monotonicity of the central bank's information sensitivity does not hold any more. If monetary policy is inflation averse ($\theta_\pi = 2.5$), the situation changes. A higher value of κ_{CB} improves the dynamics and lowers the central bank's loss versus the RE case. This is accompanied by a prolongation of the speed of convergence. The implications of the private sector's sensitivity are monotonous. A higher κ_{PA} increases the CB's loss and speeds the convergence.

Table 3.4: Impulse response functions - Inflation - Heterogeneous case

	RE dynamics				$(\lvert AL\rvert - \lvert RE\rvert)$ dynamics											
					$\kappa_{CB}, \theta_\pi = 1.3$						$\kappa_{CB}, \theta_\pi = 2.5$					
	$\theta_\pi = 1.3$		$\theta_\pi = 2.5$		0.8		1		1.2		0.8		1		1.2	
	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL
					$\kappa_{PA} = 0.8$											
$g_0 = 0, u_0 = 1$	1.7487	7	1.3387	7	0.6537	1000+	0.6019	1000+	0.5697	1000+	0.2539	39	0.2236	67	0.2124	88
$g_0 = 1, u_0 = 0$	0.5245	7	0.4016	7	0.1961	1000+	0.1643	1000+	0.1474	1000+	0.0762	39	0.0549	183	0.0539	278
$g_0 = 1, u_0 = 1$	2.2732	7	1.7403	7	1.0390	139	0.9138	665	0.8455	994	0.4016	22	0.3304	37	0.3230	44
					$\kappa_{PA} = 1$											
$g_0 = 0, u_0 = 1$	1.7487	7	1.3387	7	0.9686	137	0.8634	463	0.7924	1000+	0.4006	17	0.3260	23	0.2931	31
$g_0 = 1, u_0 = 0$	0.5245	7	0.4016	7	0.3250	38	0.2590	463	0.2194	1000+	0.1567	13	0.0978	23	0.0749	54
$g_0 = 1, u_0 = 1$	2.2732	7	1.7403	7	1.7522	18	1.4775	25	1.2776	37	0.8172	9	0.5789	13	0.4679	18
					$\kappa_{PA} = 1.2$											
$g_0 = 0, u_0 = 1$	1.7487	7	1.3387	7	1.5282	19	1.3394	24	1.1794	37	0.7021	9	0.5639	10	0.4482	14
$g_0 = 1, u_0 = 0$	0.5245	7	0.4016	7	0.5707	14	0.4490	18	0.3538	37	0.3202	7	0.2198	8	0.1345	14
$g_0 = 1, u_0 = 1$	2.2732	7	1.7403	7	3.2381	10	2.7914	10	2.3122	12	1.6431	5	1.2306	5	0.8841	7

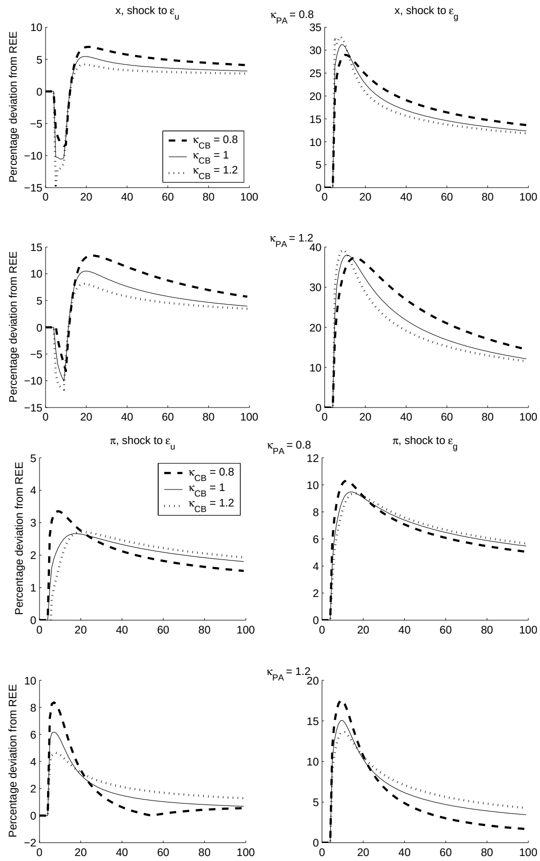
Table 3.5: Impulse response functions - Output gap - Heterogeneous case

	RE dynamics				$(\|AL\| - \|RE\|)$ dynamics											
					$\kappa_{CB}, \theta_\pi = 1.3$						$\kappa_{CB}, \theta_\pi = 2.5$					
	$\theta_\pi = 1.3$		$\theta_\pi = 2.5$		0.8		1		1.2		0.8		1		1.2	
	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL
					$\kappa_{PA} = 0.8$											
$g_0 = 0, u_0 = 1$	-0.2246	7	-0.9146	7	0.1961	1000+	0.3508	21	0.5048	11	0.6856	39	0.7270	29	0.7312	26
$g_0 = 1, u_0 = 0$	0.9326	7	0.7257	7	-0.0588	979	-0.1704	9	-0.2351	7	-0.2057	15	-0.2717	15	-0.2522	15
$g_0 = 1, u_0 = 1$	0.7080	7	-0.1889	7	-0.3117	111	-0.5013	6	-0.4447	6	1.0842	22	1.3259	16	1.1106	19
					$\kappa_{PA} = 1$											
$g_0 = 0, u_0 = 1$	-0.2246	7	-0.9146	7	0.2073	991	0.2590	463	0.5117	10	0.7973	35	0.8803	23	0.9434	20
$g_0 = 1, u_0 = 0$	0.9326	7	0.7257	7	0.1148	180	-0.0777	442	-0.2195	4	-0.2217	30	-0.2641	13	-0.3297	13
$g_0 = 1, u_0 = 1$	0.7080	7	-0.1889	7	0.3331	305	-0.4433	7	-0.3562	7	1.2828	22	1.5630	13	1.7691	10
					$\kappa_{PA} = 1.2$											
$g_0 = 0, u_0 = 1$	-0.2246	7	-0.9146	7	0.3887	146	0.2719	319	0.3538	37	0.9368	32	1.0626	20	1.2101	14
$g_0 = 1, u_0 = 0$	0.9326	7	0.7257	7	0.4207	2	0.1571	12	-0.1062	25	-0.2798	29	-0.3016	11	-0.3631	11
$g_0 = 1, u_0 = 1$	0.7080	7	-0.1889	7	1.6845	1	0.5498	14	-0.4534	4	1.5923	23	1.8752	13	2.3870	7

Table 3.6: Impulse response functions - Loss Function - Heterogeneous case

	RE dynamics				($\lvert AL\rvert - \lvert RE\rvert$) dynamics											
					$\kappa_{CB}, \theta_\pi = 1.3$						$\kappa_{CB}, \theta_\pi = 2.5$					
	$\theta_\pi = 1.3$		$\theta_\pi = 2.5$		0.8		1		1.2		0.8		1		1.2	
	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL	max	HL
					$\kappa_{PA} = 0.8$											
$g_0 = 0, u_0 = 1$	0.9866	7	1.2963	7	0.4249	1000+	0.4405	1000+	0.4488	1000+	0.2971	39	0.2726	54	0.2636	64
$g_0 = 1, u_0 = 0$	0.7286	7	0.4340	7	0.0920	1000+	0.0829	984	0.0789	982	0.0480	170	0.0479	350	0.0499	433
$g_0 = 1, u_0 = 1$	1.4906	7	1.5852	7	0.3805	1000+	0.3605	1000+	0.3500	999	0.4698	22	0.4080	29	0.4018	32
					$\kappa_{PA} = 1$											
$g_0 = 0, u_0 = 1$	0.9866	7	1.2963	7	0.5338	883	0.5612	463	0.5829	253	0.4366	19	0.3815	23	0.3532	28
$g_0 = 1, u_0 = 0$	0.7286	7	0.4340	7	0.2059	83	0.1020	1000+	0.0963	991	0.1312	9	0.0616	58	0.0501	158
$g_0 = 1, u_0 = 1$	1.4906	7	1.5852	7	1.0276	8	0.5171	1000+	0.5842	1000+	0.8349	11	0.6773	13	0.5814	16
					$\kappa_{PA} = 1.2$											
$g_0 = 0, u_0 = 1$	0.9866	7	1.2963	7	0.7906	50	0.7307	51	0.7666	37	0.6938	11	0.6027	12	0.5244	14
$g_0 = 1, u_0 = 0$	0.7286	7	0.4340	7	0.4847	6	0.3006	6	0.1238	1000+	0.2956	5	0.1809	5	0.0847	22
$g_0 = 1, u_0 = 1$	1.4906	7	1.5852	7	2.4613	4	1.6706	5	0.9294	22	1.5456	6	1.2712	6	1.0343	7

Figure 3.3: The effect of sensitivity to innovations on the impulse responses of inflation and the output gap ($\alpha = 0.1$ ***)***



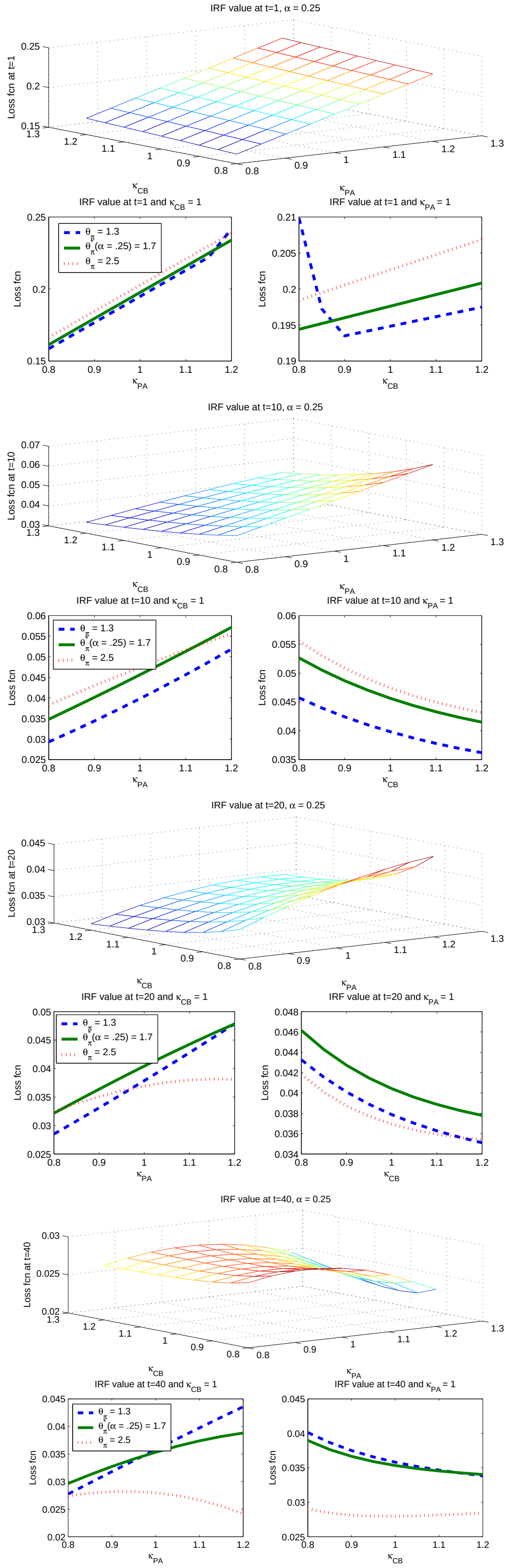
Robustness check

We ran several experiments to check the robustness of the results with respect to the shock magnitudes. The basic results remain mostly unchanged. As g_t gets bigger in relative terms we observe a polarization of the policy effect at the short and long horizon. When $t = 1$, policy that does not stress inflation stabilization helps the central bank to deliver the lowest loss. As time passes, policy stressing inflation stabilization slowly becomes dominant and by $t = 20$ it delivers the lowest central bank loss.

If u_t is the dominant shock in relative terms we get a slightly different picture. At the short horizon, there is a region where a policy fighting inflation can improve the loss function. This policy becomes dominant over time.

In Figure 3.4 we plot the central bank's loss function as it develops over time and with respect to different combinations $\{\kappa_{PA}, \kappa_{CB}\}$, and no serial combination in shocks. We observe similar results up to the autocorrelation coefficient of 0.2. Such an exercise is reasonable since learning can substitute high autocorrelation of exogenous shocks in order to deliver the persistence in inflation and the output gap often observed in the real data. This is found, for instance, by Milani (2004).

Figure 3.4: Loss function IRF at time t=1,10,20,40 ($u_t = 1, g_t = 1$)



In Figure 3.4, we observe a different effect of monetary policy, in terms of the loss function, over time. In early periods after a shock, a less inflation-responsive policy is preferred. With time a more inflation responsive policy is effective. This can be interpreted by diminishing disagreement between the private sector and central bank expectations. Over time, both groups of agents by learning converge to the same forecasting model and thus produce similar forecasts. As the expectations become homogeneous, the importance of inflation-vigilant policy rises, contributing significantly to economic stability. What is important to notice is that in the first periods this does not hold and a too responsive policy can actually considerably destabilize the economy. This suggests that when expectations are heterogeneous, monetary policy should not be too active in order to be stability improving. This is an important observation which we will elaborate on later in our discussion.

Economic Intuition

Despite their complexity, we try to provide a simple economic interpretation of the simulated results. It will help us if we rewrite the model (2.4)-(2.6) as

$$\begin{aligned} x_t &= -\varphi\theta_0 + (\hat{E}^{PA}x_{t+1} - \hat{E}^{CB}x_{t+1}) - \varphi(\theta_\pi \hat{E}^{CB}\pi_{t+1} - \hat{E}^{PA}\pi_{t+1}) + v_t \\ \pi_t &= \lambda\varphi\theta_0 + (\lambda\varphi + \beta)\hat{E}^{PA}\pi_{t+1} - \alpha\varphi\theta_\pi\hat{E}^{CB}\pi_{t+1} + (\hat{E}^{PA}x_{t+1} - \hat{E}^{CB}x_{t+1}) + u_t + \lambda v_t \end{aligned}$$

Demand shock A demand shock hits the output gap first and temporarily transmits to the inflation rate. Given that we start from the REE, agents were expecting equilibrium values of inflation and the output gap. In the RE and persistence-less environment, the shock would have just a one period impact. Under adaptive learning it transmits via expectations to subsequent periods. Given the surprise, agents update their forecasting models. The sensitivity to this innovation plays a role, and the implications differ for the central bank and the private sector. The central bank's actions should neutralize the shock. If the central bank is sensitive to a surprise, it updates its model so that it overpredicts future inflation and the output gap. A positive demand shock will cause an upward correction in the model parameters, which will yield higher predictions of inflation and the output gap for the future periods. The interest rate thus reacts to higher expected values of inflation and the output gap than there would be under RE. The monetary policy is suddenly more restrictive, and we can observe a decline in the inflation deviations from the REE as κ_{CB} increases. The policy becomes more restrictive than it would be under RE (and even under the homogeneous case), the output gap is pushed below its RE value, and the deviation increases with monetary policy restrictiveness. In terms of the deviations from RE, the monetary policy inflation vigilance and the sensitivity to new information act in the same direction.

Using the same logic, we can interpret the effect of increasing private sector sensitivity to innovations. A demand shock transmits further to the economy via expectations, but it has different implications. Private agents update their model similarly to the central bank. Their expectations, however, influence the economic dynamics directly. A positive shock motivates model updates, yielding higher inflation and output gap forecasts in the future. Higher output gap expectations imply a higher output gap and consequently higher inflation. Higher inflation expectations have a direct effect on inflation, which increases, and an indirect effect on the output gap via a decrease in the real interest rate, which positively influences the output gap. The implications of κ_{PA} for inflation and the output gap are intuitively straightforward. In the results summary, we observed, that the implications for the output gap depend on policy responsiveness. If the policy rule is set in an optimal way, preferences are neutral ($\theta^*_\pi(\alpha = 0.3) = 1.3$), and κ_{PA} increases the output gap responsiveness. When $\theta_\pi = 2.5$, the effect is inverse. The final response of the output gap thus depends on the combination of $\{\alpha, \kappa_{CB}, \kappa_{PA}\}$. If $\kappa_{CB} < \kappa_{PA}$ then the private sector's predictions of the output gap and inflation exceed the central bank's expectations. The difference in the output gap expectations has a positive effect on the contemporaneous output gap. The final effect of the inflation expectations depends on the policy. If the policy is such that $\theta_\pi \hat{E}^{CB}\pi_{t+1} < \hat{E}^{PA}\pi_{t+1}$, then we observe a positive effect on the output gap. This is the case when monetary policy is less inflation averse ($\alpha = 0.5$ and thus θ_π goes to 1). There is a higher probability of a negative contribution of inflation expectations to be observed if the policy is inflation averse and θ_π increases. This explains our observations made above.

Cost-push shock Assuming no persistence in the shock, it has an immediate impact the contemporaneous inflation and the prediction model updates, via which it transmits further. In the next period, since no other shock occurs, inflation should return to the REE. Since the agents and the central bank update their model and thus upward bias their expectations, the inflation rate and the output gap increase above the RE values. The mechanism of monetary policy is the same as in the previous shock case. An inflation averse policy pushes inflation down to the RE dynamics, and since the policy is now more aggressive than under the RE, the output gap decreases more and becomes more responsive.

The central bank's innovation sensitivity decreases the inflation rate responsiveness to a cost-push shock, but increases the responsiveness of the output gap. Again, monetary policy becomes more restrictive than under rational expectations, since the central bank predicts higher inflation (due to the model updates), it tightens the interest rate, which closes the output gap, or better put pushes the output gap to negative numbers, and the inflation rate returns to the RE dynamics. Thus by changing κ_{CB}, we can explain the decrease in the responsiveness of inflation accompanied by the increase in the responsiveness of the output gap.

The private sector's innovation sensitivity helps the cost shock to propagate to inflation. As private agents become more innovation sensitive, they anticipate higher inflation than under full knowledge, and thus increase the actual inflation rate. With increasing κ_{PA}, agents update their models more and produce higher forecasts of inflation. This immediately increases inflation due to higher anticipated inflation in the future. Agents also update their forecasts of the output gap. They will anticipate the reaction of the central bank, which they assume to employ the same expectations as themselves, which will lead to a policy rate adjustment, and the output gap drops to negative numbers. Since κ_{PA} will bias the expected monetary policy reaction upwards, private agents will assume a lower output gap than under RE. This explains why the output gap becomes more reactive if the private sector is information innovation sensitive. This phenomenon is observable particularly if the central bank prefers inflation stabilization. The message is mixed if the central bank becomes less responsive to inflation. The dynamic responses in such a case become more complex, and the implications are not monotonous.

4. Discussion and Final Remarks

The world is simpler if knowledge and beliefs are homogeneous. If knowledge is homogeneous, inflation hawkiness helps to decrease inflation variability and speed up learning.[2] If knowledge and beliefs are heterogeneous, the results suggest that policy ought not to be an inflation hawk as variability increases and the speed of convergence slows. For the central bank to play its role effectively in the heterogeneous information world and help the economy converge to the first best equilibrium, policy ought to be conservative and focus on information and knowledge homogenization in the economy. Under such a scheme the central bank's interest rate policy can be the most effective.

This finding is crucial for monetary policy based on calibrated models. If monetary policy relies on a calibrated model which is not updated with respect to new information too much, it may in theory be harmful to economic stability. This is the case, especially, if other economic agents use, for instance, simple statistical models. Such models are often updated whenever a new observation arrives.[3]

In reality, the model uncertainty is usually high. Economic agents can never be certain that their model is the only correct one. Given the model uncertainty, if a central bank insists on its model and is less willing to learn than the private sector, it leads to an increase in economic volatility by pushing the economy away from the REE towards the ”equilibrium” given by the bank's model. Moreover, if the policy is not interest rate-smoothing, i.e., interest rates are changed in an aggressive way, it may harm the economy.

Let us demonstrate this by assuming a situation in which an economy is initially in long-run equilibrium. The inflation rate is zero. Both the central bank and private agents use models that correspond to the REE model. The central bank is aware of that and thus is unwilling to change its model. Private agents are, however, uncertain about their model, and they favor the doubt. Now, an inflationary shock arrives. Both the central bank and private agents had expected the equilibrium (zero) inflation before. The central bank does not put any weight on the unanticipated inflation and sets the interest rate so that it brings inflation back to the REE equilibrium (to zero). Because the central bank believes (in this set-up) that the private agents use the same model to form their forecasts, the central bank envisages that this interest rate change delivers zero inflation in the next period. The private agents are, however, uncertain about their model, and given the unanticipated inflation, they update their model and believe that so does the central bank does so to. This leads them to expect that because inflation was high today, it is going to be high tomorrow too. No further shock arrives. The actual inflation rate is a convex combination of the central bank's and private agents' expectations. Thus, the actual inflation rate will be higher than what the central bank expects but smaller than what private agents anticipate. The adjustment/learning process continues in the same fashion until the REE is achieved eventually. Certainly, the tougher monetary policy is on inflation, the faster the convergence back to the REE will be.

[2] Ferrero (2003) provides an excellent analysis in this respect.

[3] Certainly, in reality, there is a discrepancy in model updating. The discrepancy is bigger, the less monetary policy is credible or understood by private agents.

This is an example where the central bank knows the REE. What happens, however, if the REE is not known with certainty? If a central bank insists on its model (its view of the world), while having a misspecified model, using the above logic it can harm the economy by pushing the economy to the equilibrium which is implied by the incorrect model and inconsistent with the actual equilibrium. With an increasing risk, it may also contribute to excessive instability if monetary policy is too active.

Good communication policy to gain credibility. We have drawn a conclusion that monetary policy faces a much simpler problem and has straightforward implications in the homogeneous-expectations economy. If expectations are homogeneous and monetary policy is an inflation hawk then the policy losses are relatively low. If expectations are heterogeneous and monetary policy is too active, the conclusion is inverse. Hence it seems desirable to achieve knowledge homogeneity to make monetary policy effective. How can knowledge homogeneity be achieved? We see two ways. First, the central bank adopts private agents' expectations or, second, private agents acquire the central bank's expectations. Abstracting from the theoretical world, neither of these is a simple task. The former will require reliable measures of such expectations. Central banks run surveys of the private sector's expectations about future economic development. There is a question, however, whether the information that such surveys yield is economically reliable, i.e., whether the data collected truly represent market expectations (those which are important and employed in macro models), and are not subject to biases instead (due to inaccuracy of responses, collusion-game behavior of some respondents, etc.). In fact, the central bank can never be sure, if the data being collected are useful for immediate policy decisions. In this respect, the latter seems to be more appealing and an easier task.

Forming its own expectations/forecasts, a central bank avoids the need to collect the private sector's expectations and verify their reliability. Instead, a central bank can concentrate its capacities on producing the best expectations/forecast on the market and to gain credibility of its actions. A central bank producing the best forecasts on the market, i.e. the private sector cannot systematically outperform them, appears to be the first step toward expectations homogeneity. This is not sufficient, of course. Another important element for making expectations homogeneous across the economy is policy credibility. As is argued in the standard monetary theory, an essential requirement for gaining credibility is transparency: reasonable discussion, clarification and justification of past policy errors and of future policy actions. This implicitly concerns the central bank's expectations which stand behind the policy decisions. Hence, to make the private sector adopt the central bank's views requires good communication of those. If a central bank communicates well, it gains credibility (expectations becomes homogeneous), and can contribute to economic stability by being active in its policies.

References

Bullard, J. and K. Mitra, “Learning about Monetary Policy Rules,” *Journal of Monetary Economics*, 2002, *49*, 1105–1129.

Clarida, R., J. Gali, and M. Gertler, “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *Quarterly Journal of Economics*, 2000, *115*, 147–180.

Dennis, R. and F. Ravenna, “Learning to set policy optimally,” *mimeo*, June 2005.

Evans, G.W., “Comment on ’Imperfect Knowledge, Inflation expectations, and Monetary Policy’ by A. Orphanides and J. C. Williams,” *mimeo*, March 2003, (Available at http://www.uoregon.edu/ gevans/OrphWillComNBER.pdf).

______ **and S. Honkapohja**, *Learning and Expectations in Macroeconomics*, 2 ed., New Jersey: Princeton University Press, 2001.

______ **and** ______ , “Adaptive learning and Monetary Policy Design,” *Journal of Money, Credit and Banking*, 2003, *35*, 1045–1072.

______ **and** ______ , “Expectations and the stability problem for optimal monetary policies,” *Review of Economic Studies*, 2003, *70*, 807–824.

Ferrero, G., “Monetary Policy and the Transition to Rational Expectations,” Dissertation Thesis, Universitat Pompeu Fabra 2003.

Fuerst, T., “Liquidity, Loanable Funds, and Real Activity,” *Journal of Monetary Economics*, 1992, *29*, 3–24.

Gali, J. and M. Gertler, “Inflation Dynamics: A Structural Econometrics Analysis,” *Journal of Economics Analysis*, 1999, *44* (2), 195–222.

Honkapohja, S. and K. Mitra, “Performance of Monetary Policy with Internal Central Bank Forecasting,” *Journal of Economic Dynamics and Control*, 2003, (forthcoming).

______ **,** ______ **, and G.W. Evans**, “Notes on Agents’ Behavioral Rules Under Adaptive Learning and Receant Studies of Monetary Policy,” *mimeo*, 2003, (Available at http://www.uoregon.edu/ gevans/).

Malik, H.A., “The Cost Channel in the New Keynesian Model: Comparing Inflation Targeting and Price Level Targeting,” *mimeo*, 2004.

Mankiw, N.G. and J. Wolfers, “Disagreement about Inflation Expectations,” *NBER Macroeconomics Annual*, 2003, *18*.

Milani, F., “Expectations, Learning and Macroeconomic Persistence.” PhD dissertation, Princeton University 2004.

Orphanides, A. and J.C. Williams, “Imperfect Knowledge, Inflation expectations, and Monetary Policy,” *NBER Working Paper*, 2003, (9884).

Preston, B., “Adaptive Learning, Forecast-Based Instrument Rules and Monetary Policy,” *Journal of Monetary Economics*, 2004, (forthcoming).

Ravenna, F. and C. E. Walsh, “The Cost Chanel in a New Keynesian Model: Evidence and Implications,” *mimeo*, 2003, (Available at http://ic.ucsc.edu/ fravenna/home/).

Appendix A

In this appendix we derive the model (2.1)-(2.3) from first principles. The derivation is standard to the literature; here we follow Malik (2004).

Agents

Households The households' objective is to maximize lifetime utility. The consumption bundle, c_t, and leisure, $(1 - N_t)$, deliver the utility. To meet the objective, a household not only decide about how much to consume and how much to work, but also decides about how much money to hold, since money is the means of transaction in this economy and serves consumption-smoothing purposes. Households face two constraints in their decisions. First, following Fuerst (1992), they need to hold cash in advance in order to purchase consumption goods. The decision about M_t^c is made at the end of the period $t-1$. Disposable income in period t is W_tN_t, where W_t is the nominal wage and N_t is the hours worked. A budget constraint is the second constraint the households face. It equates the current period income from labour (W_tN_t), financial assets ($M_t^c + (1 + i_t^d)M_t^d$) and the ownership of firms (Π_t^f) and banks (Π_t^b), to the value of current period consumption (P_tc_t) and the financial portfolio carried to the next period (M_{t+1}). The representative household's problem can be formally written as

$$\max_{\{c_t,N_t,M_{t+1}^c,M_{t+1}^d\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t E_0 \left(\frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp(\varepsilon_t^c) - \psi \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{A.1}$$

subject to

$$\begin{aligned} M_t^c + W_tN_t &\geq P_tc_t, &(A.2)\\ M_{t+1} + P_tc_t &= M_t^c + (1+i_t^d)M_t^d + W_tN_t + \Pi_t^f + \Pi_t^b, &(A.3)\\ M_t &= M_t^c + M_t^d. &(A.4)\end{aligned}$$

Here c_t represents the CES composite index (Dixit-Stiglitz aggregator) of real consumption, $c_t = \left(\int_0^1 c_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ with $c_t(i)$ being the consumption of differentiated good i and $\epsilon > 1$; $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the corresponding nominal price index, with $P_t(i)$ being the price of the differentiated good i. N_t is the hours worked, M_t^c is cash money, M_t^d is deposit money, Π_t^f is the profit from firm ownership, Π_t^b is the profit from bank ownership, W_t is the nominal wage, and i_t^d is the nominal return on the deposit money. ε_t^c is the preference shock, which is assumed to follow an AR(1) process $\varepsilon_t^c = \rho_c \varepsilon_{t-1}^c + \nu_t^c$, with ν_t^c being iid with zero mean and finite variance, and $0 < \rho_c < 1$. β, ϕ and ψ are scalars between 0 and 1, and $\sigma > 1$.

Setting up the Lagrangian function

$$
\begin{aligned}
L(c_t, N_t, M^c_{t+1}, M^d_{t+1}) &= \sum_{t=0}^{\infty} \beta^t E_t \left(\frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp(\varepsilon^c_t) - \psi \frac{N_t^{1+\phi}}{1+\phi} \right) + \\
&\quad + \lambda_{1,t} \left(M^c_t + W_t N_t - P_t c_t \right) \\
&\quad + \lambda_{2,t} \left[M^c_t + (1+i^d_t) M^d_t + W_t N_t + \Pi^f_t + \Pi^b_t - P_t c_t - M^c_{t+1} - M^d_{t+1} \right].
\end{aligned}
$$

and maximizing it gives a set of first order conditions

$$\frac{\partial L(.)}{\partial c_t} = \beta^t c_t^{-1/\sigma} \exp(\varepsilon^c_t) - \lambda_{1,t} P_t - \lambda_{2,t} P_t = 0 \tag{A.5}$$

$$\frac{\partial L(.)}{\partial N_t} = -\beta^t \psi N_t^{\phi} + \lambda_{1,t} W_t + \lambda_{2,t} W_t = 0 \tag{A.6}$$

$$\frac{\partial L(.)}{\partial M^d_{t+1}} = -\lambda_{2,t} + (1+i^d_{t+1}) \lambda_{2,t+1} = 0 \tag{A.7}$$

$$\frac{\partial L(.)}{\partial M^c_{t+1}} = -\lambda_{2,t} + \lambda_{1,t+1} + \lambda_{2,t+1} = 0 \tag{A.8}$$

Combining (A.5) and (A.6) gives the Euler equation for the household's labour supply

$$\frac{c_t^{-1/\sigma}}{\psi N_t^{\phi}} \exp(\varepsilon^c_t) = \frac{P_t}{W_t}. \tag{A.9}$$

Combining (A.5),(A.7) and (A.8) gives the Euler equation for consumption

$$\frac{c_t^{-1/\sigma}}{P_t} \exp(\varepsilon^c_t) = \beta(1+i^d_t) E_t \left(\frac{c_{t+1}^{-1/\sigma}}{P_{t+1}} \exp(\varepsilon^c_{t+1}) \right). \tag{A.10}$$

Having the relation for aggregate consumption, we also have to solve for the individual demand for differentiated goods $c_t(i)$. Here the household solves

$$\max_{c_t(i)} \quad c_t = \left(\int_0^1 c_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.11}$$

subject to the budget constraint

$$P_t c_t = \int_0^1 c_t(i) P_t(i) di, \tag{A.12}$$

where $P_t c_t$ are the expenditures on the consumption bundle c_t, and $P_t(i)$ is the price of an individual good. The solution to this problem is the individual good demand

$$c_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} c_t. \tag{A.13}$$

In summary, constraints (A.2)-(A.4) and equations (A.9), (A.10), and (A.13) describe the household's optimal decisions.

Firms Firms operate in a monopolistically competitive environment. As such, to maximize their profits, they choose how much to produce, what price to charge, and how much labour to demand. Following the timing in Fuerst (1992), management's decisions are taken after the shocks to the economy are realized. We assume that labour is the only production factor. To start production, a firm goes to the labour market to hire workers. Once the output is produced and sold, the labour is paid out. When the costs are covered, the firm transfers its net financial position to households.

Each firm, distinguished as firm i, produces one type of good and solves the following problem

$$\max_{\{N_t(i),P_t(i),B_t(i)\}} \qquad E_0 \sum_{t=0}^{\infty} \Phi_{t+1} \Pi_t^f(i) \tag{A.14}$$

where $\Pi_t^f(i) = P_t(i)y_t(i) - W_t N_t(i)$ is firm i's nominal profit and Φ_{t+1} is the stochastic discount factor defined as $\beta^{t+1}/(c_{t+1}P_{t+1})$.[4] $N_t(i)$ is the labour demanded by firm i, and $P_t(i)$ is the firm-specific price charged on the output $y_t(i)$. Note that the firm's problem is in fact static and thus the firm maximizes only $\Pi_t^f(i)$ subject to

$$y_t(i) = A_t N_t(i), \tag{A.15}$$

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} c_t, \tag{A.16}$$

(A.15) is the firm's production function, where labour is the only production factor. The technology associated with the labour is captured by $A_t = \exp(\varepsilon_t^A)$, where $\varepsilon_t^A = \rho_A \varepsilon_{t-1}^A + \nu_t^A$ is the aggregate technology shock, ν_t^A is iid, zero mean and finite variance disturbance, $0 < \rho_A < 1$. (A.16) is the demand function for the consumption good $c_t(i)$ the firm produces.

We substitute all the constraints into the profit function and suitably rearrange to obtain

$$\max_{P_t(i)} \qquad \Pi_t^f = P_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} c_t - \frac{W_t}{A_t}\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} c_t$$

The first order condition follows

$$\frac{d\Pi_t^f}{dP_t(i)} = (1-\epsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} c_t + \epsilon \frac{W_t}{A_t} P_t(i)^{-1}\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} c_t = 0.$$

Rearranging it and using constraints (A.15)-(A.16) gives a set of conditions characterizing the optimal behavior of the i-th firm:

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} MC_t, \tag{A.17}$$

$$\frac{W_t}{P_t(i)} = \frac{\epsilon - 1}{\epsilon} A_t \tag{A.18}$$

[4] It follows that if the firm acts in the best interests of the shareholder, the discount factor corresponds to the representative household's relative valuation of consumption across time.

MC_t are the firm's nominal marginal costs, $MC_t = \frac{W_t N_t(i)}{y_t(i)}$. (A.17) is the standard pricing rule in monopolistic competition. The price is a fixed markup over marginal costs. (A18) is the labour demand. Note that these conditions characterize the firm's optimal behavior in a frictionless environment.

To introduce persistence into the prices in the model, Calvo's pricing scheme is assumed. The production sector is monopolistically competitive and as such has control over prices. Calvo's pricing mechanism assumes that in every period only a fraction of firms, $\theta \in (0, 1)$, can adjust its price. The rest of the firms, $(1 - \theta)$, charge the same price as in the previous period. θ is often viewed as a price-stickiness measure. The higher its value, the higher the degree of price persistence. Since the pricing mechanism is well known and described in the literature, we will limit ourselves to its optimal solution.

Introducing Calvo's pricing mechanism, the firm's problem is no longer a static one. If a firm i is allowed to change price in period t, it chooses to charge the optimal price

$$P_t^*(i) = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t(MC_{t+k}), \tag{A.19}$$

which is the discounted sum of the future expected marginal cost. Since we are in a monopolistically competitive environment, note that the marginal cost here meets the first order condition (A.17). This specification fully corresponds to the one employed in Gali and Gertler (1999). β is the subjective discount factor from the households' problem. In this specification, the firm takes into account the possibility that it might not be allowed to change the price for some time from now on.

Introducing price persistence into the economy, the set of conditions (A.17)-(A18) characterizing the firm's optimal behavior in a monopolistically competitive environment is extended by the time dependent Calvo pricing rule (A.19). The firm applies it only if it wins the lottery and is allowed to change the price. Otherwise the firm charges the same price as in the previous period.

At this point, it is useful to determine the aggregate price level, since later we will be particularly interested in the aggregate dynamics. As stated above, the aggregate price level is computed as

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

The aggregate level in the sticky-price environment is a weighted average of past prices and new prices. The weights are given by the portion of firms allowed to change prices. The aggregate price level becomes

$$P_t = \left[(1 - \theta) P_t^{*(1-\epsilon)} + \theta P_{t-1}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{A.20}$$

In summary, in the frictionless environment, the optimal behavior of the firm is given by equations (A.17)-(A18). If the Calvo pricing rule is introduced, (A.19) also applies. It is employed if the firm is allowed to change its price. Otherwise, it charges the price from the last period.

Monetary Authority The monetary policy, in order to anchor the nominal side of the economy, is assumed to follow the targeting rule

$$i_t^{CB} = \theta_\pi \left(E_t \pi_{t+1} - \pi^* \right) + \theta_x E_t x_{t+1}, \tag{A.21}$$

where i_t^{CB} is the policy instrument, π_{t+1} is the inflation rate between periods t and $t+1$, x_{t+1} is the output gap in the $t+1$ period (see the definition below), and π^* is the inflation target. The target is set exogenously by the central bank and constitutes a nominal anchor for the economy (solving the nominal indeterminacy problem). According to the rule, the central bank sets its policy instrument i_t^{CB} on the basis of the expected deviation of inflation from the target in the next period, and the expected output gap. θ_π and θ_x characterize the bank's preferences with respect to inflation stabilization and/or to output gap stabilization. The higher the value of θ, the more vigilant the bank is. The reason for the choice of policy rule (A.21) is twofold. First, the choice is motivated by the empirical evidence of Clarida et al. (2000), who argue for this type of rule. Second, Bullard and Mitra (2002) find that this type of rule is robust to deliver rational expectations equilibrium determinacy and E-stability, which is required for the analysis below.

Model Equilibrium

Definition 6 *The flexible-price equilibrium is given by an allocation* $\{c_t, N_t, M_{t+1}^d, M_{t+1}^c, B_t\}_{t=0}^{\infty}$ *and a set of* $\{P_t, P_t(i), i_t^b, i_t^{CB}\}_{t=0}^{\infty}$ *such that*

1. *households maximize their lifetime welfare (A1) subject to constraints (A2)-(A4);*
2. *monopolistically competitive firms maximize their present-value profit (A14) constrained by (A.15)-(A.16);*
3. *perfectly competitive private banks maximize their profit;*
4. *the central bank meets its inflation target and zero-output-gap objectives; and*
5. *the labour market, money market, and goods market clear.*

Definition 7 *The sticky-price equilibrium is given by an allocation* $\{c_t, N_t, M_{t+1}^d, M_{t+1}^c, B_t\}_{t=0}^{\infty}$ *and a set of* $\{P_t, P_t(i), i_t^b, i_t^{CB}\}_{t=0}^{\infty}$ *such that*

1. *households maximize their lifetime welfare (A1) subject to constraints (A2)-(A4);*
2. *monopolistically competitive firms maximize their present-value profit (A14) constrained by (A.15)-(A.16), and Calvo's pricing principle allows the firm to set an optimal price according to (A.19) if it is allowed to change its price, otherwise* $P_t(i) = P_{t-1}(i)$ *;*
3. *perfectly competitive private banks maximize their profit;*
4. *the central bank meets its inflation target and zero-output-gap objectives; and*
5. *the labour market, money market, and goods market clear.*

Log-Linearized Model and Aggregate Equilibrium

From now on we focus our attention particularly on the aggregate dynamics. We log-linearize the sticky-price model and describe its aggregate-level dynamics. Because we concentrate specifically on the dynamics of output and inflation, we concentrate on the IS and Phillips curves.

First we derive the IS curve, which characterizes the dynamics of output around its steady state. The derivation is straightforward and follows the same strategy as Ravenna and Walsh (2003) and Malik (2004). We log-linearize the Euler equation from the household's problem (A10) to get

$$c_t = E_t c_{t+1} - \sigma(i_t^d - E_t \pi_{t+1}) + \sigma \varepsilon_t^c. \tag{A.22}$$

From the market clearing condition it follows that $c_t = y_t$. If we define the output gap as $x_t = y_t - y_t^f$, then (A22) becomes

$$x_t = E_t x_{t+1} - \sigma(i_t^d - E_t \pi_{t+1} - r_t^f) + \sigma \varepsilon_t^c,$$

where r_t^f is the real interest rate that arises in the frictionless equilibrium and y_t^f is the output in the frictionless equilibrium. Both are defined as

$$\begin{aligned} r_t^f &= \left(\frac{1}{\sigma}\right) E_t \left(y_{t+1}^f - y_t^f\right) + \varepsilon_t^c, \\ y_t^f &= \frac{\sigma}{1+\sigma\phi}\left[\ln\left(\frac{\epsilon - 1}{\epsilon}\right) - \ln\psi + (1+\phi)\varepsilon_t^A + \varepsilon_t^c - i_t^{d,f}\right], \end{aligned}$$

where $i_t^{d,f}$ is the nominal interest rate in the frictionless equilibrium. For computational convenience and without loss of generality, we will assume that this rate is equal to zero.

Eliminating r_t^f from the above equation for the output gap we get

$$x_t = E_t x_{t+1} - \sigma(i_t^d - E_t \pi_{t+1}) + v_t, \tag{A.23}$$

where $v_t = \frac{\sigma(1+\phi)(1-\rho_A)}{1+\sigma\phi}\varepsilon_t^A - \frac{\sigma(1+\rho_c - 2\sigma\phi)}{1+\sigma\phi}\varepsilon_t^c$. Recalling the properties of ε_t^A and ε_t^c and further assuming $\rho_A = \rho_c = \rho$, v_t follows an AR(1) process[5]. Equation (A.23) constitutes the IS curve as a function of the expected future output gap and the *ex ante* real interest rate.

Second, we derive for the New Keynesian Phillips curve. Log-linearizing and combining (A19) and (A20) we obtain

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\rho} mc_t, \tag{A.24}$$

where mc_t is the log of real marginal costs. To eliminate the marginal costs, we plug (A.17) into (A18) and divide both sides by P_t; we obtain the real marginal costs. Log-linearizing that under the perfect knowledge assumption gives

$$mc_t = w_t - p_t - \varepsilon_t^A. \tag{A.25}$$

[5] The process is $v_t = \rho v_{t-1} + \nu_t^v$, where $\nu_t^v = \frac{\sigma(1+\phi)(1-\rho_A)}{1+\sigma\phi}\nu_t^A - \frac{\sigma(1+\rho_c-2\sigma\phi)}{1+\sigma\phi}\nu_t^c$.

Substituting in (A24) for the log-linearized labour supply function (A9), gives

$$mc_t = \frac{1+\sigma\phi}{\sigma} y_t - (1+\phi)\varepsilon_t^A.$$

We deduct y_t^f from mc_t to obtain mc_t in terms of the output gap

$$mc_t = \frac{1+\sigma\phi}{\sigma} x_t + \ln\left(\frac{\epsilon-1}{\epsilon}\right) - \ln\psi + \varepsilon_t^c.$$

Substituting this expression back to (A24) gives the New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t, \tag{A.26}$$

where $\gamma = \frac{(1-\theta)(1-\theta\beta)}{\theta}$, $\lambda = \gamma\frac{1+\sigma\phi}{\sigma}$, and $u_t = \varepsilon_t^c$, assuming $\epsilon = \frac{1}{1-\psi}$.

Appendix B

MSV representation

Using the method of undetermined coefficients, we derive the exact form of the minimum state variable (MSV) representation for the model considered in the text. Starting with the reduced form (2.9) and assuming rational expectations, i.e., $\hat{E}_t^P(.) = \hat{E}_t^{CB}(.) = E_t(.)$, we get

$$y_t = M_0 + (M_1 + M_2)E_t y_{t+1} + P\epsilon_t, \tag{A.27}$$

where

$$\epsilon_t = F\epsilon_{t-1} + \varepsilon_t.$$

Now assume the MSV form takes the form

$$y_t = a + b\epsilon_t. \tag{A.28}$$

Taking the appropriate expectations needed in (A.27) one obtains

$$E_t y_{t+1} \quad = \quad a + bF\epsilon_t,$$

Plugging these expectations back into (A.27) yields

$$y_t = M_0 + (M_1 + M_2)a + [(M_1 + M_2)bF + P]\epsilon_t. \tag{A.29}$$

Using the method of undetermined coefficients, it follows that the MSV solution must satisfy

$$\begin{aligned} M_0 + (M_1 + M_2)a &= a, \\ (M_1 + M_2)bF + P &= b. \end{aligned}$$

Solving for the matrices a, and b we get

$$\begin{aligned} a &= (I - M_1 - M_2)^{-1} M_0, \\ vec(b) &= [\mathbf{I} - F' \otimes (M_1 + M_2)]^{-1} vec(P), \end{aligned} \tag{A.30}$$

Appendix C

Here we derive the matrices used in Proposition 2 on page 14.

Having the map from the PLMs to ALM

$$T[a,b] = \left[M_0 + (M_1+M_2)\hat{a}_t, P + (M_1+M_2)\hat{b}_t F)\right].$$

we take derivatives with respect to $\hat{a}_t$ and $\hat{b}_t$. Using the rules for the derivatives of matrices we get

$$\begin{aligned} DT_a(a) &= \frac{d}{d\hat{a}_t}\left[M_0 + (M_1+M_2)\hat{a}_t\right] = I \otimes (M_1+M_2), \\ DT_b(b) &= \frac{d}{d\hat{b}_t}\left[P + (M_1+M_2)\hat{b}_t\right] = F' \otimes (M_1+M_2). \end{aligned}$$

Appendix D

Optimal Expectations-Based Policy Rule

The central bank minimizes a quadratic loss function

$$\min_{\{x_t,\pi_t\}} \qquad V = \frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\alpha x_{t+i}^2 + (1-\alpha)(\pi_{t+i}-\pi^*)^2\right]\right\}$$

subject to

$$\begin{aligned} x_t &= \hat{E}_t^{CB}x_{t+1} - \sigma\left(i_t - \hat{E}_t^{CB}\pi_{t+1}\right) \\ \pi_t &= \lambda x_t + \beta\hat{E}_t^{CB}\pi_{t+1}. \end{aligned}$$

Note that the central bank assumes that the private sector trust the bank's expectations and adopts them for their own decisions. The central bank a priory assumes that monetary policy is credible. Further, we assume the bank does not observe current period exogenous shocks u_t and v_t.

The first order condition to the problem is

$$\alpha x_t + \alpha(1-\alpha)(\pi_t - \pi^*) = 0.$$

Using the FOC, the Phillips curve and IS curve to solve for i_t, we obtain the optimal policy rule under discretion. When we assume that the inflation target π^* is zero, then the expectations-based policy rule takes the form

$$i_t = \theta_0 + \theta_\pi\hat{E}_t^{CB}\pi_{t+1} + \theta_x\hat{E}_t^{CB}x_{t+1},$$

where $\theta_\pi = 1 + \frac{(1-\alpha)\lambda\beta}{\lambda^2(1-\alpha)+\alpha}$, and $\theta_x = \frac{1}{\sigma}$, $\theta_0 = 0$.

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