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Abstract

In this paper we describe a new axiomatization of quantum mechanics (QM) in which we replace the concept of the measurement by the concept of the observation. We shall describe and discuss this axiomatization in details. We show that new axioms are clear and evident and based on the common sense. Our approach is based on the idea of the observation of the individual state of the individual measuring system. We distinguish two concepts of a state: the individual state of the individual measuring system and the state of an ensemble of systems. Then we **prove** that the ontic model of QM (where the wave function describes the state of the individual system) is inconsistent. This is our main result. It implies that the "standard von Neumann's text-book QM" is inconsistent. This implies the necessity to consider "non-realistic" models for QM. Moreover we show that the proofs of Bell's theorem and of quantum nonlocality are not valid in these "non-realistic" models for QM. This implies that there is no valid proof of the quantum nonlocality.

1. **Introduction.**

Our starting point is the fact that certain form of observability must be present in any reasonable theory. Without possibility to observe something, the theory cannot be tested and cannot be related to the reality.

The main idea in this paper is the axiomatization of the concept of an observation. We give a new axiomatization of quantum mechanics (QM) based on the concept of an observation instead of the axiomatization based on the concept of the measurement.

We consider the minimal requirement of observability in the following form: it is possible to observe in which individual state the individual measuring system is (at a given moment of time). Observability means to observe the individual state of an individual measuring apparatus.

In each probability theory there is a duality between two concepts: the states of an ensemble and the states of an individual system. For example let us consider the dice. The space of individual states is $\Omega = \{1,2,3,4,5,6\}$ while the space of possible states of ensembles is the set of all probability distributions on Ω. Similarly for Brownian particle the state of an individual particle is the point in the physical space while the state of an ensemble is the probability distribution on the physical space. For QM this is analogical: there is an individual state $i \in \mathrm{Ind}\,(S)$ of the measuring system and the state $\psi \in \mathbf{H}_S$ of the ensemble.

Our axiomatization of QM (it can be considered also as the observational formulation of QM) is based on this simple idea: **what is important is to observe the individual state of the individual measuring apparatus.**

On the other hand, the concept of a measurement is not mentioned among our axioms, i.e. in the definition of QM. The description of a measurement is a completely intrinsic question inside of QM (see [4]).

In the next section we shall present the axiomatization of QM based on these ideas: the main result is this new axiomatization of QM based on the concept of an observation.

It is not reasonable to expect that states of any system could be observed. Thus we introduce the set of systems which we shall called observable systems for which their individual states can be observed.

In this way our axiomatization is a completely general approach to the ontology of quantum systems.

The main consequences of this new axiomatization of QM are:

(i) the inconsistency of the ontic model of QM,
(ii) the Bell's theorem is unproven
(iii) the quantum non-locality is unproven
(iv) the hybrid-epistemic model of QM (see [4]) is the unique consistent and reasonable model for QM

We consider our axiomatization of QM as a true objective axiomatization of QM which is clearly evident and corresponds to the practical use of QM.

In sect. 2 we present in details the new axiomatic formulation of QM (Axioms 1 – 9). In sect. 3 we prove our basic theorems and discuss their consequences. In sect. 4 we describe the possible models of QM and their interrelations. In sect. 5 we show that proofs of Bell's theorem and of quantum non-locality are not valid. In sect. 6 we discuss the principle of superposition and solve the "Schrodinger cat paradox" . In sect 7 we describe the solution of the measurement problem (see [5]). In sect. 8 we discuss out results. In sect. 9 we summarize our results. Then in Appendix A we present the simple form of our main argument.

In this paper we continue the study from [3], [4], [7], [8], [9], [10].

2. A new observational axiomatization of quantum mechanics.

At first we state the three standard axioms which are common in all axiomatizations of QM.

Axiom 1.
To each system S there corresponds (for the simplicity) the finite dimensional complex Hilbert space of states $H_S = H(S)$.

At the beginning it is necessary to define clearly the state space. Let us consider the system S and its Hilbert space $\mathbf{H}_S$. The state space of pure states is defined as a set of rays by

$$\mathbf{P}_S = \{ [\psi] \mid \psi \in \mathbf{H}_S , \| \psi \| = 1 \} , \quad \text{where } [\psi] = \{ a\psi \mid a \in \mathbf{C} , |a| = 1 \} , \| \psi \| = 1 .$$

We shall suppose that all state spaces used below will have dimension greater or equal to two.

It is assumed that the state $[\psi] \in \mathbf{P}_S$, $\| \psi \| = 1$ is the state of an ensemble of systems prepared in certain way. We shall consider always the state $[\psi] \in \mathbf{P}_S$ as a state of certain ensemble **E** of systems.

Axiom 2.
Let M and S be two systems, then there exists a composite system $T = M \oplus S$ for which we have $\mathbf{H}_T = \mathbf{H}_M \otimes \mathbf{H}_S$.

Axiom 3.
For each system S there exists the unitary group $\{ U_t \mid t \in \mathbf{R} \}$ in the space $\mathbf{H}_S$ such that the state vector $\psi(t)$ of S evolves along the standard rule $\psi(t) = U_t\, \psi(0)$. This unitary group is generated by the Hamiltonian of the system S .

We continue with new axioms describing the concept of an observation.

Definition. Observable system is such a system that its individual state (at a given moment of time) can be found by an observation of the system.

Axiom 4.

(i) For each observable system S there is defined the set of its possible individual states

$$\mathrm{Ind}\,(S) = \{i_1, i_2, \; \dots , i_n\} \; . \;\; n > 1 \; .$$

(here $n \leq \dim H_S$ and typically $n \ll \dim H_S$)

(ii) For each individual observable system S it is possible to observe (in a given moment of time) its individual state

$$\mathrm{ISt}\,(S) \in \mathrm{Ind}\,(S) \, .$$

The fact that certain individual state ISt (S) is observed is considered as a random event.

(iii) Let us consider an ensemble $\mathbf{E} = \{S_1, S_2, \; \dots , S_N\}$, $N \to \infty$, of systems prepared in a same way. Then we assume that $\mathrm{Ind}\,(S_1) = \dots = \mathrm{Ind}\,(S_N)$. We shall also assume that $H(S_1) \approx \; \dots \approx \; H(S_N)$. (Here $\approx$ denotes the isomorphism.)

(iv) We shall assume that for each S and for each $i \in \mathrm{Ind}\,(S)$ and for each $\varepsilon > 0$ there exists $\psi \in H_S$ and there exists an ensemble **E** in the state ψ such that $S \in \mathbf{E}$ and $|\{R \in \mathbf{E} \mid \mathrm{ISt}(R) = i\}| > (1\text{-}\varepsilon)\, N$. This means that the value i is not superfluous and it will have the positive probability for some $\psi \in H_S$ (this is a rather technical assumption).

Definition.

Let us consider the ensemble $\mathbf{E} = \{S_1, S_2, \; \dots , S_N\}$, N large, and let the state of **E** be $[\psi] \in \mathbf{P}_S$. In this way we obtain (by making an observation) the sequence of individual states

$$\mathrm{ISt}\,(S_1), \mathrm{ISt}\,(S_2), \; \dots , \mathrm{ISt}\,(S_N) \;\; \in \mathrm{Ind}\,(S) \, .$$

We can define a new ensembles $\mathbf{E}^{(i)}$, $i \in \mathrm{Ind}\,(S)$ in the following way:

$$\mathbf{E}^{(i)} = \{\, S \in \mathbf{E} \mid \mathrm{ISt}\,(S) = i \,\} \;\; ,$$

Then we define the relative frequency, i.e. the probability as

$$p\,(\, i \mid [\psi], \mathbf{E}\,) = N^{-1} \mid \mathbf{E}^{(i)} \mid , \quad i \;\in \mathrm{Ind}\,(S)^{1} \;\; .$$

(Here $|\,.\,|$ denotes the number of elements.)

It means that to every $[\psi]$ and **E** there corresponds a probability distribution $p\,(\,.\mid \psi, E\,)$ on the set Ind (S).

[1] The stabilization of the relative frequency is automatically assumed.

We shall assume that the resulting probability distribution on Ind (S) depends only on the state $[\psi]$ of an ensemble **E**, i.e.

Axiom 5.
For each ψ and **E** the probability distribution $p(\,.\,|\,\psi, \mathbf{E})$ depends only on $[\psi]$, i.e.
$p(\,.\,|\,[\psi], \mathbf{E}) = p(\,.\,|\,[\psi])$, for each **E** in the state $[\psi]$.

Definition.
Deterministic probability distributions on Ind (S) are distributions $p^{(1)}, \dots, p^{(n)}$, where

$$p^{(i)}(i) = 1 \text{ and } p^{(i)}(j) = 0 \quad \text{if } j \neq i \quad , i \in \text{Ind (S)}.$$

The probability distribution p on Ind (S) is deterministic, if there exists $i \in$ Ind (S) such that $p = p^{(i)}$.

Definition.
For each $i \in$ Ind (S) we define the set

$$L^{(i)} = \{ \psi \in H_S \mid p(j \mid [\psi]) = 0 \text{ , for each } j \neq i \text{ , } j \in \text{Ind (S)} \} \ .$$

We shall call any such $L^{(i)}$ the homogeneous subspace of H_S . The set of all homogeneous subspaces will be denoted by Hom (S) .
The state $[\psi]$ is a homogeneous state if $\psi \in L^{(i)}$ for some $i \in$ Ind (S) . The set of all homogeneous states is denoted hom (S).

If $[\psi]$ is a homogeneous state, then $p(\,.\,|\,[\psi])$ is a deterministic probability distribution on Ind (S) .

Axiom 6. We shall assume that $p(i \mid [\psi])$ depends quadratically on ψ (this is generally true in QM – see the Born's rule).

Proposition 1.
Let $j \in$ Ind (S) and let us assume that there exist an operator K^j , such that

$$p(j \mid [\psi]) = (\psi \mid K^j \psi) \text{ , for each } \psi \in H_S \text{ , } \|\psi\| = 1.$$

Denote

$$N^j = \{ \psi \in H_S \mid p(j \mid [\psi] = (\psi \mid K^j \psi) = 0 \} \ .$$

Then N^j is a linear subspace of H_S.

Proof.

(i) We show that the operator K^j is positive. For $\psi \in H_S$, $\|\psi\| = 1$ the positivity of $(\psi \mid K^j \psi)$ follows from the definition since $p(j \mid [\psi]) \geq 0$. For the vector $a\psi$, $a \in \mathbf{C}$, we have $(a\psi \mid K^j a\psi) = |a|^2 (\psi \mid K^j \psi) \geq 0$.

(ii) The spectral decomposition of K^j is $K^j = \Sigma_s \lambda_s \phi_s \otimes \phi_s^*$ where $\lambda_s > 0$ for each $s = 1, \dots, k \le n$. We obtain

$(\psi| K^j \psi) = \Sigma_s \lambda_s (\psi| \phi_s \otimes \phi_s^* \psi) = \Sigma_s \lambda_s (\psi | \phi_s) (\phi_s | \psi) = \Sigma_s \lambda_s |(\phi_s | \psi)|^2$.

Then $N^j = \{ \psi \mid (\phi_s | \psi) = 0$ for each $s = 1, \dots, k \}$ since $\lambda_s > 0$. (The end of proof.)

Proposition 2.
The set $L^{(i)}$ is a linear subspace of H_S for each $i \in \mathrm{Ind}(S)$.

Proof.
We have $L^{(i)} = \cap \{ N^j \mid j \neq i \}$.

Definition.
We define that $P^{(i)}$ is the orthogonal projection from H_S onto $L^{(i)}$, $i \in \mathrm{Ind}(S)$.

Clearly, $\psi \in L^{(i)}$ if and only if $P^{(i)}(\psi) = \psi$.

Thus we have a 1-1 map

$$i \in \mathrm{Ind}(S) \rightarrow L^{(i)} \in \mathrm{Hom}(S) \ .$$

The rest of axioms are standard: Born's rule, the update rule and the existence of observable systems.

Axiom 7. (Born's rule.)

For each state $[\psi] \in \mathbf{P}_S$ and for each individual state $i \in \mathrm{Ind}(S)$ the Born's rule holds

$$p (i \mid [\psi]) = \| P^{(i)} (\psi) \|^2 \ .$$

Axiom 8. (The update rule for ensembles and states.)
Let $\mathbf{E}$ be an ensemble in the state $[\psi]$ and let the observable system S be an element of $\mathbf{E}$. After the observation we found that the individual state of S is $i \in \mathrm{Ind}(S)$. Then the system S must be considered as an element of the ensemble $\mathbf{E}^{(i)}$. Thus the ensemble $\mathbf{E}$ to which S belongs must be updated to an ensemble $\mathbf{E}^{(i)}$ as a result of acquiring the new information on the individual state of S. Thus the pertaining of S into some ensemble is updated in the following way

$$S \in \mathbf{E} \ \rightarrow \ S \in \mathbf{E}^{(\mathrm{ISt}(S))} \ .$$

The change of the state of S , i. e. the update of the state of ensemble is

$$[\psi] \rightarrow \ [\ \| P^{(i)} (\psi) \|^{-1} P^{(i)} (\psi) \ ] \ , \quad \|\psi\| = 1.$$

Axiom 9. (The existence of observable systems – an evident requirement.)
For each $n \geq 2$ there exists at least one observable system satisfying $| \mathrm{Ind}(S) | = n$.

3. **Basic theorems.**

Axioms we have formulated above have important consequences. At first we shall formulate and prove theorems and their consequences and then we shall discuss their meaning.

The following Theorem 1 is the **most important consequence** of our axioms.

Theorem 1,
If $i, j \in \mathrm{Ind}\,(S)$ and $i \neq j$ then homogeneous subspaces $L^{(i)}$ and $L^{(j)}$ are orthogonal.

Proof.
At first we shall show that $P^{(j)} P^{(i)} = 0$.
Let $\psi \in L^{(i)}$, i.e. $P^{(i)}(\psi) = \psi$, $||\psi|| = 1$. Then $p\,(\,.\,|\,[\psi]\,) = p^{(i)}$, so that $p\,(\,j\,|\,[\psi]\,) = p^{(i)}\,(j) = 0$. Then using the Born's rule we obtain $0 = p\,(\,j\,|\,[\psi]\,) = ||\,P^{(j)}\,(\psi)\,||^2$, so that $P^{(j)}\,(\psi) = 0$. Then $P^{(j)} P^{(i)}(\psi) = P^{(j)}\,(\psi) = 0$.
Let ψ be orthogonal to $L^{(i)}$. Then $P^{(i)}\,(\psi) = 0$ by the definition of $P^{(i)}$. Thus we have $P^{(j)} P^{(i)}(\psi) = 0$. Together we obtain that $P^{(j)} P^{(i)} = 0$.
Let $\psi \in L^{(i)}$ and $\phi \in L^{(j)}$, $||\psi|| = ||\phi|| = 1$. We have $P^{(i)}\,(\psi) = \psi$ and $P^{(j)}\,(\phi) = \phi$ and then

$$(\phi\,|\,\psi\,) = (\,P^{(j)}\,(\phi)\,|\,P^{(i)}\,(\psi)\,) = (\,\phi\,|\,P^{(j)}\,P^{(i)}(\psi)\,) = (\phi\,|\,0\,) = 0\,. \quad \text{(The end of the proof.)}$$

Thus in our axiomatization of QM any two homogeneous states coming from distinct homogeneous subspaces must be orthogonal. Let us remark that there must exist at least two distinct (and at least one-dimensional) $L^{(i)}$'s.

In his important monograph [1] von Neumann explicitly states that each ensemble in the pure state is homogeneous. This is the basis of the standard QM in the formulation that each $[\psi] \in \mathbf{P}_S$ can be a state of an individual system.

Theorem 1 implies that this von Neumann's postulate is **inconsistent** with QM. This means that the "standard von Neumann's text-book QM" is inconsistent! But there is no problem with QM since there are others possible models of QM, where all calculations and practical results of QM are the same as in the standard QM.

Theorem 2.
Assume that Ind (S) contains at least two elements. Then the set Hom (S) of all homogeneous subspaces creates the complete orthogonal decomposition of the Hilbert space H_S .

Proof.
We have already shown the orthogonality of homogeneous subspaces. Let us assume that Hom(S) is not a complete orthogonal decomposition of H_S . Then there exists a state $[\psi] \in \mathbf{P}_S$

which is orthogonal to all elements of Hom (S) , i.e. to all homogeneous subspaces. Using Axiom 7 (Born's rule) we obtain

$$p(i \mid [\psi]) = \| P^{(i)}(\psi) \|^2 = 0, \text{ for each } i \in \mathrm{Ind}(S) .$$

This means that the function $p(\,.\mid[\psi])$ is trivially equal to 0 and this is impossible since this function is a probability distribution. Thus Hom (S) must be a complete orthogonal decomposition of H_S .

4. Models of QM and their evaluation from the point of view of our axiomatic model of QM.

At first we shall describe the list of available models of QM (in this paper we shall systematically exclude hidden variables models).

Definition.
A theory T is a model of QM if all empirical consequences of QM are also empirical consequences of T.

Proposition 3.
Our axiomatization of QM is a model of QM.

Proof.
Clearly every calculation or a demonstration from the standard QM can be reproduced on our axiomatic formulation of QM since all essential axioms of QM are reproduced in our axiomatic theory of QM. Section 4, 5, and 6 from [4] contain detailed explicit description of possible measurement processes and also a discussion of these processes. The main assumptions: the state space, the unitary evolution, the Born's formula, the projection postulate etc. are the same in both theories.

Definition.

(i) A system S is an ontic system, if Ind (S) = P_S [2].
(ii) A system S is an epistemic system, if Ind (S) = 0 .
(iii) A system S is hybrid-epistemic system if Ind (S) ≥ 2 and Hom (S) is an orthogonal decomposition of H_S
(iv) A system S is a hybrid system, if S is hybrid-epistemic system and each homogeneous subspace $L^{(i)}$ is one-dimensional. (evidently, each hybrid system is also a hybrid-epistemic system.)

A typical hybrid –epistemic system S is the composite of the hybrid system M and the epistemic system E , $S = M \oplus E$, $\mathbf{H}_S = \mathbf{H}_M \otimes \mathbf{H}_E$.

[2] Here, of course, the number of elements of Ind(S) is infinite, but the essence of our argument go through independently of this fact.

Definition.

(i) A theory T is an ontic model of QM if each system is an ontic systems
(ii) A theory T is an epistemic model of QM if each system is an epistemic systems
(iii) A theory T is a hybrid model of QM if it contains only hybrid systems
(iv) A theory T is a hybrid-epistemic model of QM if it contains systems which are either hybrid-epistemic or epistemic.

Theorems 1 and 2 from the preceding section have important consequences.

Proposition 3. There cannot exist ontic systems, i.e. any ontic model is inconsistent (Theorem 1 asserts that any two homogeneous states belonging to distinct homogeneous subspaces must be orthogonal)

(i) The purely epistemic model is not acceptable since it contains no possibility of any observation
(ii) The hybrid model is acceptable.
(iii) Each observable system must be hybrid or hybrid-epistemic (Theorem 2 asserts that Hom (S) is an orthogonal decomposition of H_S, if all homogeneous subspaces are one-dimensional then the system is hybrid, otherwise it is hybrid-epistemic).
(iv) Situation where Hom (S) is an incomplete orthogonal decomposition is impossible (Theorem 2).
(v) Following the taxonomy of Gisin [11], the hybrid model is monistic while the hybrid-epistemic model is dualistic.

The inconsistency of the ontic model of QM has important consequences

(i) The von Neumann's postulate stating that each ensemble in the pure state is homogeneous is inconsistent with QM
(ii) von Neumann's postulate is a hidden assumption in the standard QM – this postulate is also the kernel of Bohr's interpretation of QM (ψ is a possible state of an individual system)
(iii) Only the hybrid-epistemic model of QM is possible since the epistemic model is not acceptable (no observations are possible).
(iv) In our axiomatization of QM all QM should be re-written in the sense of the hybrid-epistemic model of QM
(v) Only homogeneous state can be attributed as a state to the individual system. Thus the statement "the (individual) system S is in a state ψ" is meaningless unless the state ψ is homogeneous.
(vi) The most important consequence is the invalidity of the Bell's theorem and the invalidity of the so-called quantum nonlocality, but this will be considered in the next section.

5. Bell's theorem and the quantum non-locality

Under the name of Bell's theorem we mean the following statement (see [2])

Locality of QM *implies* Bell's inequality .

Bell's inequality is clearly inconsistent with QM. The proof of Bell's theorem is usually done in the ontic model of QM. The proof is based on the calculation done with individual states from at least two different bases.

Theorem 3.
The proof of the Bell's theorem cannot go through in the epistemic, hybrid or hybrid-epistemic models of QM.

Proof.
We are not able to prove that there does not exist an another proof of Bell's theorem but we can show that the standard proof of Bell's theorem cannot go through in these models.
Let us start with the epistemic model. It is clear that the proof of Bell's theorem is based on the considerations concerning the individual states of Alice's and Bob's systems. But in the epistemic model there do not exist any individual states.
In the case of the hybrid model the situation is similar. There exist individual states but the set of individual states is too small - it contains only one orthogonal base and this is insufficient for the proof. In fact in each proof of Bell's theorem there must exist at least two distinct bases of individual states.
The case of the hybrid-epistemic model is more close to the epistemic model so that argument from the hybrid model is applicable also to the hybrid-epistemic model. (The end of the proof.)

Theorem 4.
The non-locality of QM cannot be proved in the epistemic, hybrid and hybrid-epistemic models of QM.

Proof.
The standard proof of the non-locality is based on the Bell's theorem. But in all models mentioned above the Bell's theorem cannot be proved. I.e., the known proofs of non-locality of QM cannot go through.

Remark.
Both these theorems do not state the impossibility of any proof but only that the standard proofs cannot go through. But it is sufficient for the statement that there is no reason for the validity of Bell's theorem and for the non-locality of QM. If there are no known proofs, this implies that these two assertions (Bell's theorem and the non-locality) **must be considered as unproven**.

The true meaning of the Bell's theorem is the following. If one assumes the locality of QM then the Bell's theorem implies that the "standard von Neumann's text-book QM" is inconsistent! So this was the first proof of our main result. Bell's proof requires the locality, so that it was possible to brush off this result with the excuse of possible nonlocality of QM. This was then the false start of quantum nonlocality.

6. The principle of superposition and the solution to the "Schrodinger's cat paradox"

It is clear that the principle of superposition holds for states of ensembles, since states are vectors from the Hilbert space H_S.

But for homogeneous states the principle of superposition does not hold.

Theorem 5.
In the hybrid model of QM the principle of anti-superposition holds for homogeneous states. Any non-trivial superposition of two distinct homogeneous states is not a homogeneous state.
In the hybrid-epistemic model the non-trivial superposition of two homogeneous states is a homogeneous state only if these homogeneous states are elements of the same homogeneous subspace.

Proof.
Let us consider the hybrid model. This is a direct consequence of the Theorem 1: the non-trivial superposition of two distinct homogeneous states is not orthogonal to these states.
Let us consider the hybrid-epistemic model. If two homogeneous states belong to two distinct homogeneous subspaces, then their non-trivial superposition will not be the homogeneous state.

Theorem 6. ("Schrodinger's cat paradox")
The "cat state"

$$(1/2)^{1/2} (\psi_{alive} + \psi_{dead})$$

is not a homogeneous state, so that it can be interpreted only as a state of an ensemble.

Proof.
Let us assume that the states ψ_{alive} and ψ_{dead} are homogeneous states (i.e. states applicable to individual systems). Since these two states are observably different, we can suppose that these two states belong to the different homogeneous subspaces. But then the "cat state" will not be a homogeneous state, so that it cannot be attributed to the individual system! This state can be interpreted only as a state of an ensemble. Clearly, in this ensemble some cats are alive and some are dead. There is no paradox.

7. The solution to the measurement problem

The measurement process consists in the realization of certain steps:

(i) The measuring system M is prepared in the well-defined initial state. The measured system S is an element of the ensemble in the state ψ .

(ii) These two systems interact by the interaction which is parametrized (for example) by the orthogonal base in the system's Hilbert space H_S.
(iii) After the interaction the individual state of the measuring system M is observed.
(iv) This process is repeated and the statistics of observed individual states of the measuring systems can be obtained. The result is the probability distribution $p(\,.\,|\,\psi\,)$ on the set of individual states Ind (M) of the measuring system M.
(v) The obtained empirical probability distribution is compared to the distribution predicted by QM.

The description of possible measuring process is described in details in [4], where two types of possible measuring processes are considered. Of course there may exist many other measurement schemas but we shall limit ourselves to these described in [4].

In general, by the introduction of the concept of the observation, the measurement problem is then the inner problem inside QM. The main problem of the existence of definite results of the measurement is clearly (and trivially) solved in our axiomatization – these are individual states of the individual measuring appasratus.

The typical example. (The measurement of spin.)
The measuring system has two individual states, Ind (S) = $\{ i_{up}, i_{down} \}$ while dim H_S is of an order of the Avogadro's number. Nevertheless $\{ L^{(up)}, L^{(down)} \}$ is an orthogonal decomposition of H_S.

8. The discussion

There are two contradicting positions

- The ontic model of QM (it is equivalent to the von Neumann's standard QM)
- The existence of definite outcomes of a given experiment.

The standard argumentation follows these steps:
The ontic model $\rightarrow$ the measurement $\rightarrow$ the impossibility of definite outcomes (i.e. the measurement problem [5]).

Our model is exactly opposite and follows another steps:
The postulation of the existence of definite outcomes $\rightarrow$ the observation of the individual state of the measuring system $\rightarrow$ the inconsistence of the ontic model.

In a short form:
The standard von Neumann's model *implies* the impossibility of the definite outcomes.
The postulation of definite outcomes *implies* the inconsistence of the ontic model of QM.

Thus there is a necessary choice between two alternatives:

- The acceptance of the ontic model of QM
- The existence of definite outcomes of an experiment.

The right choice is simple and evident: the existence of definite outcomes is an evident fact while the existence of an ontic model of QM is a pure speculation without any objective support. Thus our axiomatization of QM is supported by evident facts while there is no real support for the ontic model of QM.

We have to sacrifice something: to sacrifice the ontic model creates no serious problem (there are other models available) while to sacrifice the existence of definite outcomes is impossible.

The main point of our axiomatization is the Axiom 4 (ii). This implies the existence of the definite outcomes in the postulating the existence of the function ISt (S) ∈ Ind (S). Other new axioms are rather technical or generally acceptable.

We obtain again the general conclusion: the existence of definite outcomes implies the inconsistence of the ontic model of QM.

It is clear that the existence of definite outcomes is an evident and sure content of the common sense.

9. Conclusions

The main input of our study is the axiomatization of the concept of an observability in QM and the relevant consequences.

The main result is that the ontic model of QM is **inconsistent**, i.e. impossible.

The other main consequences are: the Bell's theorem is unproven, the quantum non-locality is unproven – they must be considered only as arbitrary hypotheses without any provable support.

We shall list the main consequences of the proposed axiomatization of QM

- The ontic model of QM is inconsistent. The QM based on the von Neumann's formulation is identical to the ontic model. The basic assumption of the von Neumann's formulation is the assumption that each wave function describes a possible state of an individual system and this implies that this formulation should be identified with the ontic model – thus the von Neumann's QM is inconsistent.
- As a consequence, the foundational part of the standard QM must be rebuilt. This means: the wave function describes the state of an ensemble (an Einstein's idea), the evolution is not an evolution of the state of an individual system but the evolution of a state of an ensemble etc.

- The ontic model must be rejected and the other possible models have the property that the Bell's theorem cannot be proved in these models. This means that there is no proof of the Bell's theorem.
- As a consequence we have obtained that there is no valid proof of the non-locality of QM.

But there are also positive consequences of our axiomatization

- The solution to the "Schrodinger's cat paradox" is obtained and it is quite simple and natural
- The solution to the Measurement problem in QM is obtained and it is simple and natural
- The rational and plausible solution to the problem of the superposition of macroscopic states is obtained
- The problem concerning the fact that the concept of a measurement makes part of the axiomatization of QM is solved
- In [4] there is a complete description of the internal measurement process inside QM and this implies the solution to the measurement problem
- There is no problem of collapse since now the "collapse" is considered as an update of an ensemble and of its state

We think that our new axioms (4 – 6) are so general so that they could be acceptable for most of experts in the foundations of QM.

So that we think that there is a time to change the standard formulation of QM, so that many long-standing problems will be solvable in a new way. We think that the von Neumann's assumption that the ensembles in the pure state are homogeneous (i.e. that any pure state can be a state of an individual system) is provably **wrong** and that it was the source of most foundational problems of QM and that exactly this assumption has to be changed.

Generally we think that the problem of the axiomatization of the concept of an observation in QM is recently the one of the important problems in QM and that the solution to this problem could be a way out from current problems in the foundations of QM.

Our considerations enter exactly into the heart of the Einstein – Bohr debate. Bohr preferred the ontic model and Einstein preferred the epistemic model (Einstein was the first to consider the idea that the wave function does not describe the state of an individual system but the state of an ensemble, see also Ballentine [6[). Both standpoints are unacceptable: the ontic model is inconsistent while the epistemic model has no possibility of any observation.

Nevertheless, the Einstein's standpoint is quite close to our hybrid-epistemic model: in fact, in most situations the wave function describes the state of an ensemble but not the state of an individual system.

The 85 years of the unsuccessful debate on the foundations of QM shows that there is something wrong in the standard QM and that there is a need for new ideas.

Appendix A. The simplified version of our main argument.

We think that it may be useful to give a simplified version of our argument (in the simple very regular situation) since the consequences of our results are very important and it is useful to see clearly the heart of the argument.

This simplified situation corresponds to the hybrid model of QM.

Axioms 1 – 5 are the same as above.

Axiom 6*.
For each $i \in \mathrm{Ind}\,(S)$ there exists exactly one state $[\psi_{hom}{}^{(i)}] \in \mathbf{P}_S$ such that

$$p\,(\,j \mid [\psi_{hom}{}^{(i)}]\,) = 0\ ^{3}, \text{ for each } j \neq i.$$

Thus we have a 1-1 map

$$i \in \mathrm{Ind}\,(S) \rightarrow [\psi_{hom}{}^{(i)}] \in \mathrm{hom}\,(S)$$

$$\text{where}\quad \mathrm{hom}\,(S) = \{\, [\psi] \mid [\psi] \text{ is a homogeneous state } \}\ .$$

As a consequence, the state of the ensemble $\mathbf{E}^{(i)}$ is $[\psi_{hom}{}^{(i)}]$ for each $i \in \mathrm{Ind}\,(S)$.

Axiom 7*. (Born's rule.)
For each state $[\psi] \in \mathbf{P}_S$ and for each individual state $i \in \mathrm{Ind}(S)$ the Born's rule holds

$$p\,(\,i \mid [\psi]\,) = |\,([\psi_{hom}{}^{(i)}] \mid [\psi]\,)\,|^2\ .$$

Axiom 8*. (The update rule for ensembles.)
Let $\mathbf{E}$ be an ensemble in the state $[\psi]$ and let the observable system S be an element of $\mathbf{E}$. After the observation we found that the individual state of S is $i \in \mathrm{Ind}\,(S)$. Then the system S must be considered as an element of the ensemble $\mathbf{E}^{(i)}$. Thus the ensemble E to which S belongs must be updated to an ensemble $\mathbf{E}^{(i)}$ as a result of acquiring the new information on the individual state of S. Thus the pertaining of S into some ensemble is updated in the following way

$$S \in \mathbf{E} \;\rightarrow\; S \in \mathbf{E}^{(\mathrm{ISt}(S))}\ .$$

As a consequence we obtain the change of the state of S: $[\psi] \rightarrow [\psi_{hom}{}^{(\mathrm{ISt}(S))}]$.

Theorem 1*.
If $i, j \in \mathrm{Ind}\,(S)$ and $i \neq j$ then $([\psi_{hom}{}^{(i)}] \mid [\psi_{hom}{}^{(j)}]\,) = 0$. I.e. any two distinct homogeneous states must be orthogonal.

[3] We suppose that individual states are not degenerated.

Proof. Let us consider the ensemble $E = \{ S_1, S_2, \ldots, S_N \}$ in the state $[\psi_{hom}^{(i)}]$. Then the probability distribution $p(. | [\psi_{hom}^{(i)}])$ is a deterministic probability distribution concentrated at i . I.e. $p(j | [\psi_{hom}^{(i)}]) = 0$ for each $j \neq i$. Using the Axiom 7 (Born rule) we obtain

$$0 = p(j | [\psi_{hom}^{(i)}]) = | ([\psi_{hom}^{(j)}] | [\psi_{hom}^{(i)}]) |^2 .$$

In his foundational monograph [] von Neumann explicitly states that each ensemble in the pure state is homogeneous. Theorem 1 implies that this von Neumann's postulate is wrong.

Theorem 2*.
The set hom (S) of all homogeneous states is the orthogonal base of the Hilbert space H_S .

Proof.
We have already shown that hom (S) is an orthogonal set. Let us assume that hom (S) is not an orthogonal base of H_S . Then there exists a state $[\psi] \in \mathbf{P}_S$ which is orthogonal to all element of hom (S) . Using Axiom 7 (Born's rule) we obtain

$$p(i | [\psi]) = | ([\psi_{hom}^{(i)}] | [\psi]) = 0 , \quad \text{for each } i \in \mathrm{Ind}(S) .$$

this means that the function $p(. | [\psi])$ is trivially equal to 0 and this is impossible since this function is a probability distribution. Thus hom (S) must be a base.

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