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2016

Dostupný z <http://www.nusl.cz/ntk/nusl-261504>

Dílo je chráněno podle autorského zákona č. 121/2000 Sb.

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Datum stažení: 15.11.2019

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**Academy of Sciences of the Czech Republic**

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Technical report No. V-1239

27.10.2016



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## Interval Matrices: Regularity Yields Singularity

Jiří Rohn<sup>1</sup>

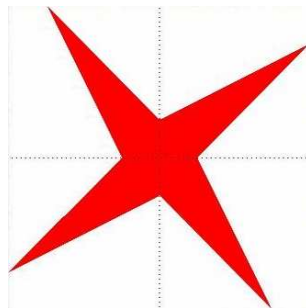
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Abstract:

It is proved that regularity of an interval matrix implies singularity of two related interval matrices.<sup>2</sup>



Keywords:

Interval matrix, regularity, singularity.

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<sup>1</sup>This work was supported with institutional support RVO:67985807.

<sup>2</sup>Above: logo of interval computations and related areas (depiction of the solution set of the system  $[2, 4]x_1 + [-2, 1]x_2 = [-2, 2]$ ,  $[-1, 2]x_1 + [2, 4]x_2 = [-2, 2]$  (Barth and Nuding [1])).

# 1 Introduction

A square interval matrix

$$[A - D, A + D] = \{ B \mid A - D \leq B \leq A + D \}$$

is called regular if each  $B \in [A - D, A + D]$  is nonsingular, and is said to be singular otherwise. In this report we show that regularity of  $[A - D, A + D]$  implies singularity of two related interval matrices. This is an atypical result which, in this author's knowledge, bears no analogy in literature.

## 2 Main result

**Theorem 1.** *If  $[A - D, A + D]$  is regular, then both the interval matrices*

$$[A^{-1}D - I, A^{-1}D + I], \tag{2.1}$$

$$[DA^{-1} - I, DA^{-1} + I] \tag{2.2}$$

*are singular ( $I$  is the identity matrix).*

*Proof.* Let  $[A - D, A + D]$  be regular. Put  $C = A^{-1}D$ . Notice that  $I - C = A^{-1}(A - D)$ , as a product of two nonsingular matrices, is nonsingular. Consider the matrix

$$(I - C)^{-1}(I + C) = (A - D)^{-1}(A + D).$$

By [3, Thm. 1.2] the matrix  $(A - D)^{-1}(A + D)$  is a  $P$ -matrix, hence so is  $(I - C)^{-1}(I + C)$ , and a theorem by Gale and Nikaido [2] implies existence of an  $\tilde{x} > 0$  satisfying

$$(I - C)^{-1}(I + C)\tilde{x} > 0. \tag{2.3}$$

Set  $x = (I - C)^{-1}\tilde{x}$ . Then  $(I - C)x = \tilde{x} > 0$ , hence

$$Cx < x, \tag{2.4}$$

and from (2.3) we have

$$\begin{aligned} 0 < (I - C)^{-1}(I + C)\tilde{x} &= (I - C)^{-1}(I + C)(I - C)x \\ &= (I - C)^{-1}(I - C^2)x \\ &= (I - C)^{-1}(I - C)(I + C)x \\ &= (I + C)x, \end{aligned}$$

which gives  $-x < Cx$  and together with (2.4)

$$-x < Cx < x,$$

which is

$$|Cx| < x. \tag{2.5}$$

This inequality shows that  $x > 0$ . Now define

$$S = C - \text{diag}(y),$$

where  $y = (y_i)$  is given by

$$y_i = (Cx)_i/x_i \quad (i = 1, \dots, n),$$

then  $|S - C| \leq I$  due to (2.5) and  $(Sx)_i = (Cx)_i - y_i x_i = 0$  for each  $i$ , hence  $Sx = 0$  and  $S$  is a singular matrix in (2.1).

Next, regularity of  $[A - D, A + D]$  implies that of its transpose  $[A^T - D^T, A^T + D^T] = \{B^T \mid B \in [A - D, A + D]\}$  which according to what has just been proved yields singularity of  $[(A^T)^{-1}D^T - I, (A^T)^{-1}D^T + I] = [(DA^{-1})^T - I, (DA^{-1})^T + I]$  and thereby also that of its transpose (2.2).  $\square$

### 3 Consequence

As a consequence we obtain the following purely linear algebraic result.

**Theorem 2.** *Let  $A$  be invertible. Then there exists a singular matrix  $S$  satisfying either*

$$|A - S| \leq I,$$

or

$$|A^{-1} - S| \leq I.$$

*Proof.* Consider the interval matrix  $[A - I, A + I]$ . If it is singular, then we are done; if it is regular, then  $[A^{-1} - I, A^{-1} + I]$  is singular by Theorem 1.  $\square$

In other words, *either  $A$  or  $A^{-1}$  can be brought to a singular matrix by shifting diagonal entries by componentwise magnitudes of at most 1.*

## Bibliography

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