The hybrid-epistemic model of quantum mechanics and the possible solution to the measurement problem

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2017

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Abstract

In this study we introduce and describe in details the hybrid-epistemic model for quantum mechanics. The main differences with respect to the standard model are following: (1) the measurement process is considered as an internal process inside quantum mechanics, i.e. it does not make a part of axioms and (2) the process of the observation of the state of the individual measuring system is introduced into axioms.
The intrinsic measurement process is described in two variants (simplified and generalized). Our model contains hybrid, epistemic and hybrid-epistemic systems. Each hybrid system contains a unique orthogonal base composed from homogeneous (i.e. ontic) states.
We show that in our model the measurement problem is consistently solvable. Our model represents the rational compromise between the Bohr's view (the ontic model) and the Einstein's view (the epistemic model).

1. Introduction

The basic open problem in quantum theory is the question what is the meaning of the quantum state. The solution of this problem is the key element to the solution of the measurement problem in quantum mechanics.

Standard quantum mechanics can be considered as a list of rules how to calculate probabilities of observed events. The main argument for this assertion is the fact that quantum mechanics treats the measurement process formally as a process external to quantum mechanics.

We do not use the concept of an interpretation of quantum mechanics since this concept is not exactly defined. We prefer the concept of a model of quantum mechanics since it means the theory whose empirical predictions agree with the empirical predictions of quantum mechanics (for example the Bohmian mechanics can be considered as a model of quantum mechanics). A model for quantum mechanics should give the clear meaning to the concept of a quantum state.

The solution of the measurement problem requires, at the first place, the change of the role of the measurement process. The measurement process cannot be the external rule defining the quantum mechanics but it must be described as a one of possible processes inside quantum mechanics.

Our aim is to define the hybrid-epistemic model of quantum mechanics and to show that in this model the measurement problem can be solved. Our hybrid-epistemic model is considered in the contrast to the well-known ontic and epistemic models.

Let us consider a pure state described by the wave function $\psi \in \mathbf{H}_S$, where $\mathbf{H}_S$ is the Hilbert space of the system S. The wave function ψ describes the ontic state if ψ is the complete description of the state of the individual system. Otherwise ψ describes the epistemic state which is considered as a state of the incomplete knowledge. In general, the incomplete knowledge means that ψ describes the state of an ensemble in which different elements may be in different individual states.

This ontic – epistemic difference in the possible meaning of ψ is the starting point of our considerations. We start from the assumption that all ψ's are epistemic, i.e. each ψ describes the state of some ensemble. (There are arguments that this – purely epistemic – view is not correct. These arguments will be considered in detail below.)

The opposite assumption that all ψ's are ontic (the standard quantum mechanics) can be also considered as not correct - mainly since the measurement problem cannot be solved in this setting.

Our approach is based on the assumption that some ψ's are epistemic and some ψ's are ontic and we shall call this model as the hybrid.model.

The hybrid-epistemic model lies between the epistemic model and the ontic model and it contains some features from both models.

In the hybrid-epistemic model all systems can be divided into two groups. The first group contains hybrid and hybrid-epistemic systems while the second group contains epistemic systems. Thus our model can be considered as a dualistic model in the sense of [12]. In dualistic models the main concept usually is the concept of the macroscopic system (as in the Copenhagen quantum mechanics). It is difficult to define what is a macroscopic system while our two groups mentioned above are consistently defined.

The problem of the superposition principle (mentioned in [12]) is consistently solved in our model by the following two assertions

- The superposition principle holds in our model
- The non-trivial superposition of two homogeneous states is a state which is not homogeneous

The main difference between the hybrid-epistemic model and the standard model of quantum mechanics lies in the following feature (we assume that the measuring system is a hybrid system while the measured system is an epistemic system).

In the typical measurement situation where the measuring system M measures the measured system S, the measured system is in a non-homogeneous **collective** state while the measuring system is in the homogeneous state, i.e. it describes the **individual** state of the measuring instrument. (This corresponds to the simple measurement model – see sect. 4 below.)

Thus the main feature (discovered in [1]) consists in the situation where the measuring system is in the **individual** state while the measured system is in the **collective** state.

The assumption that the individual state of the measuring systems **implies** the individual state of the measured system is the **main argument** in the standard argumentation. We consider this argument as not correct. This is the **main difference** between the standard model and the hybrid-epistemic model: the measuring system is in our model in the **homogeneous collective** state while the measured system is in the **non-homogeneous collective** state).

Our solution of the measurement problem is a result of a long study of the inner structure of quantum mechanics (see [8], [1], [6], [3]). The measurement problem of quantum mechanics is at least 22 years old (see [2]), probably 85 years old (see [10]).

2. Ontic, epistemic and hybrid states and systems

At the beginning we define clearly the state space. Let us consider the system S and its Hilbert space $\mathbf{H}_S$. The state space of pure states is defined as a set of rays by

$$\mathbf{P}_S = \{ [\psi] \mid \psi \in \mathbf{H}_S , |\psi| > 0 \} , \quad \text{where } [\psi] = \{ a\psi \mid a \in \mathbf{C} , |a| > 0 \} , |\psi| > 0 .$$

We shall consider (for the simplicity) only systems with the finite dimensional Hilbert space and we shall suppose that all state spaces used below will have dimension greater or equal to two.

We start from the assumption that all ψ's are epistemic, i.e. that each ψ describes a possible state of some ensemble. We shall assume that for some systems some ψ's may be ontic but we shall not make, at the moment, no hypotheses on the existence of them. Instead of the concept of an ontic state we shall use the concept of the homogeneous ensemble (see von Neumann [10]).

Definition.
Let **E** be an ensemble in the state ψ . We shall say that the state of the ensemble **E** is **homogeneous**, when all members of this ensemble are in the same individual state. In other words, ψ is homogeneous if and only if ψ can be considered as an individual state of any member from the ensemble **E**.

One can immediately see that the concept of the individual state (i.e. the state of an individual system) is used only through the above definition of the homogeneous state. In the rest of the paper there is no other use of the concept of an individual state.

This means that if the epistemic state ψ is homogeneous then it can be identified with some ontic state. Thus ψ's are divided into two groups: non-homogeneous (or epistemic) ψ's and homogeneous (or ontic) ψ's . We introduce the appropriate classification for systems.

Definition.

(i) The system S is called the **epistemic** system if ψ is epistemic and not homogeneous for any $[\psi] \in \mathbf{P}_S$.

(ii) The system S is called the **hybrid** system if there exists an orthogonal base $[\psi_1]$, … , $[\psi_n]$ in the space $\mathbf{P}_S$ such that the state $[\psi]$ is homogeneous if and only if $[\psi]$ is a member of this orthogonal base. We shall denote this orthogonal base by hom ($\mathbf{P}_S$).

(iii) The system T is called the **hybrid-epistemic** system if there exists a decomposition of this system $T = M \oplus S$ into two parts where M is hybrid and S is epistemic. M is called the hybrid component of T and S is called the epistemic component of T .

(iv) The system S is called an **ontic** system if ψ is homogeneous for each $[\psi] \in \mathbf{P}_S$, i.e. if hom ($\mathbf{P}_S$) = $\mathbf{P}_S$.

We remark (for the completeness) that states $[\psi_1]$, … , $[\psi_n]$ from the space $\mathbf{P}_S$ are orthogonal if and only if ψ_1 , … , ψ_n are orthogonal in $\mathbf{H}_S$ and $\{ [\psi_1] , \dots , [\psi_n] \}$ is a base in $\mathbf{P}_S$ if and only if $\{ \psi_1 , \dots , \psi_n \}$ is a base in $\mathbf{H}_S$.

Note that the non-trivial superposition of two homogeneous states of a hybrid system is not a homogeneous state. This means that no superposition is possible inside hom ($\mathbf{P}_S$).

The superposition principle holds, only the non-trivial superposition of two homogeneous states is not a homogeneous state.

Definition.
The model of quantum mechanics is a theory which is empirically equivalent to quantum mechanics, i.e. the theory which produces the same set of empirical predictions as quantum mechanics (this is for example the Bohmian mechanics).

Definition.

Let **M** be a model of quantum mechanics.

(i) **M** is a hybrid-epistemic model if each system in **M** is either epistemic, hybrid or hybrid-epistemic and if there exists at least one hybrid system and at least one epistemic system in **M.**

(ii) **M** is an epistemic model if each system in **M** is epistemic

(iii) **M** is a hybrid model if each system in **M** is hybrid

(iv) **M** is an ontic model if each system in **M** is ontic.

We can see that ontic systems can exist only in the ontic model. Moreover, we can state that

- The ontic model describes the standard (textbook) quantum mechanics.
- The epistemic model corresponds to the so-called statistical interpretation of quantum mechanics (preferred by Einstein, for example).
- The hybrid model corresponds to the modified quantum mechanics introduced and studied in [1], [3], [6].
- The hybrid-epistemic model is the model introduced here.
- There are also the Bohmian model, collapse models and many-world models of quantum mechanics which change the structure of quantum mechanics more seriously.

For $|\psi| > 0$ we shall identify $[\psi]$ with the1-dimensional subspace of $\mathbf{H}_S$ defined as a union of $[\psi]$ with $\{0\}$ and then we denote by $P([\psi])$ the orthogonal projection from $\mathbf{H}_S$ onto $[\psi]$.

For the decomposition of T into two systems, $T = M \oplus S$, we have $\mathbf{H}_T = \mathbf{H}_M \otimes \mathbf{H}_S$. If $[\psi] \in \mathbf{P}_M$ is the state of M and $[\phi] \in \mathbf{P}_S$ is the state of S then the state of the system $M \oplus S$ will be $[\psi \otimes \phi]$ (assuming that systems M and S are independent) since $[a\psi \otimes b\phi] = [(ab)\, \psi \otimes \phi] = ab\, [\psi \otimes \phi]$ for each $a, b \in C$ satisfying $|a|, |b| > 0$.

Let S be a hybrid system. Then to each state $[\psi] \in \mathbf{P}_S$ there is an associated 1-dimensional subspace and there is a corresponding orthogonal projection $P([\psi])$ in $\mathbf{H}_S$.

If $M \oplus S$ is a hybrid-epistemic system and M is its hybrid component and if $[\psi] \in \mathbf{P}_M$ then there is a corresponding projection defined by $P([\psi]) \otimes \mathrm{Id}(\mathbf{H}_S)$ where $\mathrm{Id}(\mathbf{H}_S)$ is the identity map in $\mathbf{H}_S$.

Let $T = M \oplus S$ and $W = U \oplus V$ be two hybrid-epistemic systems where M is the hybrid part of T and U is the hybrid part of W . Then we have the decomposition

$$T \oplus W = (M \oplus U) \oplus (S \oplus V) .$$

Here $M \oplus U$ is the hybrid part of $T \oplus W$ and $S \oplus V$ is the epistemic part of $T \oplus W$. Moreover we have $\hom(M \oplus U) = \hom(M) \times \hom(U)$.

Now we can define the concept of the measuring system.

Definition.

(i) Every hybrid system is a simple measuring system

(ii) Every hybrid-epistemic system is a general measuring system

The role of measuring systems will be clarified later in this paper.

3. Axioms for the hybrid-epistemic model of quantum mechanics

We shall modify standard axioms for quantum mechanics with respect to our setting. There will be new axioms concerning the observation process.

The solution of the measurement problem must contain two basic ingredients

- The measurement process must be an internal process in quantum mechanics
- Instead of the measurement concept the axiomatic description must contain the concept of an observation of the state of the individual measuring system.

Axiom 1.
To each system there corresponds (for the simplicity) the finite dimensional complex Hilbert space of states. It is assumed that any system is either an epistemic system, a hybrid system or a mixed system. This means that each system S can be written as sum of the hybrid system (the hybrid part of S) and the epistemic system (the epistemic part of S) and the hybrid part resp. the epistemic part can also be trivial.

Axiom 2.
Let $T = M \oplus S$ and $W = U \oplus V$ be two hybrid-epistemic systems together with their decompositions into hybrid and epistemic subsystems. Then

$$\mathbf{H}_{T\oplus W} = \mathbf{H}_T \otimes \mathbf{H}_W\ ,\quad \mathbf{H}_{M\otimes U} = \mathbf{H}_M \otimes \mathbf{H}_U\ ,\quad \mathbf{H}_{S\oplus V} = \mathbf{H}_S \otimes \mathbf{H}_V \quad \text{and}$$

$$\mathbf{H}_{T\oplus W} = \mathbf{H}_{M\otimes U} \otimes \mathbf{H}_{S\oplus V}\ .$$

This Axiom covers also the situation when some of considered systems are trivial.

Axiom 3.
For each system S there exists the unitary group $\{ U_t \mid t \in \mathbf{R} \}$ in the space $\mathbf{H}_S$ such that the state vector $\psi(t)$ of S evolves by the standard rule $\psi(t) = U_t\, \psi(0)$. This unitary group is generated by the Hamiltonian of the system S .

After having mentioned the standard axioms we shall postulate the main axioms concerning the process of an observation. The idea is that the relation between the state of an ensemble (the epistemic state of the measured system) can be related to the state of the individual measuring system only if the ensemble is homogeneous. This is related to the fact that in the measurement process it is possible to observe only the state of the individual measuring system. Thus the state of the measuring system must be homogeneous i.e. from hom (M).

There is an important fact that two (different) homogeneous states of the hybrid system are necessarily orthogonal.

There is a well-known von Neumann's infinite regress: M_1 measures S, then M_2 measures $M_1 \oplus S$, M_3 measures $M_2 \oplus (M_1 \oplus S)$, M_4 measures $M_3 \oplus (M_2 \oplus (M_1 \oplus S))$ and so on to infinity. Observation of the homogeneous state of the individual state of the individual measuring system stops the von Neumann's infinite regress.

In our theory it is not true that all systems are the same: there are epistemic systems, there are hybrid systems and there are hybrid-epistemic systems. Intuitively, epistemic systems are the measured systems, while hybrid systems are measuring systems.

There is another feature of our theory which consists in the fact that there are no axioms on measurement – measurement is considered as a process inside quantum mechanics, like other processes (this is described in following sections). But, on the other side, there is introduced the process of the observation of the individual state of the individual measurement apparatus.

Axiom 4. (Observability)
Let $A = \{t_1, \ldots t_s\}$ be a finite set of times, $t_1 < \ldots < t_s$, and let M be the hybrid system.

(i) The states $[\psi(t_1)]$ **, … ,** $[\psi(t_s)]$ of the individual hybrid system M at times $t_1 < \ldots < t_s$ can be observed. Each individual state belongs to the hom (M), i.e. it is a homogeneous state.

(ii) Let $T = M \oplus S$ be the hybrid-epistemic system where M is its hybrid subsystem. Then states $[\psi(t_1)]$ **, … ,** $[\psi(t_s)]$ of the hybrid subsystem M at times $t_1 < \ldots < t_s$ can be observed and these states belong to hom (M).

The observability is postulated only for hybrid (or hybrid-epistemic) systems. In general, up to now, there may not exist any hybrid systems (i.e. that only epistemic systems exist). We have to suppose that there are some hybrid systems.

Axiom 5.
For each n = 2, 3, … there exists at least one hybrid-epistemic system with the dimension of its hybrid part equal to n.

Let us now consider the situation where we have the hybrid-epistemic system $T = M \oplus S$ where M is its hybrid subsystem. Let $\{ [\psi_1], \ldots, [\psi_n] \}$ be the set hom (P_M) – i.e. the ontic (homogeneous) base of $\mathbf{H}_M$. To each $[\psi_i]$ there corresponds the projection $P([\psi_i])$ in the space $\mathbf{H}_M$, i = 1, … , n and then the projection

$$P([\psi_i]) \otimes \mathrm{Id}(\mathbf{H}_S)$$

in the space $\mathbf{H}_T = \mathbf{H}_M \otimes \mathbf{H}_S$, where $\mathrm{Id}(\mathbf{H}_S)$ denotes the identity map in the space $\mathbf{H}_S$.

Let the system $T = M \oplus S$ is in the state Ψ. Let us assume that we have observed the individual system M and we found it in the state $[\psi_i]$.

Axiom 6. (Born's rule)
Assume that the hybrid-epistemic system $T = M \oplus S$ is in the state $[\Psi] \in \mathbf{P}_T$.
The probability to find the individual subsystem M in the state $[\psi_i]$ is equal to

$$\mathrm{prob}\,([\psi_i]) = \| (P([\psi_i]) \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi) \|^2 .$$

Axiom 7. (the up-dating rule)
During the observation of the state of the individual system M the new information on the state of T is obtained and this requires to up-date the original state $[\Psi]$ to a new state

$$[\Psi'] = (\mathrm{prob}\,([\psi_i]))^{-1/2}\,[(P([\psi_i]) \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi)] \;,$$

assuming that the individual subsystem M is observed to be in the state $[\psi_i]$.

This rule is in the standard quantum mechanics considered as a collapse (i.e. the sudden unpredictable change) of the state of the individual system. In our model this rule is considered as a change of the ensemble (and, as a consequence, the change of the state of an ensemble) based on the acquiring a new information.

The state $[\Psi]$ is the state of an ensemble of systems T's. In this ensemble the individual states of M may vary. But after the observation of the individual state $[\psi_i]$ we know that the state $[\Psi]$ must change in such a way that the state of the subsystem M will be $[\psi_i]$.

4. The simplified model of the measurement process

Now we shall describe the internal model of the measurement process. This model is analogical to the well-known von Neumann's model [10]. We shall consider the hybrid-epistemic system $T = M \oplus S$ where M is its hybrid subsystem. Intuitively, S is the measured system and M is the measuring system.

We shall assume that spaces $\mathbf{H}_M$ and $\mathbf{H}_S$ have the same dimension equal to $n \geq 2$.

The measurement process starts with the specification of the orthogonal base in the space $\mathbf{H}_S$. Let $\phi_0, \dots, \phi_{n-1}$ be the fixed (freely chosen) orthogonal base of the space $\mathbf{H}_S$. This base is the parametrization of the measurement process (in the standard language the measurement is parametrized by the observable A such that $\phi_0, \dots, \phi_{n-1}$ is the set of eigenvectors of A).

Let $\{[\psi_0], \dots, [\psi_{n-1}]\}$ be the set hom $(\mathbf{P}_M)$ – i.e. the homogeneous base of $\mathbf{H}_M$.

We shall define the unitary map U in the space $\mathbf{H}_M \otimes \mathbf{H}_S$ by the prescription

$$U(\psi_i \otimes \phi_j) = \psi_{i\oplus j} \otimes \phi_j, \quad \text{where } i, j = 0, \dots, n-1$$

and where $i \oplus j = i+j$ if $i+j \leq n-1$, $i \oplus j = i+j-n$ if $i+j \geq n$[1]) .

Clearly, the map U is the standard "entangling" map since it transforms $\psi_0 \otimes \phi_j$ onto $\psi_j \otimes \phi_j$.

If the measuring system M is in the state $[\psi_0]$ and the measured system S is in the state $\Phi = \sum b_j \phi_j$ then the initial state $\Psi = \psi_0 \otimes \Phi$ is transformed by U into the state

$$\Psi' = \sum b_j \psi_j \otimes \phi_j \qquad \text{since} \quad U(\psi_0 \otimes \phi_j) = \psi_j \otimes \phi_j .$$

The internal measurement process is realized in two steps:

(i) The initial state $\Psi = \psi_0 \otimes \Phi$ of the system $T = M \oplus S$ is transformed to the state Ψ'

[1]) It is clear that the map U depends also on the order of sets $\{\phi_0, \dots, \phi_{n-1}\}$ and $\{[\psi_0], \dots, [\psi_{n-1}]\}$. we shall not study this dependence in more detail.

(ii) One finds what is the homogeneous state of the individual measuring system M and this implies the up-dating of the state Ψ' . Ψ' is up-dated to the state Ψ'' which is obtained using the Axiom 7. We obtain (assuming that we have observed that the individual measuring system S is in the state $[\psi_k]$ for some k)

$$\Psi'' = (\text{prob}([\psi_k]))^{-1/2} (P([\psi_k]) \otimes \text{Id}(\mathbf{H}_S)) (\Psi') = (\text{prob}([\psi_k]))^{-1/2} b_k \psi_k \otimes \phi_k$$

The probability that the state $[\psi_k]$ will be obtained is given by Axiom 6

$$\text{prob}([\psi_k]) = \| (P([\psi_k]) \otimes \text{Id}(\mathbf{H}_S)) (\Psi') \|^2 = |b_k|^2 .$$

The final result of the measurement process will be the state $\Psi'' = [\psi_k \otimes \phi_k]$ of the composed system which can be decomposed into the state $[\psi_k]$ of M and the state $[\phi_k]$ of S . This happens with the probability prob $([\psi_k])$.

5. The general model of the measurement process

The typical simplified measurement is the case of a spin system where $n = 2$. This means that the measured system's state space and also the measuring system's space are two-dimensional. But in reality the measuring system is usually the macroscopic system with a large number of degrees of freedom. In this section we describe a model for such macroscopic measuring systems.

We shall assume that the measuring system can be decomposed into two parts $M \oplus N$ where M is its hybrid part and N is its epistemic part. The measured system will be denoted S . The complete system under consideration is $M \oplus N \oplus S$ - M is its hybrid part and $N \oplus S$ is its epistemic part. . We shall assume that the dimension of M and S are same and equal to $n \geq 2$.

Now we shall construct the "entangling" map U . We assume that there are maps V^{ij} , i, j = 0, .. , n-1 in the space $\mathbf{H}_N$. Maps V^{ij} are assumed to be unitary, otherwise they are arbitrary. We choose an arbitrary orthogonal base $\{ e_1 , \dots , e_l \}$, $l = \dim(\mathbf{H}_N)$ in the space $\mathbf{H}_S$. We assume that there is the ontic base $\psi_0 , \dots , \psi_{n-1}$ in the space $\mathbf{H}_M$ and a given orthogonal base $\phi_0, \dots , \phi_{n-1}$ in the space $\mathbf{H}_S$. The set

$$\{ \psi_i \otimes e_m \otimes \phi_j \mid i, j = 0, \dots , n-1 , \ m = 1, \dots l \}$$

is the orthogonal base in the space $\mathbf{H}_M \otimes \mathbf{H}_N \otimes \mathbf{H}_S$. The map U is defined by

$$U (\psi_i \otimes e_m \otimes \phi_j) = \psi_{i\oplus j} \otimes V^{ij} (e_m) \otimes \phi_j \quad \text{where } i, j = 0, \dots , n-1 , \ m = 1, \dots l .$$

It can be simply proved that this map is unitary. It is clear that $\| U (\psi_i \otimes e_m \otimes \phi_j) \| = 1$. Let us consider two orthogonal vectors $\psi_i \otimes e_m \otimes \phi_j$ and $\psi_{i'} \otimes e_{m'} \otimes \phi_{j'}$., if j and j' are different then images of these vectors under U are orthogonal and we can assume that j = j'. If i and i' are different then $i\oplus j$ and $i'\oplus j$ are different and this implies that images under U will be orthogonal. We can suppose that also i = i' and then the statement is proved since V^{ij} is unitary.

The initial state of the system $N \oplus S$ will be $\Phi = \sum b_{mj}\, e_m \otimes \phi_j$ and the initial state of the system M is supposed to be ψ_0 . Thus the initial state of the full system will be $\Psi = \psi_0 \otimes \Phi = \sum b_{mj}\, \psi_0 \otimes e_m \otimes \phi_j$. We obtain

$$\Psi' = U(\Psi) = \sum b_{mj}\, \psi_j \otimes V^{0j}(e_m) \otimes \phi_j \ .$$

The up-dating map (projection + renormalization) will be (assuming that the state $[\psi_k]$ of M was observed)

$$\Psi'' = (\mathrm{prob}([\psi_k]))^{-1/2}\ (P([\psi_k]) \otimes \mathrm{Id}(\mathbf{H}_N) \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi')$$

and in the explicit form

$$\Psi'' = (\mathrm{prob}([\psi_k]))^{-1/2} \sum b_{mk}\, \psi_k \otimes V^{0k}(e_m) \otimes \phi_k$$

where

$$\mathrm{prob}([\psi_k]) = \| (P([\psi_k]) \otimes \mathrm{Id}(\mathbf{H}_N) \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi') \|^2 \ .$$

This model of the measurement process can accommodate the idea of the macroscopic measurement apparatus.

6. The analysis of the measurement problem

The standard definition of the measurement problem is provided by Maudlin ([2], p. 7) by the consideration of the following three statements:

(M1) The wave-function of a system is a complete description of the state, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.

(M2) The wave-function always evolves in accord with a linear dynamical equation, i.e. the linearity of quantum mechanics.

(M3) Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up).

Any two of these propositions are consistent with one another, but the conjunction of all three of them is inconsistent. This can be easily illustrated by means of Schrödinger's cat paradox (see also Esfeld [11]).

In this paper we shall consider (M2) as true, i.e. we shall not consider the non-linear collapse theories.

We shall consider an individual measurement act as an observation of the individual state of the measuring system. (The result of this observation naturally requires the up-dating of the state of the measuring and measured systems.) In fact, only in the case of an ensemble in the homogeneous state there exists a relation between the state of an ensemble and the individual state of an individual system which is a member of this ensemble.

All this implies that we consider the assumption (M3) as valid. Thus our assumptions are that (M2) and (M3) are both valid.

Thus the assumption (M1) cannot be valid. But there are many ways in which this assumption can be invalid. Nevertheless there exists a certain way (the hybrid-epistemic model proposed here) which is consistent also with other natural requirements.

It is clear that the standard (textbook) model of quantum mechanics can be identified with the ontic model in which each pure state is homogeneous (and each system is ontic) and thus (M1) is true. Thus the standard model cannot solve the measurement problem and this is one of the principal disadvantages of this model. The ontic model is the extreme case from our point of view.

The second extreme possibility is the epistemic model, where there are no homogeneous states at all. (It is well-known that Einstein was the first to propose the idea that the wave function has only the statistical meaning – i.e. that ψ always describes only the state of an ensemble.) The principal disadvantage of this epistemic model is that it does not satisfy (M3) – there are no individual states of the individual measuring system in the epistemic model.

Tim Maudlin calls this model the ensemble interpretation and he says (in [2, p.10]): "According to that interpretation, the wave-function is not intended to describe *individual systems* but only collections of systems … ". We have called this model the epistemic model.

Maudlin rejects this model by the following argument: "And since we are interested in individual cats and detectors and electrons, since it is a plain *physical fact* that some individual cats are alive and some dead, some individual detectors point to 'UP' and some to 'DOWN', a *complete* physics, which is able at least to *describe* and *represent* these physical facts, must have more to it than ensemble wave-functions. If the wave-function does not completely describe the physical states of individual cats we should seek a new physics which does".

In our hybrid-epistemic model cats and detectors are described as hybrid systems so that they have individual states (i.e. states of individual systems). In this case observed states are homogeneous, i.e. describe individual systems. (In fact, cats and detectors can be, in general, described by hybrid-epistemic systems – see the section 5 – but it does not change the presented argument.) On the other hand electrons should be described by epistemic systems but this does not create problems since we do not want describe states of individual electrons (Maudlin incorrectly requires this but it is the consequence of his hidden adherence to the ontic model).

The third possibility, which is already (almost) consistent is the hybrid model. The hybrid model assumes that every system is hybrid - i.e. that for each system there is defined an (exactly one) orthogonal base of homogeneous states in his Hilbert space. This assumption is consistent with the axioms of quantum mechanics and it was the starting point of our first papers in this direction (see the concept of the modified quantum mechanics in [1] and then in [3], [6], [8]) [2]).

There is a disadvantage of this model (but this disadvantage does not create a direct inconsistency) consisting in the fact that its assumption is, in some sense, unnatural. For example this assumes that for the standard 2-dimensional spin system there should exist a ("preferred") orthogonal base in the system's Hilbert space and this contradicts to the idea of the isotropy of the physical space. The hybrid model means that for each system there is a "preferred" base of its individual states. In the case of the spin system this seems to be unnatural. It is also clear that in this case one must to suppose that only for some systems individual states can be observed (see [3]).

The forth possibility is our hybrid-epistemic model. This model assumes that there are some hybrid systems, some epistemic systems and some hybrid-epistemic systems. (The possibility of hybrid-epistemic systems is the consequence of the fact that we can always create the composed systems.)

The hybrid-epistemic model is able to solve the measurement problem:

- The assumption (M1) is not valid so that the conclusion of the inconsistency of (M1) - (M3) – i.e. the measurement problem, is not relevant.
- In the hybrid-epistemic model all experimental predictions are equal to the experimental predictions of the standard quantum mechanics. This statement follows from the axioms of the hybrid-epistemic model and it was proved in all details in ([1]).
- In the hybrid-epistemic model the measurement problem can be solved by describing the measurement process as an internal process in quantum mechanics (see preceding sections 4 and 5).

Thus we can assert that the proposed hybrid-epistemic model solves the measurement problem and that this model is empirically equivalent to the standard quantum mechanics

At the moment (see [11], [12]) there are following possible solutions to the measurement problem [3])

- Dualism as in orthodox Copenhagen quantum mechanics
- The non-linear collapse models
- The Bohmian mechanics
- The many world theories
- The hybrid model for quantum mechanics (see [3]) [4])
- The hybrid-epistemic model for quantum mechanics presented here [5])

Let us consider these models from the point of view how they are far from the standard quantum mechanics.

- The Copenhagen quantum mechanics has many well-known internal problems
- It is clear that the Collapse model containing the non-linear evolution goes properly against the basic principle of linearity of quantum mechanics.

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[2]) The true origin of the modified quantum mechanics lies in the non-standard probability theory (see [8]).
[3]) We do not consider the many-world interpretation as a possible solution to the measurement problem since there is no consistent definition of the probability in this model
[4]) The hybrid model is the special case of the hybrid-epistemic model
[5]) The hybrid-epistemic model allows the existence of properly quantum systems like a spin

- The Bohmian mechanics introduces additional variables (positions of particles) and in this way it is rather far from the spirit of quantum mechanics. Bohmian mechanics, if expressed in variables (ψ, $\mathbf{x}$) where ψ denotes the wave function of the system and $\mathbf{x}$ denotes positions of particles, is not, in fact linear, since the superposition of such states cannot be done.
- The many world models have own problems with the interpretation of the probability
- The hybrid model (see [3]) is consistent and empirically equivalent to the standard quantum mechanics, but it suffers (at least) from problems with isotropy of the physical space. Nevertheless this model is a special case of the hybrid-epistemic model.
- The hybrid-epistemic model is rather close (in fact, maximally closed) to the epistemic model (which alone is not admissible) but there is a part of this model which solves problems of the epistemic model (the existence of hybrid-epistemic systems). Principle of superposition is valid in the hybrid-epistemic model but the non-trivial superposition of two homogeneous states is not more homogeneous. Moreover this model is a reasonable compromise between the standard Bohr's attitude (the ontic model) and the Einstein's idea (the epistemic model).

As the result of this discussion we can state that the proposed hybrid-epistemic model offers the solution to the measurement problem which is closest to the standard quantum mechanics.

Our model has a feature which is very closed to the well-known Bohr's view: Bohr always required that in quantum mechanics there is a need of classical measuring apparatuses and that quantum objects can be seen only through their interactions with the classical apparatuses. The classical apparatus was seen a priori as a macroscopic object.

In our hybrid-epistemic model there is an analogy: systems are divided into hybrid (or hybrid-epistemic) systems and epistemic systems, where former play the role of measuring systems and letter play the role of measured systems. The hybrid (or hybrid-epistemic) systems have the necessary properties to be considered as measuring systems while they need not have any property of being macroscopic systems. Bohr's idea was often criticized that the definition of the "macroscopic" object is impossible. But our model is immune to this type of criticism since the concept of a macroscopic object is not used in our model. In this way we have solved the problem of the definition of the concept of "macroscopic system" – one has to use the concept of hybrid (or hybrid-epistemic) system instead of the concept of the "macroscopic" system. These systems are well-defined in our hybrid-epistemic model so that our model does not suffer from the criticism based on the non-definiteness of the concept of the "macroscopic" system.

Let us consider the differences between the hybrid model and the hybrid-epistemic model in more details. In a sense, the hybrid model is the special case of the hybrid-epistemic model (in fact, there is no axiom requiring the existence of some epistemic systems in our model).

But the possibility of the existence of epistemic systems implies some important facts:

- Epistemic systems are, in some sense, purely quantum systems, which have no classical analogs (like the spin systems).
- Individual states of epistemic systems have no sense – they do not exist in the model.
- States of epistemic systems can be analyzed in the measurement processes (see above) but the measurement processes require the existence of hybrid (or hybrid-epistemic) systems.

- Thus the states of epistemic systems can be seen only through the interaction with the hybrid (or hybrid-epistemic) systems
- The isotropic spin system must be either epistemic or ontic [6]). This implies that in the hybrid-epistemic model the isotropic spin system must be epistemic.
- This also implies that in the hybrid model of quantum mechanics there are no isotropic spin systems. This is a considerable disadvantage of the hybrid model.

Thus our keys to knowledge of the reality of quantum systems are:

- The existence of hybrid (or hybrid-epistemic) systems
- The possibility of interactions between epistemic systems and hybrid (hybrid-epistemic) systems.

The unique information obtainable from the quantum world is by observing the hybrid-epistemic systems (as measuring apparatuses where the individual state of an individual measuring system can be observed) in the interaction with the measured epistemic systems.

In each case, the hybrid-epistemic model is an important generalization of the hybrid model (the model originally proposed as a modified the quantum mechanics – see [6]) and it is more realistic and closer to the standard quantum mechanics.

We would like to remark that our hybrid-epistemic model allows the existence of systems which have no homogeneous states, i.e. no individual states – all states are collective states. These systems (epistemic systems in our terminology) are "purely" epistemic – there are no individual states of them. This means that the concept of the state of an individual system is not applicable to them. In this sense the epistemic systems are pure quantum systems without any classical analog (the example is the spin). The state of the individual epistemic system cannot be observed.

In a certain sense the concept of the hybrid-epistemic system is a two-level compromise. The first compromise is the concept of the hybrid system which is the compromise between the ontic system and the epistemic system. And then the second compromise is constructed: the hybrid-epistemic system is the composition of the hybrid system and the epistemic system. In this sense our concept of a system could be described as (ontic-epistemic)-epistemic.

In each case our concept of a quantum state can be seen as a compromise between Bohr's (ontic) and Einstein's (epistemic) views.

7. Conclusions

In this paper we have constructed the hybrid-epistemic model of quantum mechanics (we have given its axiomatic formulation). We have shown that the empirical predictions of this model are identical to the empirical predictions of the standard quantum mechanics. We have shown that in this model it is possible to solve the measurement problem of quantum mechanics.

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[6]) If there exists a state which is ontic then using the transitivity of the orthogonal group of the physical space we obtain that many states must be ontic, so that too many systems must be ontic.

Our solution of the measurement problem is based on the careful and detailed analysis of the concept of a quantum state.

In the contrast to the ontic and epistemic models where the meaning of the wave function is either ontic or epistemic, in our model some wave functions are ontic and other wave functions are epistemic (this is the spirit of the concept of the hybrid system). The hybrid system is such a system which has a unique orthogonal base composed of ontic states and all other states are epistemic.

Then we consider also systems composed from the hybrid and epistemic parts. In our model we consider hybrid, epistemic and hybrid-epistemic systems and there are no ontic systems, so that the assumption (M1) is not satisfied.

We postulate that the individual state of the hybrid system (in the given time) can be observed (Axiom 4 above). The hybrid (or hybrid-epistemic) systems are our window into the quantum world – this is what can be seen (and what only can be seen and known).

But using this instrument we are able to measure the state of the epistemic system as well. But the result of such measurement will give only the **collective** state of an ensemble of systems. To associate the concept of the state with the individual epistemic system is not possible since the epistemic system **has no individual states** (no homogeneous states). The concept of the state of an individual epistemic system is a non-sense (in our hybrid-epistemic model of quantum mechanics).

It is shown that in the hybrid-epistemic model of quantum mechanics it is possible to solve the measurement problem of quantum mechanics. The solution is presented in two sections of this paper (the simple measurement model and the general measurement model). The solution of the measurement problem is based on two necessary ingredients

- The measurement process must be the internal process in the theory
- The observation process must be introduced into the axioms of the model.

We have shown that there are (at the moment) four possible ways how to solve the measurement problem in quantum mechanics: the non-linear Collapse model, the Bohmian mechanics, the hybrid model and the hybrid-epistemic model. We think that the hybrid-epistemic model described here is the most realistic model and that this model is closest model to the spirit of quantum mechanics.

Our model contains moreover the original Bohr's idea that (proper) quantum systems are not directly accessible and that they are accessible only through the measurement process. In fact, epistemic systems have no individual states (so that the individual state of an epistemic system cannot be observed) and their collective states are accessible only in the measurement process involving some hybrid (or hybrid-epistemic) system as a measuring apparatus.

In the hybrid-epistemic model of quantum mechanics systems are not all of the same type. We have three types of systems: hybrid ones, epistemic ones and hybrid-epistemic ones (in fact, measured systems are usually epistemic, while measuring systems are usually hybrid-epistemic). The hybrid systems (and more generally hybrid-epistemic systems) have some classical features – they have individual states (as, for example, the "pointer states" of measuring systems). The epistemic systems have no classical features, they are properly quantum. And the hybrid-epistemic systems are compositions of these two.

The ideas of the presented approach to the measurement problem are direct consequences of the approach to the quantum mechanics presented in [1] which is based on the ideas of the non-standard probability theory presented in [8].

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