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Credit Risk, Systemic Uncertainties and Economic Capital Requirements for an Artificial Bank Loan Portfolio

Alexis Derviz, Narcisa Kadlčáková, Lucie Kobzová*

Abstract

This paper analyses the impact of different credit risk-based capital requirement implementations on banks' need for capital. The capital requirements for an artificially constructed risky loan portfolio are calculated by applying the BIS approach, the two widespread commercial risk-measurement models, CreditMetrics and CreditRisk+, and, finally, an original synthetic model similar to KMV. In the first three cases we closely follow the methodologies proposed by the regulatory or credit risk models. Economic capital requirements for the latter are obtained by means of Monte Carlo simulations. In the context of CreditMetrics, we additionally perform a Monte Carlo-based stress testing of the monetary policy changes reflected in the term structure of interest rates. Our model of KMV type combines the elements of the structural and the reduced-form methods of risky debt pricing, and the possibilities of its numerical solution are outlined.

JEL Codes: G21, G28, G33.
Keywords: credit risk, economic capital, market risk, New Basel Capital Accord, systemic uncertainty.

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Nontechnical Summary

Central banks need to understand the mechanisms of loan valuation and prudential capital determination used by commercial banks for two main reasons. First, they are responsible for orderly operation of the banking sector. This aspect of financial stability is strongly influenced by the credit risk management practices of the banks under their jurisdiction. Currently, the development of global standards in this area, summarized by the New Basel Capital Accord, its extensions and amendments, is exerting systematic pressure on banks to modernize their credit risk management procedures, and the central bank regulatory role is evolving accordingly. Second, domestic credit creation in the banking sector impacts decisively on the transmission of monetary policy, which is the core central bank responsibility. Lending to the real economy and the balance sheet situation of banks are much dependent upon their attitude towards assessing, and provision-taking for, risky loans.

The present paper investigates the impact of different credit risk-based capital requirement implementations on banks' need for capital. By constructing an artificial bank loan portfolio that mimics key aggregate features of Czech corporate and household borrowing, we gauge the lending-bank economic capital requirements generated by the borrower side of the Czech economy. We start by calculating the regulatory requirement according to the BIS approach, and proceed by applying to the same portfolio the two widespread commercial credit-risk models: CreditMetrics and CreditRisk+. Economic capital for the former is obtained by means of Monte Carlo simulations. In conducting them, we took into account the monetary policy uncertainties reflected in random movements of the Czech term structure and investigated a number of scenarios of key interest rate changes and their transmission along the yield curve. The well-known shortcomings of the conventional credit risk models have to do with non-tradability of major parts of the loan portfolio, the counter-intuitive dependence of the loan value on market interest rates, as well as the pro-cyclical behavior of economic capital implied by the existing models. It was necessary to analyze the banks' way of dealing with both problems realistically. Therefore, we calculated economic capital requirements for the same artificial portfolio in our own credit risk model, which extends the well-known KMV framework and addresses the named issues of market incompleteness and pro-cyclicity.

The main findings of the paper and its contribution to the literature are the following:

1. Risky debt valuation by the traditional asset pricing methods currently in use by the banking industry tends to generate higher loan values and reduce economic capital requirements, compared to other possible regulatory and model-based risk measurement methods. Therefore, the regulator may see an effort on the part of the banks to treat different parts of loans on their balances differently in terms of economic capital. The difference will go in the direction of reducing capital allocation (and specializing collateral requirements) in those segments of the loan portfolio that exhibit strong correlation with traded risks.
2. Those methods of credit risk measurement which explicitly deal with market incompleteness (i.e. the lack of market valuation of both the loan itself and the assets of the obligor) lead to a better recognition of the role of the business cycle and other systemic macroeconomic factors in economic capital determination. Therefore, the regulator should encourage the use of methods that allow for counter-cyclical adjustments by banks of pro-cyclically biased risk-management procedures.

1. Introduction

Regular assessments of the default risk of bank clients and estimations of credit risk at the portfolio level are becoming a necessity for banks in their daily operations. The design of optimal lending contracts and the need to conform to new regulatory trends constitute at least two reasons why banks have to pay closer attention to quantitative methods for assessing the credit risk of their clients. While primarily designed for use in commercial banks, credit risk models have recently started to attract the attention of other groups of economic professionals. It is the supervisory function of central banks that is mostly triggering the interest in examining credit risk models in this environment. Beside that, an overall assessment of the creditworthiness of domestic firms has implications for the conduct of monetary policy. These and other reasons have prompted several central banks in Europe to develop and implement their own models for monitoring the financial situation of domestic firms and the lending performance of domestic banks[1].

The objective of the present paper is to develop an assessment technique for analyzing the impact of different credit risk-based capital requirement determination methods on the potential needs for capital in the Czech banking sector. For this purpose, we apply the discussed methods to capital requirement calculations for an artificially constructed risky loan portfolio. The latter reflects a number of prominent features of Czech non-financial borrowers.

When defining the creditworthiness characteristics of the tested loan portfolio, we apply Moody's method for rating private firms. To determine capital requirements for this portfolio, we use the New Basel Capital Accord (NBCA) and the CreditMetrics and CreditRisk+ models. In the context of CreditMetrics, we are able to conduct stress testing to gauge the impact of interest rate uncertainty (e.g. caused by changes in monetary policy and different reactions of the yield curve to these changes) on economic capital calculations. In addition, we describe an independent debt valuation model of KMV type and outline the techniques for its numerical implementation. The proposed model has a substantial advantage over the previously mentioned ones in that it addresses three key problems of credit risk modeling. Namely, this model, although remaining in the KMV line of analysis,

- incorporates macroeconomic systemic factors, such as position in the business cycle, interest rate and exchange rate volatility and the monetary policy stance, when deriving a valuation of bank lending risks,
- combines the features of structural and reduced-form models of debt valuation,
- offers a framework for assessing the influence of market risk factors on credit risk in a bank loan portfolio.

In the latter respect, our model advances towards an *integrated financial risk assessment methodology*, which has recently been called for in the risk analysis literature (see, for instance, Barnhill and Maxwell, 2002, or Hou, 2002).

The principal output of the paper is a comparative analysis of the predictions of these models when applied to an artificially created loan portfolio constructed using Czech data. Another

[1] Rating systems and creditworthiness-assessment models for firms have been developed, among others, by the central banks of France, Germany, Italy, Austria and the UK.

important contribution is a demonstration, even if in an incipient manner, of the way in which scarce and usually unavailable variables can be estimated or proxied to obtain the inputs required by the credit risk models. One often-mentioned drawback of credit risk modeling is the difficulty with which the credit risk analyst can access the required input data. This problem was also present in our case. Although overcome, the data problem has had negative implications for the robustness of our results. Thus, from this perspective we have to look at the paper's findings with caution. However, the insight into credit risk modeling that is offered here can be extended at a later stage when more data is available. We also hope that our findings may be of use to banking supervisors when these issues become a matter of regulatory practice.

Although credit risk models often prove useful for other purposes, their main merit rests in estimating the capital level that banks have to maintain over the given risk horizon. The outcome is called regulatory capital in regulatory terms and economic capital in terms of credit risk modeling. Both regulatory and economic capital are supposed to cover unexpected losses resulting from banks' lending operations to clients exposing different levels of default risk. Whereas holding regulatory capital is compulsory as a part of adherence to prudential regulations, holding economic capital is the banks' own choice, on condition that its level reaches at least the level of regulatory capital. Worth mentioning, however, is the regulatory tendency to come closer to credit risk modeling and to allow banks to develop their own models for determining the amount of regulatory capital to hold. These models will most probably adopt and synthesize many features of the credit risk models already in use. This is one reason why comparing regulatory and economic capital today is becoming an insightful exercise for the regulatory decisions of the future.

2. Literature Review

In June 1999, the Basel Committee on Banking Supervision released a proposal to replace the 1988 Accord with a more risk-sensitive framework. A concrete proposal in the form of a consultative document, "The New Basel Capital Accord", was presented in January 2001. This document proposed new regulatory rules for banks' capital adequacy evaluations. The main innovations related to credit and operational risk. In terms of credit risk, the New Basel Capital Accord revised the old 1988 Basel Accord by proposing a more risk-sensitive methodology for assessing the default risk of banks' clients. The risk inputs entering the final capital adequacy computations were closely related to the risk characteristics of individual bank clients. In this sense, the proposed methodology opted for the adoption of ratings (developed by external agencies or by banks themselves) in quantifying and signaling to the bank the default risk of individual borrowers. In a simpler version of the methodology (the standardized approach), ratings are directly associated with risk weights (for example, an A-rated asset would be assigned a risk weight of 50%, a BBB-rated asset would be assigned a risk weight of 100%, and so on). In a more advanced approach (the Internal Rating Based, or IRB, approach), ratings represent the basis for computing the probability of an obligor's default. Default probabilities and other risk characteristics (loss given default, exposure at default) enter more complicated formulas for determining the risk weights of individual assets in regulatory capital estimations.

The consultative document was open to comments and feedback from the banking industry until May 2001. Subsequently, the Basel Committee on Banking Supervision updated the original documents twice (in June and December 2001) and conducted three quantitative impact studies. The Committee intends to incorporate the feedback received from the banking industry and to come up with the final version of the NBCA by the end of 2003. The active implementation of the document by internationally operating banks is supposed to start after the end of 2006. The discussion with banks concerning potential changes to the original NBCA (January 2001) has revolved around several topics:

- the construction of a new risk-weight curve in the IRB approach,
- narrowing the gap between the capital estimations produced by the two versions of the IRB approach (foundation and advanced),
- adequate treatment of small and medium-sized enterprises,
- the business cycle dependence of the IRB approach,
- the necessity of implementing conservative credit-risk stress testing under the IRB approach.

In the banking industry, credit risk modeling has also been explored and extended since the release of the four major credit risk models at the end of the last decade[2]. In this paper we consider only two such models, CreditMetrics and CreditRisk+, which utilize, respectively, the structural and the reduced-form approach to modeling default risk (see Duffie and Singleton, 1998). In the structural approach, it is assumed that default is triggered when an unobserved variable (obligor's firm asset value) falls below a certain threshold level (firm's outstanding debt). CreditMetrics extends this reasoning to rating downgrades by defining rating class-specific threshold levels that mark the switch from one rating class to another in the event that the firm's standardized asset returns cross these threshold values. In the reduced-form literature, default is modeled as an autonomous stochastic process that is not driven by any variable linked to the obligor's firm capital structure or asset value. Particular formulations for the default process were considered in Jarrow and Turnbull (1995) (exponential distribution), Jarrow, Lando and Turnbull (1997) (a continuous Markov chain) and Duffie and Singleton (1998) (a stochastic hazard rate process). CreditRisk+ represents the reduced-form approach by assuming that the average number of defaults in each homogeneous class of obligors follows a Poisson distribution. The unifying element of the CreditMetrics and CreditRisk+ models is the Value at Risk (VAR) methodology used in quantifying and provisioning for credit risk at the portfolio level. Even though CreditMetrics derives the portfolio value distribution and CreditRisk+ the portfolio loss distribution at the end of the risk horizon, both models estimate economic capital such that unexpected losses are covered by the estimated economic capital within an acceptable confidence level.

The KMV model (see, for instance, Crosbie, 1999) represents another step towards market-based derivation of economic capital. Similarly to CreditMetrics, it uses the obligor's equity price statistics to derive the value distribution of a given loan. Correlations are obtained automatically from the risk factors that determine the obligor firm values (equities). However, this method requires the assumption of complete markets, the validity of risk-neutral asset valuation and

[2] We refer to JP Morgan's Credit Metrics/Credit Manager model, Credit Suisse Financial Products' CreditRisk+, KMV Corporation's KMV model, and McKinsey's CreditPortfolioView.

tradability of both the obligors' equities and their debt in the bank portfolio. The KMV team offers unspecified remedies in cases where one of these preconditions is not satisfied (Sellers and Davidson, 1998), but open sources of credit risk literature offer no general solution of these problems.

This paper proposes a way around the said difficulties in the KMV approach by resorting to the so-called pricing-kernel methods of asset pricing (comprehensive expositions can be found in, for instance, Campbell et al., 1997, and Cochrane, 2001). Asset tradability and market completeness are no longer necessary, and there are numerous possibilities for modeling default events that depend on systemic and idiosyncratic risk factors. Numerical approaches to calculating pricing-kernel-based asset values have also been developed in recent years (see, for instance, Ait-Sahalia and Lo, 2000, or Rosenberg and Engle, 2002).

3. Methodology

The paper looks at Czech banks and their commercial loan market from the potential capital requirements point of view. Three pillars make up the main structure of our analysis. First, the tested bank loan portfolio is constructed in such a way as to reflect with some degree of realism the rating distribution of a pool of Czech bank clients. Second, we take into account the random nature of interest rates and other economic fundamentals that enter the loan valuation. Among other things, this means that identifiable uncertainty factors in the loan characteristics which are usually treated in the market risk context (interest rates and exchange rates) were an integral part of the capital calculations as far as each of the tested approaches allowed. Third, when conducting model-based economic capital calculations, we follow the *market loan pricing point of view* wherever possible (i.e. when the corresponding model allows it either explicitly or implicitly). This is done because we want to identify those elements of capital requirements which may be seen differently from the credit risk modeling and regulatory perspectives.

Our analysis utilizes a hypothetical portfolio containing 30 loans. This simplified portfolio mirrors the rating structure of a real loan portfolio obtained on the basis of a pool of corporate customers of six Czech banks[3]. Since ratings are the key input in many credit risk approaches, a simplified version of Moody's rating methodology for private firms has been applied to obtain ratings in our real sample of bank clients. Estimates of other inputs required by credit risk modeling which were not available in the real bank data set were obtained using aggregate data from CNB databases.

We examined and compared the predictions of the NBCA with those delivered by the CreditMetrics and CreditRisk+ models. Following the last available consultative version of the NBCA guidelines (January 2001) we found that in our particular example the standardized approach of the NBCA predicted approximately the same level of capital as the credit risk models at the 95% confidence level. At the 99% confidence level, the internal credit risk models predicted a higher level of economic capital than the NBCA's standardized approach, but these estimates were still lower than the estimates of the NBCA's IRB approach. We obtained different results when applying the NBCA guidelines as formulated by the third quantitative impact survey, QIS 3 (October 2002). Here, the results of both NBCA approaches (standardized and IRB) were more

[3] We would like to thank Alena Buchtíková for making this data set available for our research purposes.

similar to each other, with the IRB requirement being slightly lower than the requirement of the standardized approach. The results of both regulatory approaches were even lower than the level of capital required by the various credit risk models.

We extend the analysis of economic capital by allowing both the bank lending rates and the forward zero rates used as discount factors in asset valuations to become random variables (a form of stress-testing). Floating lending rates and random changes in the forward zero curves were all implemented using Monte Carlo simulations in the context of the CreditMetrics model. As expected, more uncertainty associated with the evolution of these variables required more economic capital to be held by banks. However, the proposed changes in forward zero curves did not impose significantly different levels of credit risk-related economic capital as compared with the case of stable forward zero curves (but maintaining floating interest rates in both cases). Downward movements in the forward zero curves (translation or rotation) required higher levels of capital at all confidence levels. This is an inconvenient consequence of the existing credit models (CreditMetrics and KMV in particular), which we strive to overcome by proposing a model of our own.

The paper is structured as follows. In Section 4 we briefly describe the main characteristics of the real bank and test portfolios and their estimation. We also mention the reasons why the models could not be implemented entirely on the basis of real Czech bank data. Section 5 presents the two approaches of the NBCA (standardized and IRB) and their estimates for credit risk-related regulatory capital. In Section 6 we outline the methodologies proposed by two widespread credit risk models (CreditMetrics and CreditRisk+) and present their economic capital estimations. In Section 7 the assumption of fixed lending and forward zero rates is relaxed and consequently new estimates of economic capital are obtained. Section 8 outlines our own model of risky debt, its valuation and the resulting economic capital requirements, going along the structural lines of the original KMV. Section 9 concludes. Finally, the Appendix outlines the technical details of the artificial loan portfolio generation, the RiskCalc model embodying Moody's methodology for rating private firms, and an estimation procedure outline for our debt pricing model.

4. Description of the Test Portfolio

Our bank data set contains the balance sheets and profit and loss accounts of non-financial firms that were granted bank loans between 1994 and 2000. The CZ-NACE classification[4], legal form and CNB loan classification[5] (from 1997) were also available for each bank customer. Six Czech banks provided the data to the CNB from 1994 until 1999, of which two banks terminated cooperation in 2000. The banks reported only a fraction of their corporate portfolios. The exact selection procedure used by banks in choosing certain firms is not known. Also unknown is the proportion of reported versus unreported clients satisfying certain criteria. In this sense, we observed a certain bias of the data providers towards non-reporting of loans in the last two categories (4 and 5) but were not able to assess the direction and magnitude of this sampling bias in our results.

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[4] CZ-NACE (Czech abbreviation: OKEČ) represents the industry classification of economic activity in the Czech Republic.

[5] The CNB's loan classification ranges from 1 to 5, with category 1 meaning standard, 2 watch, 3 nonstandard, 4 doubtful and 5 loss loans.

Since our main goal was to assign ratings to banks' corporate clients, we primarily focused on their default behavior. Default was defined as a credit event in which the loan classification of a certain company migrated from the 1st or 2nd category to any of the 3rd, 4th or 5th categories over the considered risk horizon. Due to the short time length of our data set we had to focus on annual default rates. The largest number of defaults occurring over a one-year period was recorded between 1997 and 1998, representing 8% of all firms in the sample[6]. The sample-based annual default rates were 0.07% between 1998 and 1999 and 0% between 1999 and 2000. These low default rates may be partially explained by the Czech economic recovery and by more prudent bank lending behavior during 1999–2000. Nevertheless, we think that the main reason is insufficient default reporting by banks. Therefore, we preferred to restrict the reference data set only to the accounting information collected in 1997 and the default events observed in 1998, assuming that default reporting by banks in that period was closer to the reality. While analyzing a longer time period would have been highly valuable, we considered that the sample-based information over 1998–2000 painted a biased picture about corporate default and, consequently, it was not used in modeling the rating structure of the test portfolio.

The annual default rate of 8% in the reference data set was significantly higher than the average value of 1.5% usually used by Moody's in the context of Western European economies. However, volume-based information about the loan defaults of individuals and corporates in the entire banking sector revealed an annual default rate of approximately 20%. We considered that neither 1.5% nor 20% would be the appropriate annual default rate for our artificial portfolio[7] and, in general, for a typical Czech bank portfolio of corporate loans. Without any other more reliable source of information, we used the 8% default rate as indicated by our sample bank data to calibrate the probit model (see Subsection A1.1 of the Appendix).

Table 1: S&P's Rating Class-Specific One-Year Default Rates

Rating	One-year default rate (%)	Cut-off values for defining ratings in the bank portfolio (%)
AAA	0	-
AA	0	-
A	0.06	0.03
BBB	0.18	0.12
BB	1.06	0.62
B	5.2	3.13
CCC	19.79	12.495

Source*: Credit Metrics – Technical Document.*

To assign ratings to each firm we used the calculated default rates (Appendix A1.2) and the tables containing cumulative default rates published by different rating agencies. Even though Moody's rating methodology was used, we preferred to calibrate our results to Standard & Poor's (S&P)

[6] The reference sample included only bank clients that were present both in 1997 and 1998 (663 firms). To examine only one-year default behavior, those enterprises which were already in default in 1997 were also eliminated. In the end we obtained a data set containing 606 firms.

[7] These figures reflected assumptions that were not applicable in our case. For example, the 1.5% level was based on the Western experience, while the 20% level was volume-based and represented both firms' and individuals' default behavior.

ratings. This was done since (a) the New Basel Capital Accord assigned risk weights based on S&P ratings and (b) the inputs in the credit risk models were, to a great extent, based on S&P data. For calibration purposes we used a 1996 S&P cumulative one-year default rate matrix that looked like that in Table 1.

The probabilities given in the second column are rating class-specific default probabilities published by S&P. The third column contains the probabilities that mark the transition from one rating class to another in our model. They are the midpoints in the intervals determined by the one-year default probabilities given in the second column. For example, if the estimated default probability of a certain firm belonged to the interval [0, 0.03), an AA rating grade was assigned to that firm. If the estimated default probability belonged to the interval [0.03, 0.12), then an A grade was assigned and so on. Based on this mapping procedure, each firm that was present in the 2000 data set was marked with a certain rating grade. The resulting rating structure and the loan classification of the pooled bank portfolio for 2000 are presented in Table 2.

Table 2: The Pooled Bank Portfolio Structure in 2000 According to Loan Classification and Ratings (Number of Firms/Percentage)

	AA	**A**	**BBB**	**BB**	**B**	**CCC**	**Total**
1	15/1.45	5/0.48	26/2.51	87/8.41	580/56.04	89/8.60	802/77.49
2	1/0.10	0	1/0.10	19/1.84	150/14.49	23/2.22	194/18.74
3	0	0	0	0	12/1.16	11/1.06	23/2.22
4	0	0	0	0	3/0.29	3/0.29	6/0.58
5	0	0	0	0	1/0.10	90.87	10/0.97
Total	16/1.55	5/0.48	27/2.61	106/10.24	746/72.08	135/13.04	**1035/100**

Source: Own computation.

Next, we constructed an artificial portfolio incorporating as much real information as possible. Outside the ratings, the bank data set did not contain information regarding loan volumes or maturities, charged interest rates and borrower asset returns or recovery rates. Since these parameters represent required inputs into many regulatory and internal credit risk models, we constructed proxy variables based on data available at the macro level or obtained them as random drawings from known distributions. In what follows we describe the manner in which these inputs were generated. The main information source was the SUD database of the Czech National Bank supervisory body, which contains yearly information about the residual maturity of Czech bank loans, their category and the borrower CZ-NACE category. SUD also categorizes loans according to the charged interest rate.

Ratings and Exposures

We adjusted the rating structure displayed in Table 2 to reflect the following changes:

- All bank clients in loan category 5 were eliminated. Loans in this category are loss loans, which are usually covered by provisions created in the current period. Moreover, the 5th category is an absorbing state. Once a loan falls into this category, there is a high probability that it will not recover. These loans pose a vacuous problem from the risk management perspective, since the future state of these loans is associated with almost no uncertainty.

- The 8.6% of firms with a CCC rating were removed from category 1 and added to categories 3 and 4. We assumed that the 8.6% outcome reflected the imperfections of our model. Czech banks monitor the creditworthiness of their clients, thus loans falling in the first category are unlikely to be granted a CCC grade.
- The rating structure was adjusted to resemble the loan volume configuration at the end of December 2000 as closely as possible (as shown in Table 3).

Table 3: Loan Volumes by Category Granted by Czech Banks, as of End 2000

Category	Volume (CZK bn)	Proportion (%)
1	607.235	68.58
2	85.811	9.69
3	54.577	6.16
4	26.982	3.05
5	110.834	12.52

Source: *CNB.*

After all these changes, the rating structure of our artificial (test) portfolio took the form shown in Table 4. To have a fair representation of all ratings in each loan category, one needs a minimum of 30 assets in the artificial portfolio.

Table 4: Rating Structure of the Artificial Portfolio – Number of Assets in Each Loan Category and Rating Classes

Loan category	AA	A	BBB	BB	B	CCC	Total
1	1	1	2	3	14	0	21
2	0	0	0	1	2	1	4
3	0	0	0	0	1	2	3
4	0	0	0	0	0	2	2
Total	1	1	2	4	17	5	30

In our test portfolio the exposure of an asset in a certain loan category represents the ratio between the total loan volume and the number of assets in that category. For example, all assets belonging to category 1 (21 in the test portfolio) have an exposure of CZK 607.235/21=28.915 billion.

Maturities

Loans with a maturity exceeding five years are sparsely represented in the Czech bank portfolios. For this reason, we considered maturities that ranged from one to five years only. Maturity was assigned to individual assets by drawing random numbers from the interval [1,5] according to the uniform distribution and then rounding these numbers to the nearest integer.

Interest Rates

We computed the mean and standard deviation of the lending rates for each loan category (using the SUD data set). To assign interest rates to assets in our portfolio, we randomly drew numbers from normal distributions described by the estimated means. Standard deviations were in general reduced to prevent interest rates from deviating too much from these mean values.

Asset Return Correlations

We grouped firms in the bank data set according to ratings and loan classification and found the CZ-NACE category that was the most frequently represented in each group. For example, in the group of firms with rating AA and loan classification 1 the largest number of firms belonged to CZ-NACE 51. If in a certain group no dominating CZ-NACE could be found, we selected randomly the representative figure from those that were represented in that group. The resulting CZ-NACE structure was mapped to the test portfolio. Having assigned a CZ-NACE label to each asset in the test portfolio, we used the price index characteristic of the corresponding branch as a proxy variable for that asset's returns8. Asset return correlations were determined by computing correlations among price indices.

Finally, having generated collateral and recovery rates (Appendix A1.3) we obtained a portfolio of 30 loans, whose characteristics are displayed in Table 5.

Table 5: Portfolio Composition and Individual Assets' Characteristics

Asset	CZ-NACE	Regulatory loan class	Rating	Loan volume	Maturity	Interest rate (%)	Type of collateral	Recovery rate (%)
1	51	1	AA	28.916	3	6.5	6	0
2	36	1	A	28.916	1	7.3	6	0
3	74	1	BBB	28.916	3	7.5	6	0
4	31	1	BBB	28.916	5	7.6	6	0
5	74	1	BB	28.916	5	8.2	4	71.43
6	20	1	BB	28.916	5	8.5	1	94.34
7	51	1	BB	28.916	1	8.7	6	0
8	28	1	B	28.916	3	8.8	2	100
9	15	1	B	28.916	4	9.5	6	0
10	51	1	B	28.916	2	10.5	1	94.34
11	52	1	B	28.916	2	11.0	5	50
12	29	1	B	28.916	1	11.1	6	0
13	70	1	B	28.916	1	11.3	5	50
14	74	1	B	28.916	2	11.6	4	71.43
15	50	1	B	28.916	2	11.7	1	94.34
16	24	1	B	28.916	1	11.9	5	50.00
17	45	1	B	28.916	2	12.1	1	94.34
18	60	1	B	28.916	2	12.2	1	94.34
19	40	1	B	28.916	3	12.5	3	89.29
20	25	1	B	28.916	2	12.6	1	94.34
21	65	1	B	28.916	4	13.0	4	71.43
22	51	2	BB	21.452	2	8.4	1	94.34
23	25	2	B	21.452	5	10.4	1	94.34
24	29	2	B	21.452	3	11.5	6	0
25	55	2	CCC	21.452	3	13.5	5	50
26	45	3	B	18.192	2	11.9	1	94.34
27	28	3	CCC	18.192	5	12.4	1	94.34
28	21	3	CCC	18.192	4	12.2	1	94.34
29	21	4	CCC	13.491	5	14.3	4	71.43
30	37	4	CCC	13.491	2	15.4	1	94.34

Source: *Own computation.*

[8]At the outset, all price indices were deflated by the PPI in order to eliminate the systemic inflationary influence in their evolution.

5. The BIS Approach to Economic Capital Determination

At the time when the present research was conducted, the latest officially released consultative document describing the New Basel Capital Accord approach was the one from January 2001. Since the release of this version, much further discussion has been going on and the Committee has proposed various modifications. The issuance of another consultative version is expected in the second quarter of 2003. At this moment the last available version of this document is in the form of technical guidance to the third quantitative survey (QIS 3). This section compares both of the above-mentioned versions of the Basel document.

5.1 The Standardized Approach According to the January 2001 Consultative Document

When computing the capital requirements for credit risk according to the standardized approach, we applied the comprehensive treatment of collateral with the standard supervisory haircuts. The comprehensive approach according to the January 2001 NBCA Consultative Document means that banks apply haircuts to the market value of collateral in order to protect against price volatility. A weight (floor factor) is applied to the collateralized portion of the exposure after adjusting for the haircut.

All claims in the simulated portfolio were assigned to the group of exposures "Claims on corporates" and were given the risk weights corresponding to their rating (Table 6).

Table 6: Risk Weights for the Group of Exposures "Claims on Corporates"

	AAA to AA-	**A+ to A-**	**BBB+ to BB-**	**Below BB-**	**Unrated**
Risk weights	20%	50%	100%	150%	100%

Source: *Basel Committee on Banking Supervision (January 2001).*

When setting the risk weights and risk-weighted assets, the quality and the amount of collateral were also considered. The NBCA defines the eligible collateral and specifies the appropriate haircuts. The eligible collateral consists of bank deposits with the lending bank, BB- or above-rated securities issued by sovereigns and public sector entities, BBB- or above-rated bank and corporate securities, equities included in a main index and gold. The haircuts are used to compute the adjusted value of the collateral according to the formula

$$\mathbf{CA = C / (1 + H_V + H_C + H_{FX}),}$$

where H_V, H_C and H_{FX} are, respectively, the exposure, the collateral and the currency haircut.

The value of risk-weighted assets with adjusted collateral is computed according to the following equation:

$$\mathbf{RWA = RW \times (V - (1\text{-}w) \times CA),}$$

where

- RW is the risk weight of the exposure (of the obligor)
- V is the exposure amount
- CA is the adjusted value of collateral
- w is a floor factor (in this case 0.15).

When the assets are covered by bank guarantee, the value of risk-weighted assets is given by:

$$\mathbf{RWA = V \times (w \times r + (1\text{-}w) \times g),}$$

where

- r is the risk weight of the obligor
- g is the risk weight of the guarantor.

For other assets not covered by acceptable collateral or bank guarantee, the risk-weighted value is **RWA = V x RW.**

The capital requirement is set as 8 per cent of the sum of all risk-weighted assets in the portfolio:

$$\mathbf{CR = 0.08 \times \sum_i RWA_i = \sum_i (0.08 \times RWA_i)\,.}$$

In our case the type of collateral offered was taken into account in only two cases (Assets 8 and 19) and influenced the risk-weighted assets through the adjusted value of the collateral. In the case of those assets which were covered by bank guarantee (Assets 6, 10, 15, 17, 18, 20, 22, 23, 26–28 and 30), the calculation took into account the risk weight of the guarantor (assuming g = 0.2). For the remaining assets, the collateral offered did not belong to the accepted collateral and the claims kept the risk weights given in Table 9.

Table 7 presents the partial results for the individual claims in the portfolio. The total capital requirement using this method is CZK 51.8 billion.

Table 7: Capital Requirement Computation – Standardized Approach (January 2001)

Asset	Rating	Volume (CZK billion)	Risk Weights (RW) without guarantee influence	Risk-Weighted Assets (RWA) with collateral and guarantee influence	Adjusted value of collateral (CA)	Capital requirement
1	AA	28.92	0.20	5.78	0	0.46
2	A	28.92	0.50	14.46	0	1.16
3	BBB	28.92	1.00	28.92	0	2.31
4	BBB	28.92	1.00	28.92	0	2.31
5	BB	28.92	1.00	28.92	0	2.31
6	BB	28.92	1.00	9.25	influence on RWA	0.74
7	BB	28.92	1.00	28.92	0	2.31
8	B	28.92	1.50	6.51	28.92	0.52
9	B	28.92	1.50	43.37	0	3.47
10	B	28.92	1.50	11.42	influence on RWA	0.91
11	B	28.92	1.50	43.37	0	3.47
12	B	28.92	1.50	43.37	0	3.47
13	B	28.92	1.50	43.37	0	3.47
14	B	28.92	1.50	43.37	0	3.47
15	B	28.92	1.50	11.42	influence on RWA	0.91
16	B	28.92	1.50	43.37	0	3.47
17	B	28.92	1.50	11.42	influence on RWA	0.91
18	B	28.92	1.50	11.42	influence on RWA	0.91
19	B	28.92	1.50	10.46	25.82	0.84
20	B	28.92	1.50	11.42	influence on RWA	0.91
21	B	28.92	1.50	43.37	0	3.47
22	BB	21.45	1.00	6.86	influence on RWA	0.55
23	B	21.45	1.50	8.47	influence on RWA	0.68
24	B	20.38	1.50	30.57	0	2.45
25	CCC	21.45	1.50	32.18	0	2.57
26	B	18.19	1.50	7.19	influence on RWA	0.58
27	CCC	18.19	1.50	7.19	influence on RWA	0.58
28	CCC	18.19	1.50	7.19	influence on RWA	0.58
29	CCC	13.49	1.50	20.24	0	1.62
30	CCC	13.49	1.50	5.33	influence on RWA	0.43
						51.84

Source: *Own computation.*

Note: *For the purposes of computing capital adequacy under the NBCA approaches, the volumes of assets were lowered by the amount of specific provisions which would be created following the CNB regulatory norms (this only happened in the case of Asset 24, for which the amount of specific provisions would be approximately CZK 1.07 billion).*

5.2 The Internal Rating Based (IRB) Approach According to the January 2001 Consultative Document

When applying the Internal Ratings Based (IRB) approach to compute the capital requirements, we followed the foundation approach. This means that the bank may use its own estimates for the default probabilities of individual obligors. However, the assigned PD may not be lower than 0.03%. The other risk components (LGD and EAD) are set according to the NBCA guidelines.

In our portfolio, the default probabilities of the individual assets were taken from the S&P matrix. Only in the case of one asset (Asset 1) was the probability of default lower than the minimum NBCA level; therefore, we had to change it to the minimum accepted value of 0.03%.

Loss Given Default (LGD) was set by taking into account the collateral of the individual claims:

- Claims secured by collateral type 1 (bank guarantee) were assigned LGD = 0.5. In the case of bank guarantees, the risk weight (RW) of the guarantor is taken into account when calculating RWA.
- Claims fully secured by collateral types 2 and 3 (cash and securities respectively) LGD* = w x LGD, where LGD = 0.5.
- Claims secured by collateral type 4 (commercial real estate) were assigned LGD = [1-(0.2 x (C/V)/1.4)] x 0.5
- Claims secured by collateral type 5 (other) or type 6 (unsecured) were assigned LGD = 0.5.

We will use the following notation:

- PD – probability of default
- M – maturity
- b – sensitivity of the maturity adjustment factor to M
- N(x) – cumulative normal distribution function
- G(z) – inverse of the cumulative normal distribution function.

Further, BRW is defined as the benchmark risk weight for corporate exposures. It is a function the exposure's PD:

$$\mathbf{BRW\ (PD) = 976.5 \times \mathit{N}\,(1.118 \times G(PD) + 1.288) \times (1 + .0470 \times (1 - PD)/PD^{0.44}).}$$

The baseline risk weight formula is

$$\mathbf{RW = min\ \{(LGD/50)\ x\ BRW\ (PD),\ \ 12.5\ x\ LGD\}.}$$

In cases where the claim maturity is explicitly known, it is necessary to include this piece of information in the risk weight calculation. This is done according to the following formula:

$$\mathbf{RW = min\ \{(LGD/50)\ x\ BRW\ (PD)\ x\ [1 + b\ (PD)\ x\ (M - 3)],\ \ 12.5\ x\ LGD\}.}$$

In our simulated portfolio the maturity of all claims was known. However, because at the time of computation the NBCA had no specifications regarding coefficient "b", we had to apply the first rule for setting RW.

The capital requirement is determined, in the same way as in the standardized approach, as 8 per cent of the risk-adjusted capital: $\mathbf{CR} = \mathbf{0.08} \times \sum_{\mathbf{i}} \mathbf{RWA_i}$.

Table 8 presents the partial results for the individual claims of the simulated portfolio. The total capital requirement that we obtain using this approach is CZK 165.46 billion.

Table 8: Capital Requirement Computation – IRB Approach (January 2001)

Asset	Rating	Volume (CZK billion)	PD (S&P)	PD (NBCA)	BRW	LGD (NBCA)	(LGD/50) x BRW(PD)	12.5 x LGD	RWA	Capital requirement
1	AA	28.92	0.0000	0.0003	14.07	0.50	0.14	6.25	4.07	0.33
2	A	28.92	0.0006	0.0006	21.37	0.50	0.21	6.25	6.18	0.49
3	BBB	28.92	0.0018	0.0018	42.21	0.50	0.42	6.25	12.21	0.98
4	BBB	28.92	0.0018	0.0018	42.21	0.50	0.42	6.25	12.21	0.98
5	BB	28.92	0.0106	0.0106	129.67	0.43	1.11	5.36	32.14	2.57
6	BB	28.92	0.0106	0.0106	129.67	0.50	1.30	6.25	37.50	3.00
7	BB	28.92	0.0106	0.0106	129.67	0.50	1.30	6.25	37.50	3.00
8	B	28.92	0.0520	0.0520	338.83	0.08	0.51	0.94	14.70	1.18
9	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
10	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
11	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
12	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
13	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
14	B	28.92	0.0520	0.0520	338.83	0.43	2.90	5.36	83.98	6.72
15	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
16	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
17	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
18	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
19	B	28.92	0.0520	0.0520	338.83	0.08	0.51	0.94	14.70	1.18
20	B	28.92	0.0520	0.0520	338.83	0.50	3.39	6.25	97.98	7.84
21	B	28.92	0.0520	0.0520	338.83	0.43	2.90	5.36	83.98	6.72
22	BB	21.45	0.0106	0.0106	129.67	0.50	1.30	6.25	27.82	2.23
23	B	21.45	0.0520	0.0520	338.83	0.50	3.39	6.25	72.69	5.82
24	B	20.38	0.0520	0.0520	338.83	0.50	3.39	6.25	69.05	5.52
25	CCC	21.45	0.1979	0.1979	665.20	0.50	6.65	6.25	134.08	10.73
26	B	18.19	0.0520	0.0520	338.83	0.50	3.39	6.25	61.64	4.93
27	CCC	18.19	0.1979	0.1979	665.20	0.50	6.65	6.25	113.70	9.10
28	CCC	18.19	0.1979	0.1979	665.20	0.50	6.65	6.25	113.70	9.10
29	CCC	13.49	0.1979	0.1979	665.20	0.43	5.70	5.36	72.27	5.78
30	CCC	13.49	0.1979	0.1979	665.20	0.50	6.65	6.25	84.32	6.75
										165.46

Source: *Own computation.*

Note: *For the purposes of computing capital adequacy under the NBCA (January 2001) approach, the volumes of assets were lowered by the amount of specific provisions which would be created following the CNB regulatory norms (this only happened in the case of Asset 24, for which the amount of specific provisions would be approximately CZK 1.07 billion).*

5.3 The Standardized Approach According to Technical Guidance to QIS 3, October 2002

All claims in the simulated portfolio were assigned to the group of exposures "Claims on corporates" and were given the risk weights corresponding to their rating grade. At this stage both the rating groups and the risk weights assigned to them remained the same as in the consultative document from January 2001 (see Subsection 5.1).

Calculating the capital requirement, we followed the comprehensive approach. The comprehensive approach here means that banks calculate their adjusted exposure to a counterparty for capital adequacy purposes by taking into account the effects of that collateral. Banks adjust both the amount of the exposure to the counterparty and the value of any collateral received to take account of future fluctuations in the value of each. (Basel Committee on Banking Supervision, QIS3, October 2002.)

Accordingly, the amount of exposure after risk mitigation for a collateralized loan is given by

$$\mathbf{E^* = max\ \{\ 0, [E\ x\ (1 + H_V) - C\ x\ (1 - H_C - H_{FX})]\},}$$

Where the haircut coefficients are defined as before and

- E* is the exposure amount after risk mitigation,
- E is current value of the exposure,
- C is the current value of the collateral received.

The exposure amount was adjusted (due to the quality and amount of the collateral) in the case of Assets 8 and 19 only, because only these two were covered by eligible collateral. In the other cases, the collateral was either not considered or the quality of the collateral (bank guarantee) was taken into account when assigning the risk weights. In the case of bank guarantees the protected portion was assigned the risk weight of the protection provider (assuming RW = 0.2). This was the case for Assets 6, 10, 15, 17, 18, 20, 22, 23, 26–28 and 30.

For each claim in the portfolio, the risk-weighted asset value can be computed as the product of the risk weight of the exposure and the exposure amount (after risk mitigation if applicable):

$$\mathbf{RWA = RW\ x\ V.}$$

The capital requirement is taken equal to 8 per cent of the risk-weighted asset sum, as is usual in the standardized and IRB approaches: $\mathbf{CR} = \mathbf{0.08} \times \sum_{\mathbf{i}} \mathbf{RWA_i}$.

Table 9 presents the partial results for the individual claims in the portfolio. The total capital requirement using this method is CZK 46.9 billion.

Table 9: Capital Requirement Computation – Standardized Approach (October 2002)

Asset	Rating	Volume (CZK billion)	Risk Weights (RW) without guarantee influence	Exposure amount after risk mitigation (E*)	Risk-Weighted Assets (RWA) with collateral and guarantee influence	Capital requirement
1	AA	28.92	0.20	28.92	5.78	0.46
2	A	28.92	0.50	28.92	14.46	1.16
3	BBB	28.92	1.00	28.92	28.92	2.31
4	BBB	28.92	1.00	28.92	28.92	2.31
5	BB	28.92	1.00	28.92	28.92	2.31
6	BB	28.92	1.00	guarantee	5.78	0.46
7	BB	28.92	1.00	28.92	28.92	2.31
8	B	28.92	1.50	0	0.00	0.00
9	B	28.92	1.50	28.92	43.37	3.47
10	B	28.92	1.50	guarantee	5.78	0.46
11	B	28.92	1.50	28.92	43.37	3.47
12	B	28.92	1.50	28.92	43.37	3.47
13	B	28.92	1.50	28.92	43.37	3.47
14	B	28.92	1.50	28.92	43.37	3.47
15	B	28.92	1.50	guarantee	5.78	0.46
16	B	28.92	1.50	28.92	43.37	3.47
17	B	28.92	1.50	guarantee	5.78	0.46
18	B	28.92	1.50	guarantee	5.78	0.46
19	B	28.92	1.50	3.47	5.20	0.42
20	B	28.92	1.50	guarantee	5.78	0.46
21	B	28.92	1.50	28.92	43.37	3.47
22	BB	21.45	1.00	guarantee	4.29	0.34
23	B	21.45	1.50	guarantee	4.29	0.34
24	B	20.38	1.50	21.45	32.18	2.57
25	CCC	21.45	1.50	21.45	32.18	2.57
26	B	18.19	1.50	guarantee	3.64	0.29
27	CCC	18.19	1.50	guarantee	3.64	0.29
28	CCC	18.19	1.50	guarantee	3.64	0.29
29	CCC	13.49	1.50	13.49	20.24	1.62
30	CCC	13.49	1.50	guarantee	2.70	0.22
						46.90

Source: *Own computation.*

5.4 The Internal Rating Based (IRB) Approach According to Technical Guidance to QIS 3 (from October 2002)

When applying the Internal Ratings Based (IRB) approach to the capital requirement computations, we followed the same approach as in Subsection 5.2 – the foundation approach. That is, the bank may use its own estimates for the default probabilities of individual obligors; the other risk components (LGD and EAD) are set according to the NBCA guidelines.

As in the case mentioned above, the default probabilities of the individual assets were taken from the S&P matrix. Only in the case of one asset (Asset 1) was the probability of default lower than the minimum NBCA level; therefore, we had to change it to the minimum accepted level of 0.03%.

However, the calculation of risk-weighted assets (followed in the case of all assets in our simulated portfolio) is different from that used under the January 2001 Consultative Document:

Risk-weighted assets (RW) = K x 12.50 x EAD,

where

Capital requirement (K) = LGD x N [(1 – R)$^{-0.5}$ x G (PD) + (R / (1 – R))$^{0.5}$ x G (0.999)] x (1 – 1.5 x b (PD))$^{-1}$ x (1 + (M – 2.5) x b (PD))

Maturity adjustment b(PD) = (0.08451 – 0.05898 x log (PD))^{2}

Correlation (R) = 0.12 x (1 – EXP (-50 x PD)) / (1 – EXP (-50)) + 0.24 x [1 – (1 - EXP (-50 x PD)) / (1 – EXP (-50))].

Loss Given Default was set by taking into account the collateral of the individual claims:

- Claims secured by collateral type 1 (bank guarantee) were assigned LGD = 0.45. In the case of bank guarantees, the risk characteristics (PD and LGD) of the guarantor are taken into account when calculating RWA.
- Claims fully secured by collateral types 2 and 3 (cash and securities respectively) LGD* = Max {0, LGD x [(E* / E)]}, where LGD = 0.45.
- Claims secured by collateral type 4 (commercial real estate) were assigned LGD = 0.35
- Claims secured by collateral type 5 (other) or type 6 (unsecured) were assigned LGD 0.45.

The capital requirement is determined as usual: $\mathbf{CR} = \mathbf{0.08} \times \sum_{\mathbf{i}} \mathbf{RWA_i}$ **,** which is equal to CZK 44.8 billion in our example. Table 10 presents the partial results for the individual claims in the portfolio.

Table 10: Capital Requirement Computation – IRB Approach (October 2002)

Asset	Rating	Volume (CZK billion)	PD (S&P)	PD (NBCA)	LGD (QIS3)	Exposure amount after risk mitigation (E*)	E*/ V	Correlation (R)	Maturity adjustment (b)	Capital requirement (K)	Risk-weighted assets (RW)	Capital requirement
1	AA	28.92	0	0.0003	0.45	28.92	1.00	0.24	0.09	0.01	2.24	0.18
2	A	28.92	0.0006	0.0006	0.45	28.92	1.00	0.24	0.08	0.01	3.82	0.31
3	BBB	28.92	0.0018	0.0018	0.45	28.92	1.00	0.23	0.06	0.02	8.39	0.67
4	BBB	28.92	0.0018	0.0018	0.45	28.92	1.00	0.23	0.06	0.02	8.39	0.67
5	BB	28.92	0.0106	0.0106	0.35	28.92	1.00	0.19	0.04	0.05	18.25	1.46
6	BB	28.92	0.0106	0.0106	0.45	guarantee	x	0.24	0.08	0.01	3.82	0.31
7	BB	28.92	0.0106	0.0106	0.45	28.92	1.00	0.19	0.04	0.06	23.47	1.88
8	B	28.92	0.052	0.052	0	0	0.00	0.13	0.03	0.00	0.00	0.00
9	B	28.92	0.052	0.052	0.45	28.92	1.00	0.13	0.03	0.13	47.28	3.78
10	B	28.92	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	3.82	0.31
11	B	28.92	0.052	0.052	0.45	28.92	1.00	0.13	0.03	0.13	47.28	3.78
12	B	28.92	0.052	0.052	0.45	28.92	1.00	0.13	0.03	0.13	47.28	3.78
13	B	28.92	0.052	0.052	0.45	28.92	1.00	0.13	0.03	0.13	47.28	3.78
14	B	28.92	0.052	0.052	0.35	28.92	1.00	0.13	0.03	0.10	36.77	2.94
15	B	28.92	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	3.82	0.31
16	B	28.92	0.052	0.052	0.45	28.92	1.00	0.13	0.03	0.13	47.28	3.78
17	B	28.92	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	3.82	0.31
18	B	28.92	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	3.82	0.31
19	B	28.92	0.052	0.052	0.054	3.47	0.12	0.13	0.03	0.02	5.67	0.45
20	B	28.92	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	3.82	0.31
21	B	28.92	0.052	0.052	0.35	28.92	1.00	0.13	0.03	0.10	36.77	2.94
22	BB	21.45	0.0106	0.0106	0.45	guarantee	x	0.24	0.08	0.01	2.83	0.23
23	B	21.45	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	2.83	0.23
24	B	20.38	0.052	0.052	0.45	21.45	1.00	0.13	0.03	0.13	35.08	2.81
25	CCC	21.45	0.1979	0.1979	0.45	21.45	1.00	0.12	0.02	0.27	71.93	5.75
26	B	18.19	0.052	0.052	0.45	guarantee	x	0.24	0.08	0.01	2.40	0.19
27	CCC	18.19	0.1979	0.1979	0.45	guarantee	x	0.24	0.08	0.01	2.40	0.19
28	CCC	18.19	0.1979	0.1979	0.45	guarantee	x	0.24	0.08	0.01	2.40	0.19
29	CCC	13.49	0.1979	0.1979	0.35	13.49	1.00	0.12	0.02	0.21	35.18	2.81
30	CCC	13.49	0.1979	0.1979	0.45	guarantee	x	0.24	0.08	0.01	1.78	0.14
												44.79

Source: *Own computation.*

6. Commercial Risk-Measurement Models

6.1 CreditMetrics

In the CreditMetrics model, risk is associated with changes in the portfolio value caused by changes in the credit quality of individual obligors (downgrades or default) over the considered risk period (usually taken as one year). The risk analysis undertaken by CreditMetrics consists of two distinct parts. The analytical part derives primary risk measures such as means, variances and standard deviations at the asset and portfolio level. Monte Carlo simulation generates a simulated portfolio value distribution at the risk horizon. Based on this distribution, estimates of economic capital can be obtained at different confidence levels. Because the aim of this paper is to estimate economic capital, we focus on the application of the Monte Carlo method[9].

Monte Carlo Simulation

The Monte Carlo simulation is a numerical method that makes inferences about unknown parameters of a stochastic process based on a large number of random draws from the distributions supposed to drive the given process. Intuitively, it consists in obtaining a large set of potential realizations of the real process and consequently in aggregating the statistical information thus obtained to estimate the unknown parameters. In CreditMetrics, this is done to estimate the portfolio value distribution at the risk horizon. Because changes in asset and portfolio value are caused by changes in the credit quality of individual assets (upgrades and downgrades) the Monte Carlo simulation is a tool that randomly generates potential changes in the actual rating structure of the portfolio.

More precisely, the Monte Carlo simulation performs a large number of random draws from a multinomial normal distribution ("scenarios", in CreditMetrics terminology). Each scenario contains the same number of components (real numbers) as the number of assets in the portfolio. Each component of each scenario is compared with pre-defined, rating class-specific threshold values marking the switch from one rating class to another. In this way, within each scenario, a new rating is assigned to each asset (obligor) in the portfolio.

Moreover, in case of non-default, each asset *i* is revalued according to the formula:

$$V_i^{g'} = \sum_{t=1}^{T_i - 1} \frac{r_i}{\left(1 + f_t^g\right)^t} + \frac{r_i + F_i}{\left(1 + f_{T_i}^g\right)^{T_i}}, \tag{1}$$

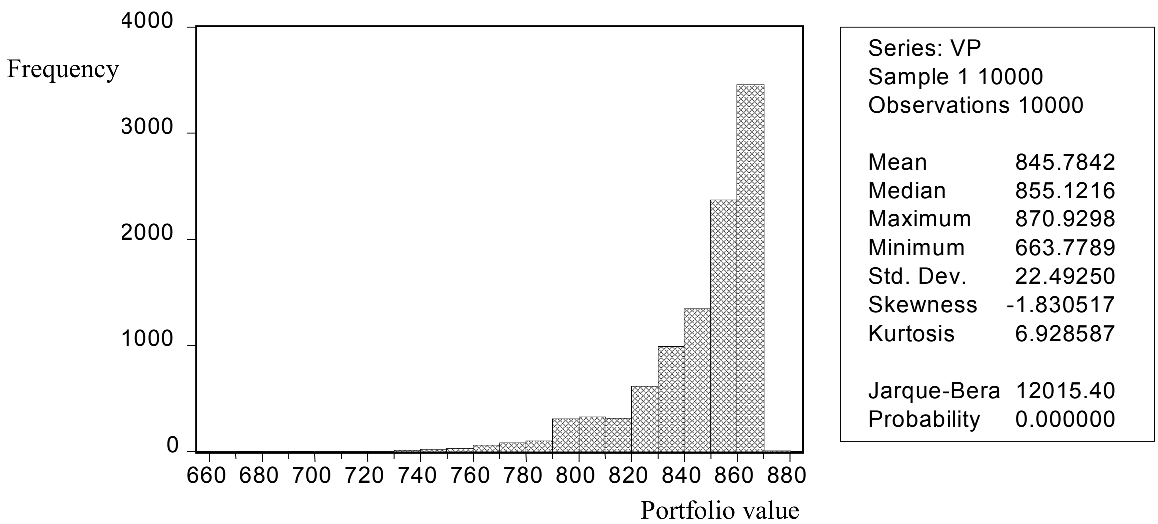
where r_i and F_i are the loan interest payments and the face value of the loan respectively, and f_t^g are the annualized forward zero rates for the years 1 to T_i, applicable to the rating class g' (here T_i is the maturity of the loan). In this specification it is assumed that the present rating changes from g to g' over the one-year period. In the case of default the present value of the loan is computed as the product of the face value of the loan and a recovery rate.

[9] In general, we do not go into a detailed coverage of the methodologies proposed by these models. This was done in our previous CNB working paper (see Derviz and Kadlčáková, 2001) specially dedicated to the subject. Here, our aim is to derive estimations of economic capital according to different models.

Note that the way of defining the future loan value as a random variable implies that the distribution of this variable would be taken with respect to the *risk-neutral probability* (RNP). CreditMetrics works with the assumption that such a probability is well-defined for the studied economy (cf. a more general view of the subject in our own model, particularly Subsection 7.1). Under the risk-neutral probability, in contrast to the “physical” one, zero forward rates are unbiased estimates of the future spot interest rates. That is, the capital requirements under CreditMetrics are also derived from RNP. This might lead to certain discrepancies between the CreditMetrics interpretation by the market (which is based on RNP) and its interpretation by the regulator (based on the physical probability).

To obtain the portfolio value distribution and derive the economic capital requirement in accordance with the CreditMetrics model, we conducted a Monte Carlo simulation with 10,000 random draws. Each scenario contained 30 correlated random draws from the standard normal distribution. Each element of each scenario represented a standardized return corresponding to one of the 30 assets belonging to the portfolio. Comparing the elements of the scenario with the threshold values characteristic of each rating category, new ratings were assigned to each of the 30 assets. Summing up the values of the 30 assets thus obtained, a new portfolio value resulted for each scenario. Since eight potential new ratings were possible for each of the 30 assets at the end of the year, the total number of potential portfolio values was 8^{30}. In practice, this number was far lower, as some rating class migrations had a zero probability of realization. Figure 1 shows the distribution of our portfolio value expected at the end of 2000 for the year 2001. On the horizontal axis are the non-overlapping intervals within which the portfolio value falls, while on the vertical axis are the frequencies with which these portfolio value realizations occurred within each interval in our simulation.

Figure 1: Outcomes of the Monte Carlo Simulation for the Loan Portfolio Value



Source: *Own computation.*

Note: *To the right are statistics that characterize the portfolio value distribution. In addition to basic descriptive statistics, the table displays skewness, kurtosis and the Jarque-Bera test statistic value.*

Economic capital is obtained as the difference between the mean of the portfolio value and a p-percentile (p is usually assumed to be 1%, 2% or 5%):

Economic Capital = Mean of the Portfolio Distribution – p-percentile.

In the case of a discrete portfolio distribution, the p-percentile is obtained by looking at the lowest portfolio value whose cumulative frequency exceeds p%. An interpretation of the p- percentile is that in p% of cases we can expect the portfolio value to take values lower than the p-percentile over the one-year period. For example, in our case we can expect that only 100 times in 10,000 cases could the portfolio value reach a value lower than CZK 767.89 billion (in other words we know with 99% probability that the portfolio value will be higher than CZK 767.89 billion at the year-end). To cover this high loss, however unlikely it is, the bank must keep economic capital of CZK 77.88 billion.

The 1% and 5%-percentiles take the values CZK 767.89 billion and CZK 796.62 billion respectively. Based on these estimations, the bank's need for economic capital at different confidence levels is:

1%-percentile	5%-percentile	Mean	99% Economic capital	95% Economic capital
767.89	796.62	845.78	77.88	49.16

6.2 CreditRisk+

CreditRisk+ is suitable for assessing credit risk in portfolios containing a large number of obligors with small default probabilities. The model groups bank customers according to their common exposure. The common exposure of an obligor represents the ratio between his bank exposure and a selected unit of exposure (CZK 1 billion in our case). Bank clients can be grouped in homogeneous "bands" that contain obligors with the same common exposure.

For obligor i, we introduce the following notations, taken from the CreditRisk+ technical document: L_i – exposure, P_i – default probability, $\nu'_i = \frac{L_i}{L}$ – common exposure, ν_i – rounded common exposure, $\varepsilon_i = \nu'_i \times P_i$ – expected loss. In addition, at the given Band j-level, ν_j is the common exposure in units of L, $\varepsilon_j = \sum_{i \in \text{Band } j} \varepsilon_i$ – expected loss in Band j in units of L, $\mu_j = \frac{\varepsilon_j}{\nu_j}$ – expected number of defaults in Band j.

The risk assessment at the asset level consists in estimating the expected loss (ε_i). At the band level, the model estimates the average number of defaults (m_j) as the ratio of the total expected loss in the band (ε_j) to the common exposure characteristic to the obligors from that band (ν_j).

By assumption, the distribution of the number of defaults in each band is of the Poisson type:

$$P_j = P(\text{number of defaults in band } j = k) = \frac{m_j^k e^{-m_j}}{k!} \qquad k = 0, 1, \ldots$$

Further, the model employs statistical tools to estimate the distribution of loss at the portfolio level. Using the properties of probability-generating functions, the model estimates recursively the probabilities that the portfolio loss reaches values expressed as multiples of the unit of exposure.

For our portfolio, the analytical risk assessments at the asset and band level are contained in Tables 11 and 12.

Table 11: Risk Assessment at the Asset Level According to the CreditRisk+ Model

Asset i	Exposure (billion CZK)	Common exposure (ν_i')	Common exposure rounded to multiples of CZK 1 billion (ν_i)	Probability of default (P_i)	Expected loss ($\varepsilon_i = \nu_i' \times P_i$)
1	28.92	28.92	29	0	0.00
2	28.92	28.92	29	0.0006	0.02
3	28.92	28.92	29	0.0018	0.05
4	28.92	28.92	29	0.0018	0.05
5	28.92	28.92	29	0.0106	0.31
6	28.92	28.92	29	0.0106	0.31
7	28.92	28.92	29	0.0106	0.31
8	28.92	28.92	29	0.052	1.50
9	28.92	28.92	29	0.052	1.50
10	28.92	28.92	29	0.052	1.50
11	28.92	28.92	29	0.052	1.50
12	28.92	28.92	29	0.052	1.50
13	28.92	28.92	29	0.052	1.50
14	28.92	28.92	29	0.052	1.50
15	28.92	28.92	29	0.052	1.50
16	28.92	28.92	29	0.052	1.50
17	28.92	28.92	29	0.052	1.50
18	28.92	28.92	29	0.052	1.50
19	28.92	28.92	29	0.052	1.50
20	28.92	28.92	29	0.052	1.50
21	28.92	28.92	29	0.052	1.50
22	21.45	21.45	22	0.0106	0.23
23	21.45	21.45	22	0.052	1.12
24	20.38	20.38	22	0.052	1.12
25	21.45	21.45	22	0.1979	4.25
26	18.19	18.19	19	0.052	0.95
27	18.19	18.19	19	0.1979	3.60
28	18.19	18.19	19	0.1979	3.60
29	13.49	13.49	14	0.1979	2.67
30	13.49	13.49	14	0.1979	2.67

Source*: Own computation.*

Four different bands have thus been obtained, with rounded common exposures of 14, 19, 22 and 29. Table 12 illustrates the partition of the portfolio in bands and the risk characteristics of each band.

Table 12: Band Partition and Risk Assessment at the Band Level According to the CreditRisk+ Model

Band j	**Rounded common exposure in Band j (ν_j)**	**Number of obligors in Band j**	**Expected loss in Band j (ε_j)**	**Expected number of defaults in Band j ($m_j=\varepsilon_j/\nu_j$)**	**Probability-generating function for Band j**
1	14	2	5.340	0.381	$\exp(-0.381+0.381z^{14})$
2	19	3	8.147	0.429	$\exp(-0.429+0.429\, z^{19})$
3	22	4	6.704	0.305	$\exp(-0.305+0.305z^{22})$
4	29	21	22.092	0.762	$\exp(-0.762+0.762z^{29})$

Source: *Own computation.*

For each band a probability-generating function is given by

$$G_j(z)=\sum_{n=0}^{\infty}\Pr\{L_j=n\}z^n=\sum_{k=0}^{\infty}P(k\text{ defaults})z^{k\nu_j}=\sum_{k=0}^{\infty}\frac{m_j^k e^{-m_j}}{k!}z^{k\nu_j}=e^{-m_j+m_j z^{\nu_j}}\ .$$

The probability-generating function for the entire portfolio is the product of the individual probability-generating functions displayed in the last column of Table 17. In this particular example we get

$$G(z)=e^{-\sum_{j=1}^{m}m_j+\sum_{j=1}^{m}m_j z^{\nu_j}}=e^{-1.877+0.381\,z^{14}+0.429z^{19}+0.305\,z^{22}+0.762z^{29}}\ .$$

To derive the probabilities that loss equals multiples of the unit of exposure, CreditRisk+ constructs a recurrence relationship:

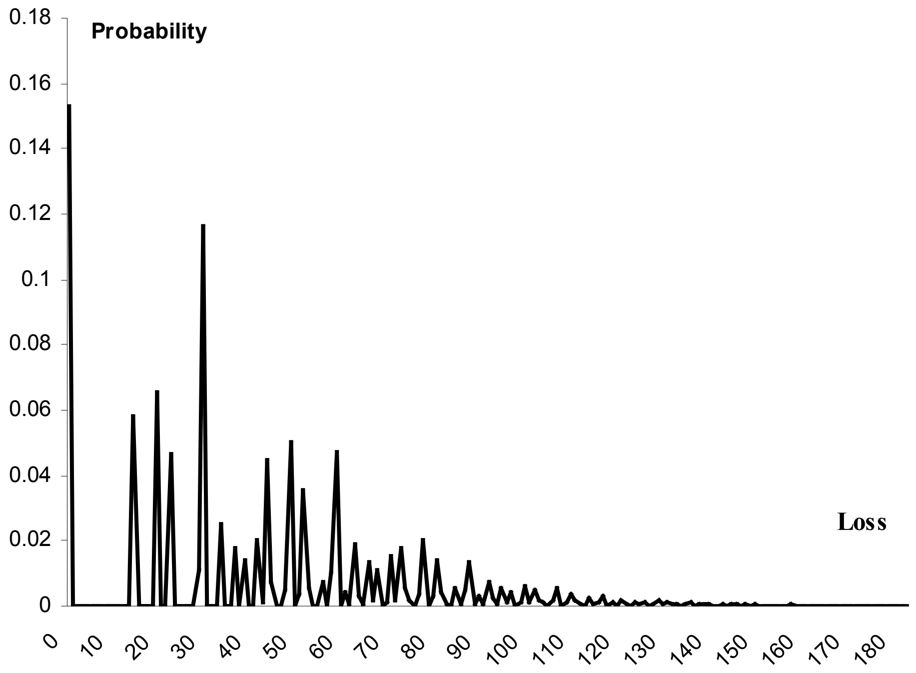
$$P_n=\sum_{\substack{j\\ \text{s.t.}\,\nu_j\le n}}\frac{\nu_j\times m_j}{n}P_{n-\nu_j}$$

that starts with the probability of no loss:

$$P_0=P(\text{No Loss})=e^{-m}=e^{-\sum_{j=1}^{m}m_j}=e^{-1.877}=0.153$$

The resulting loss distribution in the case of our portfolio is shown in Figure 2.

Figure 2: Loss Distribution Based on the CreditRisk+ Model



***Source**: Own computation.*

In CreditRisk+, economic capital is given by the difference between the p-percentile and the expected mean of the loss distribution.

Economic Capital = p-percentile – Expected Loss

Applying the CreditRisk+ approach to our portfolio, the estimation of risk capital at different confidence levels ended up with the values expressed below.

1%-percentile	5%-percentile	Expected loss	99% Capital	95% Capital
133	101	42.18	90.82	58.82

7. Monte Carlo Simulation with Floating Lending Rates and Fixed Forward Zero Curves

We assume that the mechanism of future changes in lending interest rates is as follows:

$$1+\frac{r_t^{g'}}{100}=\left(1+\frac{r_t}{100}\right)*e^{\sigma_t}$$

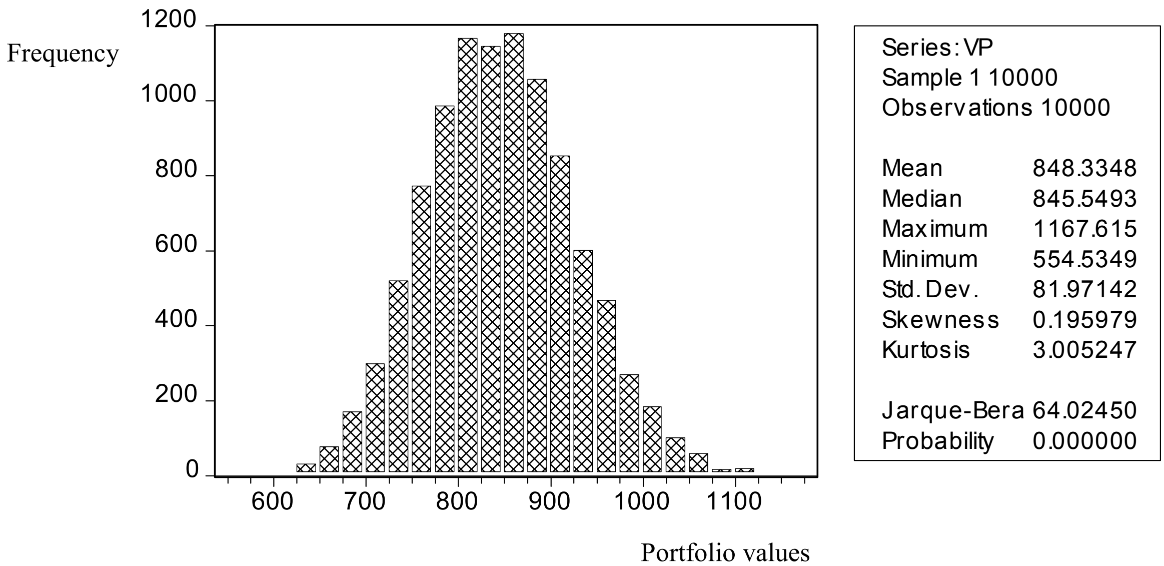
Here σ is a normally distributed random variable, so the exponential follows a log-normal distribution.

In this formulation, interest rates on loans preserve the mark-ups that capture obligor-specific risk or liquidity premia (already incorporated in the old interest rates), but also contain a random component reflecting uncertainty related to the future course of the money market rate (PRIBOR). The random component does not vary across obligors. In our simulations it was obtained as a random draw from a log-normal distribution. The parameters (mean and standard deviation) describing the log-normal distribution were estimated on actual Czech data (1Y PRIBOR over the year 2000, a period of relative rate stability and no monetary policy changes).

7.1 Monte Carlo Simulation with Floating Interest Rates

A total number of 10,000 random draws from the standard log-normal distribution were performed to determine the random component of the PRIBOR and thus the loans' interest rates. Due to the floating nature of the interest rates, the valuations of each asset and of the portfolio varied in line with the particular values of the random draws. A total number of 10,000 portfolio values at the end of the risk horizon were thus obtained. Figure 3 shows the relative frequencies of these values at the year-end.

Figure 3: The Empirical Distribution of the Portfolio Value with Floating Interest Rates



Source: *Own computation.*

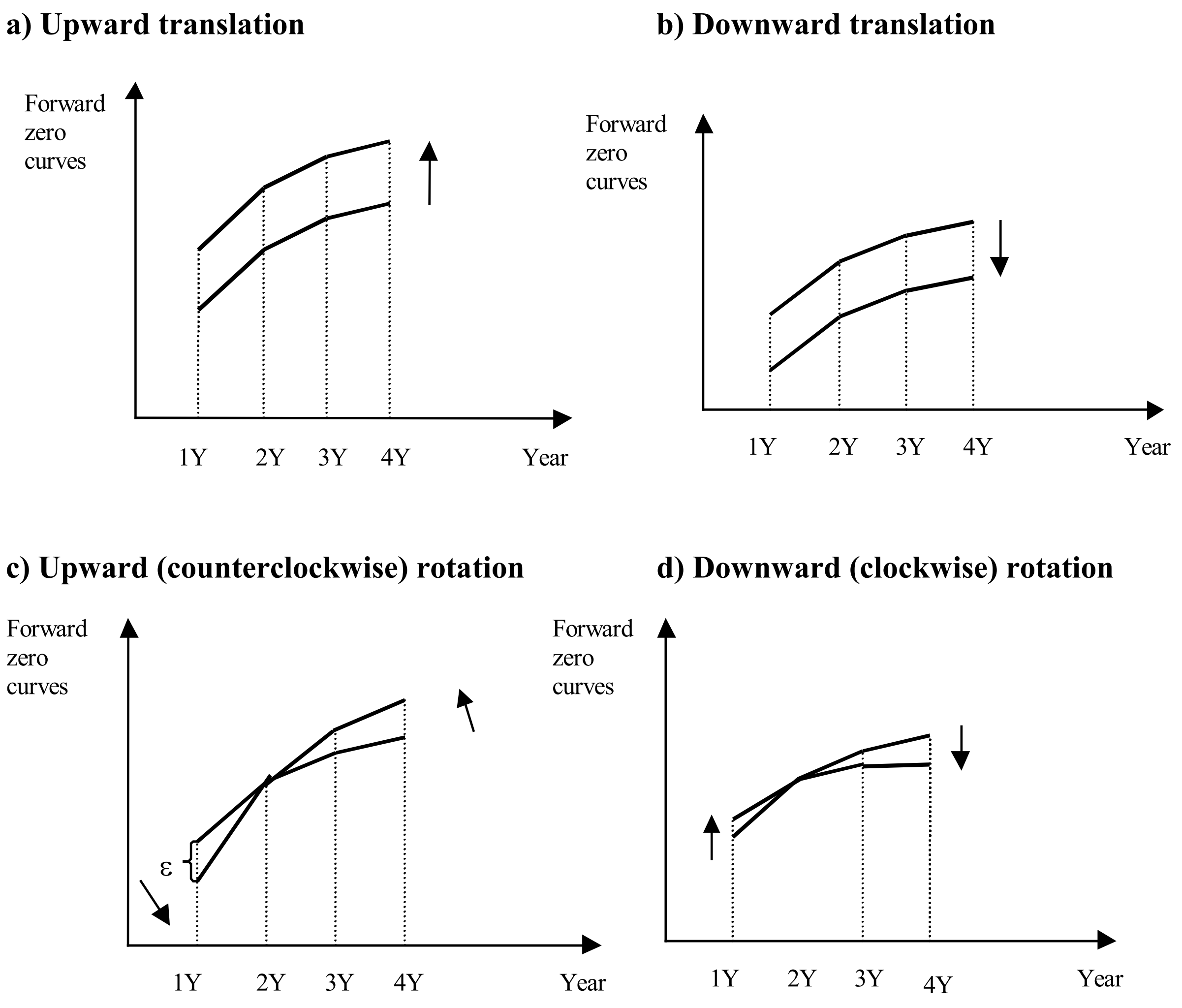
Note: *The horizontal axis shows non-overlapping intervals that cover the entire range of the estimated portfolio values, and the vertical axis shows the frequencies with which the portfolio values fell into those intervals. The descriptive statistics to the right are the same as in Fig. 1.*

The estimation of economic capital in this case is given in Table 13.

7.2 Monte Carlo Simulation with Floating Interest Rates and Stochastic Forward Zero Curves

The next four cases retain the assumption that floating interest rates were charged by the bank on its loans. Additional uncertainty is added with regard to changes in forward zero curves (the discount factors entering the loan valuations) over the one-year period. We analyzed the impact on the bank's need for economic capital under the following changes in the forward zero curves: upward translation, downward translation, clockwise rotation and counter-clockwise rotation. Rotations of the forward zero curves are around the point determined by the two-year forward rate. These changes are illustrated in Figure 4.

Figure 4: Simulated Changes in the Forward Zero Curves



We assumed that these changes reflected subjective expectations concerning the evolution of the Czech forward zero curves over a one-year period. They became rating class-specific by adding the US forward spread (the difference between the US rating class-specific and the US forward zero curves). We incorporated these changes into the forward zero curves in the model according to the formula:

$$1+\frac{f_t^{g'}}{100}=\left(1+\frac{f_t}{100}\right)*\left(1+\frac{s_t^g}{100}\right)*e^{\varphi_t},\ t=1,2,3,4 \qquad (2)$$

Here f_t is the original Czech forward zero rate at year t, $s^g{}_t$ is the spread in forward zero rates characteristic of the g-th rating class at time t, and φ_t is a random draw from the normal distribution (thus $e^{\varphi}{}_t$ is log-normally distributed).

The proposed changes in the forward zero curves were captured in the model by considering particular random variables φ_t in (2):

$\phi_t = \mu + \varepsilon$, t=1,2,3,4, for an upward translation,

$\phi_t = -\mu + \varepsilon$, t=1,2,3,4, for a downward translation

$\varphi_1 = -\mu + \varepsilon$, $\varphi_3 = \mu + \varepsilon$, $\varphi_4 = 2\mu + \varepsilon$, for an upward rotation

$\varphi_1 = \mu + \varepsilon$, $\varphi_3 = -\mu + \varepsilon$, $\varphi_4 = -2\mu + \varepsilon$, for a downward rotation.

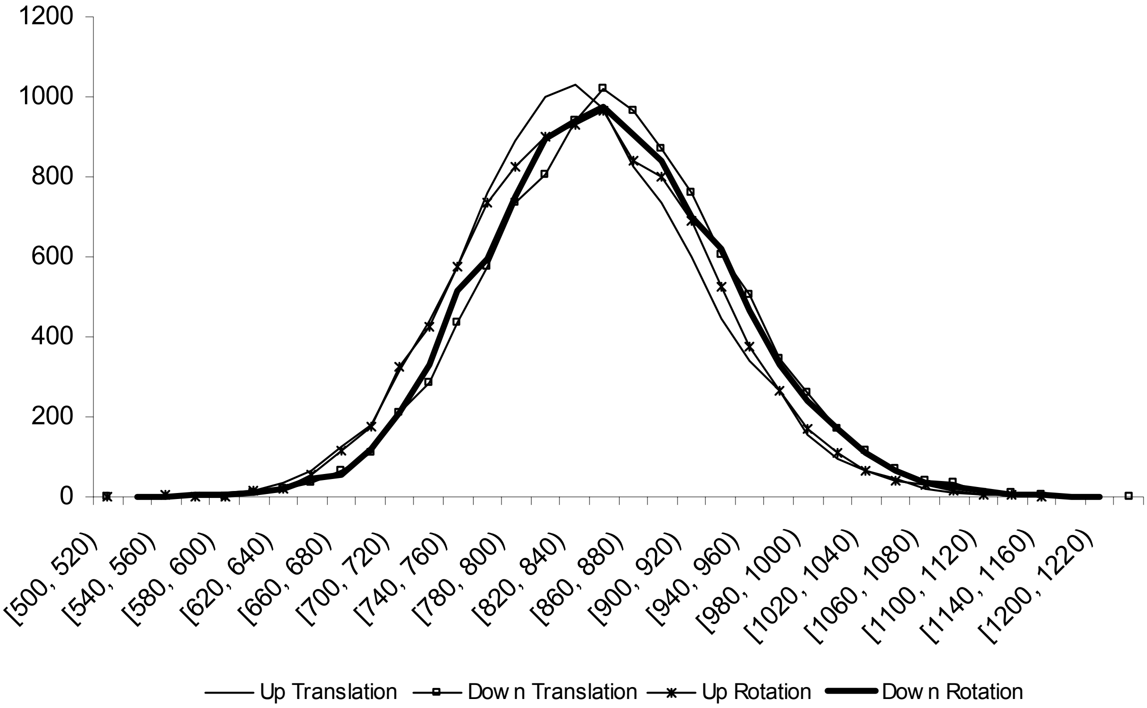
The overall effect of an upward shift in the forward zero curve is a decrease in the present value of all loans in the portfolio. If the bank heavily discounts the future, the opportunity cost of granting loans increases, since alternative assets may provide higher returns in the future. Accordingly, the present value of the cash flows accrued from the loans is lower compared with the case where the forward zero curves remained unchanged. The effect of a downward shift of the forward zero curve is the opposite of the one mentioned above. Rotations of the forward zero curve affect assets' valuations depending on maturity. For example, the counter-clockwise (upward) rotation discounts assets with a short maturity less and assets with a long maturity more. Therefore, the valuation of the portfolio is very sensitive to the portfolio composition. If more assets fall into the long maturity category the present value of the portfolio tends to fall, while if they fall into the short maturity category the present value of the portfolio tends to increase.

In all previous formulations of the φ_t distribution, the parameter μ determined the magnitude of the deterministic change in forward zero curves and ε added random deviations. We wanted changes in the deterministic part of the discount factors not exceeding one percent (thus, if for a given maturity the forward zero rate was 3.4%, we wanted it to deviate upward to 4.4% only). This assumption implied a value for μ of 0.01. In each case, ε was assumed to follow a normal distribution with mean $-\frac{\sigma^2}{2}$ and standard deviation σ, so that the mean of the log-normally distributed factor e^{ε} is equal to 1. Under these conditions φ became normally distributed with mean μ+m. The standard deviation σ of ε was estimated by computing the standard deviation of the log(1+1Y PRIBOR/100) variable using daily observations over the year 2000 after removing the trend. The same estimated values of the parameters μ and σ were used in all cases of random changes in forward zero curves.

We performed Monte Carlo simulations containing 10,000 scenarios that simultaneously accounted for random changes in interest rates and forward zero curves. Shown next are the portfolio value distributions and the estimates of economic capital based on simulations that incorporated the proposed changes into the forward zero curves.

Figure 5 displays the portfolio value distributions when the four FZC change cases discussed above are compared.

Figure 5: Portfolio Distributions under Different Assumptions Regarding Changes in FZC



Source: *Own computation.*

Economic capital estimations at different confidence levels are displayed in Table 13. The downward translation and the clockwise rotation of the forward zero curves impose the highest requirements of economic capital in this particular example. However, economic capital seems to converge towards the fixed forward zero curves (with floating lending rates) case when the confidence level is reduced.

Table 13: Capital Requirements Assuming Different Changes in Interest Rates and Forward Zero Curves (fzc)

10,000 random draws	**1%-percentile**	**5%-percentile**	**Mean**	**99% Ec. capital**	**95% Ec. capital**
Fixed interest and fixed fzc	767.90	796.62	845.78	77.89	49.16
Floating interest and fixed fzc	669.65	718.48	848.33	178.69	129.85
Upward translation of fzc	655.15	705.70	834.25	179.11	128.56
Downward translation of fzc	672.44	724.32	856.94	184.50	132.62
Counter-clockwise (upward) rotation of fzc	668.83	722.92	839.14	170.31	116.22
Clockwise (downward) rotation of fzc	668.83	722.92	853.01	184.18	130.09

Source: *Own computation.*

8. A Structural Model of Risky Debt with a Random Default Arrival

We next give an outline of a model of risky debt, its valuation and the resulting economic capital requirements, going along the "structural" lines of the original KMV model and its ramifications. The term "structural" means that we make the default explicitly dependent on loan and obligor characteristics. However, we borrow an additional element from the so-called "reduced-form" models of default (for a survey of both types of model, see Bohn, 1999), by working with a default process in Poissonian form. This technique was introduced by Jarrow and Turnbull, 1995. We follow the variant utilized by Madan and Unal, 1998, 1999, in that the default event arrival rate becomes a function of the same obligor fundamentals as the ones that drive the asset prices. However, this principle is developed in a way to establish a link between the default process of the abstract reduced-form models and the empirics inspired by the Expected Default Frequency notion of KMV. The proposed model allows one to deal with a loan portfolio with correlated defaults in a natural way.

8.1 Definitions

Consider a portfolio of n loans issued by n different companies (obligors). Loan j pays a coupon c_t^j at time t and the coupon plus the face value F^j at maturity T^j (t=1,…,T^j). The value of the loan to firm j at time t is denoted by V_t^j. Then, $B_t = \sum_{j=1}^{n} V_t^j$ is the value of the loan portfolio. We are interested in finding B_0.

There are N assets traded in the market, with prices P_t^k at time t (k=1,…,N). These assets represent all sources of aggregate uncertainty in the economy independent of the actions of obligors defined above. In this sense, the financial markets outside the considered borrower set are complete. These uncertainty factors will be represented by random variables S^k. By S, we denote the vector with components S^k. Let S be a Markov process with a given transitional probability.

The loans are risky. If firm j generates period t-cash flow $Y_t^J = f^J(S_t)$ net of all other debt service obligations, then the probability of default π_t^J on the loan is an inverse function of the difference $Y_t^J - C_t^J$: $\pi^j(S) = \pi(Y^j - C^j)$. Here, $C_t^J = c_t^J$ if $t<T^j$, and $C_t^J = c_t^J + F^J$ if $t=T^j$. Hence, the variable driving the default rate in our model is an analogue of the *distance-to-default* measure used in KMV. One should think of π as approaching unity when the distance to default falls to minus infinity, and approaching zero when it increases to plus infinity.

The space of the random events in our model is formed by pairs ω=(s,b), where s is a realization of S and b=[b^1,…,b^n], b^j=S if there is no default (survival) and b^j=D if the firm defaults on theloan. The arrival *fact* of the default event itself is assumed to be independent of S, i.e. only *the probability value* of the default is S-dependent, through the cash flow variable Y^j.

For each loan, there is collateral that is tradable and depends on the same sources of uncertainty as the basic assets. That is, the collateral price for loan j is equal to $Z^j = \xi^j(\omega)$. If the loan defaults, the bank seizes the collateral, i.e. receives the value of Z^j.

There are two important cases to be distinguished with regard to the collateral prices. One possibility is to allow ξ^j to depend on both s and b. That is, this collateral is worth different amounts depending on whether the debt has defaulted or not. This would be the case if there were a separate structural factor behind the realization of b, correlated with the market risk factors S. The same factor should be responsible for the value of the collateral. This situation would allow loan j to be priced in accordance with the risk-neutral valuation principles (see an example of such a valuation in Derviz and Kadlčáková, 2002, Section 3). It may occur when the collateral is very obligor-specific.

However, an equally legitimate case is that of the collateral being totally unrelated to the operation of the firm (e.g. securities in its investment portfolio). Then $Z^j=\zeta^j(s)$ and a unique risk-neutral valuation of the loan is impossible. In that case, one must resort to pricing techniques based on explicit individual portfolio optimization. This is done next.

8.2 The Individual Loan and the Portfolio Value

Let us consider an optimizing investor in discrete time who decides upon allocating his/her wealth between the existing marketable assets, i.e. the n traded securities, the n collateral assets and the n company loans (all defined above). Let this be an optimization problem under uncertainty in discrete time with transaction costs, of the type discussed in Derviz, 2000.

Let g^j be the stream of coupons/dividends paid out by the basic security j, and h^j the same thing for the collateral security j. These values are unknown at the beginning of each period, when the investor makes the portfolio allocation decisions. Define y^j as the current yield on the basic security and z^j as the current yield on the collateral:

$$1+y^j_{t+1}=\frac{g^j_{t+1}+P^j_{t+1}}{P^j_t},\ 1+z^j_{t+1}=\frac{h^j_{t+1}+Z^j_{t+1}}{Z^j_t}.$$

i_{t+1} will denote the risk-free short rate between periods t and $t+1$. The period utility function of the investor is a function of two variables: current cash holdings and the dividend rate withdrawn after the investment strategy gains are realized: $u=u(x,\rho)$. The ρ-dependence is of the standard Inada form, and the x-dependence is strictly increasing and concave, with the limit $-\infty$ when x goes to $-\infty$, for every ρ. If the time preference rate of the investor is $\beta\in(0,1)$, the pricing kernel (stochastic discount factor, see Campbell et al., 1997, or Cochrane, 2001) is given by

$$\Lambda^{t+1}_t=\frac{\beta u_\rho(x_{t+1},\rho_{t+1})}{u_\rho(x_t,\rho_t)},\ \Lambda^\tau_t=\prod_{k=t}^{\tau}\Lambda^{k+1}_k,\ \tau>t,\ \Lambda^t_t=1$$

(subscripts by u denote partial derivatives).

The information available to the investor at time t consists of the trajectories of g, h, P and Z as well as the default event realizations b, all up to time t. The no-default up to time t subset of the event space for security j will be denoted by N^J_t. Let the S-dependent statistics of survival, $K^J_{t,\tau}$, between times t and $\tau\geq t+1$, be defined as

$$K_{t,\tau}^{j} = \prod_{k=t+1}^{\tau}\left(1-\pi_k^j\right).$$

Now we apply the standard asset pricing theory results. The optimal investor behavior implies the following asset pricing formulae (special cases of the discrete time consumption-based CAPM):

$$E_t\left[\Lambda_t^{t+1}(1+y_{t+1}^j)\right]=1,\ E_t\left[\Lambda_t^{t+1}(1+z_{t+1}^j)\right]=1,\ E_t\left[\Lambda_t^{t+1}\right](1+i_{t+1}) = 1-\frac{u_x(x_t,\rho_t)}{u_\rho(x_t,\rho_t)}, \tag{3}$$

$$Z_t^j = E_t\left[\sum_{\tau=t}^{\infty}\Lambda_t^\tau h_\tau^j\right], \tag{4}$$

$$V_t^j = Z_t^j + E_t\left[\sum_{\tau=t}^{T^j}\Lambda_t^\tau N_\tau^j\left(C_\tau^j - h_\tau^j\right)\right]. \tag{5}$$

In view of our assumptions about the default arrival independence of *S*, equation (5) can be rewritten in the form

$$V_t^j = Z_t^j + E_t^*\left[\sum_{\tau=t}^{T^j}K_{t,\tau+1}^j\Lambda_t^\tau\left(c_\tau^j - h_\tau^j\right)\right] - E_t^*\left[K_{t,T^j+1}^j\Lambda_t^{T^j+1}Z_{T^j+1}^j\right], \tag{6}$$

where now the conditional expectation E_t^* is taken only with respect to the market-wide risk factors. Thus, we have eliminated the firm-specific default events from the debt pricing formulae (5). Also note that the asset pricing equations (3) could be written with expectation E_t^* instead of E_t from the outset.

Now one can utilize the previously made market completeness assumption to note that the value of the individual loans and the loan portfolio as a whole can be calculated as soon as one reconstructs a formula for the pricing kernel Λ_t^τ from the pricing equations (3). Standard orthogonal basic decomposition methods known from numerical mathematics can be used to do this. Examples in the literature include Ait-Sahalia and Lo, 2000, Jackwerth, 2000, and Rosenberg and Engle, 2002. The necessary inputs are the price and yield processes for the basic assets and collateral assets, both available as market data.

8.3 Application to Economic Capital Calculations and Simulations

The model outlined above is likely to be free of certain deficiencies typical of the two standard ones, whose application to economic capital calculations was described in detail in Sections 5–7. For instance, a counterintuitive negative dependence of the capital requirement on the market interest rate is an unfortunate feature of the way CreditMetrics works with the relation between the debt value and the economic capital. That model does not have any link between interest rates and the firm's ability to repay the loan. On the contrary, our model is able to do so, because the net cash flow Y^j will usually be negatively related to the market rates of interest. Therefore, a reduction of the latter (such as a downward translation of the forward zero curve) increases Y^j and thereby reduces the default event rate.

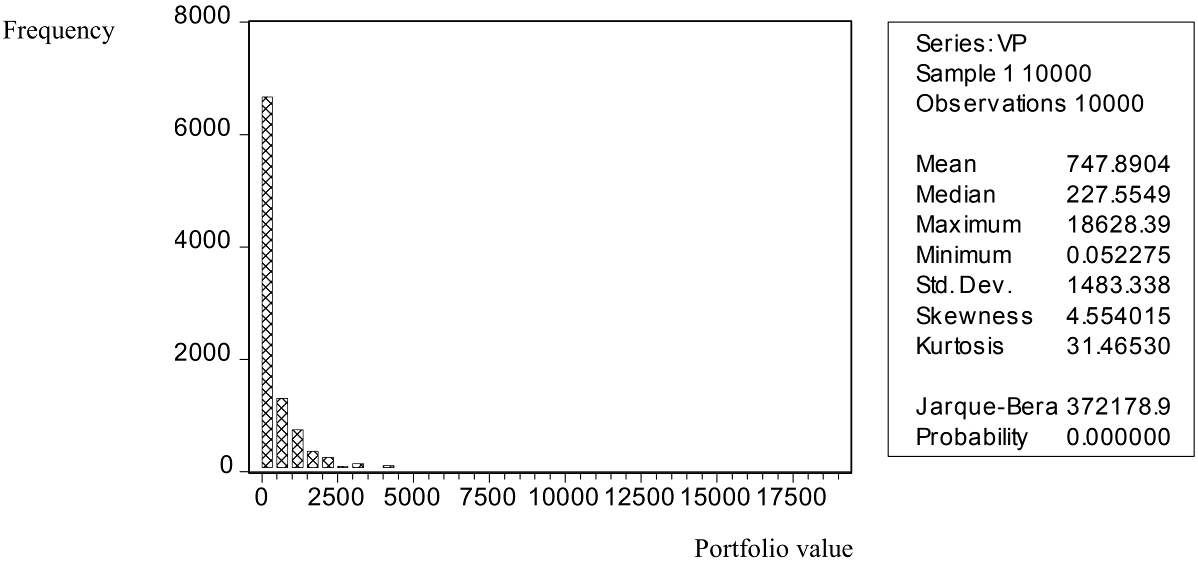
Another important advantage of the model is its ability to handle correlated defaults in a natural way. This is because the default correlation in the model is not an exogenously given property or an *ad hoc* assumption, but instead follows by construction from the dependence of the default rates on the common risk factors *S*.

A certain difficulty lies in the necessity to calculate the pricing kernel recursively for multi-period loan contracts. However, the calculations themselves are routine and are based on well-developed numerical techniques, allowing one to apply relatively standard software.

Once the distribution of the loan portfolio market price has been calculated, it can be used to derive the bank-internal measure of economic capital. The latter will subsequently be compared with the prudential capital values derived directly from the statistics of *S* and *b* according to regulatory principles. The difference will tell us the degree of discrepancy between internal risk measurement and regulatory mechanisms of risk-based capital allocation by the bank.

Details of the numerical procedures used to simulate our artificial portfolio value according to the presented model are given in Appendix A3.

Figure 6: Portfolio Value Arbitrage-Based (Risk-Neutral) Distribution According to the Pricing-Kernel Loan Valuation Model



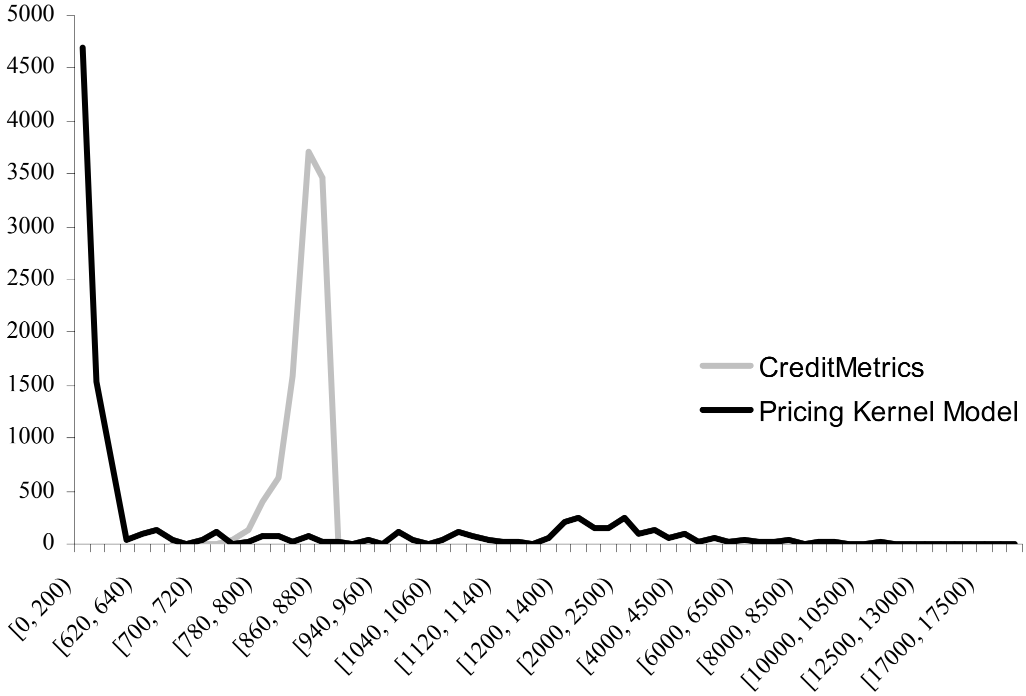
The corresponding economic capital requirements are given in Table 14.

Table 14: Capital Requirements According to the Pricing-Kernel Model

Riskless value of the portfolio	Mean	Economic capital
832.86	747.89	84.97

If one compares the above valuation with the one obtained by means of the CreditMetrics model (Section 5), the increase in the portfolio valuation coming from the pricing-kernel approach is substantial. It is illustrated in Figure 7 below.

Figure 7: Portfolio Value Distributions Based on the Monte Carlo Simulation of CreditMetrics and the Pricing-Kernel Model



The reasons for this difference are twofold. First, the CreditMetrics valuation uses a correlation structure taken over automatically from the representative equity prices of firms in the considered obligor classes. This correlation structure is necessarily inaccurate, particularly in reflecting the role of systemic risk factors (cf. the comment on the market interest rate impact in Section 6). Our model incorporates systemic factors directly, which means that macro-developments are transmitted directly into the whole portfolio and change its value in accordance with intuition. Second, we have used a somewhat different collateral valuation in the pricing-kernel model than in the CreditMetrics simulations. A collateralized loan in our understanding always pays out a recovery rate in case of default, even though this rate is subject to random fluctuations around the face value of the debt. On the contrary, our CreditMetrics exercise worked with an understanding that was close to the Czech regulatory one, i.e. certain collateral categories were given zero recovery rates. This happens even though the initial value of every item of collateral is equal to the face value of the loan. Altogether, the market collateralized debt valuation in terms of standard asset pricing theory is naturally biased toward reducing the differences between default and no-default realizations of uncertain factors (the risk-neutral probability of an asset value upside/downside shift is lower/higher than the actual probability). Therefore, this theory subsumes such asset prices that agents become nearly indifferent between holding loans and collateral. In complete asset markets, the indifference is perfect. Here, we are working with non-traded loans, which generate market incompleteness, but the pricing-kernel valuation would still alleviate the loan-collateral demand discrepancy to the extent that the default events and collateral values are dependent on common systemic risk factors.

The basic properties of the pricing kernel-based valuation can be summarized as follows.

A. Every loan value is dependent on the market attitude towards risk.
B. The arbitrage-free (i.e. financial market-based) loan value distribution gives a higher weight (prices higher) to recovery value payments of the collateral, and a lower one to the original risk adjusted-loan service payments, than the actual probability.
C. For each macroeconomic or other systemic risk factor, loans with default probability/cash flow negatively/positively correlated with this factor obtain a valuation increase from that factor. So, for instance, bank claims on exporters are valued higher when the exchange rate is increasing (the national currency is depreciating).

These properties cannot be obtained from the CreditMetrics- or CreditRisk+-based valuation, a fact which introduces considerable bias into their results. The KMV model probably works with the same phenomena as described in this section. However, according to existing public sources on that model, it does not overcome the structural vs. reduced-form modeling dilemma for private borrowers. This is one of the main original contributions of the present pricing kernel-based model.

9. Conclusion

The following table sums up the credit risk-related estimations of regulatory and economic capital in all the cases considered. The impact of lending rate variations (market risk) is partially reflected in some of the capital requirement estimations.

Table 15: Summary of Capital Estimations

	1%-percentile	5%-percentile	Mean	Capital requirement			
				99% Economic capital	95% Economic capital	Non VaR economic capital	Regulatory capital
CreditMetrics	767.90	796.62	845.78	77.89	49.16		
CreditRisk+ (Loss)	133	101	42.18	90.82	58.82		
Pricing-kernel model			747.8904			84.97	
Floating interest and fixed fzc	669.65	718.48	848.33	178.69	129.85		
Upward translation of fzc	655.15	705.70	834.25	179.11	128.56		
Downward translation of fzc	672.44	724.32	856.94	184.50	132.62		
Counter-clockwise (upward) rotation of fzc	668.83	722.92	839.14	170.31	116.22		
Clockwise (downward) rotation of fzc	668.83	722.92	853.01	184.18	130.09		
NBCA – Standardized approach (January 2001)							51.84
NBCA – IRB approach (January 2001)							165.46
NBCA – Standardized approach (October 2002)							46.90
NBCA – IRB approach (October 2002)							44.79

Accordingly, the main conclusions of this study can be summarized as follows.

- The capital requirements according to the January 2001 NBCA consultative document and October 2002 QIS 3 technical guidance differed significantly. The newer guidance was less demanding on the capital level and made the use of the IRB approach more convenient to the banks in the sense of the capital level required.

- The regulatory capital requirement resulting from the January 2001 NBCA guidance was much lower in the NBCA's standardized approach than in the IRB approach. Estimated capital was similar in the standardized approach and the modeling part when a 95% confidence level was selected in the models. Although at a 99% confidence level the models imposed higher capital requirements, they were still lower then the estimated capital in the IRB case of the regulatory requirements.

- Following the QIS 3 technical guidance from October 2002, the regulatory requirements of the two NBCA approaches were quite similar. The capital requirements according to both regulatory approaches were lower than the level of capital required by the credit risk models, but the difference was not significantly higher.

- When floating interest rates and changes in yield curves were modeled, the estimate of economic capital was generally higher. This result is intuitive, as increased uncertainty should generally impose higher levels of capital that banks must hold.

- The particular changes in the forward zero curves analyzed in this paper did not impose significantly different levels of economic capital than the case with stable forward zero curves. The case when forward zero rates went downward (both translation and clockwise rotation) required slightly more capital, and this situation persisted at all levels of confidence selected.

- The risky debt valuation obtained by conventional incomplete market asset pricing methods tended to generate higher loan values and reduce the economic capital requirements compared to the other regulatory and model-based risk measurement methods. Therefore, the regulator may encounter an effort on the part of the banks to treat different parts of the loans on their balances differently in terms of economic capital. The difference will go in the direction of reducing capital allocation (and specializing collateral requirements) in those segments of the loan portfolio which exhibit strong correlation with traded risks.

- Asset pricing methods of risk measurement may lead to a better recognition of the role of the business cycle and other systemic macroeconomic factors in economic capital determination. The pro-cyclical nature of many existing risk management procedures may therefore give way to natural (and desirable) counter-cyclical adjustments.

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Appendix

A.1 Generating the Unobservable Characteristics of the Artificial Bank Loan Portfolio

A1.1 Predictors of Default

We applied Moody's guidelines (see Section A.2 for a methodology outline) to obtain ratings for the firms present in our pool of Czech bank customers. Based on accounting data for 1997, we constructed financial ratios and tested their power in predicting default in 1998. In this sense, we performed a series of univariate probit regressions containing as dependent a variable taking the value 0 if a firm defaulted and 1 if a firm did not default between 1997 and 1998. We looked for candidates for the explanatory variables belonging to five out of the six categories proposed by the RiskCalc model: profitability, leverage, liquidity, size and activity[10]. The variables that were tested within each category are displayed in Table A1. Marked with a star are those variables that were found significant at least at the 90% confidence level in the univariate probit tests.

Table A1: Explanatory Variables of the Univariate Probit Regressions

Category	Variable
Profitability	- EBIT/Assets
	- Net Income/Equity
	- Net Income/Assets*
	- Operating Profit Margin
	- Total Debt/Total Assets*
Leverage	- *Debt Service Coverage Ratio*
	- Bank Debt Ratio*
	- Equity Ratio
Size	- Total Assets/PPI97
	- Sales/PPI97
Liquidity	- Quick Ratio*
	- Cash/Total Assets
	- Short Term Debt/Total Debt
	- Current Ratio*
	- Cash Position Ratio*
	- Net Indebtedness*
Activity	- Sales/Total Assets*
	- Inventories/COGS
	- Interest Expense/Sales

The only significant variable (at the 90%, but not at the 95%, confidence level) in the profitability category was Net Income/Assets. In the activity category we also found only one significant variable (Sales/Total Assets). In the leverage category two variables were significant: Bank Debt Ratio and Total Debt/Total Assets. The Bank Debt Ratio dominated the Total Debt/Total Assets variable, so the former was the only variable chosen in the leverage category. The majority of the variables in the liquidity category were significant. To select only one or two we applied the

[10] Measures of growth such as sales or asset growth were not used, because only slightly more than 300 firms were recorded in 1996 out of the 606 firms considered between 1997 and 1998. Thus, the use of lagged variables would have reduced the number of observations, restricting the available information about defaults and the degrees of freedom of the regression.

technique proposed by Moody's. Starting with the most significant one (Cash Position Ratio) we added liquidity-representative variables in this probit regression until they proved insignificant or had a counterintuitive sign. Actually, none of the remaining variables proved more powerful than the Cash Position Ratio, so this was the only variable chosen. Size variables showed no significance in relation to default, so we did not include any variable from this category.

Overall, the variables that were found significant in the simple probit tests were Net Income/Assets, Bank Debt Ratio, Sales/Total Assets and Cash Position Ratio. Consequently, we ran a probit regression using all these four explanatory variables. The outcome is displayed in Table A2.

Table A2: The Probit Regression Outcome Based on four Explanatory Variables

Variables (X_j^{1997})	**Coefficient** (c_j^{1997})	**t-statistics**
Constant	1.19	7.43
Net Income/Assets	0.006	1.06
Cash Position Ratio	0.83	2.62
Bank Debt Ratio	-0.46	-1.45
Sales/Total Assets	0.114	1.85

A1.2 Probability of Default Estimation

We used the 1997–1998 database as the reference data set. As our bank data set extended until 2000 and because we wanted to use the most recent information available, we assumed that the structural relationship between default and the variables given in Table 2 was stable over time and therefore applicable in 2000[11]. Based on this assumption it was possible to estimate the one-year probability of default for each firm in the data set for 2000 according to the relation:

$$P_i^{Default2000} = \Phi(-c_0^{1997} - \sum_{j=1}^{4} c_j^{1997} X_{ij}^{2000}).$$

Here Φ is the standard normal cumulative distribution function, *c*s are the estimated coefficients based on the 1997 data (see Table 2) and X_{ij} are the four financial ratios computed for each of the 1,035 firms present in the 2000 data set. These predictions of the annual default rates are further used to assign dot.PD-like ratings.

A1.3 Types of Collateral and Recovery Rates

The type of collateral was assigned randomly to the individual assets by drawing a random number from the distribution displayed in Table 7. The support of this distribution consists of numbers from 1 to 6. They codify the types of collateral accepted by Czech banks (1=bank

[11] Although this assumption is not completely out of the question, we had to rely on it due to the unavailability of more reliable default information in 2000.

guarantee, 2=cash, 3=securities, 4=commercial real estate, 5=other, 6=no collateral). The probabilities with which the choices were made mirrored the ratios of the volume of the individual types of collateral offered to the total volume of classified assets. To calculate these ratios, we used the information about the structure of classified assets of the Czech banks' portfolios at the end of 2000 as given by CNB sources. We assumed that each asset in our simulated portfolio was either fully covered by specific collateral or not covered at all.

Due to unavailability of own data, the recovery rates of the individual assets were computed, based on their collateral quality, as

Recovery rate = 100/(1 + He + Hc),

where He and Hc are the "haircut" coefficients corresponding, respectively, to the loan and the collateral volatility. They reflect subjective assessments that took into account the collateral quality (see Table A3 below). The computation was based on the NBCA methodology for setting the value of adjusted collateral (from January 2001, cf. Subsection 5.1).

Table A3: Collateral Type Distribution and Haircut Coefficients

Collateral	Type	Share	Coefficient
1.	bank guarantee	0.368	0.03
2.	cash	0.0125	0.0
3.	securities	0.006	0.06
4.	commercial real estate	0.183	0.2
5.	other	0.12	0.5
6.	no collateral	0.31	X

Source: *Own computation.*

A.2 RiskCalc™ for Private Companies: Moody's Default Model

Moody's RiskCalc™ model predicts private firms' default probabilities based on selected financial indicators. Unlike public firms, which are quoted and regularly traded on a stock exchange, private firms' securities are infrequently traded. This fact restricts the access to market information for these firms, implying that such information can be hardly incorporated into a default model. There are other differences between public and private firms that necessitate a special default model for private firms. As documented in Moody's proprietary databases, private firms typically have smaller size and higher retained earnings, and maintain more current debt and belong to the medium range in terms of asset size. These and other factors would cause an analysis of the default behavior based on financial statement data to produce disparate results when applied to public and private enterprises. Comparative studies performed by Moody's on private and public companies confirmed this outcome.

In devising, implementing and validating its default model for private firms, Moody's used three large databases:

- A proprietary database called Moody's Credit Research Database (CRD), containing accounting and default-related information on private, middle-size market firms.

- Moody's Default Database (MDD), containing information on US and Canadian public firms that have defaulted since 1980.
- This latter database was extended with financial data for non-defaulted public firms available in Compustat's Research Insight database.

At the time the technical document was published (May 2000) the content of these databases allowed one to obtain a reliable construct and evaluation of Moody's RiskCalc model. Some aspects relating to these data sets are displayed in Table 1.

Table A4: Characteristics of the Data Sets Used by Moody's

	Database Summary			
	Time Span	**Number of Firms**	**Number of Defaults**	**Number of Financial Statements**
Private Firms	1989–1999	24,718	1,621	115,351
Public Firms	1980–1999	15,805	1,529	130,019

Source: *Moody's RiskCalc™ for Private Companies.*

Moody's model for rating private firms has been also applied in a number of European countries: Belgium (2002), France (2002), Germany (2001), the Netherlands (2002), Portugal (2002), Spain (2002) and the United Kingdom (2002). Table 2 presents the main features of the data sets used in these countries. The last row contains similar information about the data set we used in the Czech Republic. Our data is poorer than the data sets used in other European countries. However, our goal was not to maintain the complexity and robustness of Moody's rating process, but to use its methodology to build our own test portfolio. For this objective, the data set we used was relevant. Recently, the Czech National Bank officially launched a Central Register of Loans, in cooperation with the Czech Banking Association. The register will enable banks to exchange information on loans provided to businesses. Commercial banks had been calling for the creation of a credit register for several years, but its instigation was not possible until the private data protection issue had been resolved.

Table A5: Characteristics of the Data Sets Used by Moody's in Europe and of the Data Set Used in the Case of the Czech Republic

Country	**Time Span**	**Number of Firms**	**Number of Defaults**	**Number of Financial Statements**
Belgium	1992–1998	102,594	6,658	523,057
France	1990–1999	253,268	25,229	1,323,754
Germany	1987–1999	24,866	1,485	111,427
Netherlands	1992–1999	19,327	436	79,696
Portugal	1993–2000	18,137	416	69,765
Spain	1992–1999	140,790	2,265	569,181
UK	1989–2000	64,531	4,723	283,511
Czech Republic	1997–1998	606	51	606

Source: *Moody's RiskCalc™ for Private Companies.*

Any default prediction analysis must start with a definition of default. Moody's formal treatment of a default event is triggered by any of the following changes in the loan status of a bank client:

- a loan is 90 days past due;
- *the credit is written down (it is placed in the regulatory classifications of substandard, doubtful or loss);*
- the loan is classified as non-accrual; or
- bankruptcy is declared.

Moody's rating methodology comprises the following basic steps:

- **Single Factor Analysis**: the aim is to select the most significant input variables for the subsequent analysis and to describe the shape of the relationship to default of each relevant factor.
- **Model Specification and Estimation**: those variables which were found to be powerful predictors of default in single factor tests are combined into multinomial logit or probit models. Typically, such models deliver scores and model-based probabilities of default for each bank client.
- **Mapping**: the estimated scores and default probabilities can be mapped into standard, alphanumeric Moody's ratings.

A2.1 Single Factor Analysis

Without a structural model dictating the choice of the explanatory variables to be included in the model, variable selection follows a forward selection process. The selection of the explanatory variables starts by constructing financial ratios representative of the following risk categories: profitability, leverage, firm size, liquidity, activity and growth. In each category the model selects the most powerful predictors of default based on univariate tests. Subsequently, it adds less significant variables to the model until they prove no additional predictive power in multivariate tests. As for the number of variables to be included in the model, Moody's has reached the conclusion that seven represents the optimal choice. Without inflicting this conclusion on any model application, Moody's warns against the tendency to overfit the model by including too many variables. While the inclusion of additional explanatory variables always increases the fit of the regression, the risk of multicolinearity is also present. Multicolinearity has negative consequences for the reliability of the estimated coefficients and deteriorates the out-of-sample performance of the model.

Moody's search for powerful predictors of default starts by defining the broad categories in which to look for candidate financial ratios. The impact that each of these categories is likely to have on default and the variables chosen by Moody's in each category are listed below.

- *Profitability:* normally, a firm that makes profits (that is, covers its own costs by own revenues) is able to honor its liability contracts and does not default. Thus, the higher the profitability of the firm the lower the default risk is expected to be. The RiskCalc model examined the following variables in this category: EBIT/Assets, Net Income/Equity, (Net Income - Extraordinary Items)/Assets and Operating Profit Margin[12].

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[12] EBIT = Earnings Before Interest and Taxes. Operating Profit Margin = (Sales - Cost of Goods Sold, General and Administrative Expenses) / Sales.

- *Leverage:* expresses the ability of the firm to pay debt from its own sources. More indebted firms pose a higher risk to lenders, so leverage is positively related to default. The variables considered by Moody's in this category were: Total Liabilities/Tangible Assets, Total Debt/Total Assets, Total Liabilities/Total Assets, Total Debt/Net Worth and Debt Service Coverage Ratio (EBIT/Interest).
- *Size:* bigger firms are usually better diversified (that is, are less subject to idiosyncratic risk) and hold a stronger market position (a feature used in underwriting). Normally, size is negatively related to default. The size variables considered were real sales or assets (nominal values divided by the CPI) and a measure of the market value of the firm's equity.
- *Liquidity:* measures a firm's ability to cover current (short-term) debt using its own current assets. While low liquidity may act as a signal of default (since default is the result of a lack of liquidity) it can also reflect active investment by the firm (cash and equivalents are thus quickly invested in other assets). For this reason, Moody's questioned whether liquidity predicts default in a timely manner, since signaling default one month in advance has less relevance for creditors than signaling it one or two years in advance. The variables included in this category were: Quick Ratio, Working Capital/Total Assets, Cash/Total Assets, Short Term Debt/Total Debt and Current Ratio.
- *Activity:* it is assumed that a firm with a higher turnover has a higher likelihood of paying its financial obligations. Thus, the variables in this category are expected to be negatively related to default. Among the variables included here one can mention the following: Accounts Receivable/COGS, Accounts Receivable/Sales, Sales/Total Assets, Inventory/COGS and Accounts Payable/COGS (COGS=cost of gold sold).
- *Growth:* the variables considered in this group are sales and asset growth over a one-year or two-year period. Growth is the only risk factor that displays a strong non-monotone relationship with default. While firms with low sales growth have bad financial prospects, firms that grow too quickly are also risky because such growth is often fueled by increased external debt. Even though the model builders recommend giving preference to monotone relationships, growth was found to be a significant predictor of default and was therefore included.

Variable selection proceeds on the basis of empirical tests assessing the statistical power of the individual variables. By statistical power, Moody's means the ability of individual financial ratios to distinguish defaulted firms from non-defaulted. Statistical power is examined on the basis of default frequency Fig.s and power curves.

Another aspect addressed within the single factor analysis is a search for those functional forms which best describe the nonlinear relationships between default and the individual financial indicators (which are in general nonlinear, especially in the case of powerful predictors of default. The transformations are not known a priori (exponential, polynomial, etc.). In handling this problem, Moody's turned to a nonparametric approach. Nonparametric regression extends the traditional regression technique by avoiding any special assumptions about the functional form of the regression curve and about the statistical distributions of the error terms. The shape of the regression curve is estimated simultaneously with the unknown coefficients. Nonparametric techniques are used to shape the individual relationships between financial ratios and future defaults. The result is a set of transformations that closely resemble the individual default frequency representations when displayed in a Fig.ical form. The transformed variables are then normalized and used as inputs in the multivariate probit or logit model.

A2.2 Model Specification and Estimation

RiskCalc combines the normalized transformed variables into the estimation of a multivariate probit or logit model. The dependent variable in such a model is a dichotomous variable Y, taking the values 0 and 1 in the case of default and non-default respectively.

The causality relationship between the dependent variable Y and the *n* explanatory variables X_j (normalized transformed financial ratios) is captured indirectly through a latent variable specification. Suppose that Y* is a latent variable whose rapport with Y is given by

$$Y_i = \begin{cases} 0, & \text{if } Y_i^* < 0 \\ 1, & \text{if } Y_i^* > 0 \end{cases}.$$

Y* is assumed linearly related to X_j as in

$$Y_i^* = c_0 + \sum_{j=1}^{n} c_j X_{i,j} + \varepsilon_i .$$

Here ε is the error term, assumed to follow a standard normal distribution in the probit model and a logistic distribution in the logit model.

The probability that firm *i* defaults given a certain realization of predictors $X_{i,j}$ can be expressed as

$$P(\text{Firm i defaults}) = P(Y_i = 0) = P\left(Y_i^* < 0 \mid X_{i,j}\right) = P\left(\varepsilon_i < -\left(c_0 + \sum_{j=1}^{n} c_j X_{i,j}\right)\right),$$

or

$$P(\text{Firm i defaults}) = F\left(-\left(c_0 + \sum_{j=1}^{n} c_j X_{i,j}\right)\right) \qquad \text{(A1)}$$

Here F is the cumulative distribution of the error terms, i.e. the cumulative standard normal distribution in the probit model (Φ in the usual notation) and $F(x) = \frac{e^x}{1+e^x}$ in the logit model.

The coefficients c_0, c_i, i=1,n are estimated using a maximum likelihood method. This method is commonly available in any statistical and econometric software package. Once the coefficients of the regression are estimated, the default probabilities for each firm are computed according to (A1).

The output of the multinomial probit or logit model is a set of raw estimates of the default probabilities for the firms in the sample. Based on look-up tables (which also have a proprietary character) these initial estimates are adjusted to produce Expected Default Frequency (EDF)-like default probabilities. These estimates are the final output of the RiskCalc used further to map firms into alphanumeric ratings.

A2.3 Mapping to Alphanumeric Ratings

Standard Moody's ratings, known under their alphanumeric representation as Aaa, Aa1,…, C, capture the default risk of long-term corporate bonds. In its bond default study[13] Moody's

[13] Available on the joint Moody's and KMV web site at ***http://www.moodyskmv.com.***

provides the theoretical background for computing historical average bond default rates for each rating category. By comparing a firm's predicted default probability with these rating class-specific bond default rates, Moody's constructs dot-PD ratings for private firms (Aaa.pd,…,C.pd). While not entirely equivalent, bond ratings and dot-PD ratings are related, since each represents a form of corporate debt. Mapping corporate default rates into bond ratings can prove useful in several circumstances, including credit risk assessments performed on bank loan portfolios. Input data in credit risk models, such as migration matrices and forward zero curves, are published regularly by rating agencies for bonds. However, there exist no analogues directly applicable to valuing bank loans granted to companies. Thus, bond-based variables can act as proxy variables in credit risk modeling conducted for private firms.

The main differences between rating long-term bonds and private firms are listed in Table A6. Rating bonds requires information in excess of default probabilities, primarily estimations of the loss given default (which depend on the characteristics of the bonds: secured versus unsecured, senior versus subordinate) and the issuer's migration risk to other rating classes. Rating long-term bonds also requires qualitative assessments that go beyond financial ratio analysis, e.g. assessment of the management. RiskCalc PDs are more volatile (reflecting changes in the financial ratios during the business cycle) and have a well-defined risk horizon (contrary to bond ratings, which are valid over a nonspecific time horizon).

Table A6: The Main Differences Between RiskCalc PDs and Moody's Long-Term Bond Ratings

Similarities & Differences Beteen RiskCalc Pds And Moody's Long-Term Bond Ratings		
Characteristics	**RiskCalc PDs**	**Moody's Long-Term Bond Ratings**
Unit of Study	Obligor	Obligation and/or Obligor
Time Horizon	Specific, one or five years	Non-specific, long term
Risk Dimension	One dimensional: Probability of default	Multi-dimensional: Probability of default, severity of default & transition risk
Information Requirements	Large, reliable, electronic datasets	Robust to poor quality or missing data
Volatility	High	Low- maintained through the cycle
Cost	Low	High
Support	Technical	Technical + Analyst contact & insight
Scale	Continuous/Abolute	21 Risk Buckets/Relative
Structure	Simple, codified analysis of few variables	Flexible as situation may requre

Source: *Moody's RiskCalc™ For Private Companies.*

For mapping purposes, RiskCalc uses standard Moody's default tables that contain 1-year and multi-year cumulative default rates characteristic of Moody's rating grades. The later are default rates unadjusted for withdrawals observed over the 1983–1999 period.

Moody's proposes several mapping procedures:

- Use the averages of the model output per Moody's grade: if the derivation sample contains firms already rated by Moody's, then one can derive an average default in each rating class. If the average default rate is 0.05% for Aaa-rated firms and 0.2% for Aa-rated firms, the cut-off probability value distinguishing these two rating classes is found between the 0.05% and 0.2%

values (the middle point, for example). Then, any firm in the prediction sample with an estimated default rate of less than 0.075% could be assigned to the Aaa rating class.

- Estimate an ordered probit model on the ratings themselves. This is a natural extension of the binary logit or probit model. In this case the dependent variable Y takes the values 0, 1,..., *n*, where *n* is the number of rating classes. The model implicitly estimates the cutoffs for various categories and the coefficients of the model. The output of the model is given by default rates for individual firms and average default rates in different rating classes.
- Impose the condition that ratings in the sample are in the same proportion as that observed within the entire Moody's rated universe. Thus, if 10% of all Moody's-rated firms are rated Aaa, then the 10% of firms with the lowest estimated default rates are mapped to Aaa. This procedure is repeated for all rating classes.
- Link the model's default rate predictions to ratings based on standard Moody's default rates by rating and horizon. This mapping procedure is similar to the first one, only that in this case Moody's ratings in the derivation sample are not required.

A3 Numerical Implementation of the Pricing-Kernel Model

Define by $L(s)>0$ the pricing-kernel exponent ($\Lambda=e^L$) as a function of the risk factor vector s. We represent L as

$$L(s) = h^0 + \sum_{\alpha} h_\alpha g^\alpha(s),$$

where h^0 is a constant and g^α are basis functions, such as orthogonal polynomials (Tchebyshev, Hermite, etc.) or other mutually orthogonal functions. The exact choice of basis g depends on the model for risk factor statistics. For the purposes of the present study, we are taking the risk factor space to be the product $\prod_{k=1}^{n}[-1,1]$ of n identical intervals between –1 and 1 (a subset of $\mathbf{R}^n$) and functions g^α to be indicator functions $s^k \mapsto \chi_{[\frac{m-3}{2},\frac{m-2}{2}]}(s^k)$, m=1,2,3,4, k=1,…,n. In other words, we are looking for a particular linear combination, for each k, of the four functions of the risk factor s^k that are identically 1 on a selected sub-interval $\left[\frac{m-3}{2},\frac{m-2}{2}\right]$ of length ½ inside [-1,1], m=1,2,3,4, and zero elsewhere, across the risk factors k. Altogether, the pricing kernel is given by

$$\Lambda_t^{t+1} = e^{h^0 + \sum_{k=1}^{n}\sum_{m=1}^{4} h_m^k \chi_{\left[\frac{m-3}{2},\frac{m-2}{2}\right]}(s^k)}.$$

The basic property of the pricing kernel can be formulated in terms of the current returns on the marketable assets as

$$E_t^*\left[\Lambda_t^{t+1}(1+y_{t+1}^j)\right] = 1,\ j=0,1,\ldots,N, \tag{A2}$$

(with y_{t+1}^0 equal to r^0, the risk-free 1Y market interest rate between years t and t+1). We will assume that $1+y_{t+1}^j = e^{w^j}$ for j>0, $1+r_{t+1}^0 = e^{i^0}$, and that the continuously accounted returns, w^j, are linear functions of the risk factors:

$$w = a_0 + As\,,\; w^j = a_0^j + \sum_{k=1}^{n} a_k^j s^k\,.$$

Note that, in accordance with this definition, $a_0^0 = i^0$.

We also assume that the density of s^k is a simple triangular function $s \mapsto \varphi(s)$ equal to 1+*s* on [-1,0], 1-*s* on [0,1] and zero elsewhere. Then equation (A2) can be rewritten as

$$J^j = e^{h^0} \prod_{k=1}^{n} I_k^j = e^{h^0 + a_0^j} \prod_{k=1}^{n} \int_{-1}^{1} e^{\sum_{m=1}^{4} h_m^k \chi_{\left[\frac{m-3}{2},\frac{m-2}{2}\right]}(s) + a_k^j s} \varphi(s) ds = 1\,. \tag{A3}$$

Let us define

$$q_1(a) = \frac{1}{a^2}\left(e^{-a} + \left[\frac{a}{2} - 1\right] e^{-\frac{a}{2}} \right),\; q_2(a) = \frac{1}{a^2}\left(a - 1 + \left[1 - \frac{a}{2}\right] e^{-\frac{a}{2}} \right),$$

$$q_3(a) = \frac{1}{a^2}\left(-1 - a + \left[1 + \frac{a}{2}\right] e^{\frac{a}{2}} \right),\; q_4(a) = \frac{1}{a^2}\left(-\left[1 + \frac{a}{2}\right] e^{\frac{a}{2}} + e^{a} \right),$$

$$q_{km}^j = \begin{cases} q_m(a_k^j), & j \ge 1 \\ 1/8, m = 1,4, \quad 3/8, m = 2,3, j = 0 \end{cases}.$$

Then it can be shown that

$$I_k^j = \sum_{m=1}^{4} q_{km}^j e^{h_m^k}\,. \tag{A4}$$

Our goal is to find coefficients h^0 and h_m^k, m=1,2,3,4, k=1,…,n, such that conditions (A2) are satisfied with maximum precision, e.g. by defining the quadratic error minimization criterion

$$M = \sum_{j=0}^{N} \left[J^j - 1\right]^2 = \sum_{j=0}^{N} \left[e^{h^0 + a_0^j} \prod_{k=1}^{n} I_k^j - 1 \right]^2 = \sum_{j=0}^{N} \left[e^{h^0 + a_0^j} \prod_{k=1}^{n} \sum_{m=1}^{4} q_{km}^j e^{h_m^k} - 1 \right]^2, \tag{A5}$$

to be minimized with respect to parameters h^0 and h_m^k. Formally, this objective can be treated as that of a non-linear regression problem, with a^j being observations of the *n*-dimensional independent variables, replicas of unity (for all *j*) – the dependent variable observations, and h_m^k – the estimated coefficients.

A properly chosen minimization procedure should result in the choice of parameters h_m^k and h^0 that makes the latter parameter a function of the former, such that the term in (A5) corresponding to j=0 is equal to zero exactly. Indeed, for any choice of h_m^k, one can achieve zero error for j=0 by setting

$$H = E_t\left[e^{\sum_{k=1}^{n}\sum_{m=1}^{4} h_m^k \chi_{\left[\frac{m-3}{2},\frac{m-2}{2}\right]}(S^k)}\right] = \prod_{k=1}^{n}\int_{-1}^{1} e^{\sum_{m=1}^{4} h_m^k \chi_{\left[\frac{m-3}{2},\frac{m-2}{2}\right]}(s)}\varphi(s)ds\,,\ h^0 = -i^0 - \log H\,, \qquad \text{(A6)}$$

always reducing the overall quadratic error magnitude in (A5).

As soon as these coefficients are found, the estimated pricing kernel Λ can be used to calculate the collateralized loan portfolio value in (6), Section 8. For instance, loan j maturing in year $T^j>t$ and having the residual service schedule $\left(C_{t+k}^{T^j}\right)_{k=1}^{T^j-t}$ (recall that $C_{T^j}^{T^j} = c_{T^j}^{T^j} + F^j$ with small c being coupons and F the face value) has the time t-value

$$V_t^j = E_t^*\left[\sum_{k=1}^{T^j}\Lambda_t^{t+k}K_{t,t+k}^j C_{t+k}^j + \sum_{k=1}^{T^j}\Lambda_t^{t+k}K_{t,t+k-1}^j \pi_{t+k}^j h_{t+k}^j\right]. \qquad \text{(A7)}$$

All terms inside the conditional expectation operator in (A7) are known functions of risk factors s. The pricing kernels $\Lambda_t^{t+\tau}$ for different maturity years $t+\tau$ will have the same set of coefficients h_m^k (we assume a stationary distribution of risk factor variables S^k under annual evaluation frequency). Only constants h^0 are different, reflecting different discounting of individual future periods.

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